

Title: Strong Gravity Lecture - 230316

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Collection: Strong Gravity (2022/2023)

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## Foliation

$$\mathcal{M} = \{ \Sigma_t \}_{t \in \mathbb{R}}$$

$\mathcal{M}$  is globally hyperbolic

$$\Sigma_t = \{ x^a \in \mathcal{M} \mid f(x^a) = t \}$$

$f$  smooth,  $\nabla_\mu f \neq 0$

$\Sigma_+, \Sigma_-$

ally hyperbolic

$$n_a = -K \nabla_a t$$

$$n_a n^a = -1$$

$$n_a = (-\alpha, \vec{0})$$

lapse

$$-1 = g^{++} n_t^2$$

$$\alpha = \sqrt{\frac{-1}{g^{++}}}$$

$$n^a = \frac{1}{\alpha} (1, \beta^i)$$

shift

$$-\frac{\beta^i}{\alpha} = n_i g^{+i}$$

$$\beta^i = \alpha^2 g^{+i}$$

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$$\alpha = \frac{d\tau}{dt}, \quad -\beta^i = \frac{dx^i}{dt}$$

$$\Sigma_+, \bar{\Sigma}_+$$

$$\varphi(1, \beta')$$

$$\alpha = \frac{d\tau}{dt}, -\beta' =$$

$$\mathcal{L}_n \gamma_{ab} = n^c \nabla_c \gamma_{ab} + \gamma_{cb} \nabla_c n^a + \gamma_{ac} \nabla_c n^b$$

$$(**) \nabla_a n^c = -K_a^c - \underbrace{(n^d \nabla_d n^c)}_{a^c} n_a$$

$$\nabla_c \gamma_{ab} = \nabla_c (n_a n_b)$$

$$\mathcal{L}_n \gamma_{ab} = -2K_{ab}$$

$$\mathcal{L}_n \gamma_{ab} = -2\alpha K_{ab}$$

$$(3) R_{abc}{}^d U_d = D_a D_b U_c - D_b D_a U_c$$

$$D_a D_b U_c = \gamma_a^d \gamma_b^e \gamma_c^f \nabla_d (\gamma_e^g \gamma_f^h \nabla_g U_h)$$

$$\gamma_{ab} = g_{ab} + n_a n_b$$

$${}^3 R^c{}_{dab} = \gamma_a^e \gamma_b^f \gamma_h^c \gamma_d^g R^h{}_{gef} - K_a^c K_{db} + K_b^c K_{ad}$$

$$\gamma^{ab} {}^3R_{db} + K K_{db} - K^a_b K_{ad} = \gamma^f_b \gamma^e_h \gamma^g_d R^h_{gef} \quad (\diamond)$$

$${}^3R + K^2 - K_{ab} K^{ab} = R + 2R_{ab} n^a n^b \quad (*)$$

$$\left. \begin{aligned} R^c_{dab} n^d &= (\nabla_a \nabla_b - \nabla_b \nabla_a) n^c \\ \gamma^c_d n^e \gamma^f_a \gamma^g_b R^d_{efg} &= D_b K^c_a - D_a K^c_b \end{aligned} \right\}$$

$$n^e \gamma^g_b R_{eg} = D_b K - D_a K^a_b \quad (\smile)$$

$$d_b + K K_{db} - K_b^a K_{ad} = \delta_b^f \delta_h^e \delta_d^g R^h_{gef} \quad (\square)$$

$$R + K^2 - K_{ab} K^{ab} = R + 2 R_{ab} n^a n^b \quad (*)$$

$$\left. \begin{aligned} d_{ab} n^d &= (\nabla_a \nabla_b - \nabla_b \nabla_a) n^c \\ \delta_d^c n^e \delta_a^f \delta_b^g R^d_{efg} &= D_b K_a^c - D_a K_b^c \end{aligned} \right\}$$

$$n^e \delta_b^g R_{eg} = D_b K - D_a K_b^a \quad (\text{smiley face})$$

Gauss-Codazzi

$$\gamma^4 R, \gamma^3 n R, \gamma^2 n^2 R$$

$$n^3 R$$

$$\gamma_{ac} n^d \gamma_b^e R^c{}_{fed} n^f = \gamma_{ac} n^d \gamma_b^e (\nabla_e \nabla_d n^c - \nabla_d \nabla_e n^c)$$

$$\left( \nabla_b n_a = -K_{ab} - D_a \ln \alpha n_b \right)$$

$$a^a = D_a \ln(\alpha)$$

$$\rightarrow = -K_{af} K_b^f + \frac{1}{\alpha} D_b D_a \ln \alpha + \gamma_a^c \gamma_b^e \gamma n^f \nabla_f K_{ce}$$

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}$$

$$E =$$

$$E = T_{ab} n^a n^b \quad \text{energy density}$$

$$p^a = -\gamma^a_c n^b T^c_b \quad \text{momentum density}$$

$$S_{ab} = \gamma^c_a \gamma^d_b T_{cd} \quad \text{matter stress-tensor}$$

$$\text{Fluid: } T_{ab} = (\rho + P) u_a u_b + P g_{ab}$$

$$E = (1+P)W^2 - P, \quad W = -n_a U^a$$

$$P^a = (E+P)U^a, \quad U^a = \frac{1}{W} \delta^a_b U^b$$

$$S_{ab} = (E+P)U_a U_b + P \delta_{ab}$$

$\nabla_c n^c$

$-D_a \ln(\alpha)$

$\nabla_f K_{ce}$

$$\left\{ \begin{array}{l} \gamma_{ab} n^d \gamma_b^e R^c{}_{fed} n^f = \frac{1}{\alpha} \mathcal{L}_n K_{ab} + \frac{1}{\alpha} D_a D_b \alpha + K_{af} K_b^f \end{array} \right. \quad ($$

$$R_{ab} n^a n^b + \frac{1}{2} R = 8\pi E$$

$$3R + k^2 - K_{ab} K^{ab} = 16\pi E$$

Hamiltonian constraint

$$R_{ab} \delta_c^a n^b - \frac{1}{2} R n_c \delta_c^a = -8\pi p_c$$

$$(\odot) \Rightarrow D_a K_b^a - D_b K = 8\pi p_b$$

Momentum constraint

$$R_{ab} = 8\pi \left( T_{ab} - \frac{1}{2} T g_{ab} \right)$$

$$\gamma^a_c \gamma^b_d R_{ab} = 8\pi \left[ S_{cd} - \frac{1}{2} (S - E) \gamma_{cd} \right]$$

$$(\diamond, \Delta) \Rightarrow$$

$$-\frac{1}{2} \mathcal{L}_m K_{cd} - \frac{1}{2} D_c D_d \alpha + \mathcal{R}_{cd} + K K_{cd} - 2 K_{ca} K^a_d = 8\pi \{ \dots \}$$

$$K_{ab}, \gamma_{ab}$$

$$\nabla \cdot \vec{E} = \rho, \quad \nabla \cdot \vec{B} = 0 \quad \text{Constraints}$$

$$\partial_t E = \nabla \times \vec{B} - \vec{J}, \quad \partial_t \vec{B} = -\nabla \times \vec{E} \quad \text{evolution eqns.}$$

$$\partial_t (\nabla \cdot \vec{B}) = 0$$

$$\partial_t (\nabla \cdot \vec{E}) = -\nabla \cdot \vec{J}$$

$$\partial_t \rho + \nabla \cdot \vec{J} = 0 \quad \Rightarrow \quad \partial_t (\nabla \cdot \vec{E} - \rho) = 0$$

$$\gamma^{ab} {}^3R_{ab} +$$

$${}^3R +$$

$$R^c{}_{dab}$$

$$\gamma^c{}_d n^e$$