Title: Strong Gravity Lecture - 230306

Speakers: William East

Collection: Strong Gravity (2022/2023)

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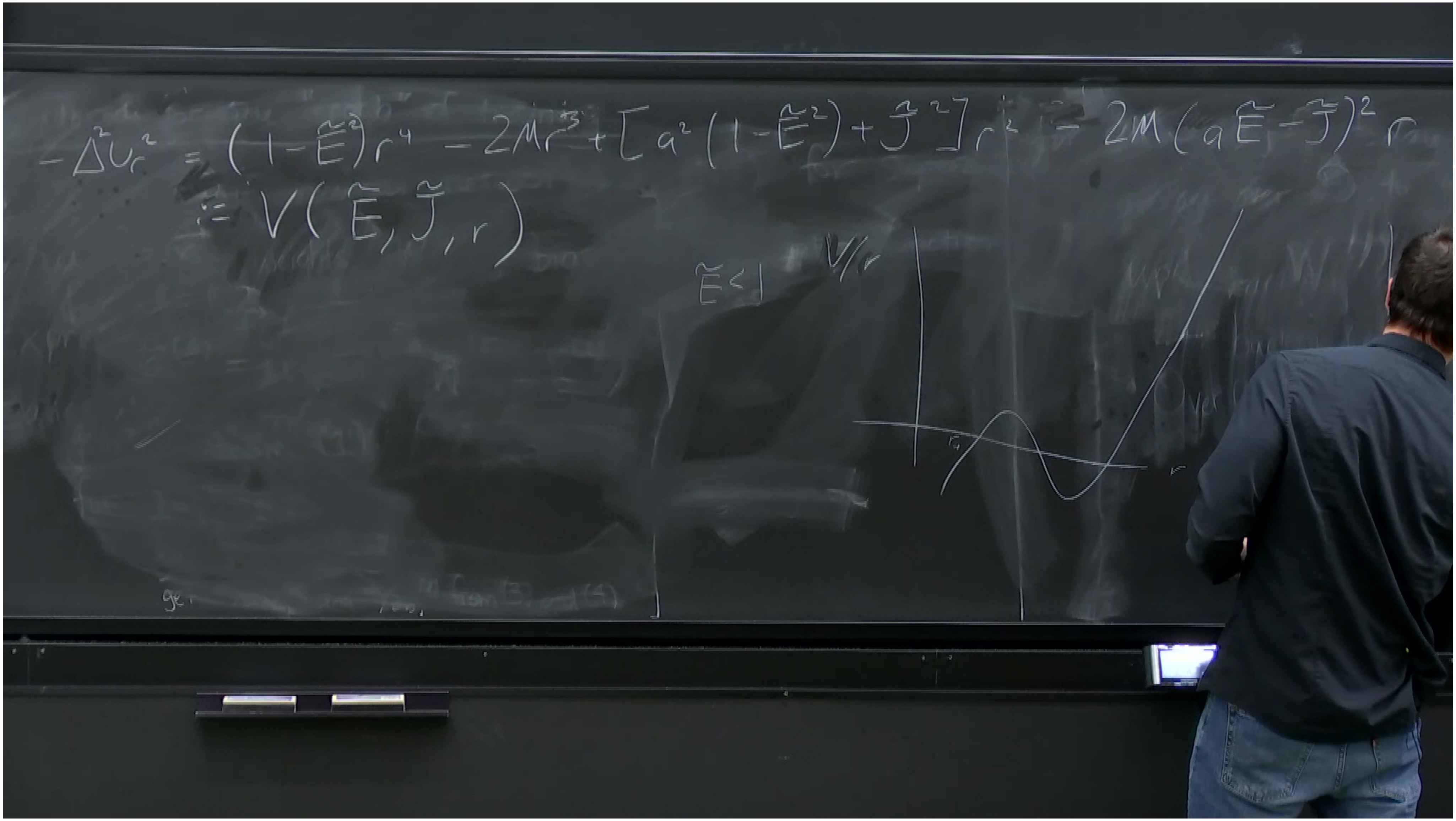
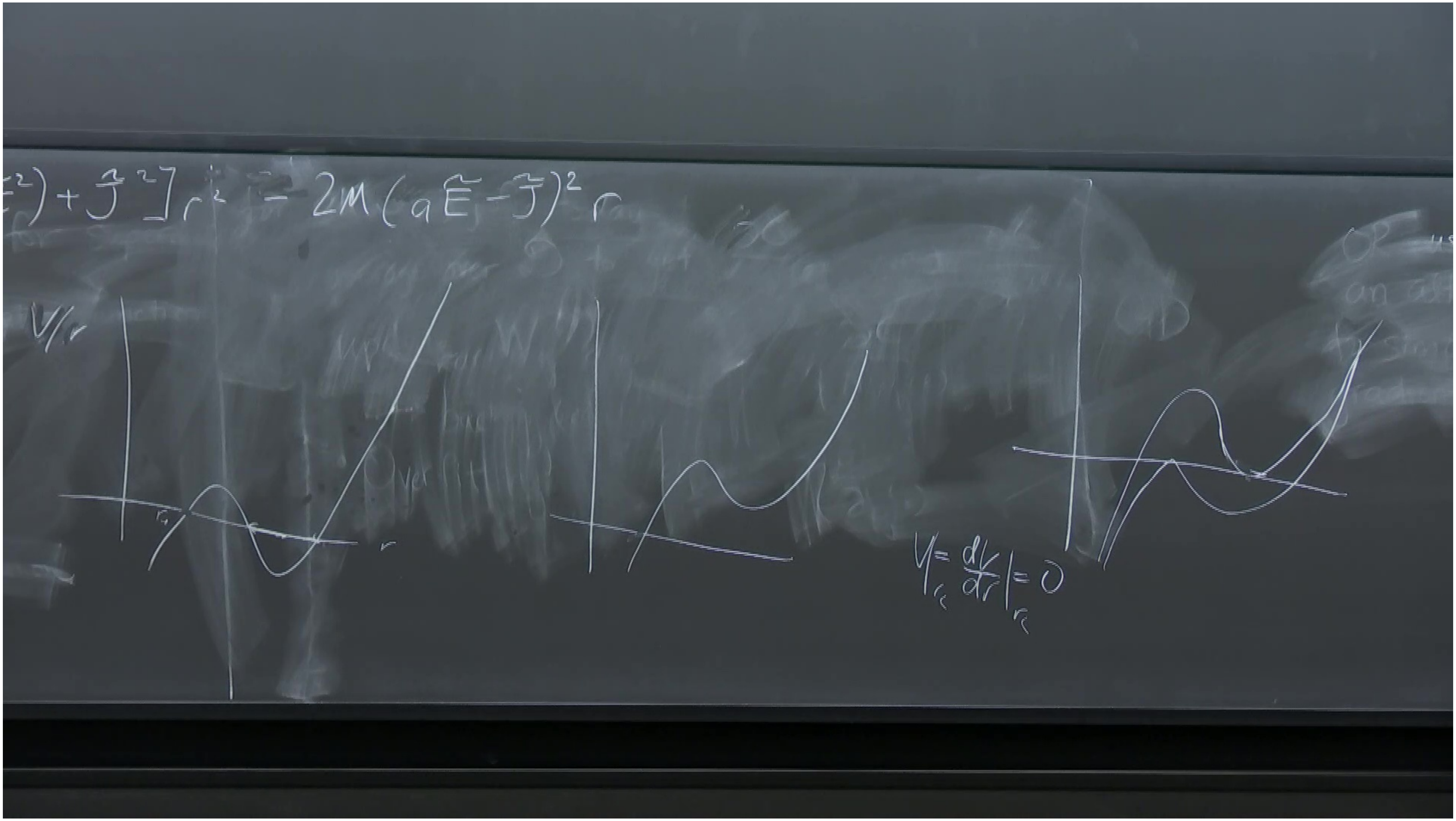
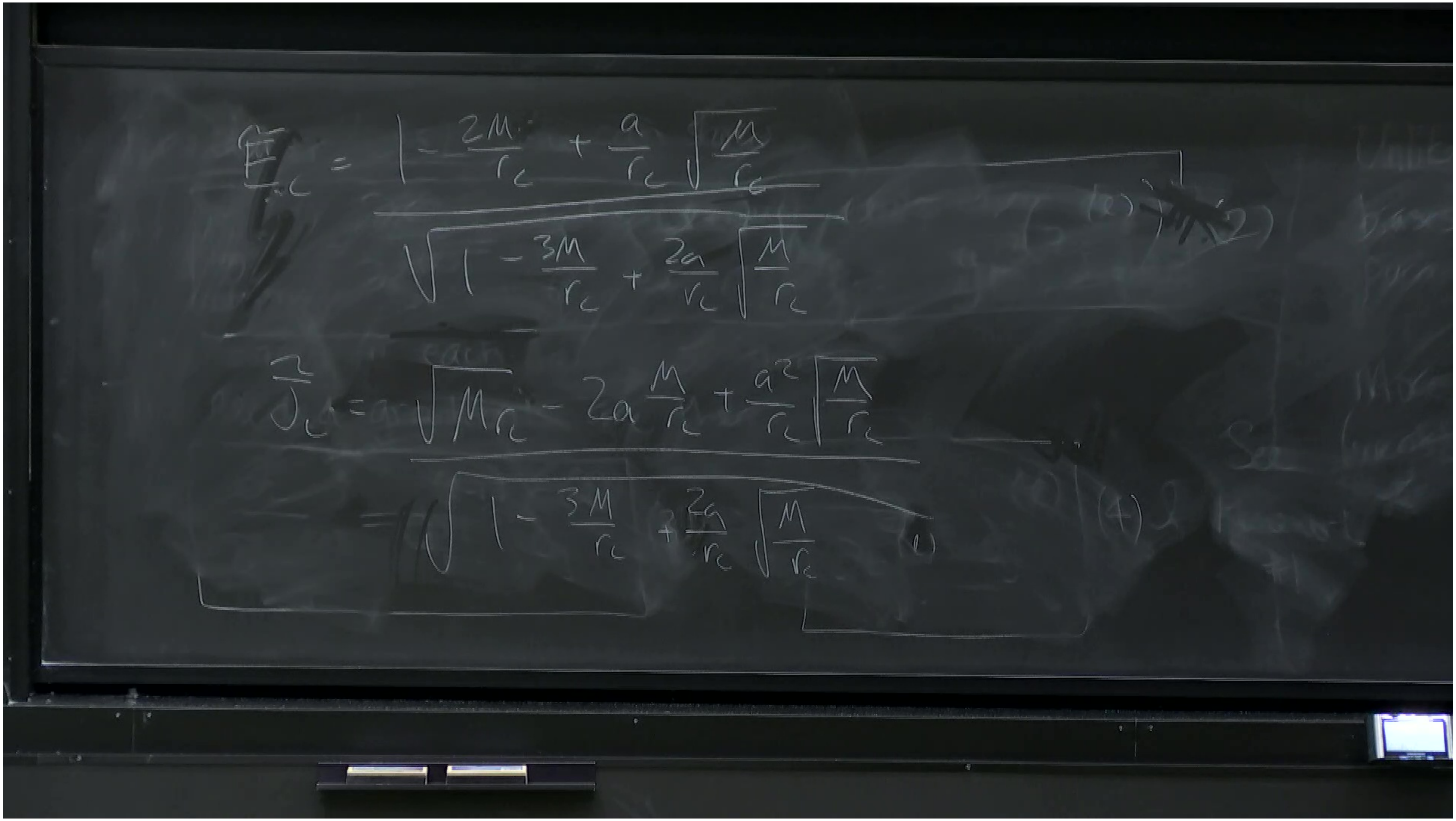
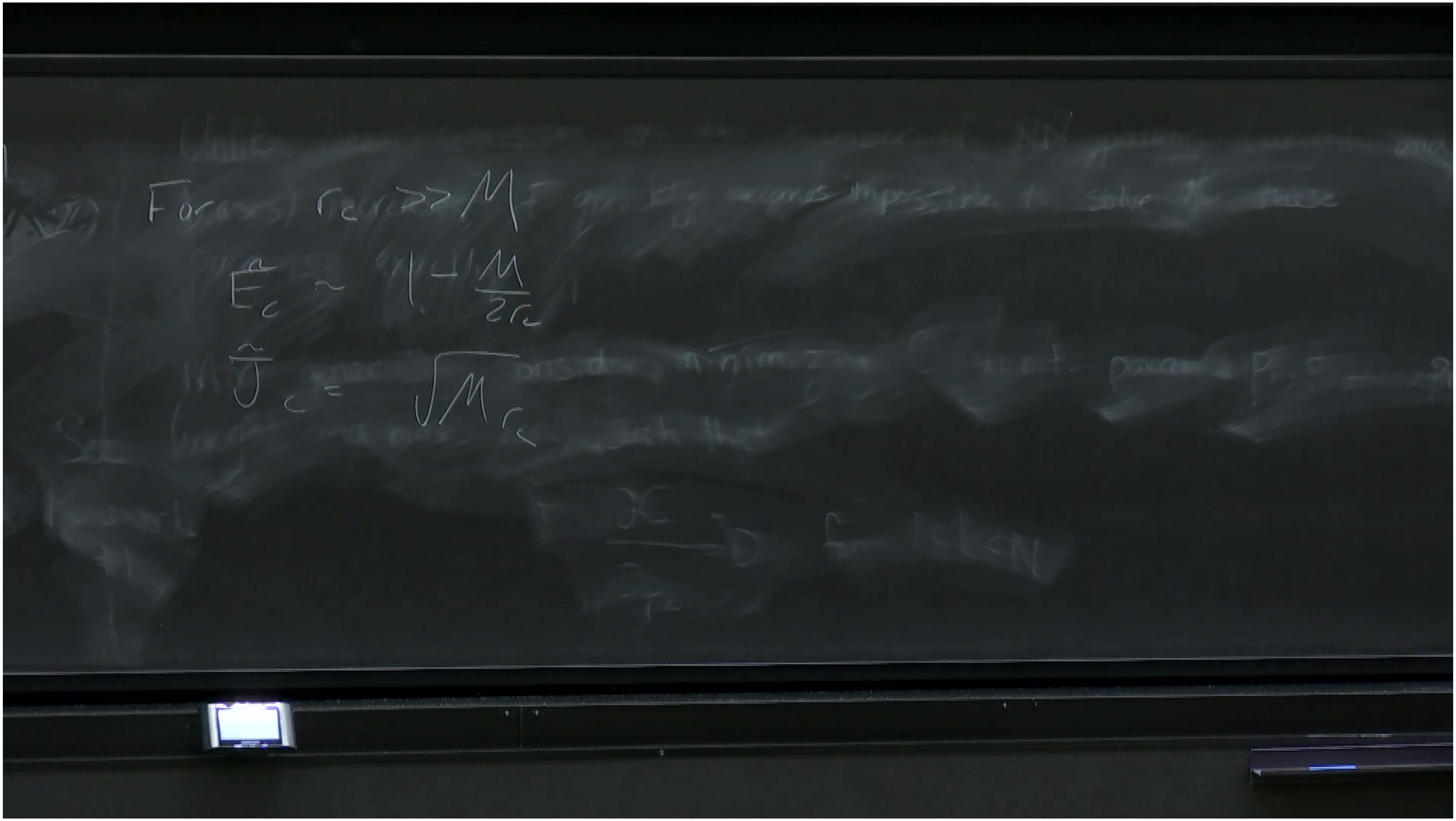
URL: https://pirsa.org/23030051

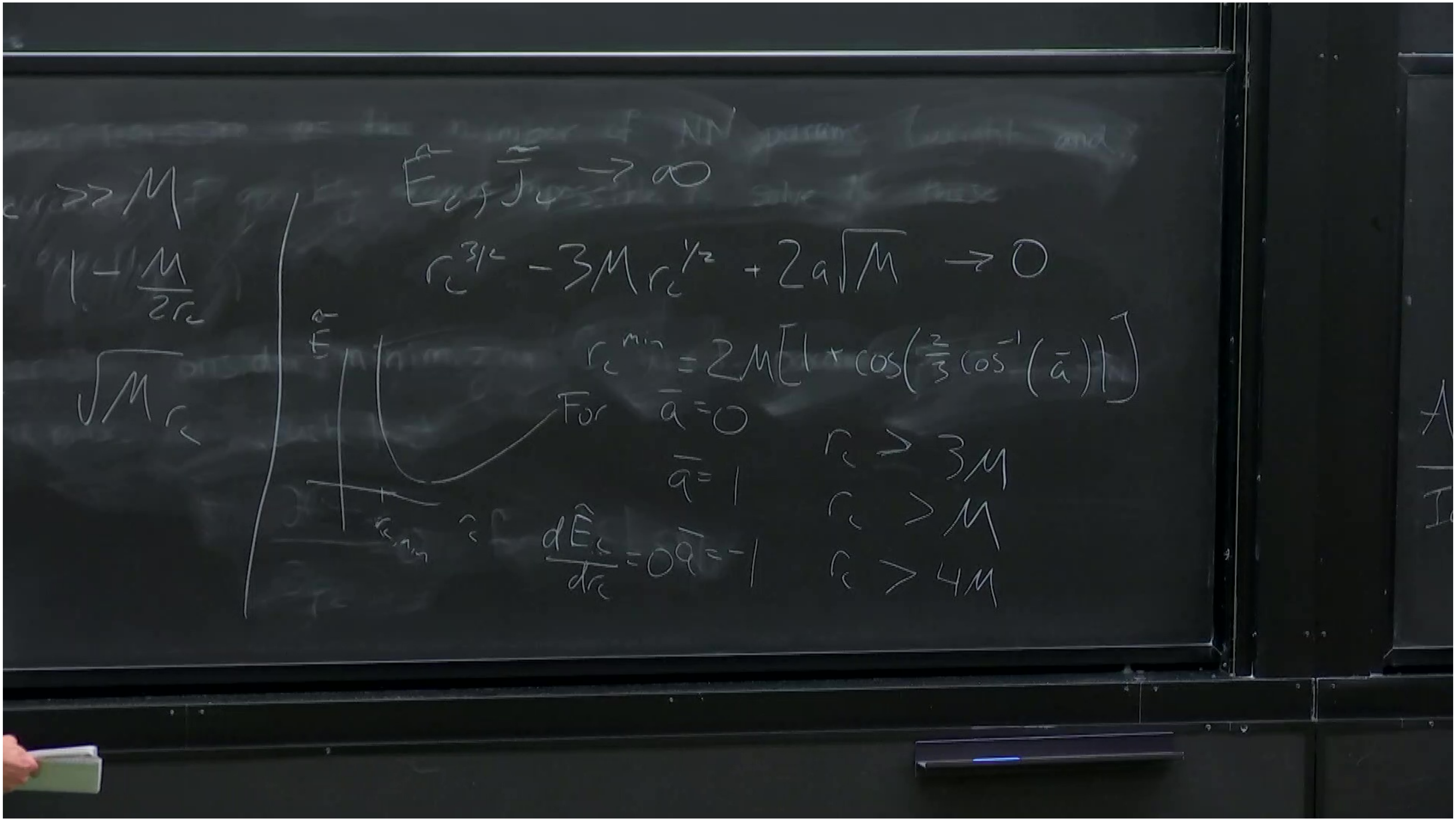

$-\Delta^2 U_r^2 = (1-\tilde{E}^2) r^4 - 2M r^3 + [a^2(1-\tilde{E}^2) + \tilde{J}^2] r^2 - 2M(a\tilde{E} - \tilde{J})^2 r$
$\equiv V(\tilde{E}, \tilde{J}, r)$
$\tilde{E} < 1$
V/r
r_4
r


$\tilde{E}^2) + \tilde{J}^2] r^2 - 2M(a\tilde{E} - \tilde{J})^2 r$
V/r
r
$V|_{r_c} = \frac{dV}{dr}|_{r_c} = 0$



$$\tilde{E}_c = \frac{1 - \frac{2M}{r_c} + \frac{a}{r_c}\sqrt{\frac{M}{r_c}}}{\sqrt{1 - \frac{3M}{r_c} + \frac{2a}{r_c}\sqrt{\frac{M}{r_c}}}}$$
$$\tilde{J}_c = \frac{\sqrt{M r_c} - 2a\frac{M}{r_c} + \frac{a^2}{r_c}\sqrt{\frac{M}{r_c}}}{\sqrt{1 - \frac{3M}{r_c} + \frac{2a}{r_c}\sqrt{\frac{M}{r_c}}}}$$


For $r_c \gg M$
$\tilde{E}_c \sim 1 + \frac{M}{2r_c}$
$\tilde{J}_c = \sqrt{M r_c}$

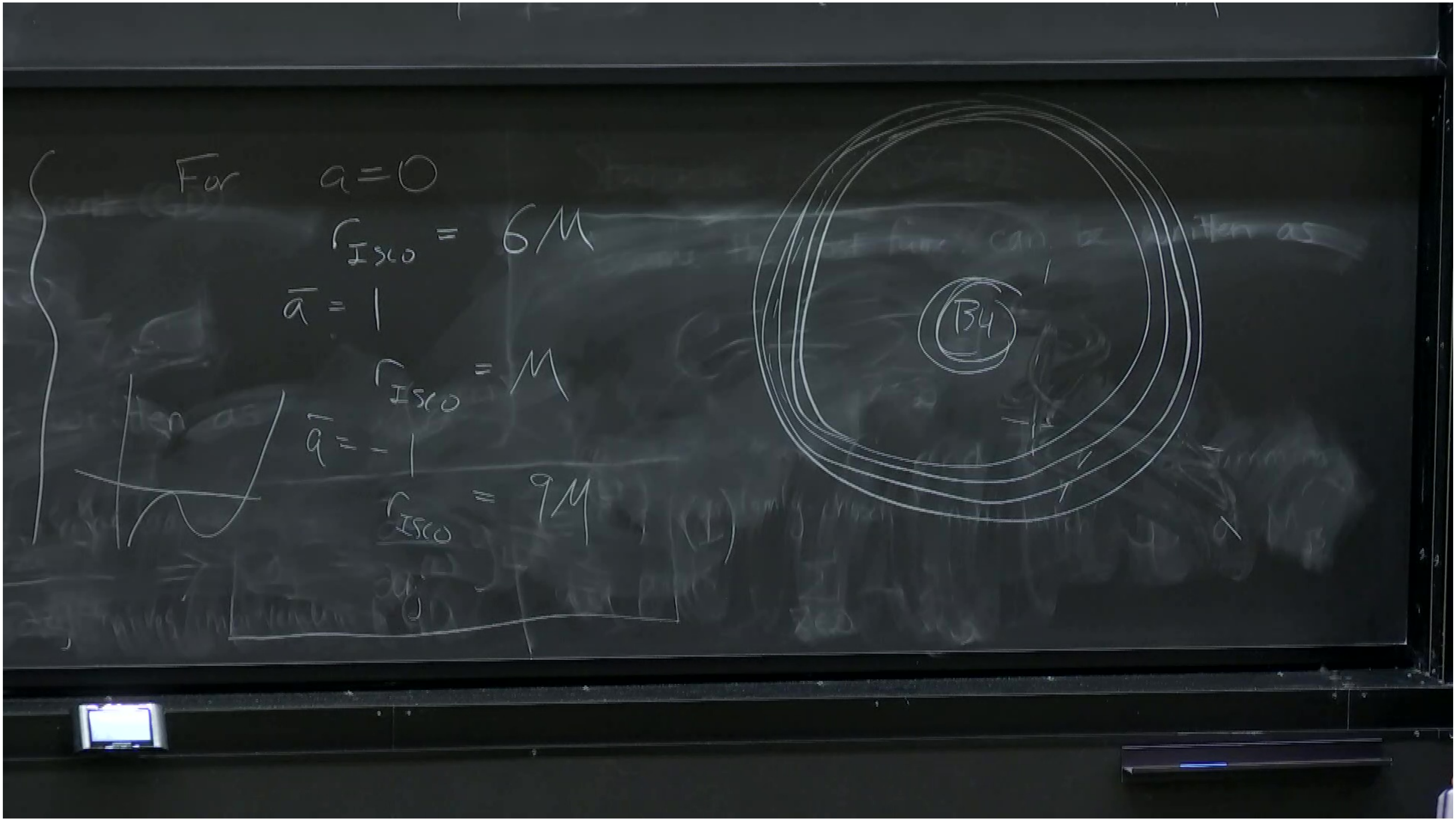

$\hat{E}_c, \bar{J}_c \to \infty$
$r_c^{3/2} - 3M r_c^{1/2} + 2a\sqrt{M} \to 0$
$r_c^{min} = 2M[1 + \cos(\frac{2}{3}\cos^{-1}(\bar{a}))]$
For $\bar{a} = 0$ $r_c > 3M$
$\bar{a} = 1$ $r_c > M$
$\bar{a} = -1$ $r_c > 4M$
$\frac{d\hat{E}_c}{dr_c} = 0$
$\hat{E}$
$r_{c,min}$
$\gg M$
$1 - \frac{M}{2r_c}$
$\sqrt{M r_c}$

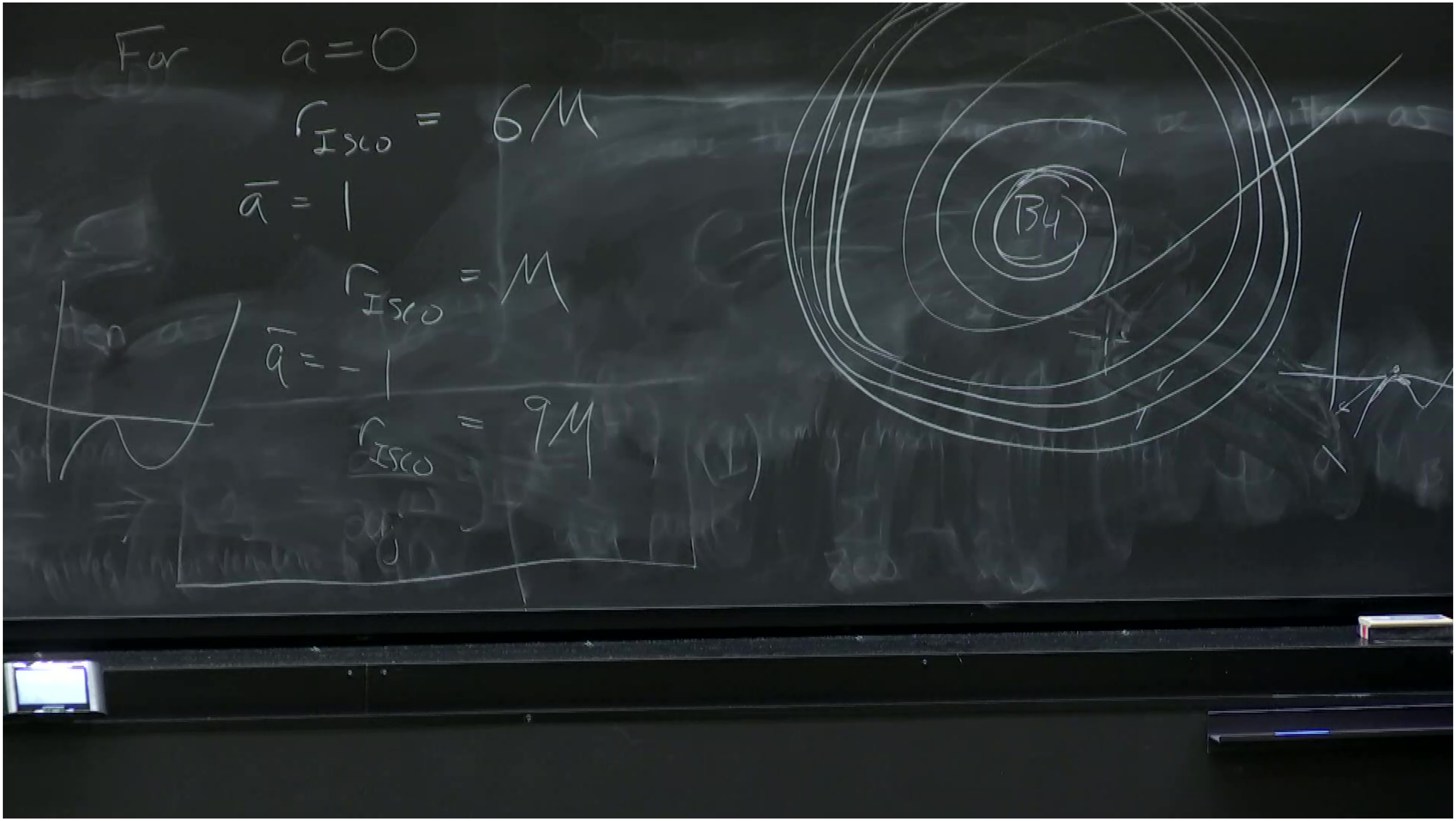
$$r_{ISCO}^2 - 6M r_{ISCO} + 8a\sqrt{M}\, r_{ISCO}^{1/2} - 3a^2 = 0$$

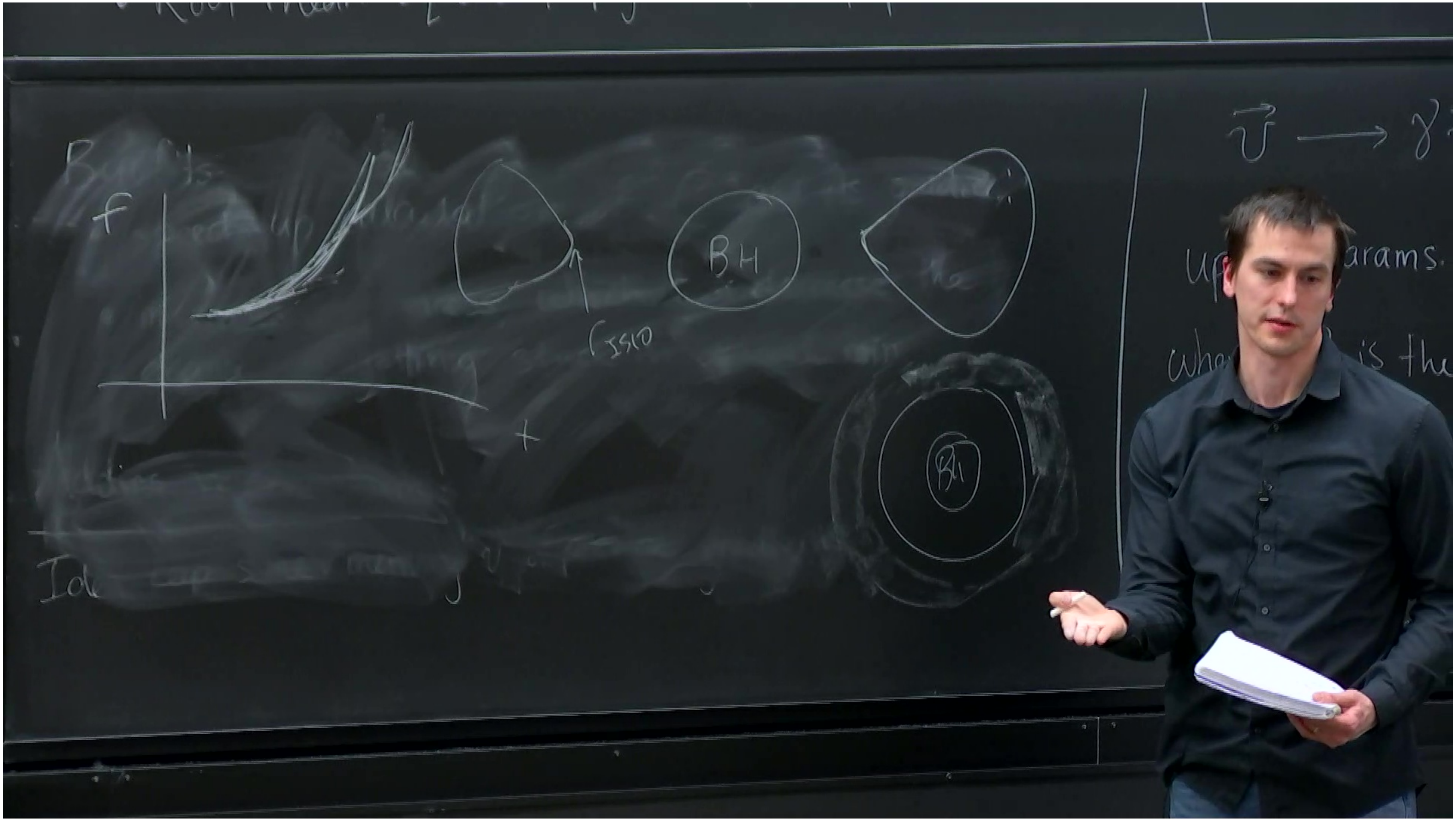
$$r_{ISCO}/M = 3 + B \mp \sqrt{(3-A)(3+A+2B)}$$

$$A = 1 + (1-\bar{a}^2)^{1/3}\left[(1+\bar{a})^{1/3} - (1-\bar{a})^{1/3}\right]$$

$$B = \sqrt{3\bar{a}^2 + A^2}$$


For a=0
r_ISCO = 6M
ā = 1
r_ISCO = M
ā = -1
r_ISCO = 9M
BH


For $a=0$
$r_{ISCO} = 6M$
$\bar{a} = 1$
$r_{ISCO} = M$
$\bar{a} = -1$
$r_{ISCO} = 9M$
BH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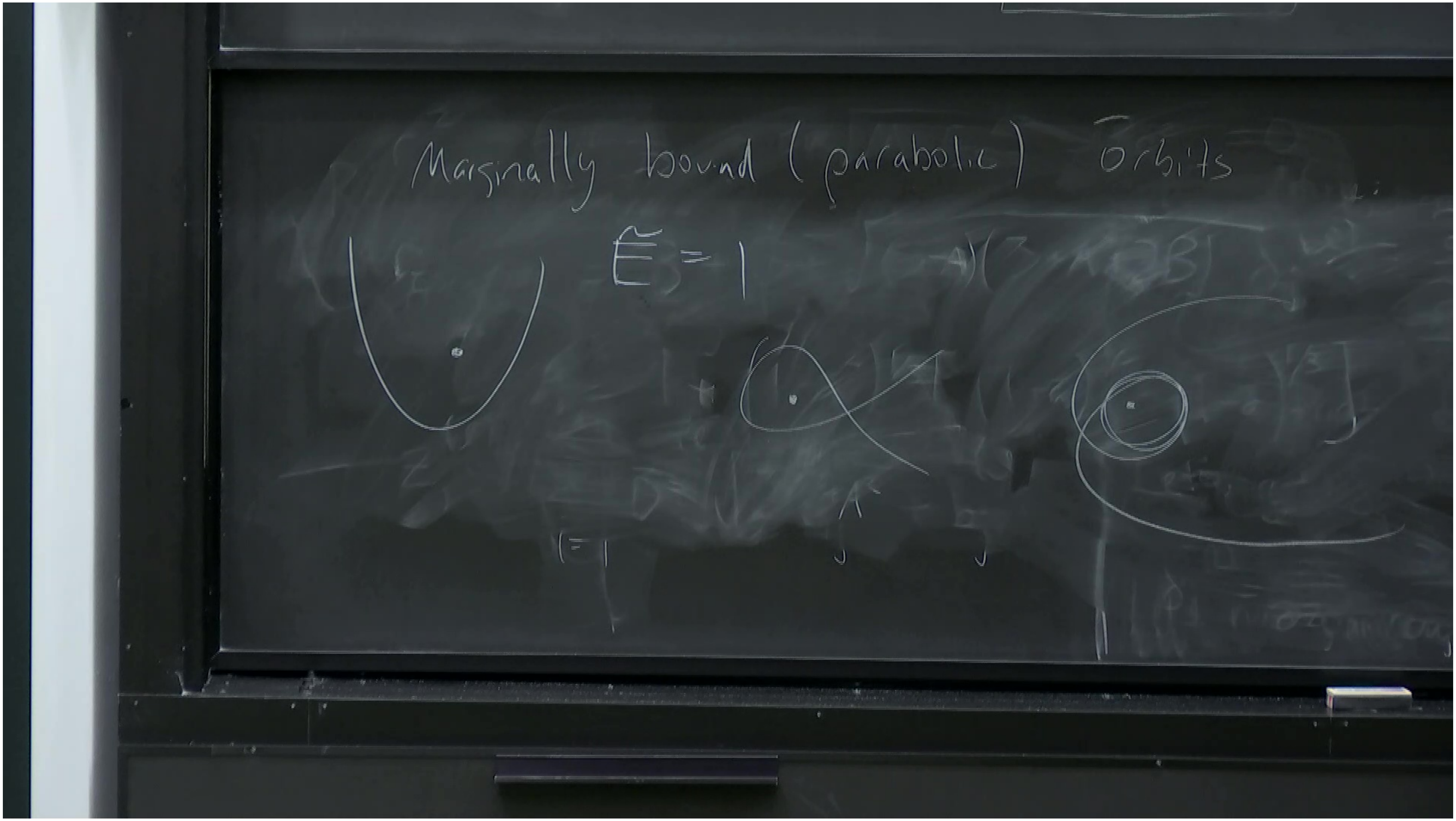
Marginally bound (parabolic) orbits
E = 1

$$V = -2Mr\left[r^2 - \frac{\tilde{J}^2}{2M}r + (a-\tilde{J})^2\right]$$

B4

$$r_p = \frac{\tilde{J}^2/2M \pm \sqrt{(\tilde{J}^2/2M)^2 - 4(a-\tilde{J})^2}}{2}$$

V/r

r

$$\tilde{J}_m = 2M\left(1+\sqrt{1-\bar{a}}\right)$$

$$r_p(\tilde{J}=\tilde{J}_m) = M\left(2-\bar{a}+2\sqrt{1-\bar{a}}\right)$$

(1) $\bar{a}=1$, $r_p = M$ $\quad a=0$, $r_p = 4M$

$\bar{a}=-1$, $r_p = M(3+2\sqrt{2}) \approx 5.8M$