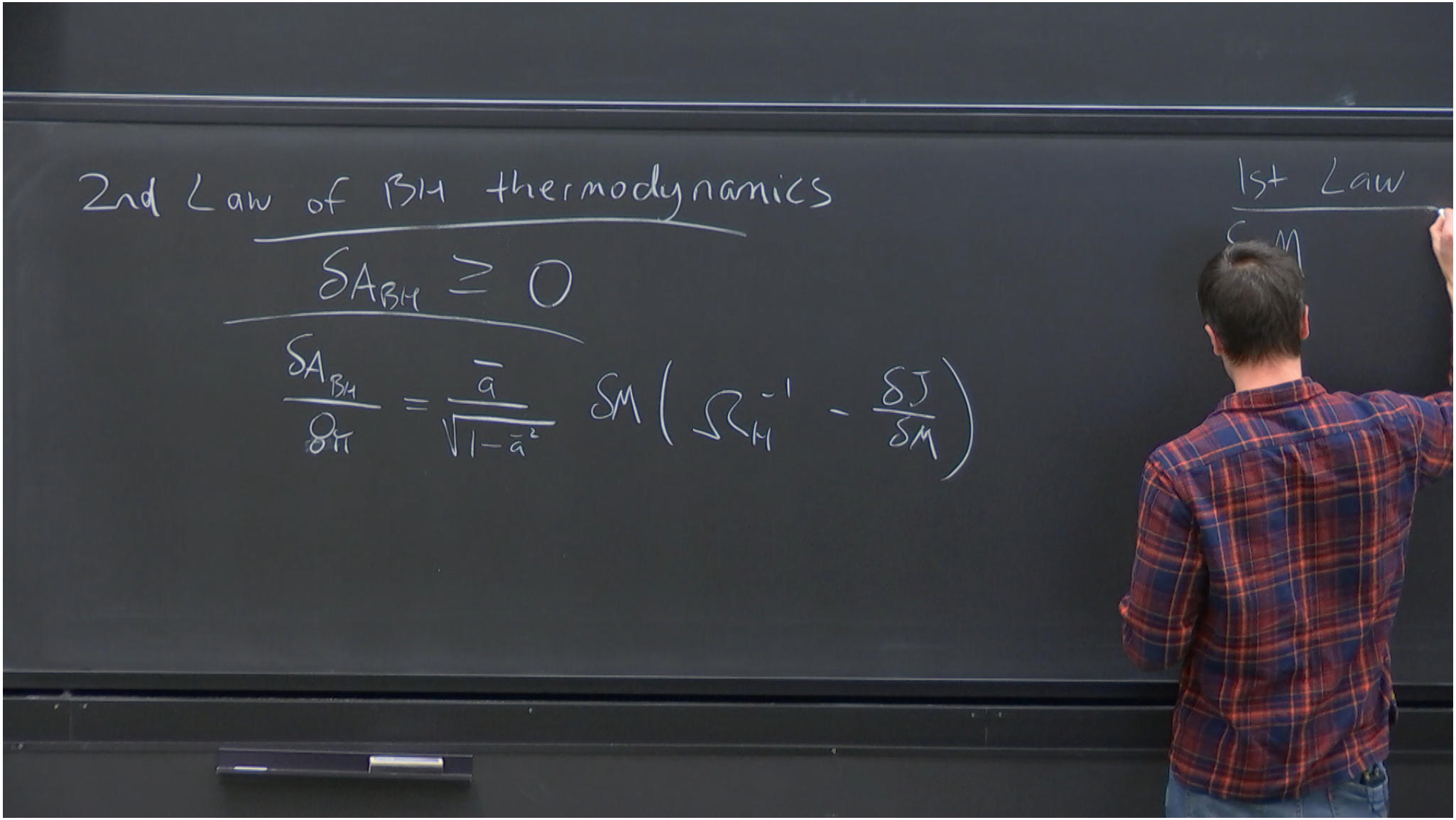
Title: Strong Gravity Lecture - 230309

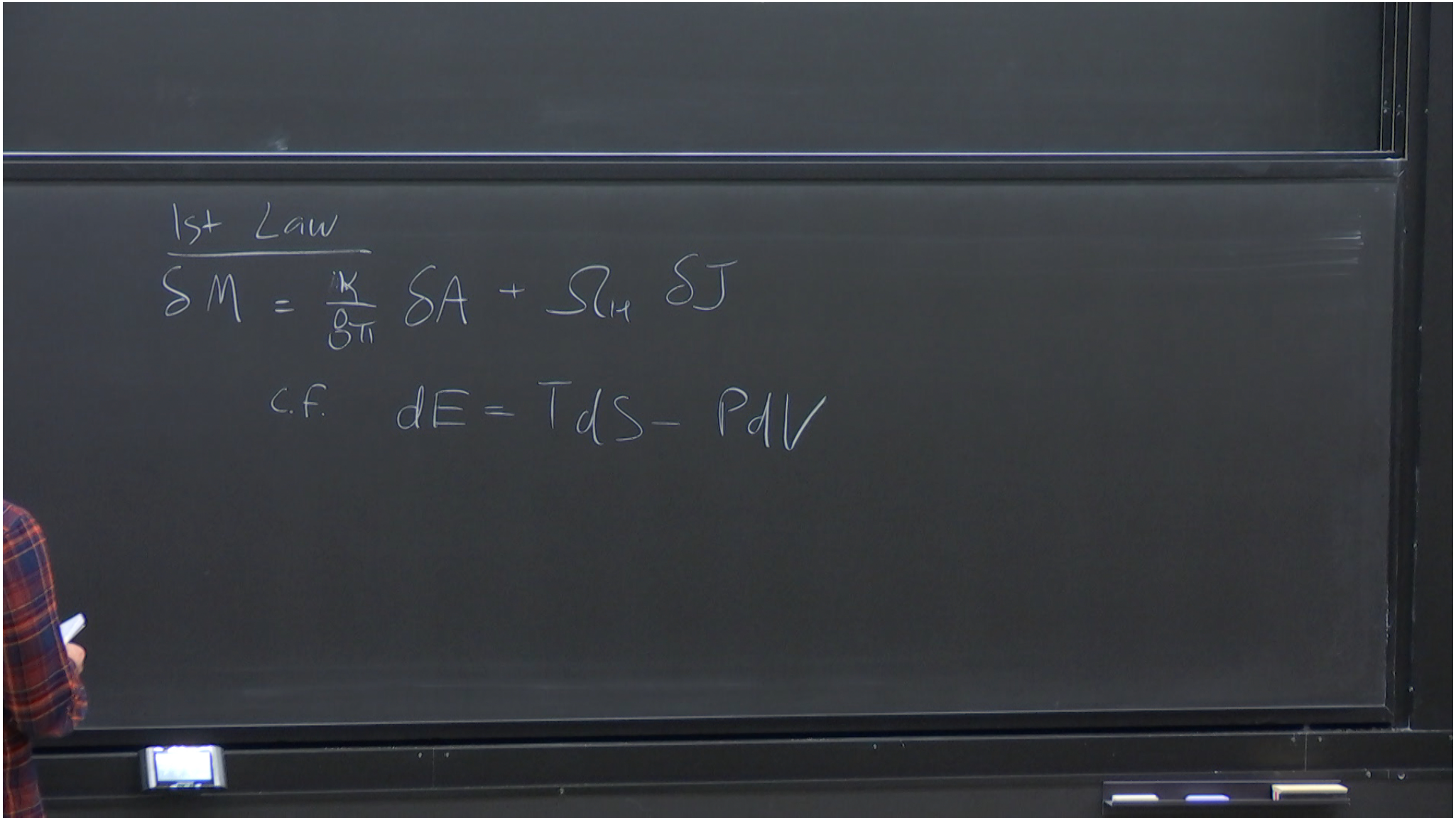
Speakers: William East

Collection: Strong Gravity (2022/2023)

Date: March 09, 2023 - 10:15 AM

URL: https://pirsa.org/23030044


2nd Law of BH thermodynamics
$\delta A_{BH} \geq 0$
$\frac{\delta A_{BH}}{8\pi} = \frac{\bar{a}}{\sqrt{1-\bar{a}^2}} \delta M \left(\tilde{R}_H^{-1} - \frac{\delta J}{\delta M} \right)$
1st Law
δM


1st Law
$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega_H \delta J$
c.f. $dE = TdS - PdV$

1st Law

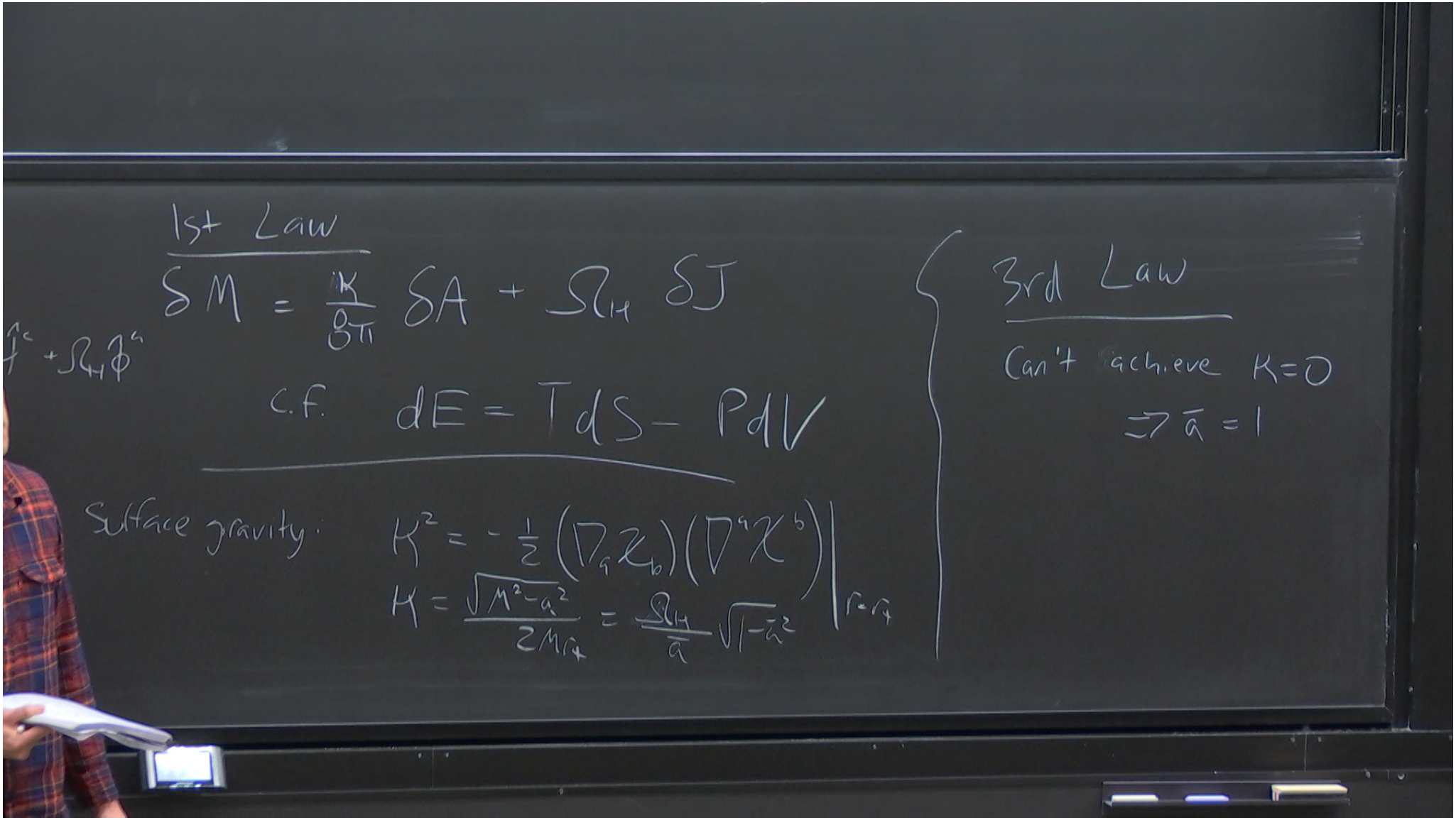
$$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega_H \delta J$$

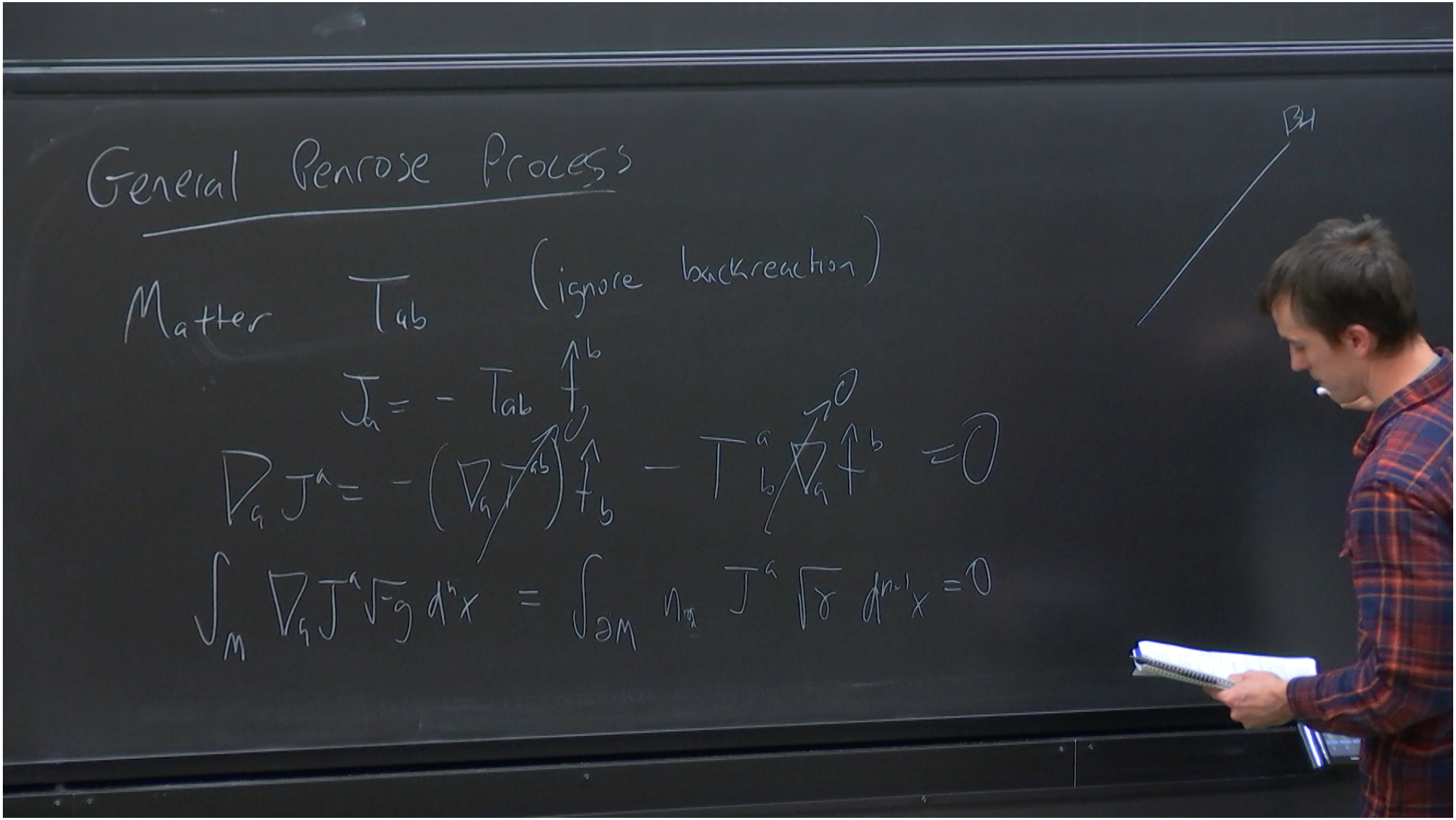
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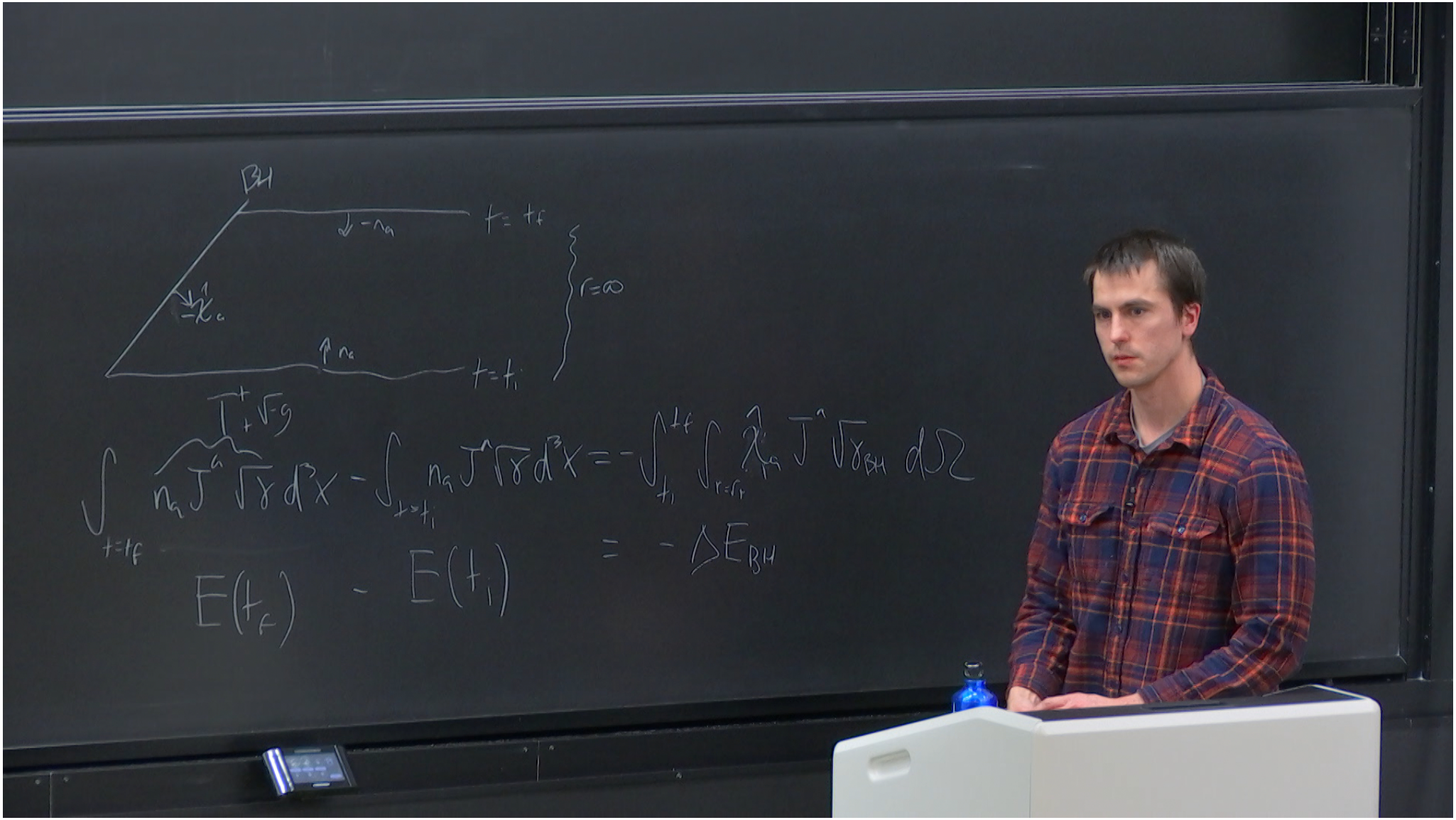
Surface gravity: $\kappa^2 = -\frac{1}{2}(\nabla_a \chi_b)(\nabla^a \chi^b)\Big|_{r=r_+}$

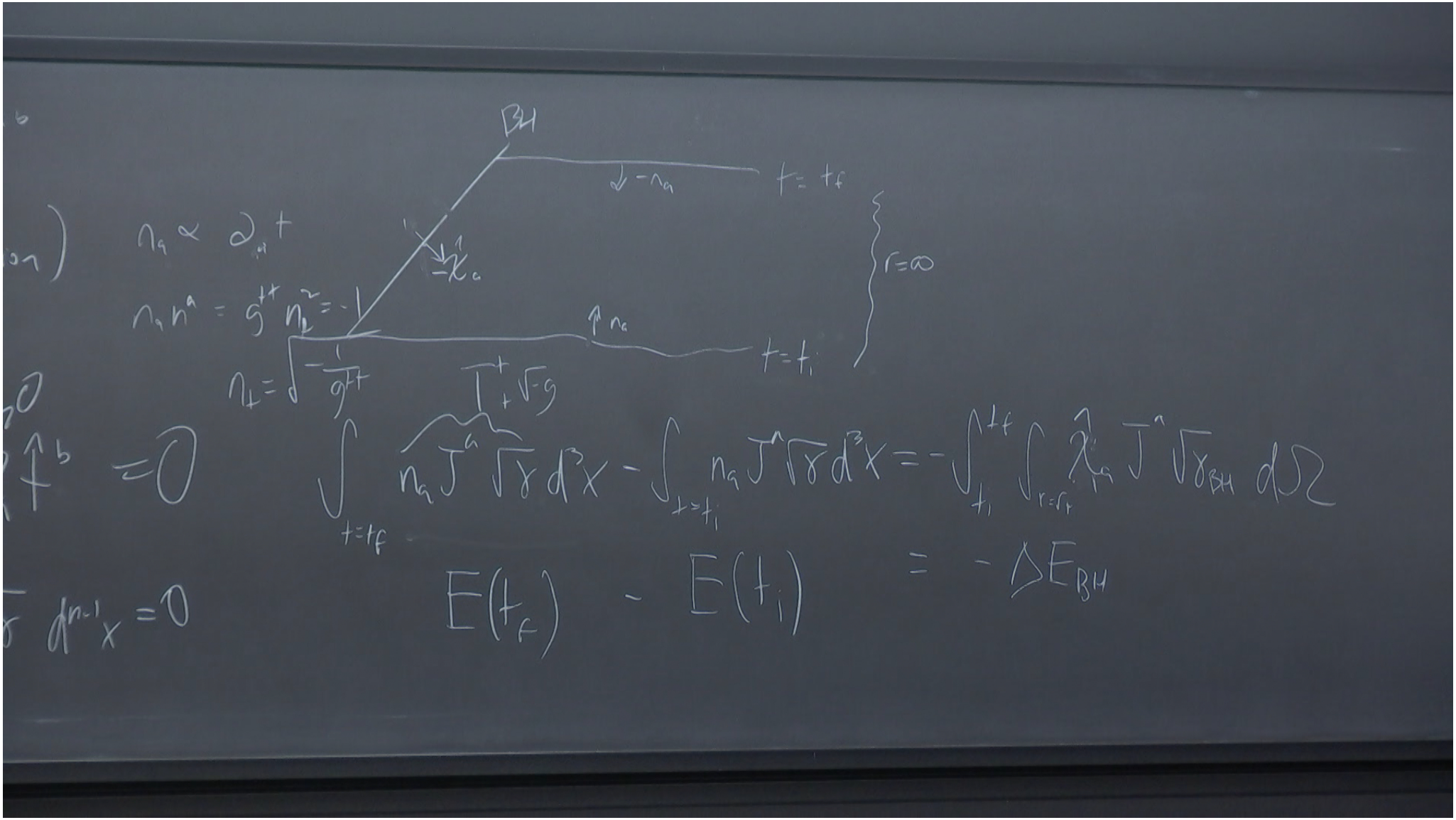
$$\kappa = \frac{\sqrt{M^2 - a^2}}{2Mr_+} = \frac{\Omega_H}{a}\sqrt{-\tilde{a}^2}$$

{ 3rd Law


1st Law
$\delta M = \frac{\kappa}{8\pi} \delta A + \Omega_H \delta J$
c.f. $dE = TdS - PdV$
Surface gravity: $\kappa^2 = -\frac{1}{2}(\nabla_a \chi_b)(\nabla^a \chi^b)\Big|_{r=r_+}$
$\kappa = \frac{\sqrt{M^2 - a^2}}{2Mr_+} = \frac{\Omega_H}{a}\sqrt{1-\bar{a}^2}$
3rd Law
Can't achieve $\kappa = 0$
$\Rightarrow \bar{a} = 1$


General Penrose Process
Matter T_{ab} (ignore backreaction)
$J_a = -T_{ab} f^b$
$\nabla_a J^a = -(\nabla_a T^{ab}) f_b - T^a{}_b \nabla_a f^b = 0$
$\int_M \nabla_a J^a \sqrt{-g}\, d^n x = \int_{\partial M} n_a J^a \sqrt{\gamma}\, d^{n-1} x = 0$
BH

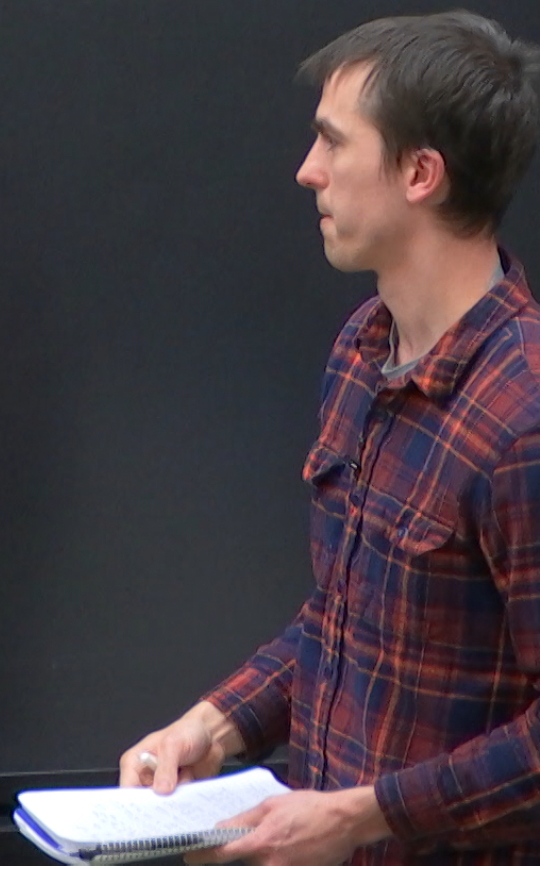

BH
$t = t_f$
$r = \infty$
$t = t_i$
$\int_{t=t_f} n_a J^a \sqrt{\gamma}\, d^3x - \int_{t=t_i} n_a J^a \sqrt{\gamma}\, d^3x = -\int_{t_i}^{t_f} \int_{r=r_f} \vec{\chi}_a J^a \sqrt{\gamma_{BH}}\, d\Omega$
$E(t_f) - E(t_i) = -\Delta E_{BH}$


BH
$t = t_f$
$r = \infty$
$t = t_i$
$n_a \propto \partial_a t$
$n_a n^a = g^{tt} n_t^2 = -1$
$n_t = \sqrt{-\frac{1}{g^{tt}}}$
$\int_{t=t_f} n_a J^a \sqrt{\gamma}\, d^3x - \int_{t=t_i} n_a J^a \sqrt{\gamma}\, d^3x = -\int_{t_i}^{t_f} \int_{r=r_+} \lambda_a J^a \sqrt{\gamma_{BH}}\, d\Omega$
$E(t_f) - E(t_i) = -\Delta E_{BH}$

$$J^b_a = T_{ab}\,\phi^b$$

$$J^\phi(t_f) - J^\phi(t_i) = -\Delta J_{BH}$$

$$0 \le T_{ab}\,\chi^a\chi^b = \left(-J_b + S_{BH}\,J^\phi_b\right)\chi^b$$



$$J^b_a = T_{ab}\,\phi^b$$

$$J^\phi(t_f) - J^\phi(t_i) = -\Delta J_{BH}$$

$$0 \le T_{ab}\,\chi^a\chi^b = \left(-J_b + \Omega_H J^\phi_b\right)\chi^b$$

$$0 \le \Delta E_{BH} - \Omega_H \Delta J_{BH}$$

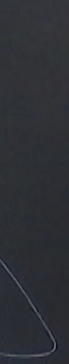


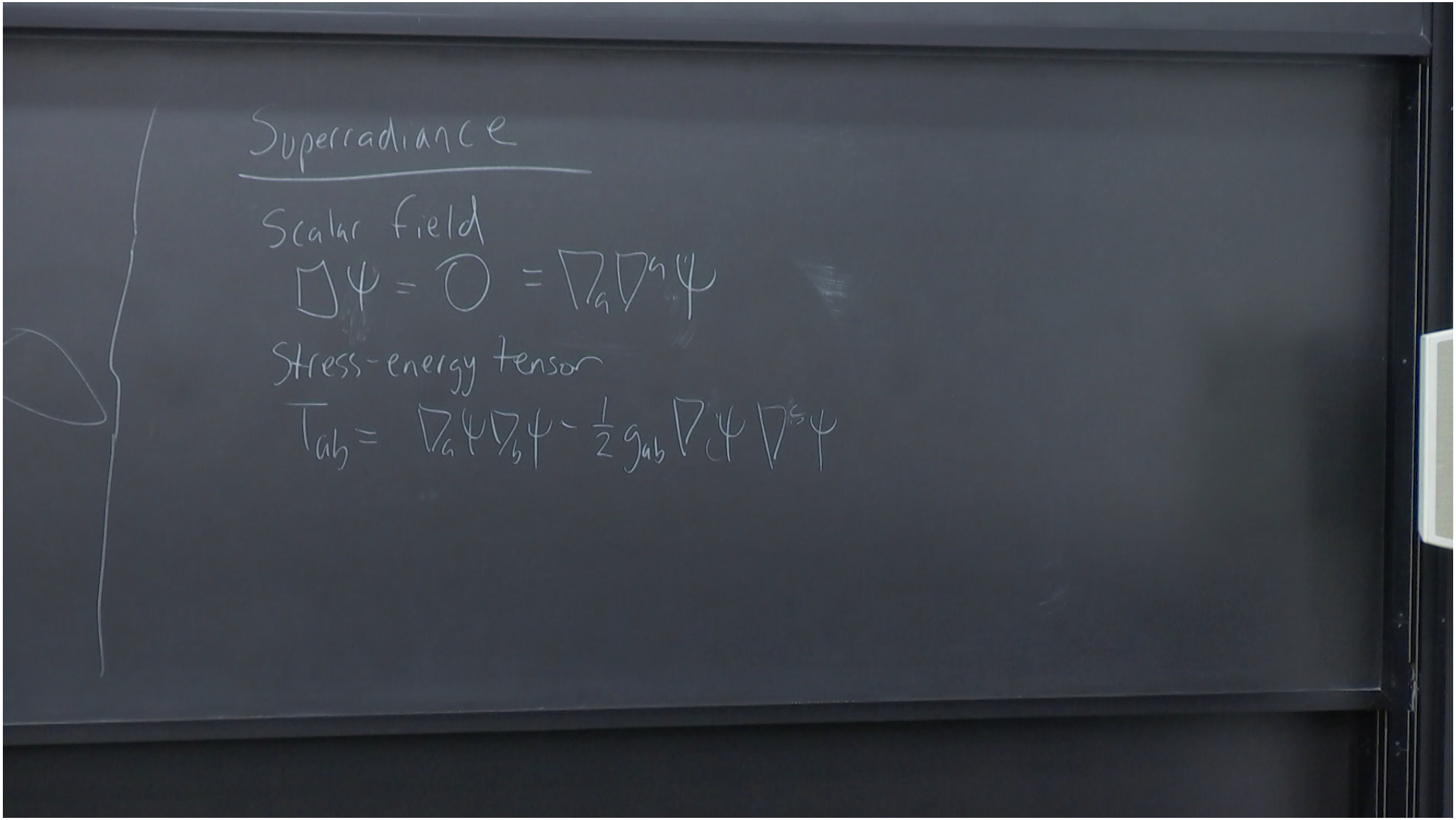
$$J^b_a = T_{ab}\,\phi^b$$

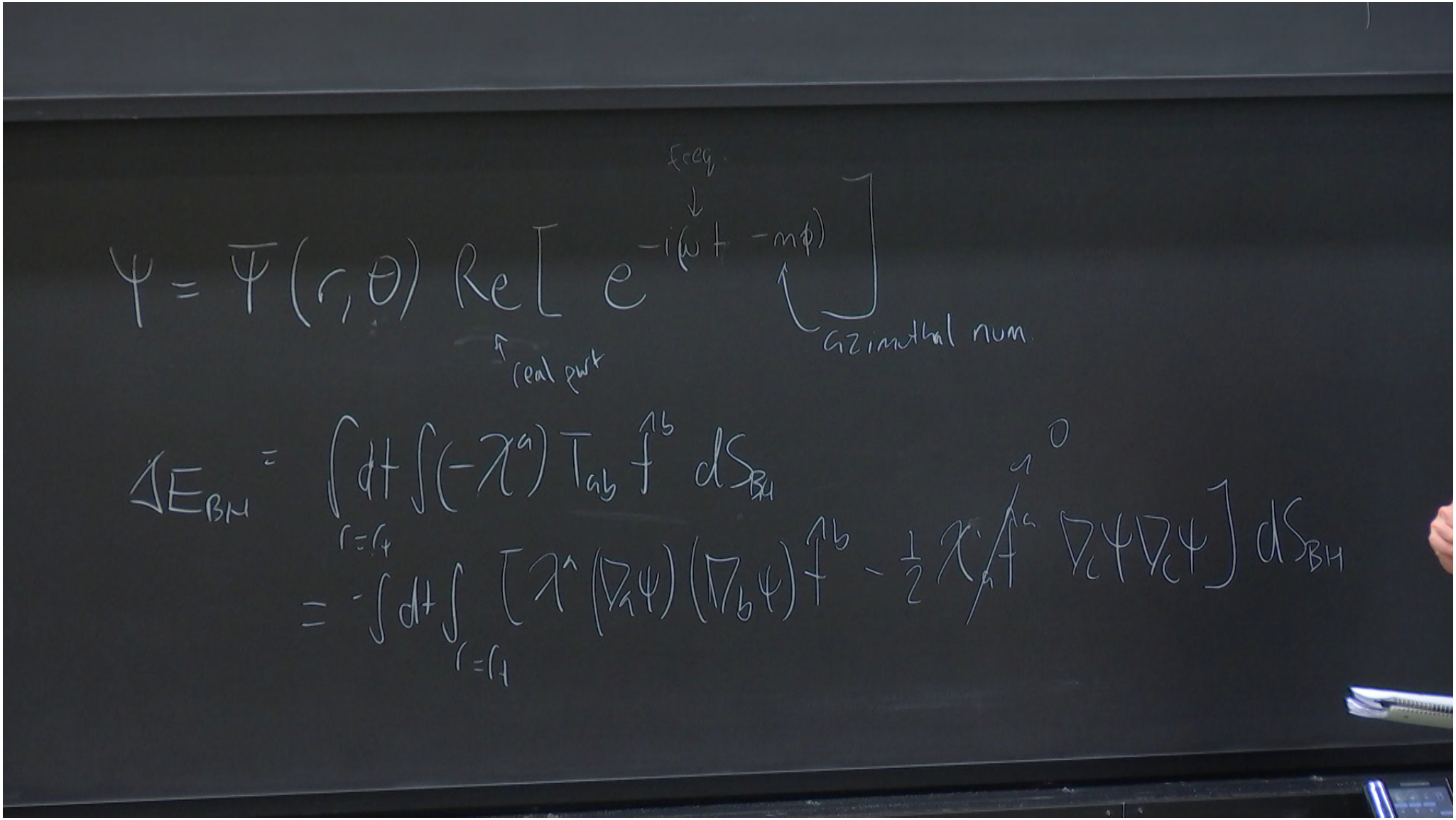
$$J^\phi(t_f) - J^\phi(t_i) = -\Delta J_{BH}$$

$$0 \le T_{ab}\,\chi^a\chi^b = \left(-J_b + \Omega_H J^\phi_b\right)\chi^b$$

$$0 \le \Delta E_{BH} - \Omega_H \Delta J_{BH}$$



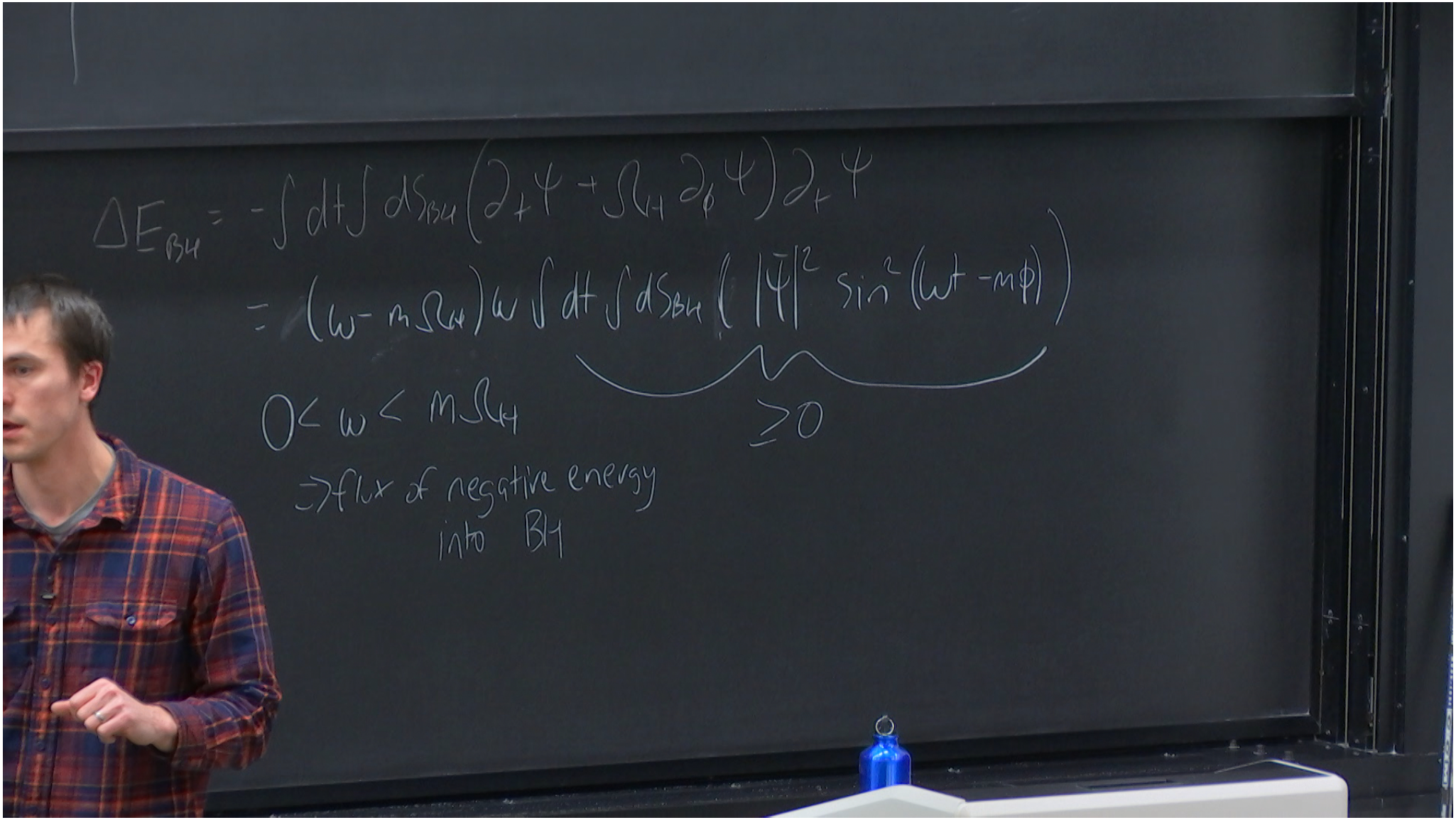

Superradiance
Scalar field
$\Box\psi = 0 = \nabla_a \nabla^a \psi$
Stress-energy tensor
$T_{ab} = \nabla_a \psi \nabla_b \psi - \frac{1}{2} g_{ab} \nabla_c \psi \nabla^c \psi$

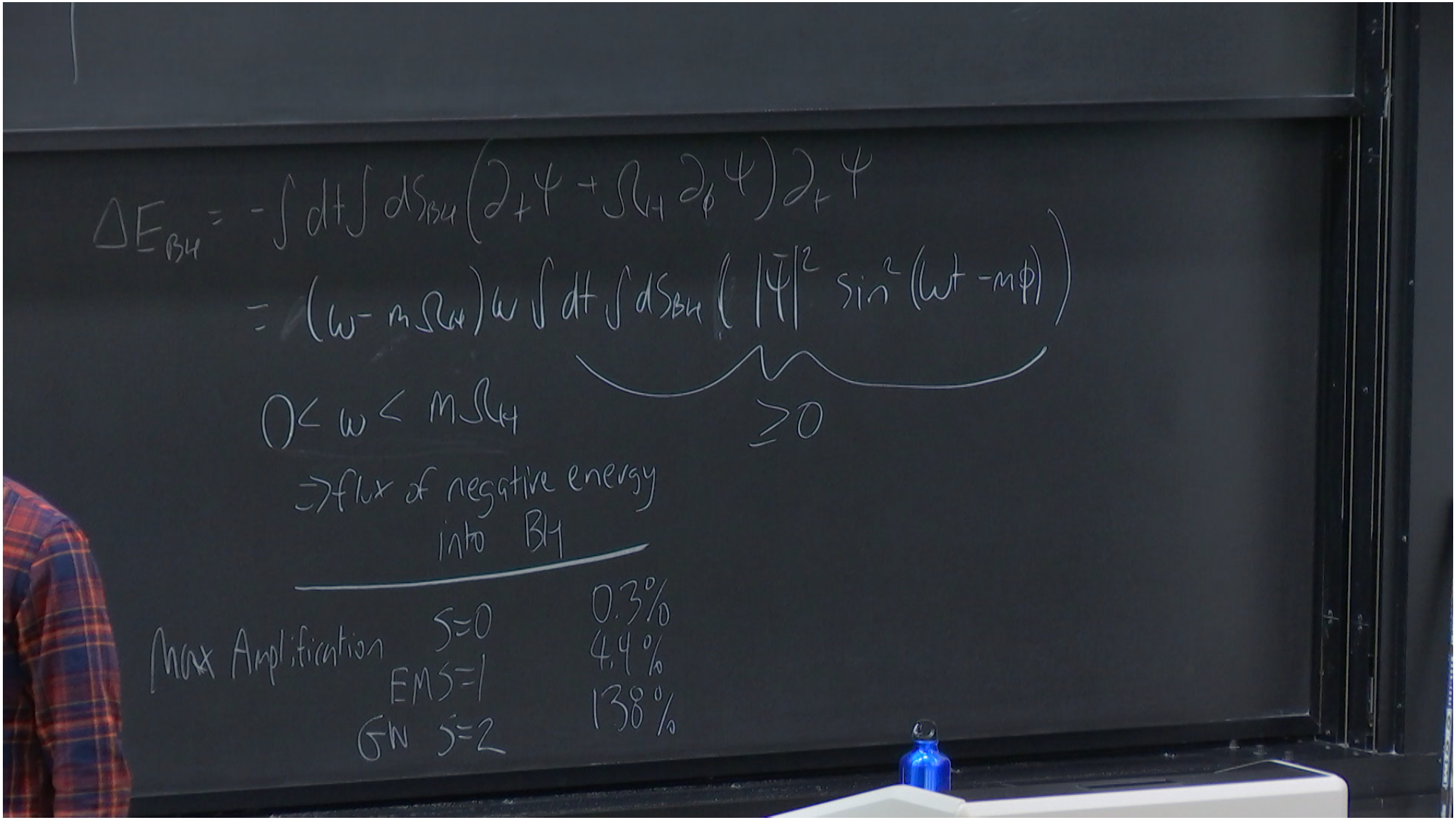


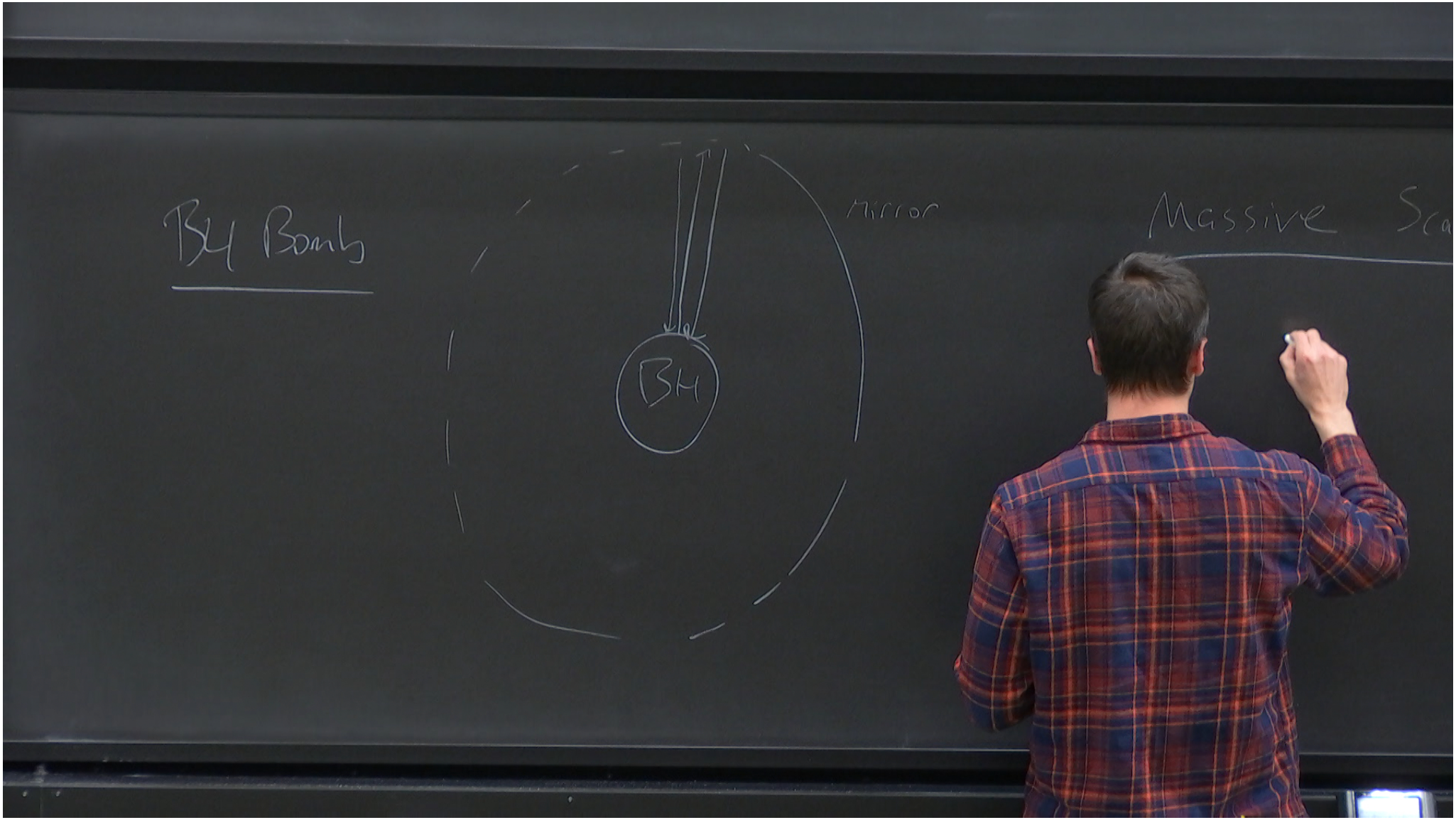
$$\Psi = \bar{\Psi}(r,\theta)\, \mathrm{Re}\left[e^{-i(\omega t - m\phi)} \right]$$

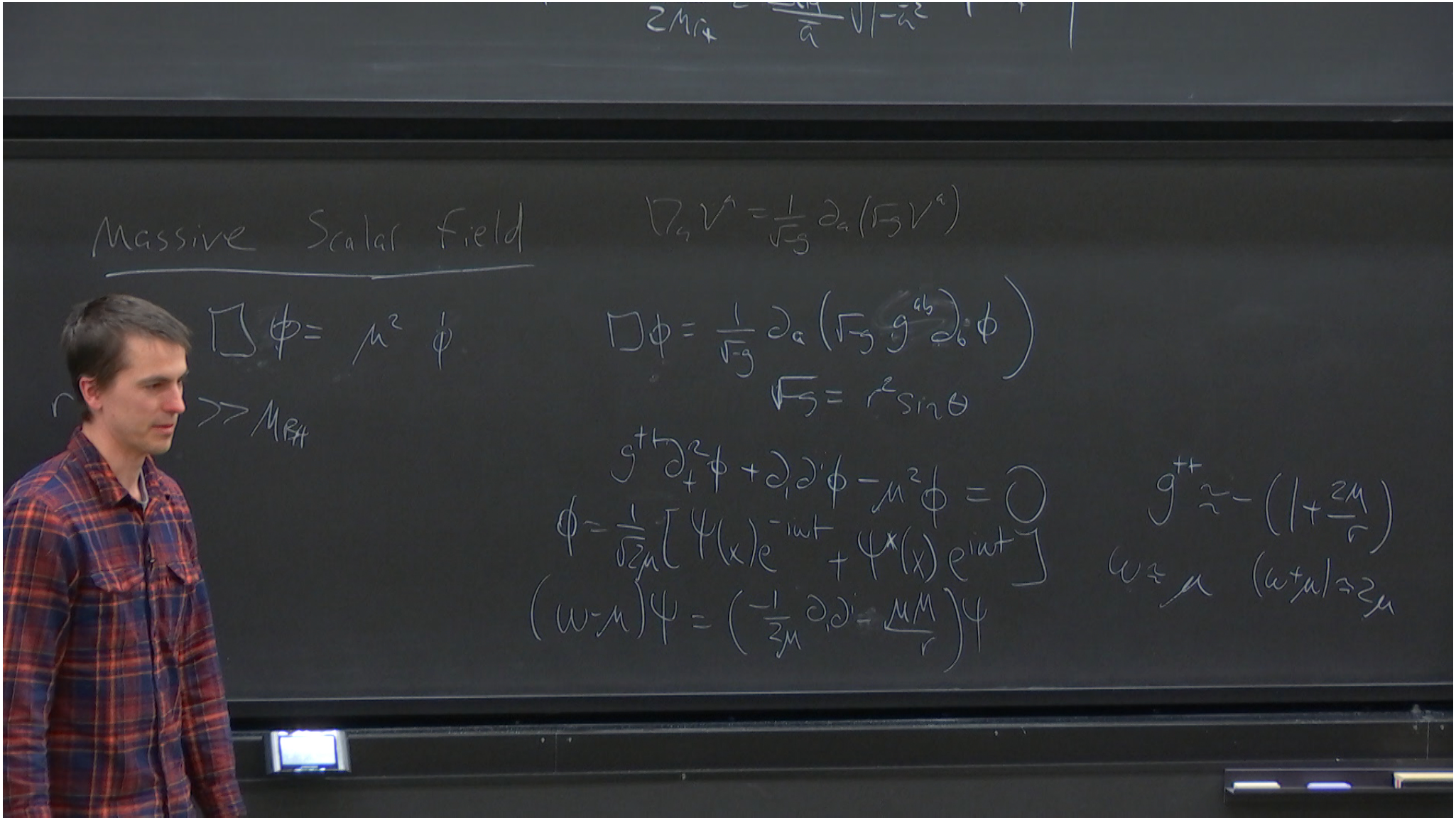
$$\Delta E_{BH} = \int_{r=r_+} dt \int (-\chi^a)\, T_{ab}\, \hat{f}^b\, dS_{BH}$$

$$= -\int dt \int_{r=r_+} \left[\chi^a (\nabla_a \Psi)(\nabla_b \Psi) \hat{f}^b - \tfrac{1}{2} \chi^a \hat{f}^a \nabla_c \Psi \nabla_c \Psi \right] dS_{BH}$$


$\Delta E_{BH} = -\int dt \int dS_{BH} \left(\partial_t \psi + \Omega_H \partial_\phi \psi\right) \partial_t \psi$
$= (\omega - m\Omega_H)\omega \int dt \int dS_{BH} \left(|\bar{\psi}|^2 \sin^2(\omega t - m\phi) \right)$
≥ 0
$0 < \omega < m\Omega_H$
$\Rightarrow$ flux of negative energy into BH


$\Delta E_{BH} = -\int dt \int dS_{BH} (\partial_t \Psi + \Omega_H \partial_\phi \Psi) \partial_t \Psi$
$= (\omega - m\Omega_H)\omega \int dt \int dS_{BH} (|\bar{\Psi}|^2 \sin^2(\omega t - m\phi))$
$0 < \omega < m\Omega_H$
≥ 0
$\Rightarrow$ flux of negative energy into BH
Max Amplification
$S=0$ 0.3%
EMS=1 4.4%
GW $S=2$ 138%


BH Bomb
mirror
BH
Massive


Massive Scalar field
$\nabla_a V^a = \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} V^a)$
$\Box \phi = \mu^2 \phi$
$\Box \phi = \frac{1}{\sqrt{-g}} \partial_a \left(\sqrt{-g}\, g^{ab} \partial_b \phi \right)$
$\sqrt{-g} = r^2 \sin\theta$
$r \gg M_{BH}$
$g^{tt} \partial_t^2 \phi + \partial_i \partial^i \phi - \mu^2 \phi = 0$
$\phi = \frac{1}{\sqrt{2\mu}} \left[\psi(x) e^{-i\omega t} + \psi^*(x) e^{i\omega t} \right]$
$(\omega - \mu)\psi = \left(-\frac{1}{2\mu} \partial_i \partial^i - \frac{\mu M}{r} \right) \psi$
$g^{tt} \simeq -\left(1 + \frac{2M}{r}\right)$
$\omega \simeq \mu \quad (\omega + \mu) \simeq 2\mu$

ar field

$\mu^2 \phi$

$$\nabla_a V^a = \frac{1}{\sqrt{-g}} \partial_a (\sqrt{-g} V^a)$$

$$\Box \phi = \frac{1}{\sqrt{-g}} \partial_a \left(\sqrt{-g}\, g^{ab} \partial_b \phi \right)$$

$$\sqrt{-g} = r^2 \sin\theta$$

$$g^{tt} \partial_t^2 \phi + \partial_i \partial^i \phi - \mu^2 \phi = 0$$

$$\phi = \frac{1}{\sqrt{2\mu}} \left[\psi(x) e^{-i\omega t} + \psi^*(x) e^{i\omega t} \right]$$

$$(\omega - \mu)\psi = \left(\frac{-1}{2\mu} \partial_i \partial^i - \frac{\mu M}{r} \right) \psi$$

$$\alpha = M\mu$$

$$\text{Energy} = \omega - \mu \approx -\frac{\alpha^2}{2n^2}$$

$$\text{Bohr radius:} \quad r_B = (\alpha\mu)^{-1}$$

$$g^{tt} \approx -\left(1 + \frac{2M}{r}\right)$$

$$\omega \approx \mu \quad (\omega + \mu) \approx 2\mu$$