Title: Strong Gravity Lecture - 230330

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Energy and angular momentum in GWs

EFEs to second order:

$$G^{(1)}_{ab}[h^{(2)}_{cd}] = 8\pi t_{ab} := -G^{(2)}_{ab}[h^{(1)}_{cd}]$$

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$$G^{(1)}_{ab}\left[h^{(2)}_{cd}\right] = 8\pi t_{ab} := -G^{(2)}_{ab}\left[h^{(1)}_{cd}\right]$$

$$t_{ab} = \frac{1}{32\pi}\left\langle \partial_a \bar{h}_{cd}\partial_b \bar{h}^{cd} - \frac{1}{2}\partial_a \bar{h}\partial_b \bar{h} - 2\left(\partial_a \bar{h}_{b)c}\right)\left(\partial_d \bar{h}^{cd}\right)\right.$$

TT gauge: $t_{ab} = \frac{1}{32\pi}\left\langle \partial_a h^{TT}_{cd}\partial_b h^{cd}_{TT}\right\rangle$

$$\langle \partial_a X \rangle = 0$$

$$\langle Y \partial_a X \rangle = -\langle (\partial_a Y) X \rangle$$

$$\mathrm{cd}) \rangle$$

$$E_{GW} = \int_{\Sigma_t} t_{00}\, d^3x$$

For plane wave:

$$E_{GW} = \frac{\omega^2}{32\pi} \langle h_+^2 + h_\times^2 \rangle$$

$$\frac{dE_{GW}}{dt} = \int_S t_{0i}\, \hat{n}^i\, dS$$

← unit normal

$$\frac{dJ^i_{GW}}{dt} = \int \varepsilon^{ijk} (\hat{n}_j r)\, t_{0k}\, dS$$

$$\frac{dE_{GW}}{dt} = \frac{1}{5}\left\langle \dddot{\mathcal{I}}_{ij}\, \dddot{I}^{ij} \right\rangle \Big|_{t=t_r}$$

$$\mathcal{I}_{ij} = I_{ij} - \frac{1}{3}\delta_{ij}\,\delta^{kl} I_{kl}$$

$$\frac{dJ^i_{GW}}{dt} = \frac{2}{5}\varepsilon^{ijk}\left\langle \ddot{\mathcal{I}}_{lj}\, \dddot{\mathcal{I}}_k^{\ l} \right\rangle$$

$\Big\rangle_{t=t_r}$

$\delta_{ij}\,\delta^{kl} I_{kl}$

$\Big\langle \dddot{I}_k^{\ l} \Big\rangle$

Perturbations to black holes

Scalar (test) field

$$g^{ab}_{\text{Kerr}} \nabla_a \nabla_b \varphi = 0$$

↑ B.L. coordinates

Ansatz: $\varphi = R(r)\,\Theta(\theta)\,e^{im\phi}\,e^{-i\omega t}$

$\frac{d}{dr}$

$\Delta = (r-r_+)(r-r_-)$

(*) $$\frac{d}{dr}\left(\Delta \frac{dR}{dr}\right) + \left[\frac{\omega^2(r^2+a^2) - 4Mam\omega r + m^2a^2}{\Delta} - (\omega^2a^2+\Lambda)\right]R = 0$$

(**) $$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) + \left[a^2\omega^2\cos^2\theta - \frac{m^2}{\sin^2\theta} + \Lambda\right]\Theta = 0$$

separation constant

$a = 0$ $\quad \Theta \to P_{\ell m}(\cos\theta)$ $\quad \Theta e^{im\phi} = Y_{\ell m}(\theta, \phi)$

spherical harmonics

$$\Lambda = \ell(\ell+1)$$

$e^{-i\omega t}$

$$\Theta = S_{\ell m}(\cos\theta; c) \qquad c = a\omega$$

$a=0$, $(*) \Rightarrow \dfrac{d^2R}{dr_*^2} + (\omega^2 - V)R = 0$

Gen

tortise coördinate: $r_* = r + 2M\ln\left(\frac{r}{2M} - 1\right)$

$$V = \left(1 - \frac{2M}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^2}\right]$$

General solution: $\psi = \sum_{l,m} \int e^{i\omega t} R_{lm\omega}(r) S_{lm}(\theta, c) e^{im\phi}$

Boundary conditions: Ingoing at horizon

$r \to r_+ , \; r_* \to -\infty \qquad R \sim e^{-i\omega r_*}$

Outgoing radiation:

$r_*, r \to \infty \qquad R \sim e^{i\omega r_*}$

$$^{t} R_{\ell m \omega}(r)\, S_{\ell m}(\theta, c)\, e^{im\phi}$$

zon

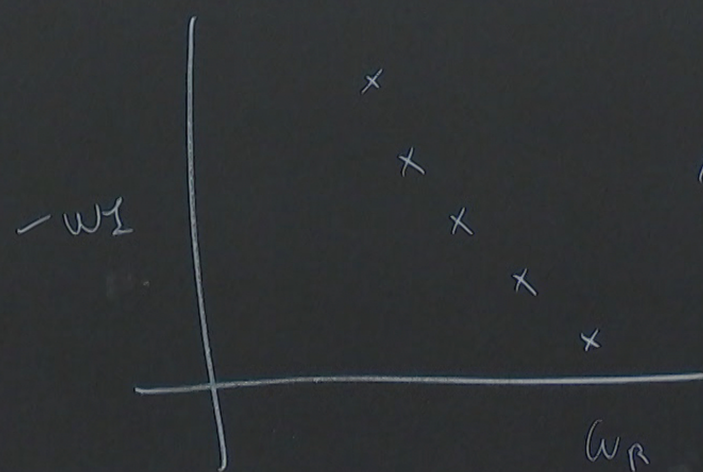
$$R \sim e^{-i\omega r_*}$$

$$e^{i\omega r_*}$$

Quasinormal modes (QNMs)

labelled (n, ℓ, m)

overtone



$$\sim e^{-i\omega t}$$

$$= e^{-i\omega_R t + \omega_I t}$$

Weyl tensor:

$$R_{abcd} = C_{abcd} + \frac{1}{2}\left(g_{a[c}R_{d]b} - g_{b[c}R_{d]a}\right) - \frac{1}{3} R\, g_{a[c}\,g_{d]b}$$

Newman-Penrose scalars

l, n, m, m^*

$(\hat{t}+\hat{r})$ $(\hat{t}-\hat{r})$ $\quad m = \frac{1}{\sqrt{2}}\left(\hat{\theta} + i\hat{\phi}\right)$

$$\Psi_4 = C_{abcd}\, n^a m^{*b} n^c m^{*d}$$

In TT, wave zone

$$\Psi_4 = \frac{1}{2}\left(\ddot{h}_{\hat\phi\hat\phi} - \ddot{h}_{\hat\theta\hat\theta}\right) - i\,\ddot{h}_{\hat\theta\hat\phi} = \ddot{h}_+ - i\,\ddot{h}_\times$$

d Jb

$\ddot{h}_+ - i\ddot{h}_x$

General solution:

$$\Psi_4 = \rho^4 \sum_{l,m} \int d\omega \; {}_{-2}S_{lm}(\theta, c)\, e^{im\phi}\, R_{lm\omega}(r)$$

$$\rho^{-1} = r - ia\cos\theta$$

QNMs labelled by (n, l, m) (for given BH spin)

Least damped QNM $\quad \ell = m = 2 \quad (n=0)$

$$M\omega^R_{22} \simeq 1.53 - (1.16)\left(1 - a/M\right)^{0.13}$$

$$Q_{22} = \frac{\omega_R}{2\omega_I} \simeq 0.7 + 1.42\left(1 - a/M\right)^{-0.5}$$

$a/M = 0.7 \qquad M\omega^R_{22} \simeq 0.53 \qquad \omega_I \simeq 0.081$