Title: Strong Gravity Lecture - 230328

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Collection: Strong Gravity (2022/2023)

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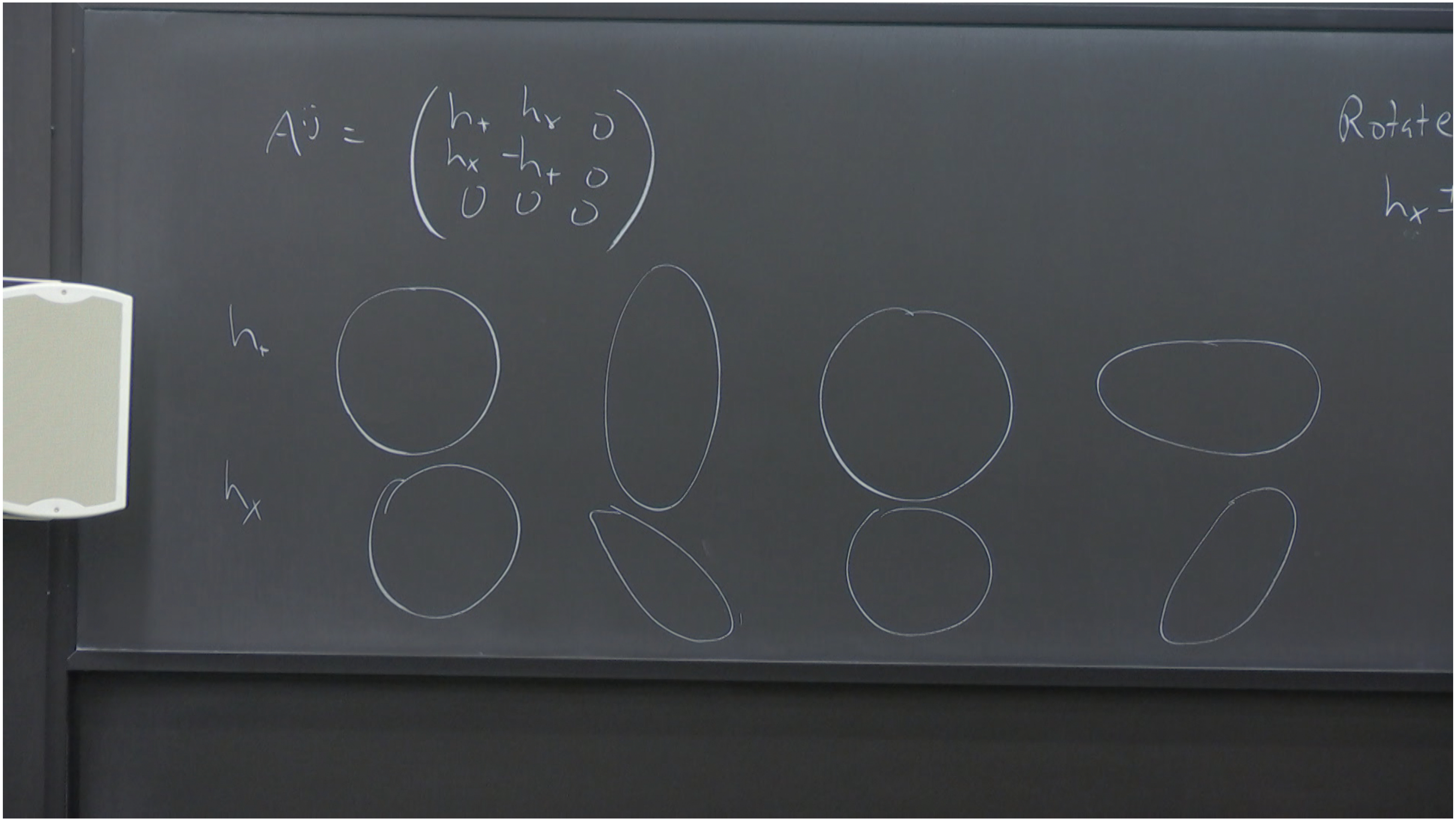
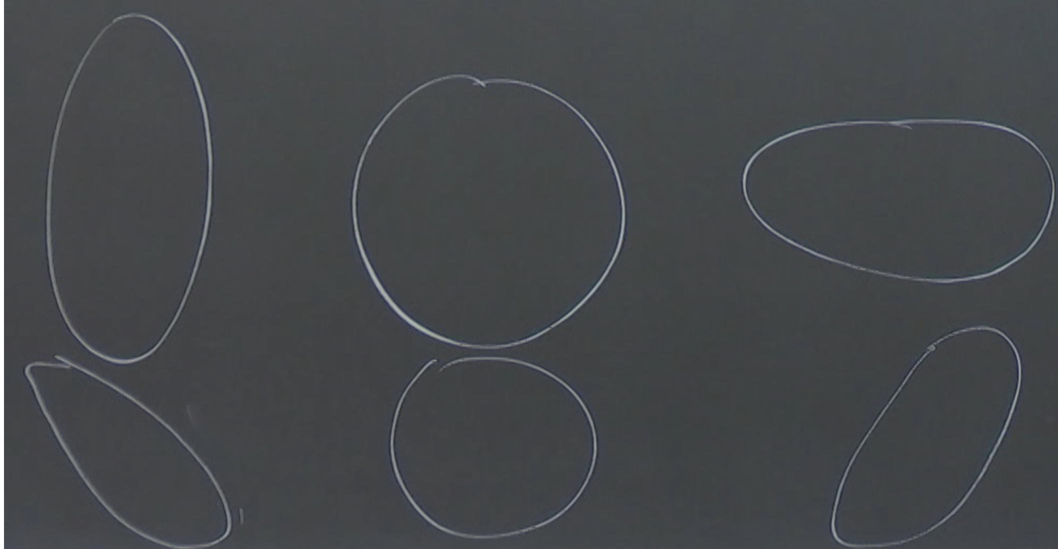

$A^{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$
h_+
$h_\times$
Rotate
$h_\times =$

Rotate by ϕ around z-axis

$$h_x \pm i h_+ \longrightarrow (h_x \pm h_+) e^{\mp 2i\phi}$$



Generation of GWs in the weak field regime

$$\Box \bar{h}_{ab} = -16\pi T_{ab}$$

Green function $\Box G(x^a - \bar{x}^a) = \delta^4(x^a - \bar{x}^a)$

$$G(x^a - \bar{x}^a) = \frac{-1}{4\pi |x^i - \bar{x}^i|} \delta\left(|x^i - \bar{x}^i| - (x^0 - \bar{x}^0) \right) \Theta(x^0 - \bar{x}^0)$$

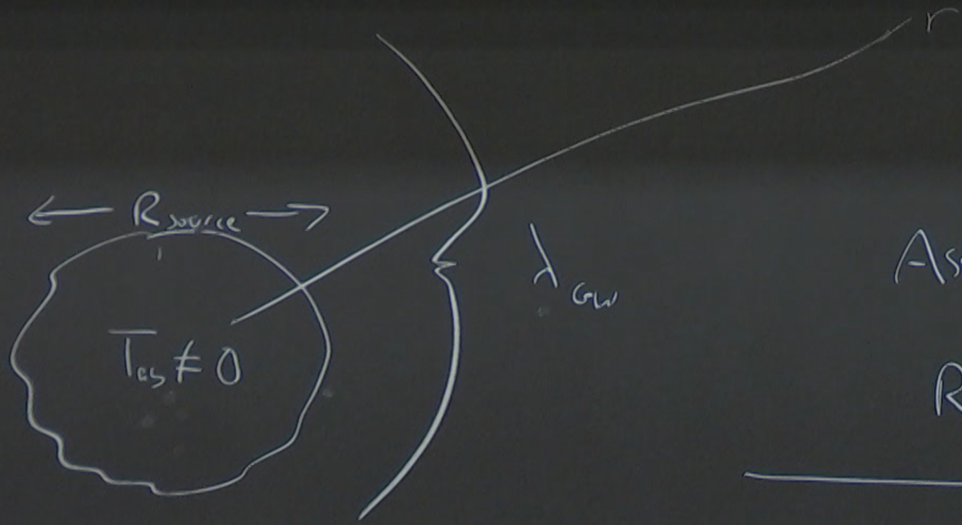
Transform to Fourier space

$$\tilde{\bar{h}}_{ab} = \int dt\, e^{i\omega t}\, \bar{h}_{ab} \qquad t = t_r - |x' - \bar{x}'|$$

$$= 4 \int dt_r \int d^3x \, \frac{T_{ab}(t_r, \bar{x}')}{|x' - \bar{x}'|}\, e^{i\omega\left(t_r + |x' - \bar{x}'|\right)}$$

$$= 4 \int d^3\bar{x}\, \frac{\tilde{T}_{ab}}{|x' - \bar{x}'|}\, e^{i\omega(x' - \bar{x}')}$$

Lorenz condition: $\tilde{h}^{0a} = \frac{i}{\omega}\partial_i \tilde{h}^{ia}$



Assume $r \gg \lambda_{GW} \sim \frac{1}{\omega}$ (wavezone)

$R_{source} \ll \lambda_{GW} = \frac{1}{\omega}$

$$\frac{e^{i\omega|\vec{x}'-\vec{x}'|}}{|\vec{x}'-\vec{x}'|} \approx \frac{e^{i\omega r}}{r}$$

$$\int d^3\bar{x}\,\tilde{T}^{ij} = \int \Big[\underbrace{\partial_k\left(\bar{x}^i \tilde{T}^{kj}\right)}_{0} - \underbrace{\partial_k\left(\tilde{T}^{kj}\right)}_{= i\omega \tilde{T}^{0j}} \bar{x}^i \Big] d^3\bar{x}$$

$$= \frac{i\omega}{2} \int \left(\tilde{T}^{0j}\,\bar{x}^i + \tilde{T}^{j0}\,\bar{x}^j \right) d^3\bar{x}$$

$$= \frac{i\omega}{2} \int \partial_\ell\left(\bar{x}^i \bar{x}^j\, \tilde{T}^{0\ell}\right) - \bar{x}^i \bar{x}^j \left(\partial_\ell \tilde{T}^{0\ell}\right) \Big] d^3x$$

$$= \frac{-\omega^2}{2} \int \underbrace{\bar{x}^i \bar{x}^j\, \tilde{T}^{00}}_{\tilde{I}^{ij}}\, d^3x$$

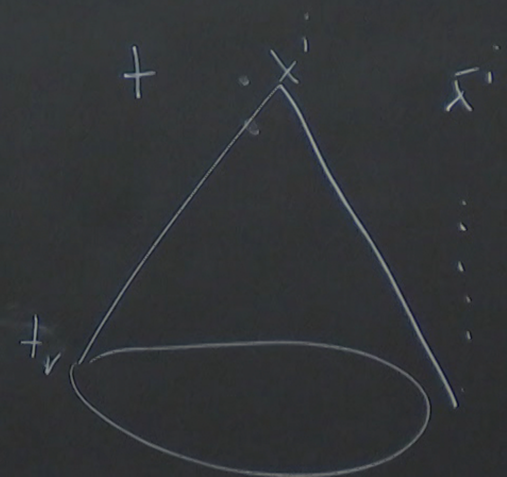
(quadropole moment tensor)

$$= \begin{cases} 1 & \text{when } x^0 - \bar{x}^0 > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{h}_{ab}(x^a) = -16\pi \int G(x^a - \bar{x}^a)\, T_{ab}(\bar{x}^a)\, d^4\bar{x}$$

$$= 4 \int \frac{T_{ab}(t - |x^i - \bar{x}^i|, \bar{x}^i)}{|x^i - \bar{x}^i|}\, d^3\bar{x}$$

$$t_r = t - |x^i - \bar{x}^i|$$



$\left.\right) \bar{x}^j \left.\right] d^3\bar{x}$

$\left.\bar{x}^j\right) d^3\bar{x}$

$\left(\partial_\ell \tilde{T}^{0\ell}\right)\left.\right] d^3x$

...e moment tensor)

$$\tilde{\bar{h}}_{ij} = -2\omega^2 \frac{e^{i\omega r}}{r} \tilde{I}_{ij}$$

$$\bar{h}_{ij} = \frac{2}{r} \ddot{I}_{ij}(t-r)$$

Lower order moment of $\rho = T^{00}$

Monopole : $M(t) = \int d^3x \, \rho(x)$

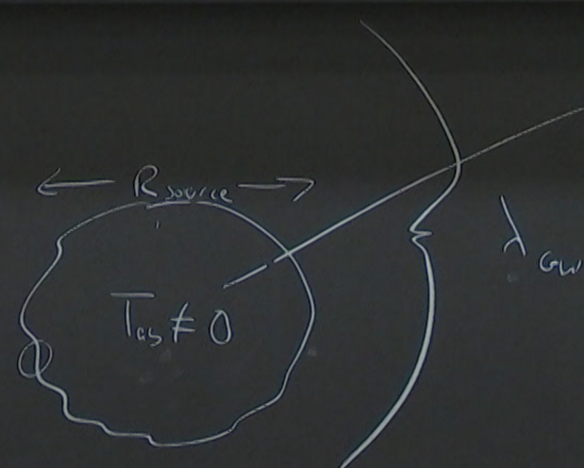
Dipole moment $D^i(t) = \int d^3x \, x^i \rho(x)$

$$|x^i-\bar{x}^i| - (x^0-\bar{x}^0)) \left(\frac{1}{\ }\right) (x^0-\bar{x}^0)$$

$$t_r = t - |x^i - \bar{x}^i|$$

$$t = t_r - |x^i - \bar{x}^i|$$

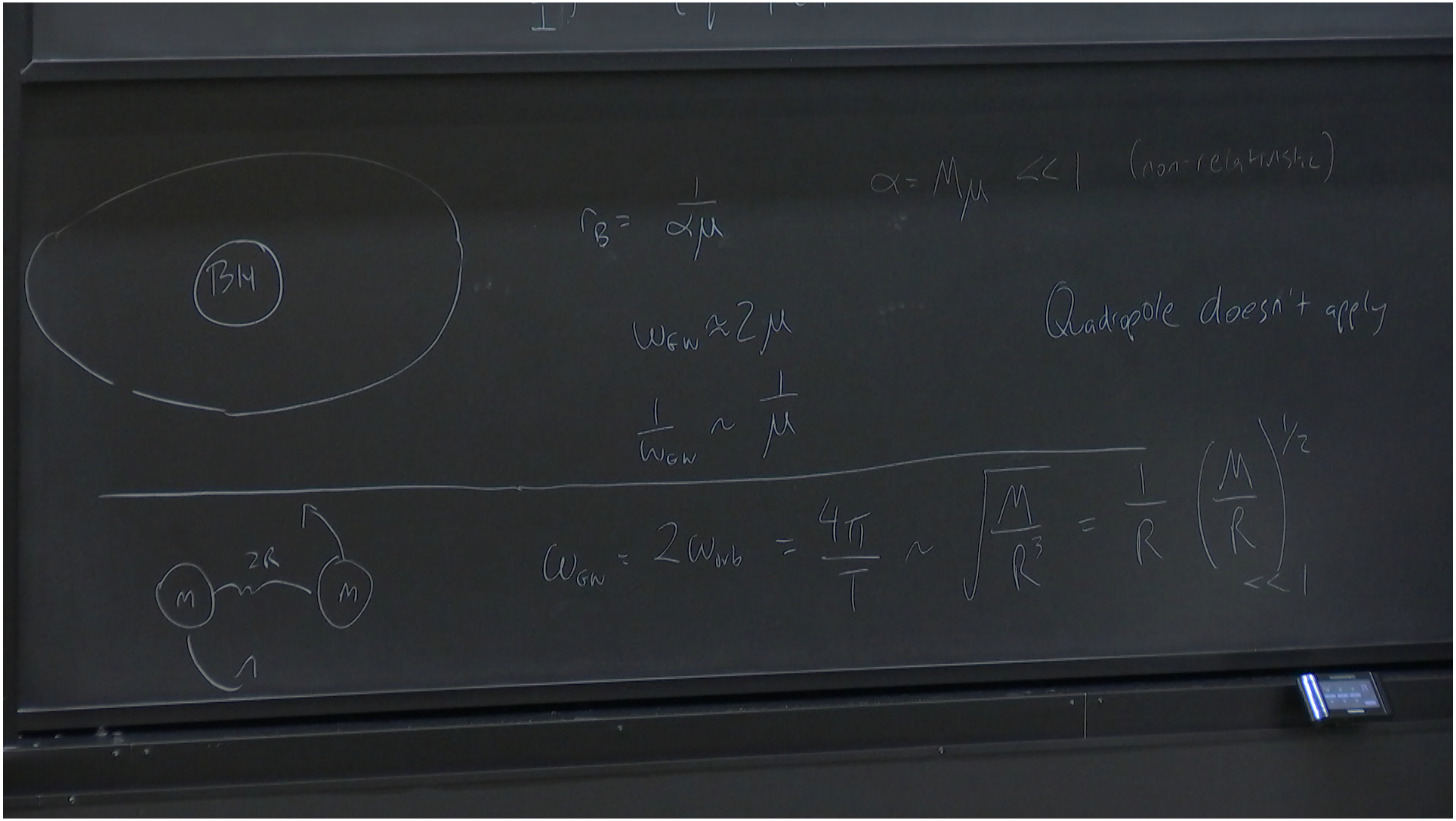
$$\omega(t_r + |x^i - \bar{x}^i|)$$



Assume $r \gg \lambda_{GW}$

$R_{source} \ll \lambda_{GW}$ =

$$\frac{e^{i\omega|x^i-\bar{x}^i|}}{|x^i-\bar{x}^i|} \approx \frac{e^{i\omega r}}{r}$$


BH
$r_B = \frac{1}{\alpha\mu}$
$\alpha = M\mu \ll 1$ (non-relativistic)
$\omega_{GW} \approx 2\mu$
$\frac{1}{\omega_{GW}} \sim \frac{1}{\mu}$
Quadropole doesn't apply
2R
M
M
$\omega_{GW} = 2\omega_{orb} = \frac{4\pi}{T} \sim \sqrt{\frac{M}{R^3}} = \frac{1}{R}\left(\frac{M}{R}\right)^{1/2}$
$\ll 1$

Energy & Angular momentum in GWs

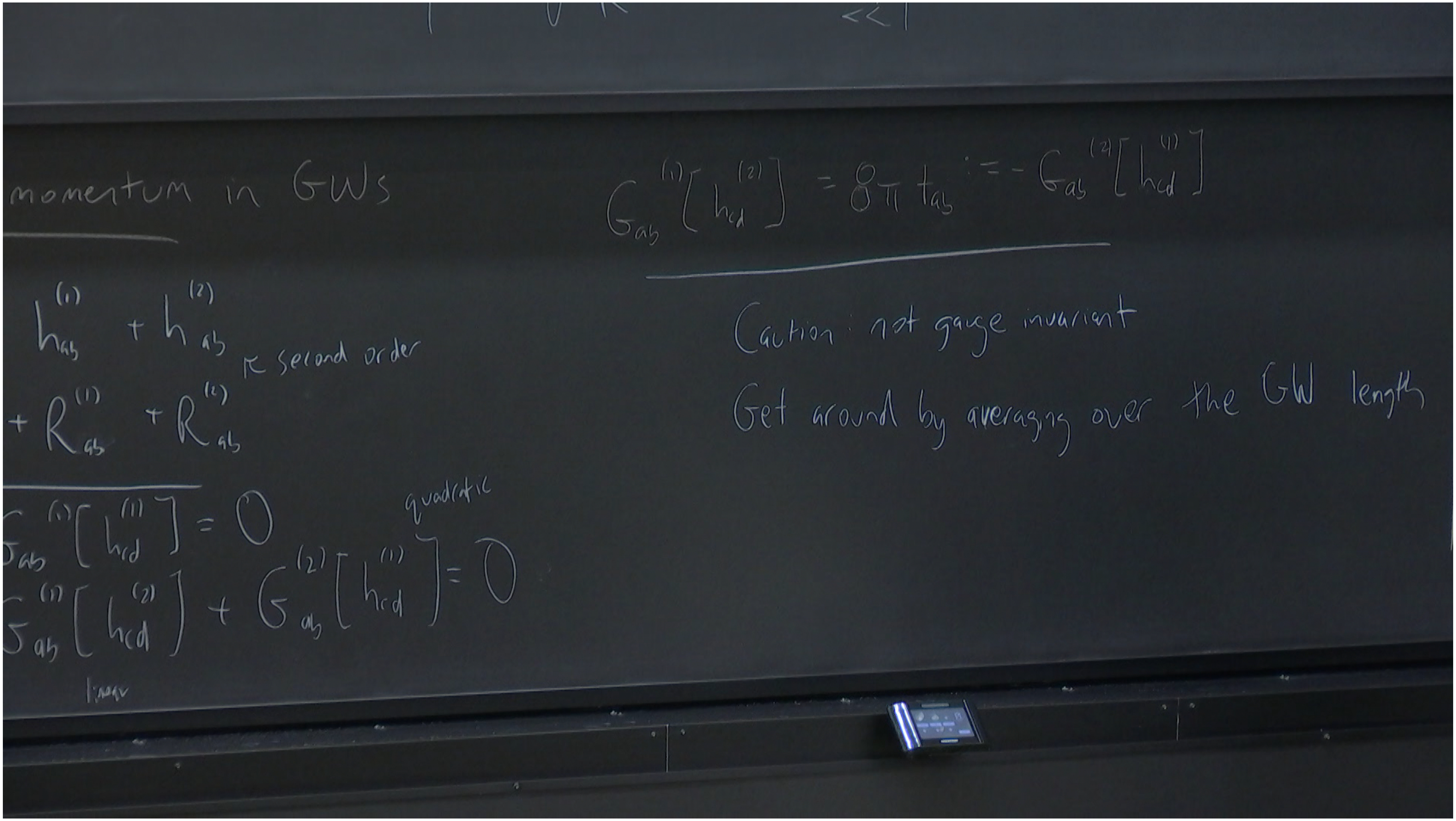
$$g_{ab} = \eta_{ab} + h^{(1)}_{ab} + h^{(2)}_{ab}$$ ← second order

$$R_{ab} = \cancel{R^{(0)}_{ab}}^{=0} + R^{(1)}_{ab} + R^{(2)}_{ab}$$

First order : $G^{(1)}_{ab}[h^{(1)}_{cd}] = 0$

quadratic

Second order : $\underbrace{G^{(1)}_{ab}[h^{(2)}_{cd}]}_{\text{linear}} + G^{(2)}_{ab}[h^{(1)}_{cd}] = 0$


momentum in GWs
$h_{ab}^{(1)} + h_{ab}^{(2)}$ ↖ second order
$+ R_{ab}^{(1)} + R_{ab}^{(2)}$
$G_{ab}^{(1)}[h_{cd}^{(1)}] = 0$
quadratic
$G_{ab}^{(1)}[h_{cd}^{(2)}] + G_{ab}^{(2)}[h_{cd}^{(1)}] = 0$
linear
$G_{ab}^{(1)}[h_{cd}^{(2)}] = 8\pi t_{ab} := -G_{ab}^{(2)}[h_{cd}^{(1)}]$
Caution: not gauge invariant
Get around by averaging over the GW length