Title: Strong Gravity Lecture - 230323

Speakers: William East

Collection: Strong Gravity (2022/2023)

Date: March 23, 2023 - 10:15 AM

URL: https://pirsa.org/23030048

Conformal transverse traceless Eqns

$$\tilde{D}_i\tilde{D}^i\psi - \frac{1}{8}\tilde{R}\psi + \frac{1}{8}\left[(\tilde{\mathbb{L}}X)_{ij} + \hat{A}_{ij}^{TT}\right]^2\psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi\tilde{E}\psi^{-3} = 0$$

$$(\tilde{\Delta}_L X)^{ij} - \frac{2}{3}\psi^6\tilde{D}^i K = 8\pi\tilde{p}^i$$

Free data:

γ_{ij}, K_{ij}

DOF: 12

Constraints: 4

Conformal transverse traceless Eqns

$$\tilde{D}_i\tilde{D}^i\psi - \frac{1}{8}\tilde{R}\psi + \frac{1}{8}\left[(\tilde{\mathbb{L}}X)_{ij} + \hat{A}_{ij}^{TT}\right]^2\psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi\tilde{E}\psi^{-3} = 0$$

$$(\tilde{\Delta}_L X)^{ij} - \frac{2}{3}\psi^6\tilde{D}^i K = 8\pi\tilde{p}^i$$

γ_{ij}, K_{ij}

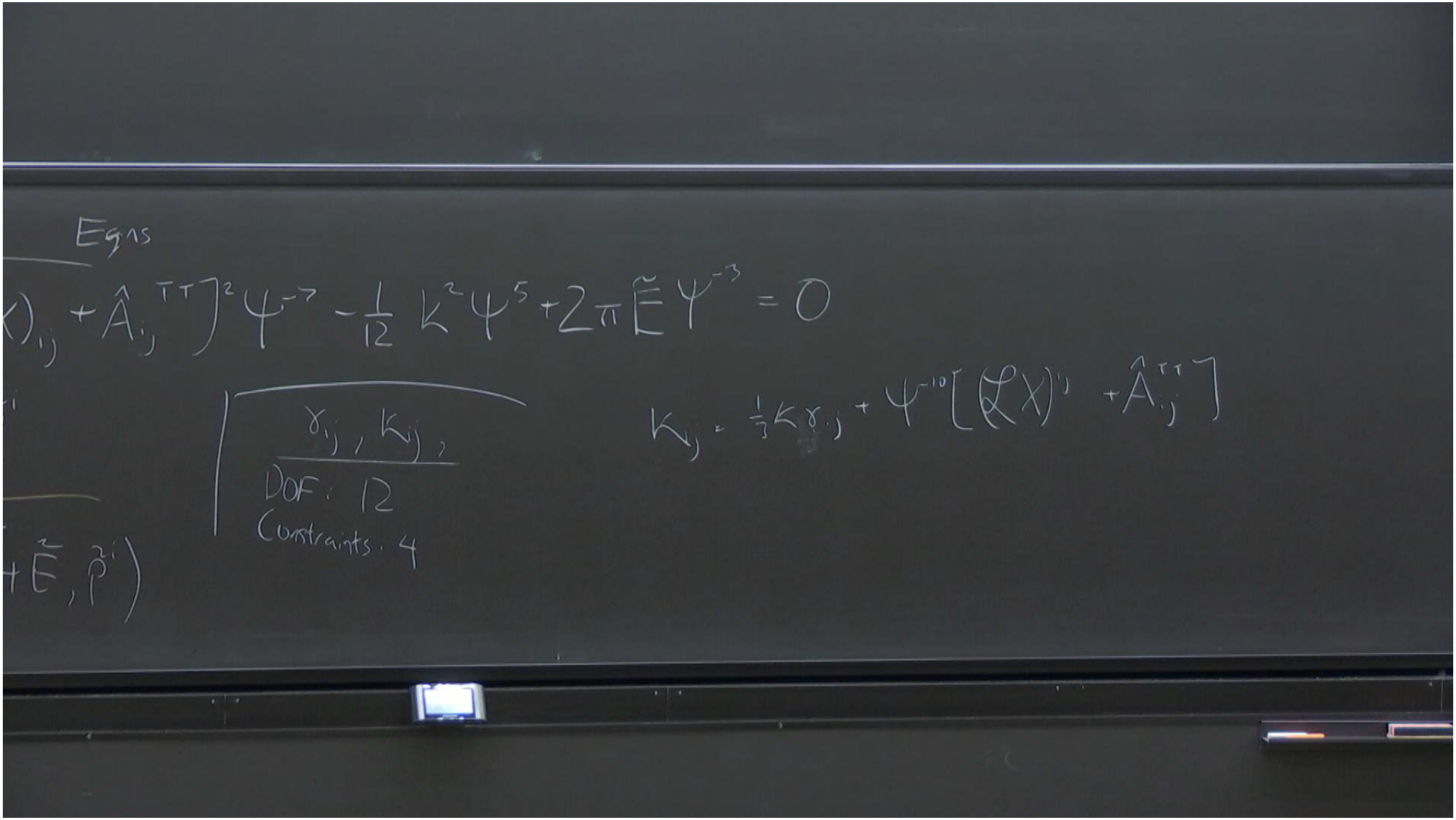
DOF: 12

Constraints: 4

$K_{ij} = \frac{1}{3}K\gamma_{ij}$

Free data: $\hat{A}_{ij}^{TT}$, K, $\tilde{\sigma}_{ij}$, $(\tilde{E}, \tilde{p}^i)$

Constrained: ψ, X^i


Eqns
γ_{ij}, K_{ij}
DOF: 12
Constraints: 4
$K_{ij} = \frac{1}{3} K \gamma_{ij} + \psi^{-10} [(\mathbb{L}X)_{ij} + \hat{A}^{TT}_{ij}]$

Conformal transverse traceless Eqns

$$\tilde{D}_i\tilde{D}^i\psi - \frac{1}{8}\tilde{R}\psi + \frac{1}{8}\left[(\hat{\ell}X)_{ij} + \hat{A}_{ij}^{TT}\right]^2\psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi\tilde{E}$$

$$(\tilde{\Delta}_L X)^{ij} - \frac{2}{3}\psi^6\tilde{D}^i K = 8\pi\tilde{p}^i$$

Free data: $\hat{A}_{ij}^{TT}$ (2) + 1 + 5 = 8

Constrained: ψ, X^i, K, $\tilde{\sigma}_{ij}$, $(+\tilde{E}, \tilde{p}^i)$

γ_{ij}, K_{ij}

DOF: 12

Constraints: 4

K

6

Simple example

$$\psi^{-7} - \frac{1}{12} K^2 \psi^5 + 2\pi \tilde{E} \psi^{-3} = 0$$

γ_{ij}, K_{ij}

DOF: 12

Constraints: 4

$$K_{ij} = \frac{1}{3} K \gamma_{ij} + \psi^{-10} \left[(\mathbb{L}X)_{ij} + \hat{A}^{TT}_{ij} \right]$$

6 = 1 + 3 + 2

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$$

Simple example

Vacuum : $\tilde{\rho} = E = 0$

Maximal + "Waveless" : $K = \hat{A}^{TT}_{ij} = 0$

Conformally flat : $\delta_{ij} = f_{ij}$

BCs: $\psi \to 1$, $X^i \to 0$ as $r \to \infty$

$$\partial_i \partial^i \psi + \frac{1}{8} (\tilde{\mathcal{L}} X)^{ij} (\tilde{\mathcal{L}} X)_{ij} \psi^{-7} = 0$$

$$\Delta_L X^i = 0$$

Domain $\mathbb{R}^3 / \mathbb{B}_R$

at $r = R$ require $D_i s^i = 0 = \tilde{D}_i (\psi^4 \tilde{s}^i)$

$$\frac{1}{r^2} \partial_r (\psi^4 r^2) = 0$$

$$\partial_r \psi + \frac{\psi}{2r} = 0 \text{ at } r = R, \quad \psi \to 1 \quad r \to \infty$$

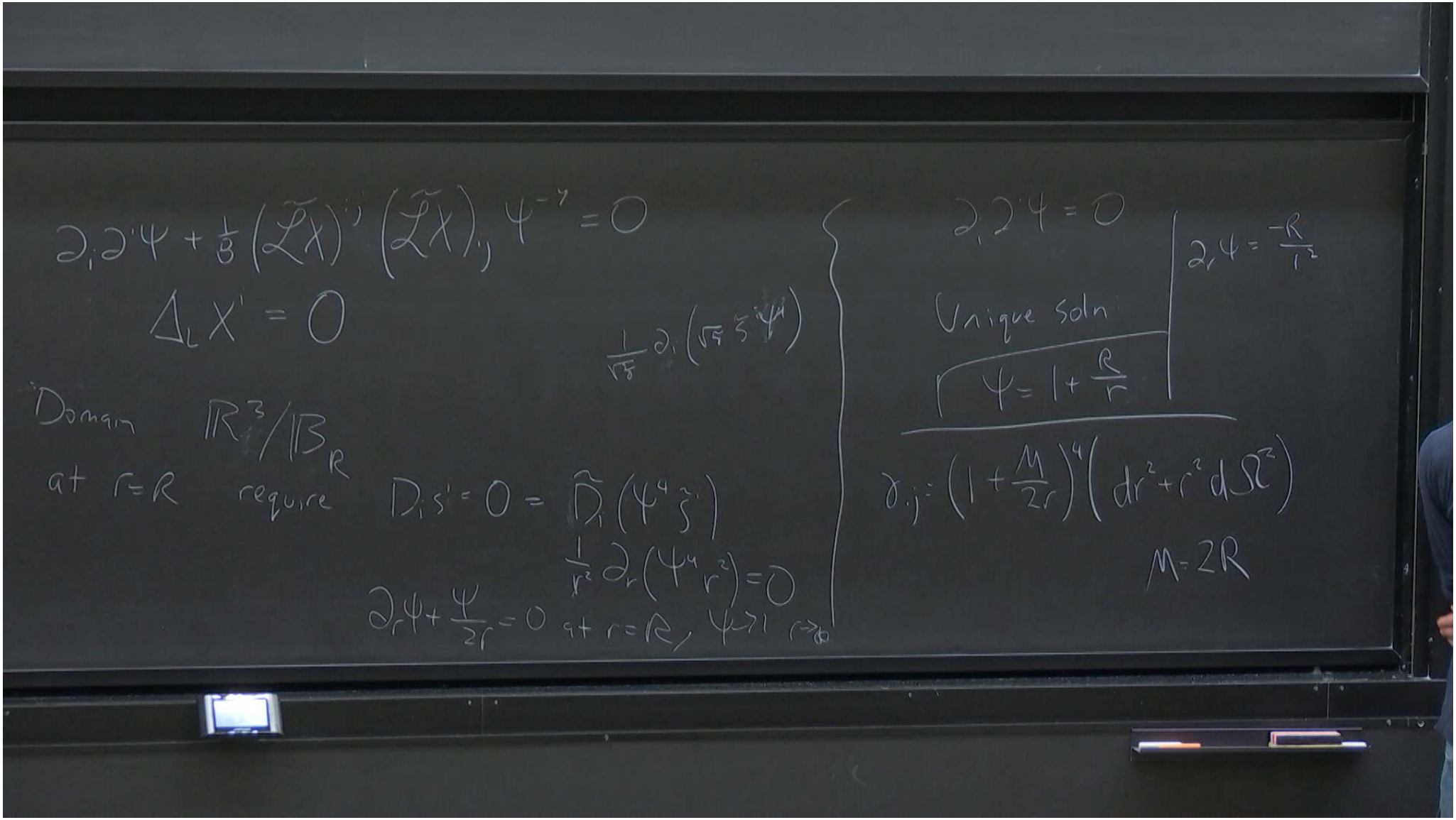
$$\partial_i \partial^i \psi = 0$$

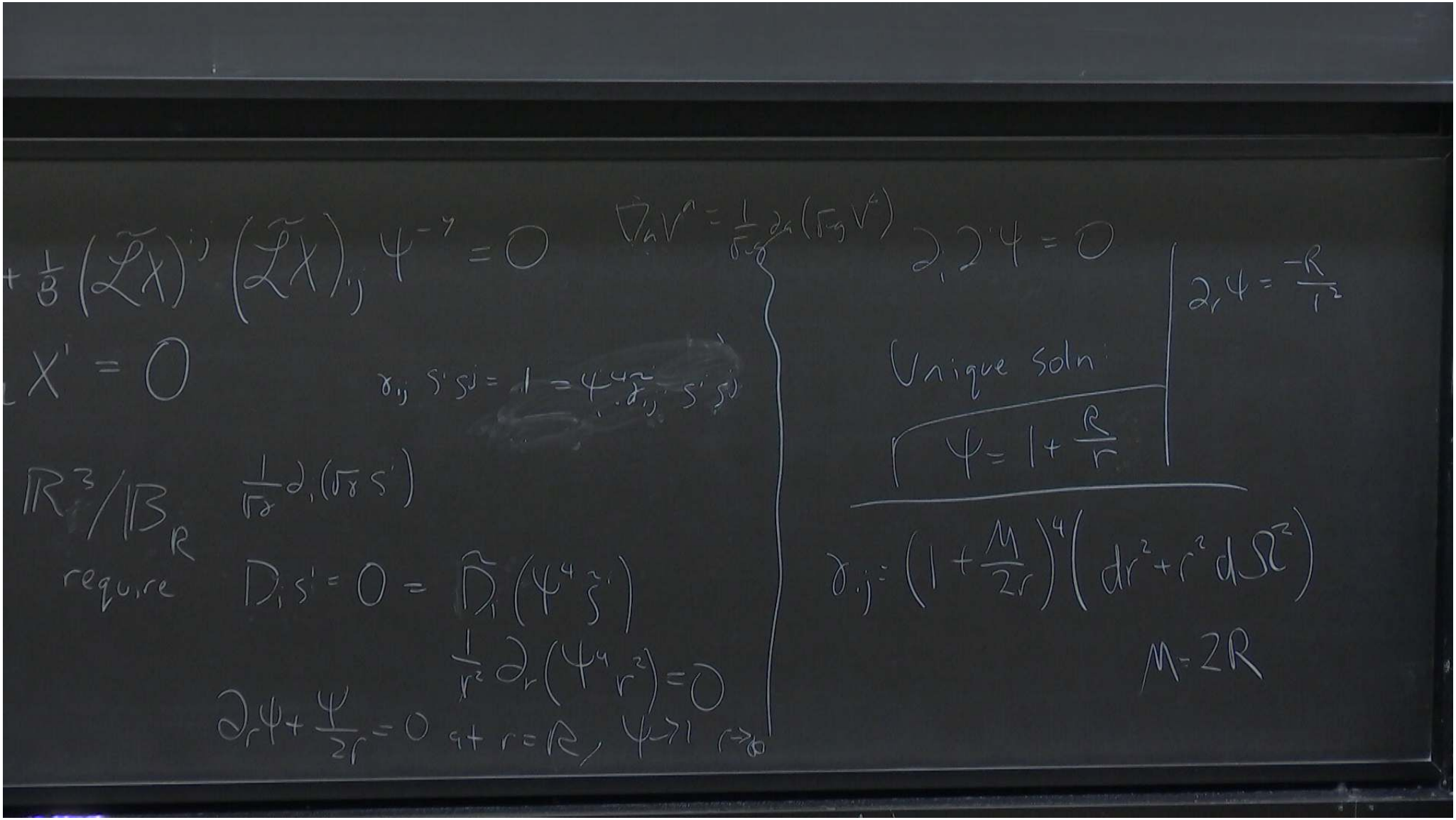
Unique soln:

$$\psi = 1 + \frac{R}{r}$$

$$\gamma_{ij} = \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

$$M = 2R$$


$\partial_i\partial^i\psi + \frac{1}{8}(\tilde{\mathcal{L}}X)^{ij}(\tilde{\mathcal{L}}X)_{ij}\psi^{-7} = 0$
$\Delta_L X^i = 0$
$\frac{1}{\sqrt{q}}\partial_i(\sqrt{q}\,q^{ij}\psi^4)$
Domain $\mathbb{R}^3/\mathbb{B}_R$
at $r=R$ require $D_i s^i = 0 = \hat{D}_i(\psi^4 \tilde{s}^i)$
$\frac{1}{r^2}\partial_r(\psi^4 r^2) = 0$
$\partial_r\psi + \frac{\psi}{2r} = 0$ at $r=R$, $\psi \to 1$ $r\to\infty$
$\partial_i\partial^i\psi = 0$
Unique soln
$\psi = 1 + \frac{R}{r}$
$\partial_i\partial^i\psi = \frac{-R}{r^3}$
$g_{ij} = \left(1+\frac{M}{2r}\right)^4(dr^2 + r^2 d\Omega^2)$
$M = 2R$

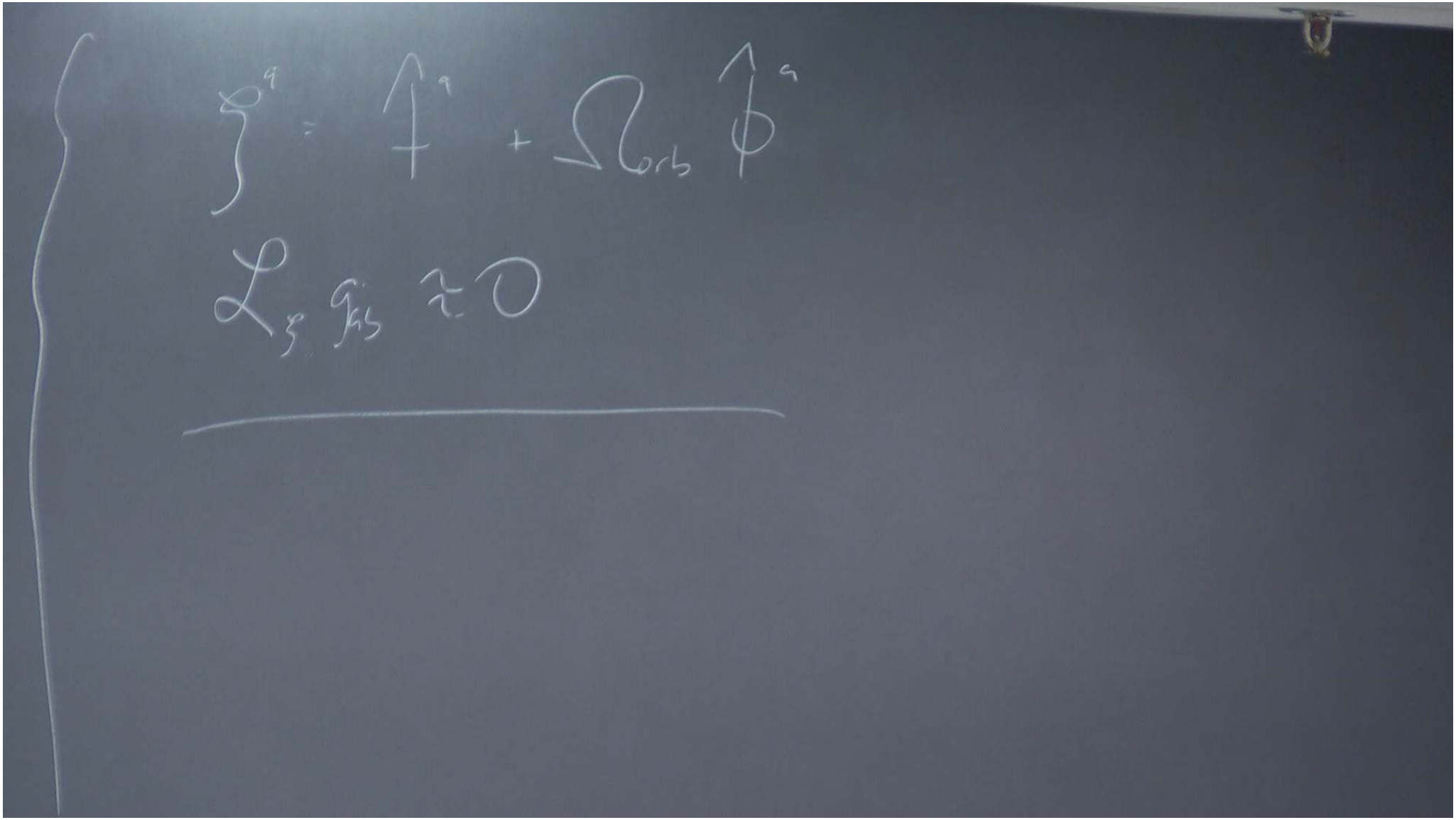

$+\frac{1}{8}(\tilde{\mathcal{L}}X)^{ij}(\tilde{\mathcal{L}}X)_{ij}\psi^{-7}=0$
$\nabla_a V^a = \frac{1}{\sqrt{\gamma}}\partial_a(\sqrt{\gamma}V^a)$
$\tilde{\Delta}\psi=0$
$\tilde{\Delta}\psi=\frac{-R}{\psi^5}$
$X'=0$
$\gamma_{ij}s^i s^j = 1 = \psi^4\tilde{\gamma}_{ij}s^i s^j$
Unique soln
$\psi = 1+\frac{R}{r}$
$\mathbb{R}^3/B_R$
$\frac{1}{\sqrt{\gamma}}\partial_i(\sqrt{\gamma}s^i)$
require $D_i s^i = 0 = \tilde{D}_i(\psi^4 s^i)$
$\gamma_{ij} = \left(1+\frac{M}{2r}\right)^4\left(dr^2+r^2 d\Omega^2\right)$
$\frac{1}{r^2}\partial_r(\psi^4 r^2)=0$
$M=2R$
$\partial_r\psi+\frac{\psi}{2r}=0$ at $r=R$, $\psi\to 1$ $r\to\infty$

Other formulations of the constraint Eqns

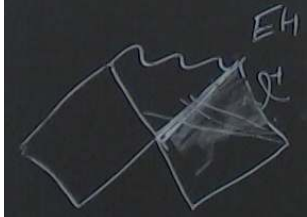
	Free	Constrained data
Conformal thin-sandwich (CTS)	$\tilde{\gamma}_{ij}, K, \tilde{\alpha}, \partial_t \tilde{\gamma}_{ij}$	ψ, β^i
Extended CTS	$\tilde{\gamma}_{ij}, K, \partial_t K, \partial_t \tilde{\gamma}_{ij}$	ψ, α, β^i

Remarks on choosing free data

Strategies: $\tilde{\gamma}_{ij} = \gamma^{(1)}_{ij} + \gamma^{(2)}_{ij} - f_{ij}$


$\xi^a = f^a + \Omega_{orb}\,\phi^a$
$\mathcal{L}_\xi g_{ab} = 0$

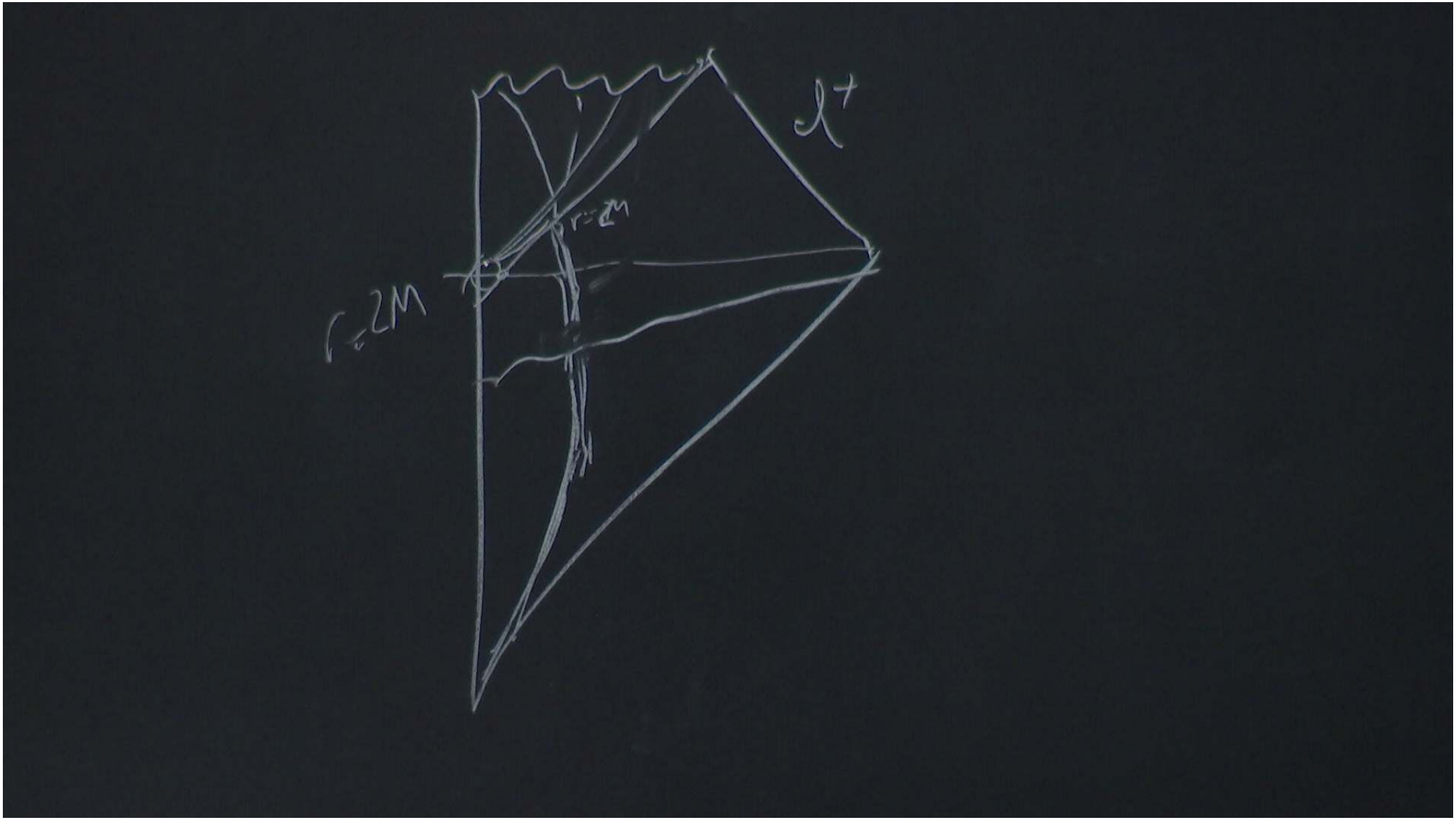
Event horizon - the boundary of the causal past of future null infinity



BH: $B = M - J^-(\mathcal{I}^+)$, EH ∂B

Features of event horizon

1) Requires knowledge of whole spacetime
2) Is "teleological" ie anticipates the future


r=2M
r=2M