Title: Strong Gravity Lecture - 230323

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Collection: Strong Gravity (2022/2023)

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Conformal transverse traceless Eqns

$$\tilde{D}_i \tilde{D}^i \psi - \frac{1}{8} \tilde{R} \psi + \frac{1}{8} \left[(\tilde{\mathbb{L}} X)_{ij} + \hat{A}_{ij}^{TT} \right]^2 \psi^{-7} - \frac{1}{12} K^2 \psi^5 + 2\pi \tilde{E} \psi^{-3} = 0$$

$$(\tilde{\Delta}_L X)^{i} - \frac{2}{3} \psi^6 \tilde{D}^i K = 8\pi \tilde{p}^i$$

Free data:

γ_{ij}, K_{ij}

DOF: 12

Constraints: 4

Conformal transverse traceless Eqns

$$\tilde D_i \tilde D^i \psi - \frac{1}{8} \tilde R \psi + \frac{1}{8} \left[(\tilde{\mathbb{L}} X)_{ij} + \hat A_{ij}^{TT} \right]^2 \psi^{-7} - \frac{1}{12} K^2 \psi^5 + 2\pi \tilde E \psi^{-3} = 0$$

$$(\tilde\Delta_L X)^{ij} - \frac{2}{3} \psi^6 \tilde D^i K = 8\pi \tilde p^i$$

Free data: $\hat A^{TT}_{ij}$, $\overset{1}{K}$, $\overset{5}{\tilde\gamma_{ij}}$, $(+\tilde E, \tilde p^i)$

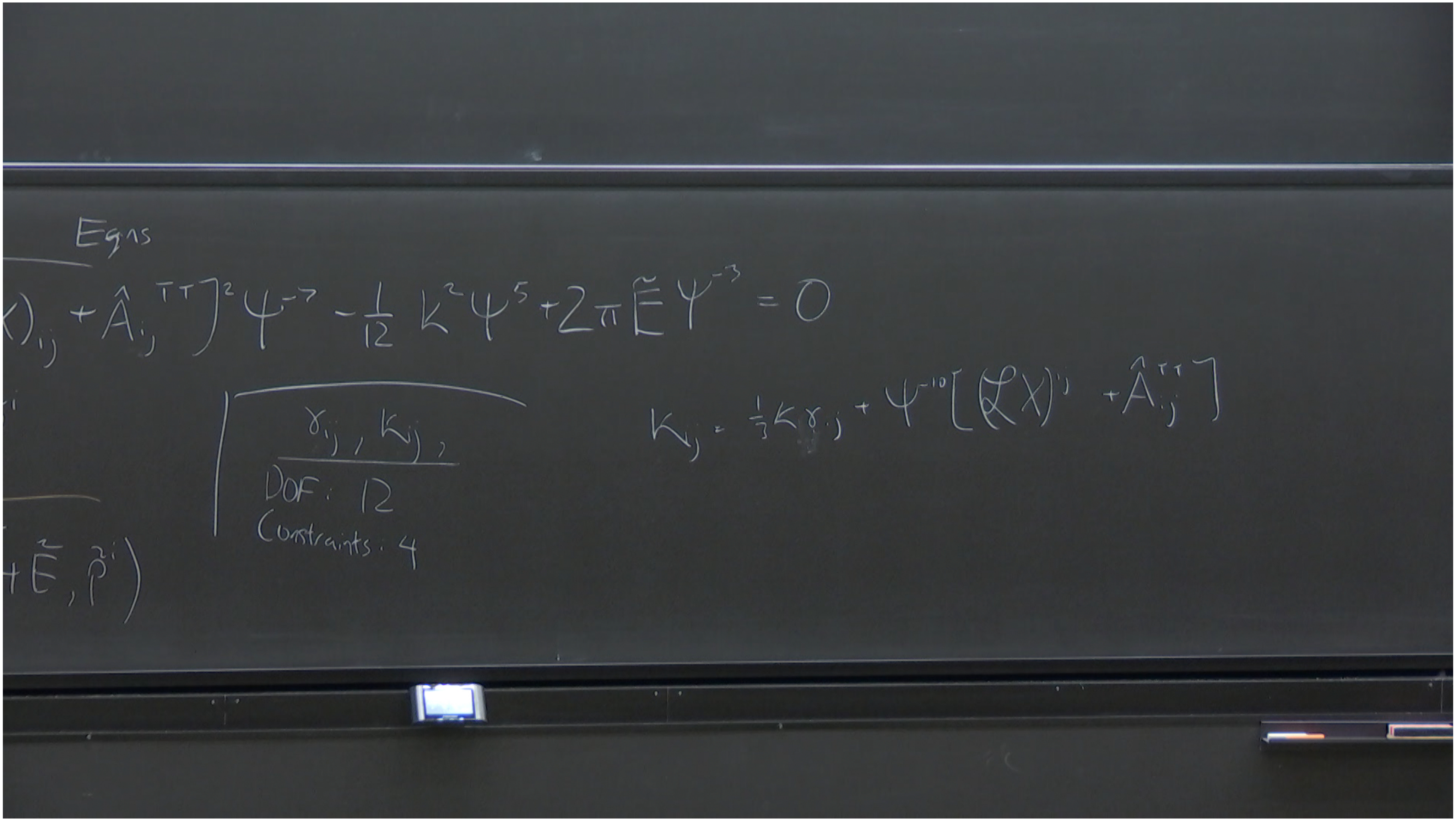
Constrained: ψ, X^i

γ_{ij}, K_{ij},

DOF: 12

Constraints: 4

$K_{ij} = \frac{1}{3} K \gamma_{ij}$


Eqns
$X)_{,ij} + \hat{A}_{ij}^{TT}]^2 \psi^{-7} - \frac{1}{12} K^2 \psi^5 + 2\pi \tilde{E} \psi^{-3} = 0$
$\gamma_{ij}, K_{ij},$
DOF: 12
Constraints: 4
$K_{ij} = \frac{1}{3} K \gamma_{ij} + \psi^{-10}[(\mathcal{L}X)^{ij} + \hat{A}^{TT}_{ij}]$
$+\tilde{E}, \tilde{p}^i)$

Conformal transverse traceless Eqns

$$\tilde{D}_i\tilde{D}^i\psi - \frac{1}{8}\tilde{R}\psi + \frac{1}{8}\left[(\tilde{\mathbb{L}}X)_{ij} + \hat{A}_{ij}^{TT}\right]^2\psi^{-7} - \frac{1}{12}K^2\psi^5 + 2\pi\tilde{E}$$

$$(\tilde{\Delta}_L X)^{ij} - \frac{2}{3}\psi^6\tilde{D}^i K = 8\pi\tilde{p}^i$$

Free data: $\hat{A}^{TT}_{ij},\ K,\ \tilde{\gamma}_{ij},\ (+\tilde{E},\tilde{p}^i)$ — $2+1+5=8$

Constrained: ψ, X^i

$\gamma_{ij},\ K_{ij},$

DOF: 12

Constraints: 4

K

6

Simple example

$$\psi^{-7} - \frac{1}{12} K^2 \psi^5 + 2\pi \tilde{E} \psi^{-3} = 0$$

$\gamma_{ij}, K_{ij},$

DOF: 12

Constraints: 4

$$K_{ij} = \frac{1}{3} K \gamma_{ij} + \psi^{-10}\left[(\mathcal{L}X)^{ij} + \hat{A}^{TT}_{ij} \right]$$

$$6 \qquad 1 + \psi^4 \tilde{\gamma}_{ij} \qquad 3 \qquad 2$$

$$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$$

Simple example

Vacuum : $\tilde{\rho} = \tilde{E} = 0$

Maximal + "Waveless" : $K = \hat{A}^{TT}_{ij} = 0$

Conformally flat : $\tilde{\gamma}_{ij} = f_{ij}$

BCs: $\psi \to 1$, $X^i \to 0$ as $r \to \infty$

$$\partial_i \partial^i \psi + \frac{1}{8} (\tilde{\mathcal{L}} X)^{ij} (\tilde{\mathcal{L}} X)_{ij} \psi^{-7} = 0$$

$$\Delta_L X^i = 0$$

Domain $\mathbb{R}^3 / \mathbb{B}_R$

at $r = R$ require $D_i s^i = 0 = \tilde{D}_i (\psi^4 \tilde{s}^i)$

$$\frac{1}{r^2} \partial_r (\psi^4 r^2) = 0$$

$$\partial_r \psi + \frac{\psi}{2r} = 0 \text{ at } r = R, \; \psi \to 1 \; r \to \infty$$

$$\partial_i \partial^i \psi = 0$$

Unique soln:

$$\psi = 1 + \frac{R}{r}$$

$$\gamma_{ij} = \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

$$M = 2R$$

$$\partial_i \partial^i \psi + \frac{1}{8} (\tilde{\mathcal{L}} X)^{ij} (\tilde{\mathcal{L}} X)_{ij} \psi^{-7} = 0$$

$$\Delta_L X^i = 0$$

$$\frac{1}{\sqrt{g}} \partial_i \left(\sqrt{g}\, s^i \psi^4 \right)$$

Domain $\mathbb{R}^3 / \mathbb{B}_R$

at $r = R$ require $D_i s^i = 0 = \tilde{D}_i (\psi^4 \tilde{s}^i)$

$$\frac{1}{r^2} \partial_r (\psi^4 r^2) = 0$$

$\partial_r \psi + \frac{\psi}{2r} = 0$ at $r = R$, $\psi \to 1$ $r \to \infty$

$$\partial_i \partial^i \psi = 0$$

$$\partial_r \psi = \frac{-R}{r^2}$$

Unique soln:

$$\boxed{\psi = 1 + \frac{R}{r}}$$

$$g_{ij} = \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2 \right)$$

$$M = 2R$$

$$+\frac{1}{8}(\tilde{\mathcal{L}}X)^{ij}(\tilde{\mathcal{L}}X)_{ij}\psi^{-7}=0$$

$$X^i=0$$

$$\tilde{\nabla}_a V^a=\frac{1}{\sqrt{g}}\partial_a(\sqrt{g}V^a)$$

$$\gamma_{ij}s^is^j=1=\psi^4\tilde{\gamma}_{ij}s^is^j$$

$$\mathbb{R}^3/\mathbb{B}_R \qquad \frac{1}{\sqrt{\gamma}}\partial_i(\sqrt{\gamma}s^i)$$

require $\quad D_is^i=0=\tilde{D}_i(\psi^4\tilde{s}^i)$

$$\frac{1}{r^2}\partial_r(\psi^4 r^2)=0$$

$$\partial_r\psi+\frac{\psi}{2r}=0 \text{ at } r=R,\ \psi\to 1 \quad r\to\infty$$

$$\partial_i\partial^i\psi=0$$

Unique soln:

$$\boxed{\psi=1+\frac{R}{r}}$$

$$\partial_r\psi=\frac{-R}{r^2}$$

$$\gamma_{ij}=\left(1+\frac{M}{2r}\right)^4\left(dr^2+r^2d\Omega^2\right)$$

$$M=2R$$

Other formulations of the constraint Eqns

	Free	Constrained data
Conformal thin-sandwich (CTS):	$\tilde{\gamma}_{ij}, K, \tilde{\alpha}, \partial_t \tilde{\gamma}_{ij}$;	ψ, β^i
Extended CTS	$\tilde{\gamma}_{ij}, K, \partial_t K, \partial_t \tilde{\gamma}_{ij}$	ψ, α, β^i

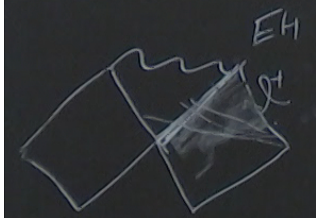
Remarks on choosing free data

Strategies: $\tilde{\gamma}_{ij} = \gamma^{(1)}_{ij} + \gamma^{(2)}_{ij} - f_{ij}$

$$\xi^a = \hat{t}^a + \Omega_{orb}\,\hat{\phi}^a$$

$$\mathcal{L}_\xi\, g_{ab} \simeq 0$$

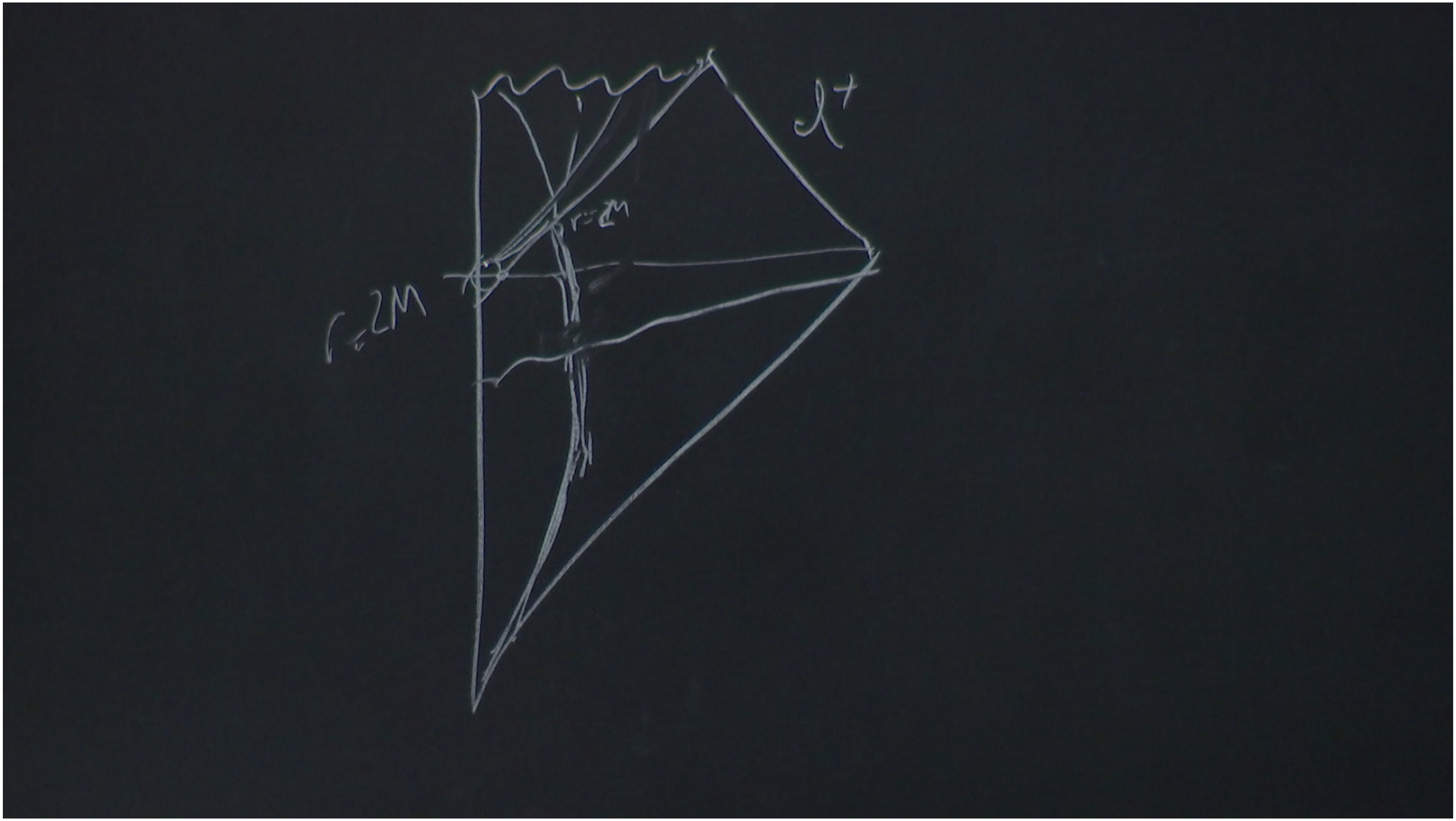
Event horizon - the boundary of the causal past of future null infinity



BH: $B = M - J^-(\mathcal{I}^+)$, EH ∂B

Features of event horizon

1) Requires knowledge of whole spacetime
2) Is "teleological" i.e. anticipates the future



$\mathscr{I}^+$
$r=2M$
$r=2M$