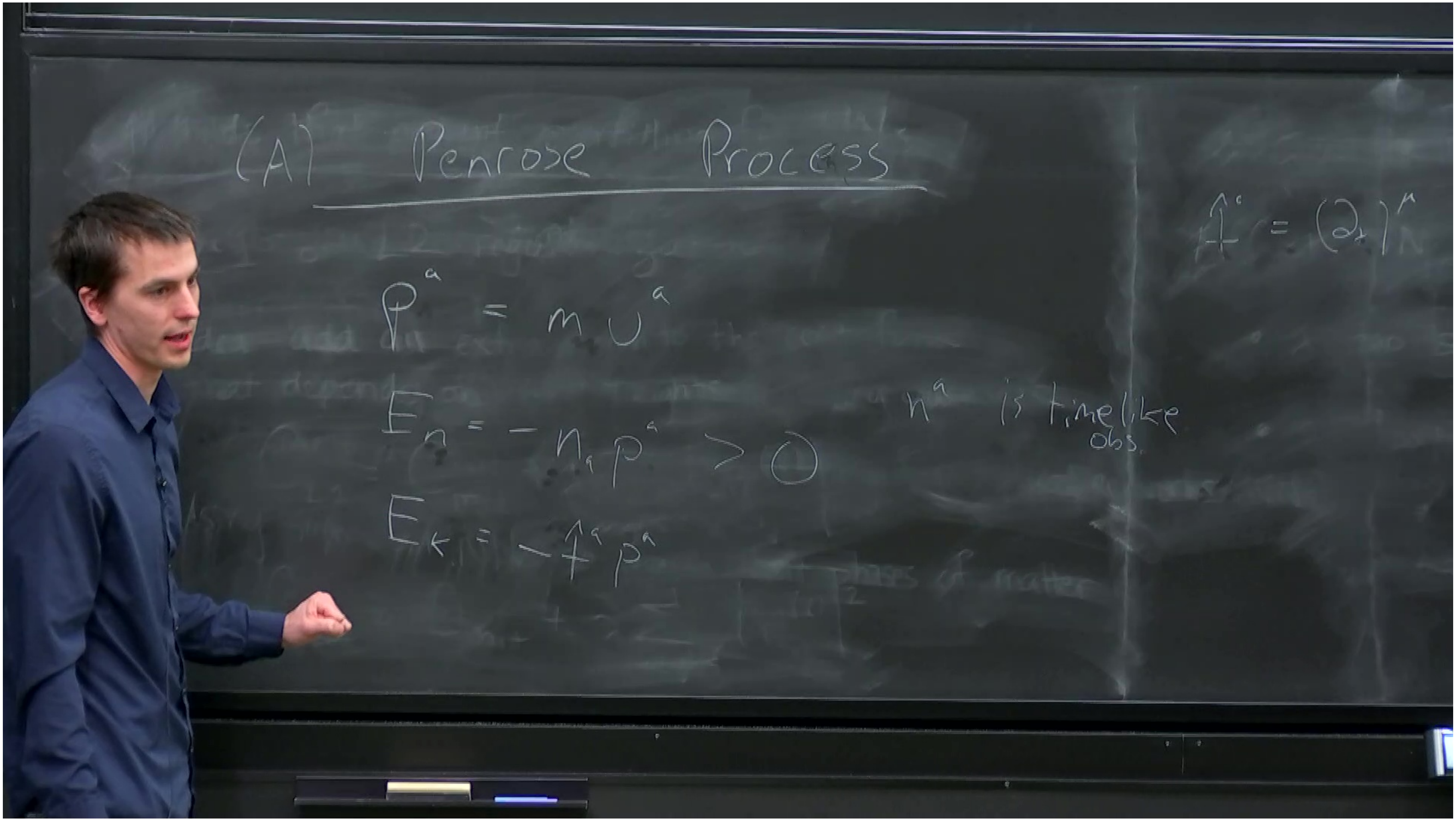
Title: Strong Gravity Lecture - 230307

Speakers: William East

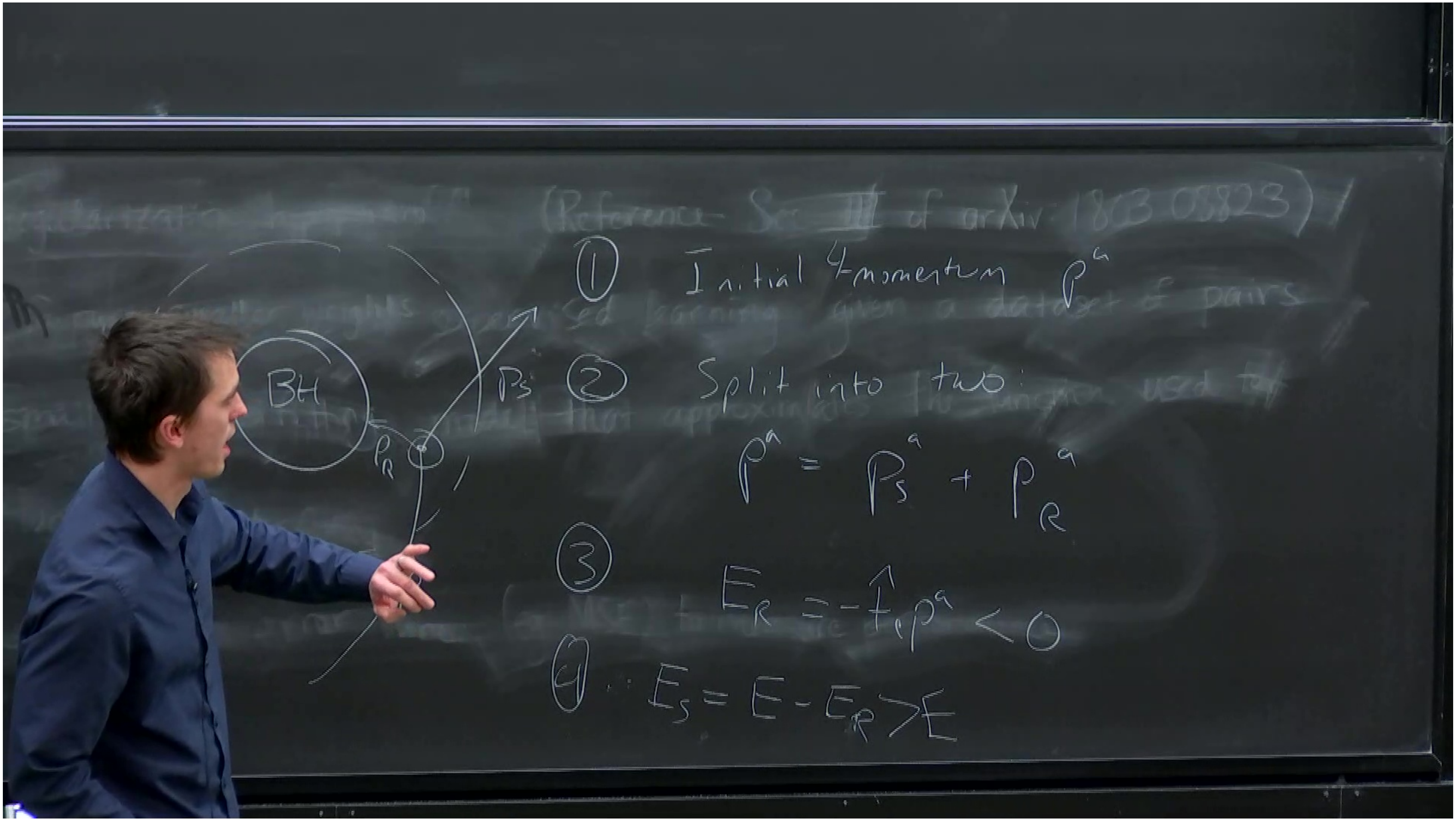
Collection: Strong Gravity (2022/2023)

Date: March 07, 2023 - 10:15 AM

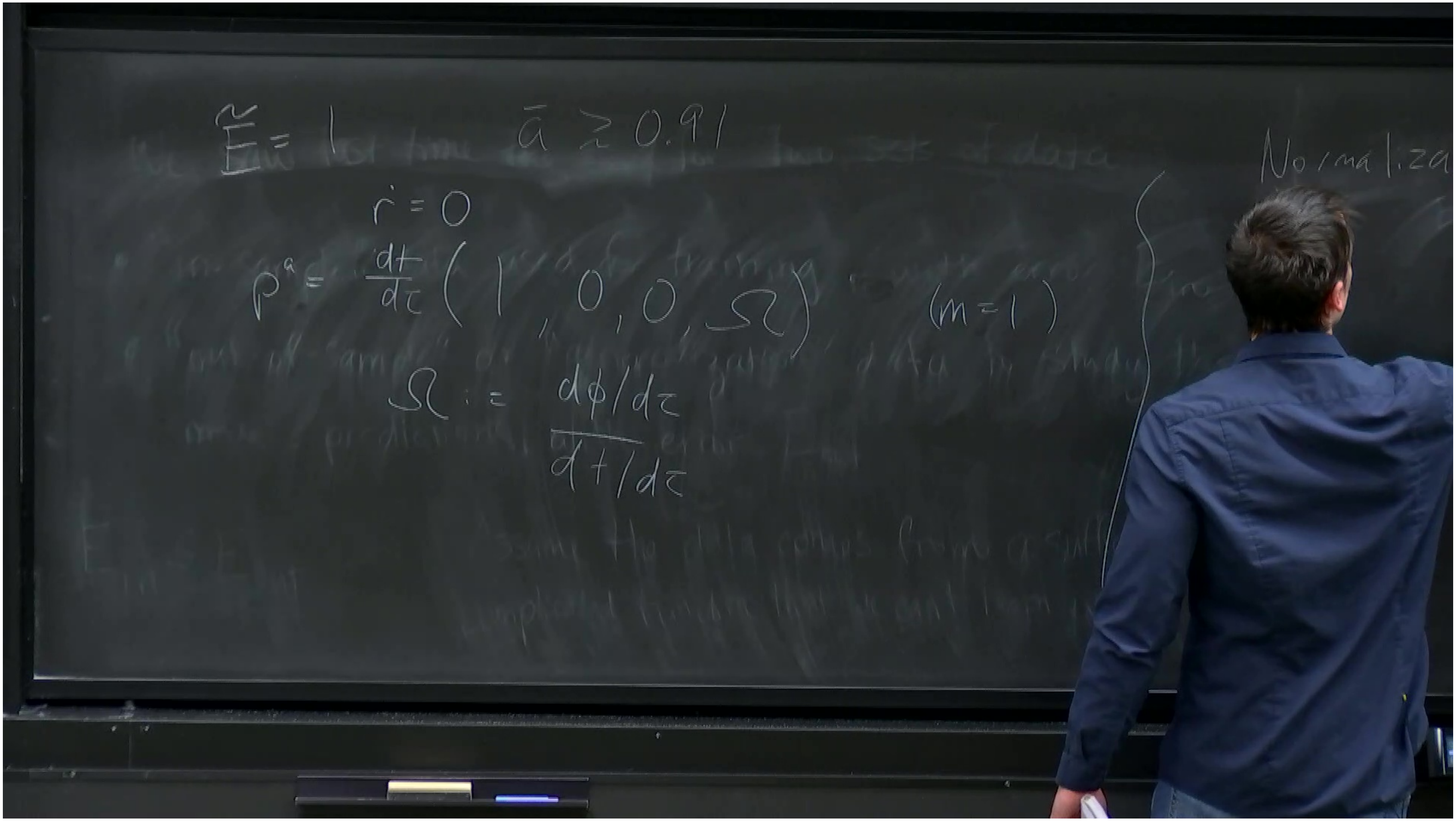
URL: https://pirsa.org/23030043

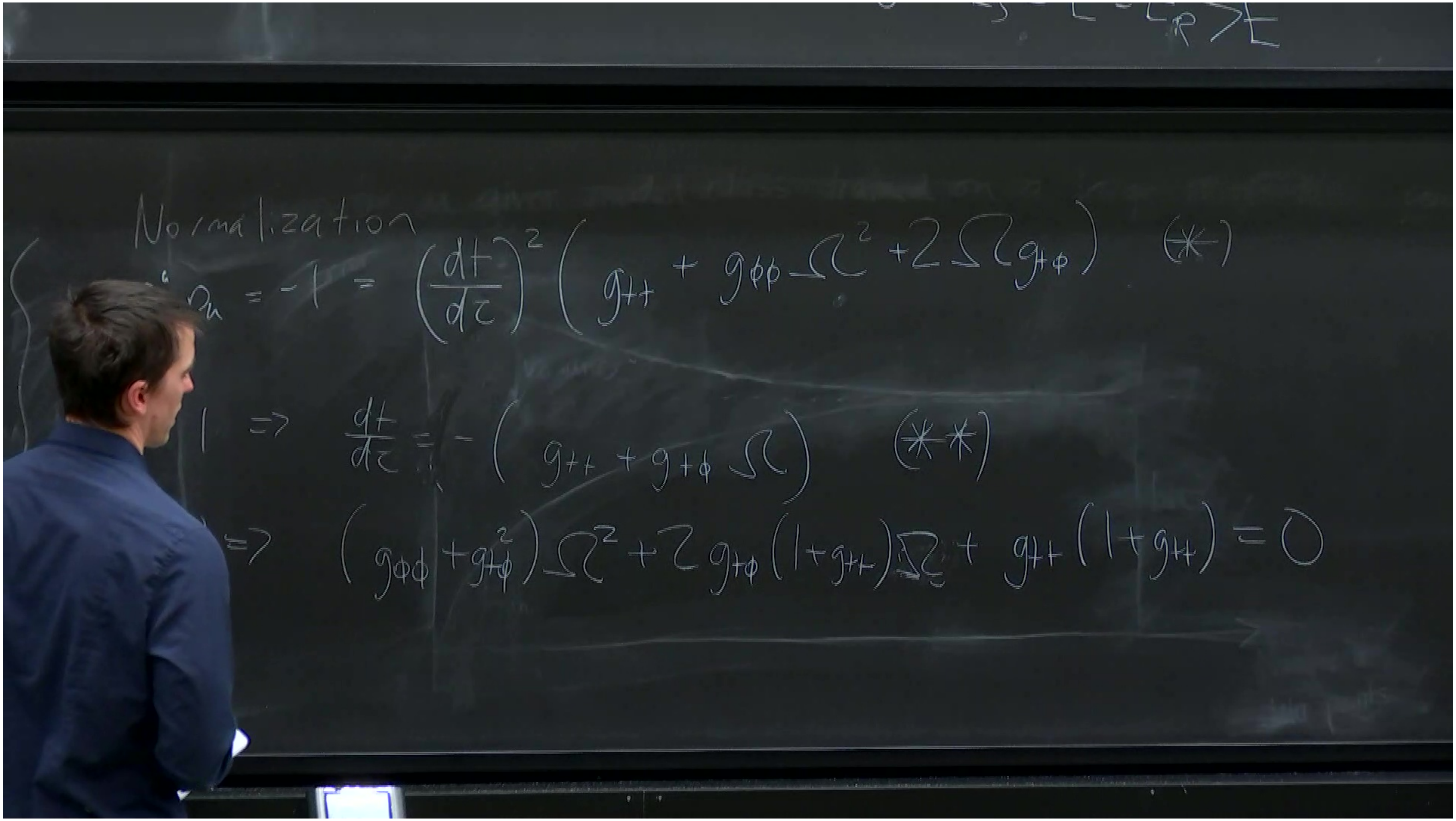

(A) Penrose Process
$p^a = m u^a$
$E_n = -n_a p^a > 0$
n^a is timelike obs.
$E_t = -\hat{t}^a p^a$
$\hat{t}^a = (\partial_t)^\mu$


Process
is timelike
obs.
BH

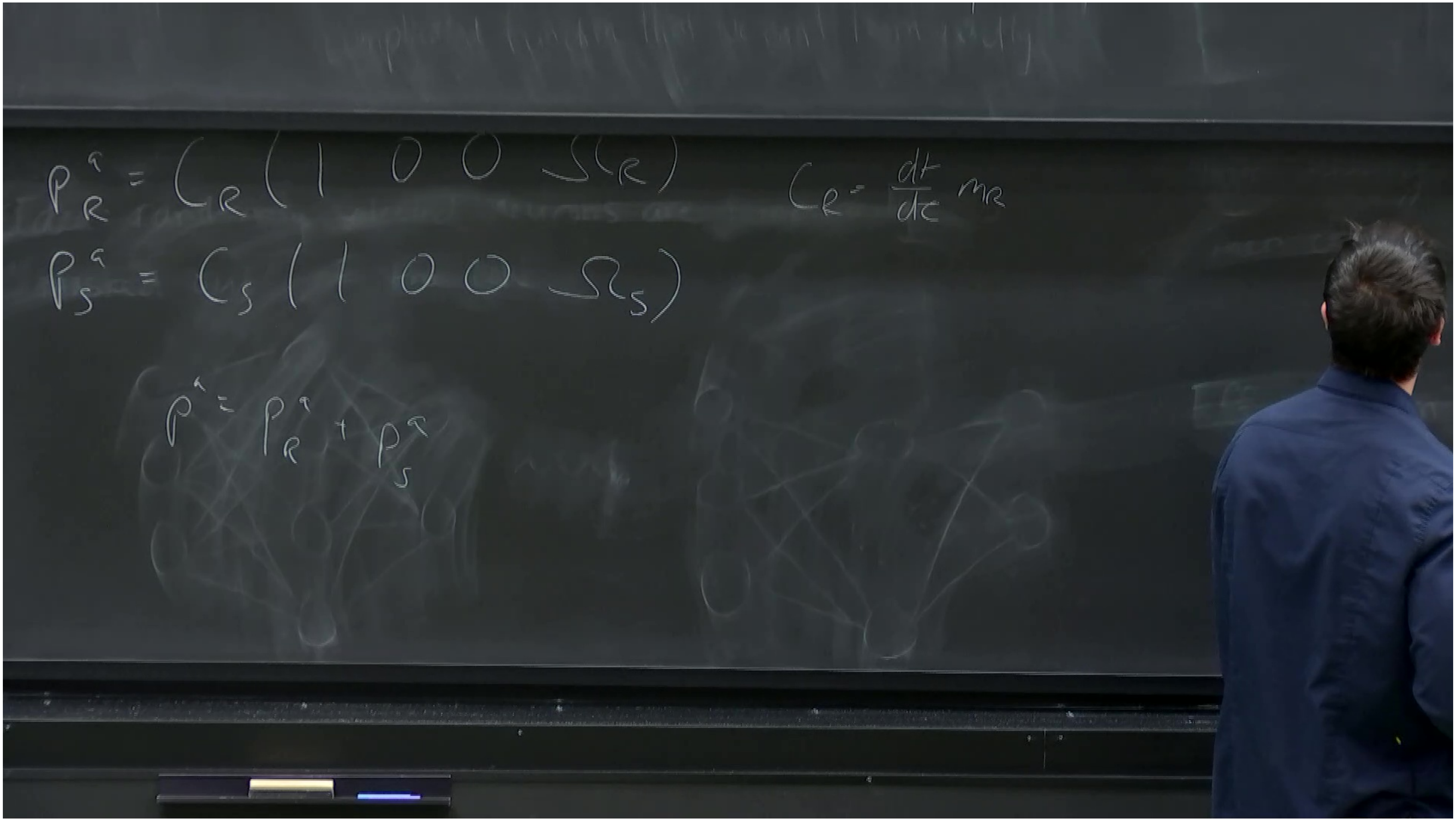


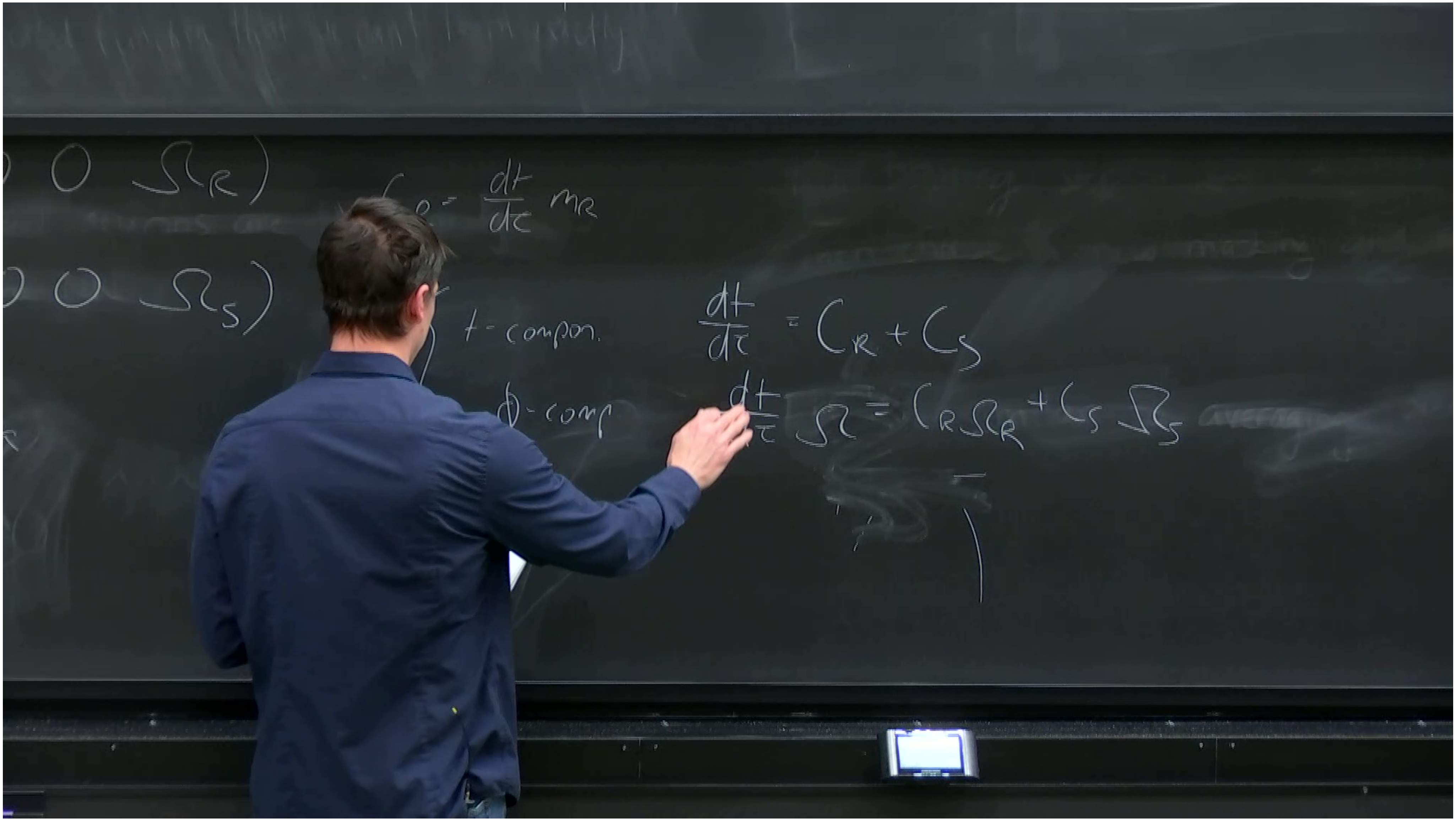
BH
p_s
p_R
1 Initial 4-momentum p^a
2 Split into two:
p^a = p_s^a + p_R^a
3 E_R = -t_a p^a < 0
4 ∴ E_s = E - E_R > E


$\tilde{E} = 1$
$\bar{a} \gtrsim 0.91$
$\dot{r} = 0$
$p^a = \frac{dt}{d\tau}(1, 0, 0, \Omega)$
$(m=1)$
$\Omega := \frac{d\phi/d\tau}{dt/d\tau}$
Normaliza

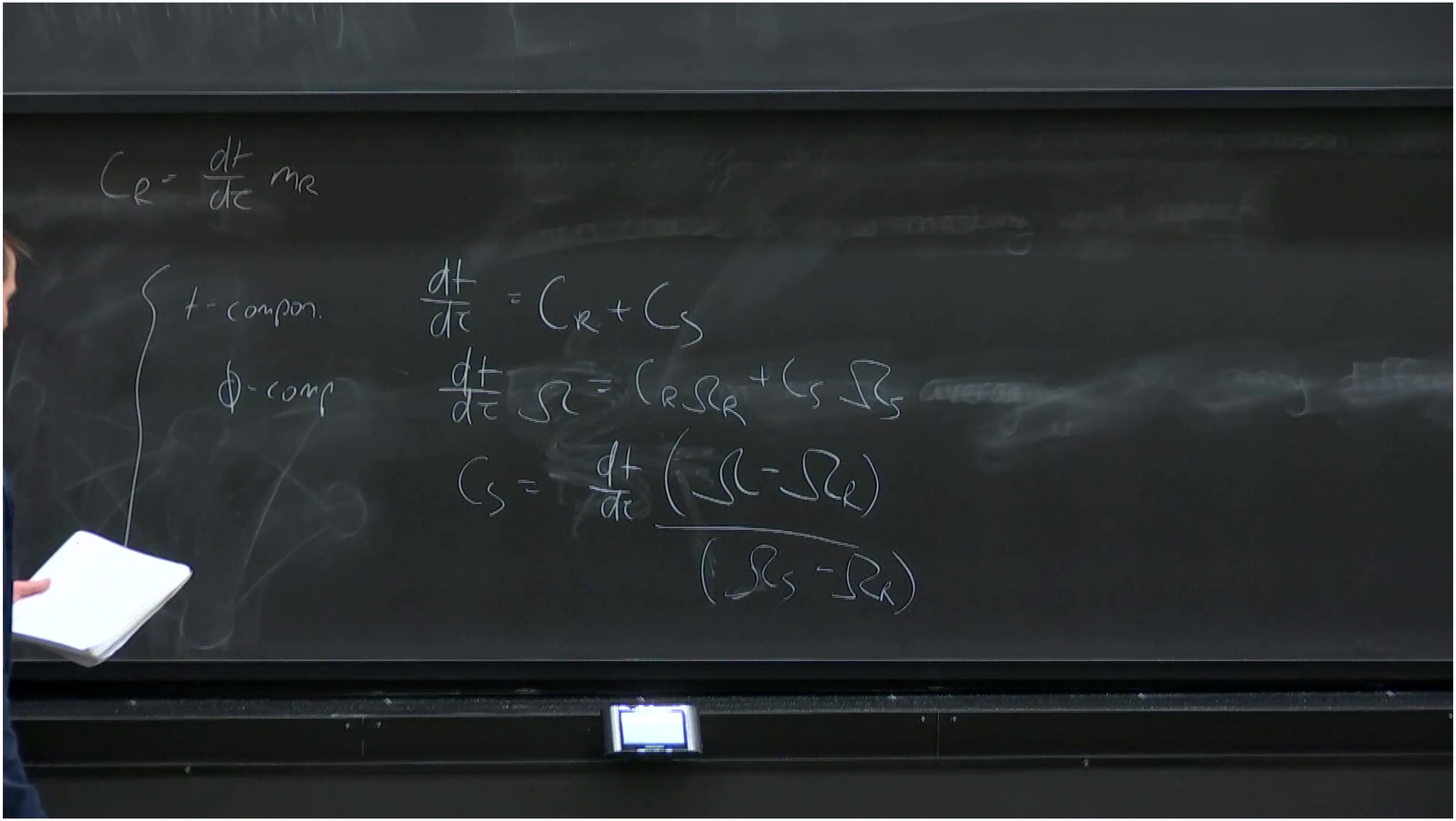


Normalization
$u^\mu u_\mu = -1 = \left(\frac{dt}{d\tau}\right)^2 \left(g_{tt} + g_{\phi\phi}\Omega^2 + 2\Omega g_{t\phi}\right)$ (*)
$\Rightarrow \frac{dt}{d\tau} = -\left(g_{tt} + g_{t\phi}\Omega\right)$ (**)
$\Rightarrow \left(g_{\phi\phi} + g_{t\phi}^2\right)\Omega^2 + 2 g_{t\phi}(1+g_{tt})\Omega + g_{tt}(1+g_{tt}) = 0$


$\rho_R^a = C_R(1 \;\; 0 \;\; 0 \;\; \Omega_R)$
$\rho_S^a = C_S(1 \;\; 0 \;\; 0 \;\; \Omega_S)$
$C_R = \frac{dt}{d\tau} m_R$
$\rho^a = \rho_R^a + \rho_S^a$



$C_R = \frac{dt}{d\tau} m_R$
t-compon.
$\frac{dt}{d\tau} = C_R + C_S$
ϕ-comp
$\frac{dt}{d\tau}\Omega = C_R\Omega_R + C_S\Omega_S$


$C_R = \frac{dt}{d\tau} m_R$
t-compon. $\frac{dt}{d\tau} = C_R + C_S$
ϕ-comp $\frac{dt}{d\tau}\Omega = C_R \Omega_R + C_S \Omega_S$
$C_S = \frac{dt}{d\tau} \frac{(\Omega - \Omega_R)}{(\Omega_S - \Omega_R)}$

$$C_S$$

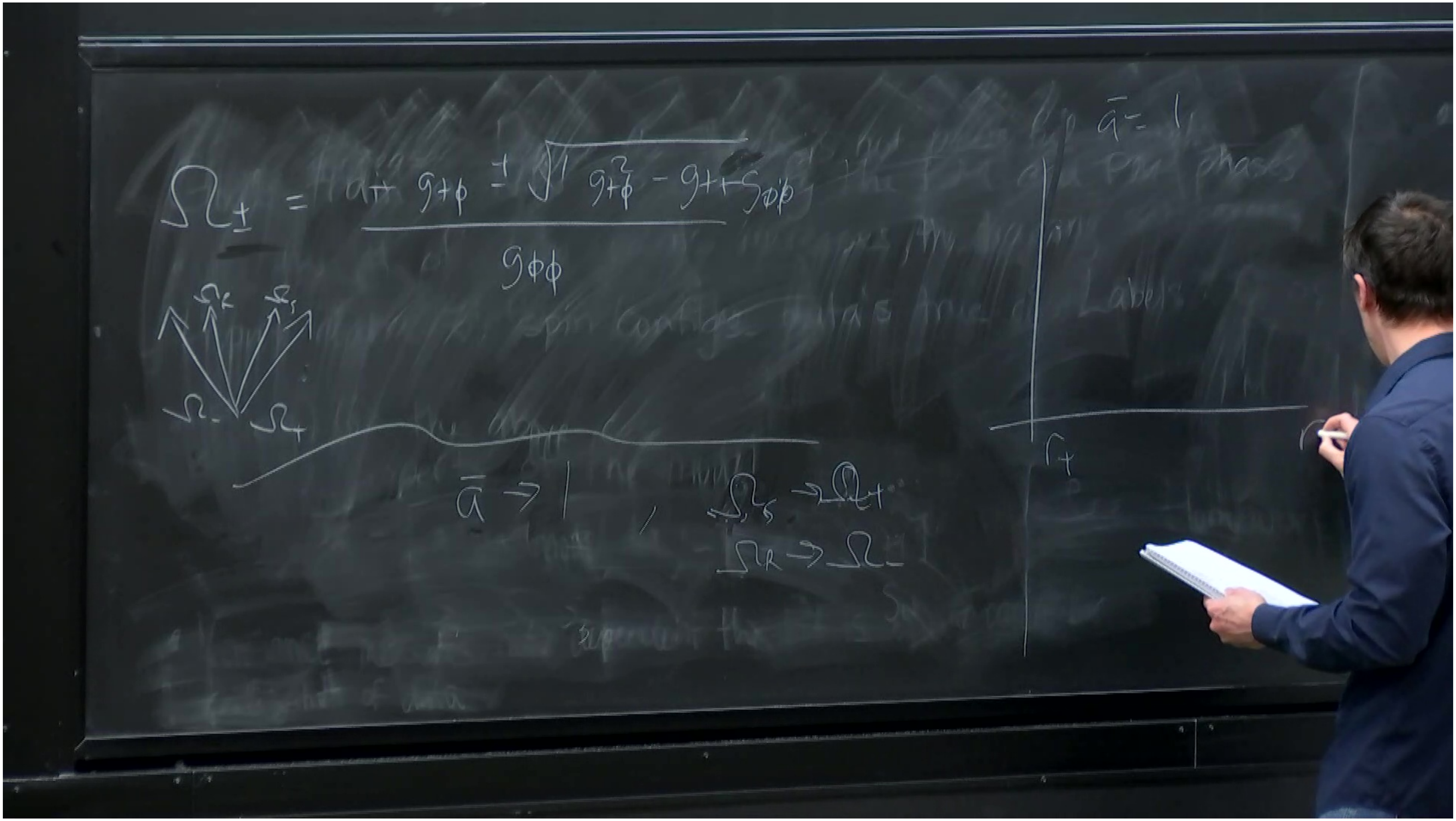
$$C_R \Omega_R + C_S \Omega_S$$

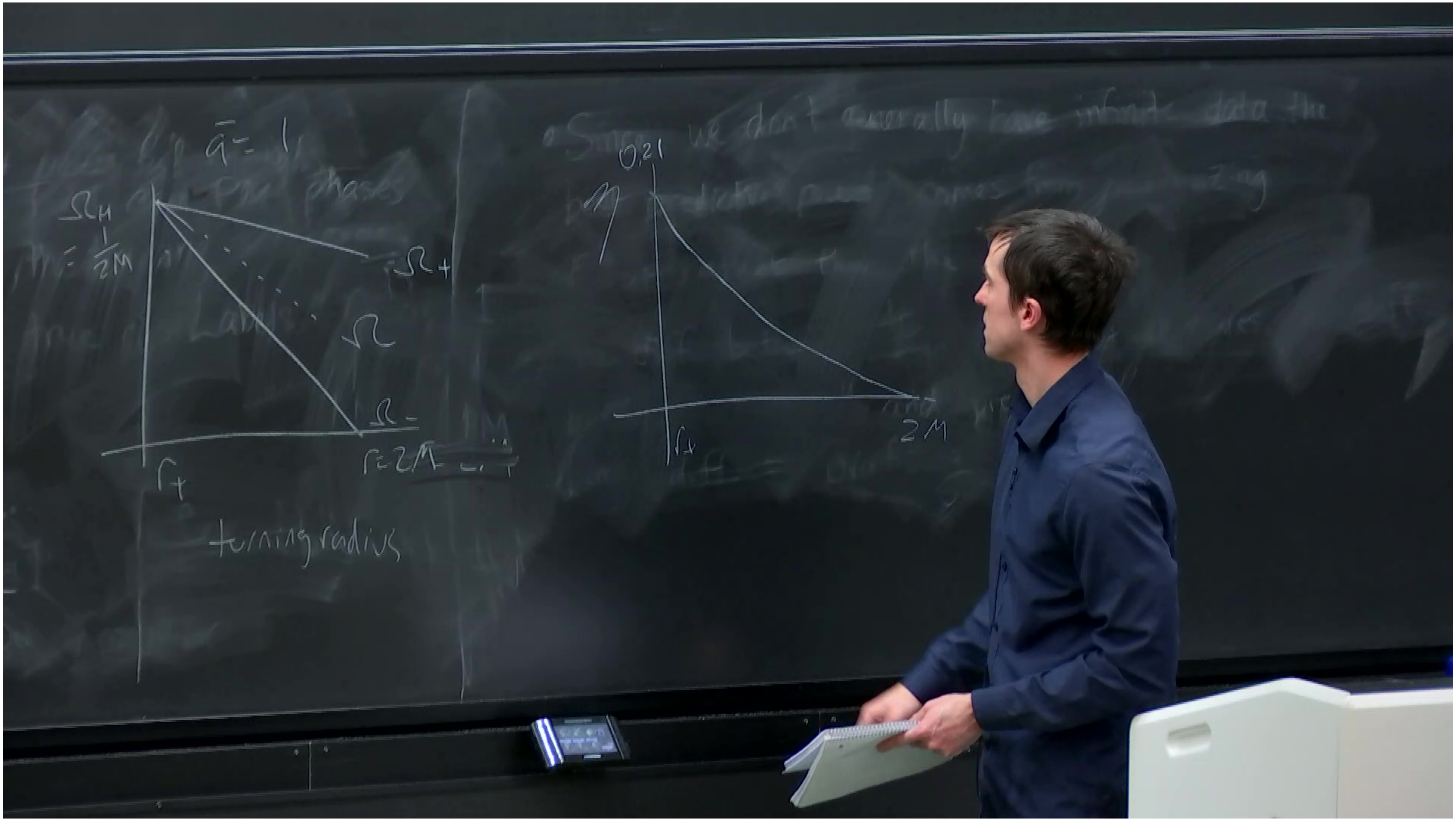
$$\frac{(\Omega - \Omega_R)}{(\Omega_S - \Omega_R)}$$

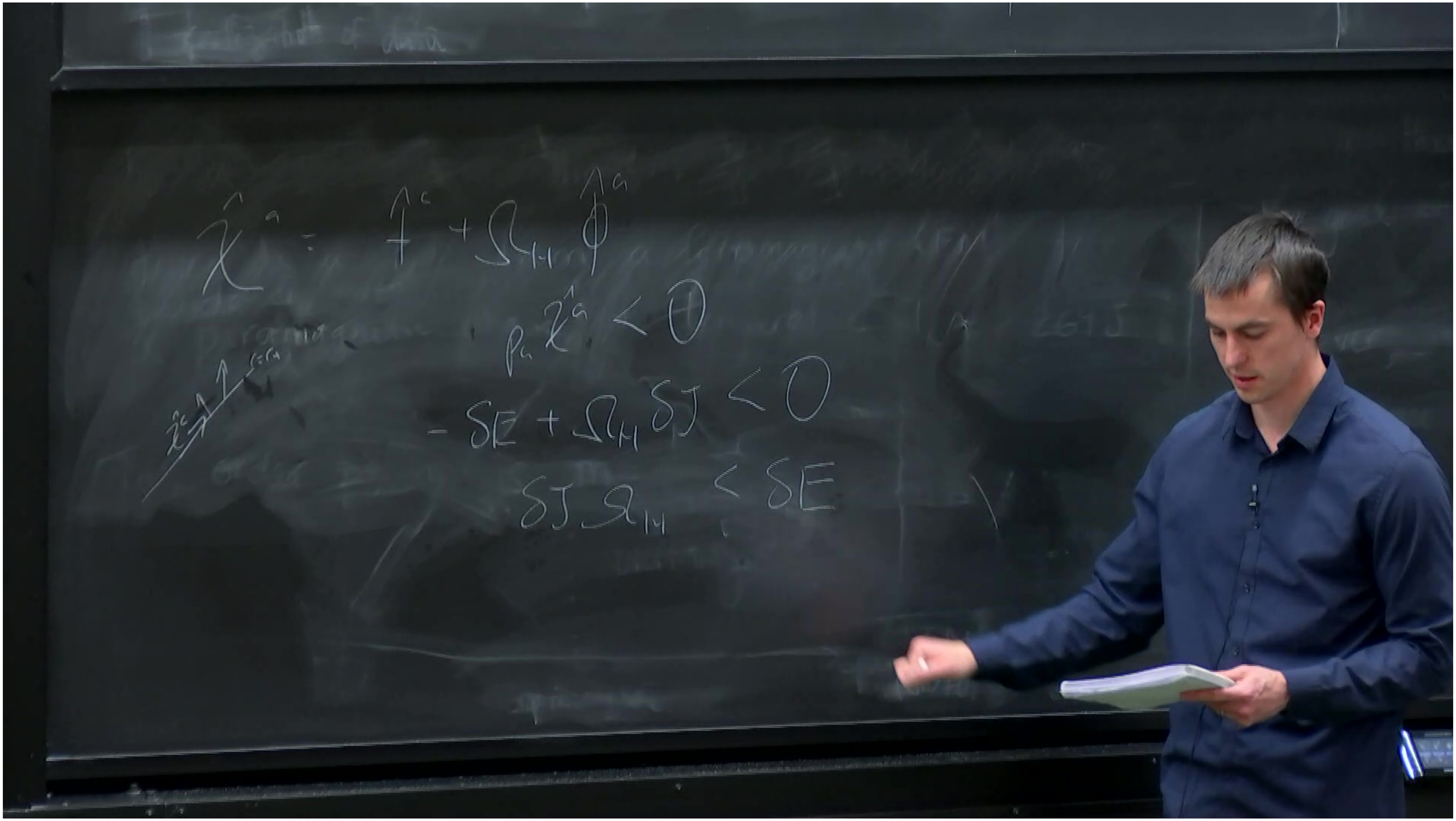
$$E_S = -\xi_a \rho_S^a = -C_S \left(g_{tt} + g_{t\phi} \Omega_S \right)$$

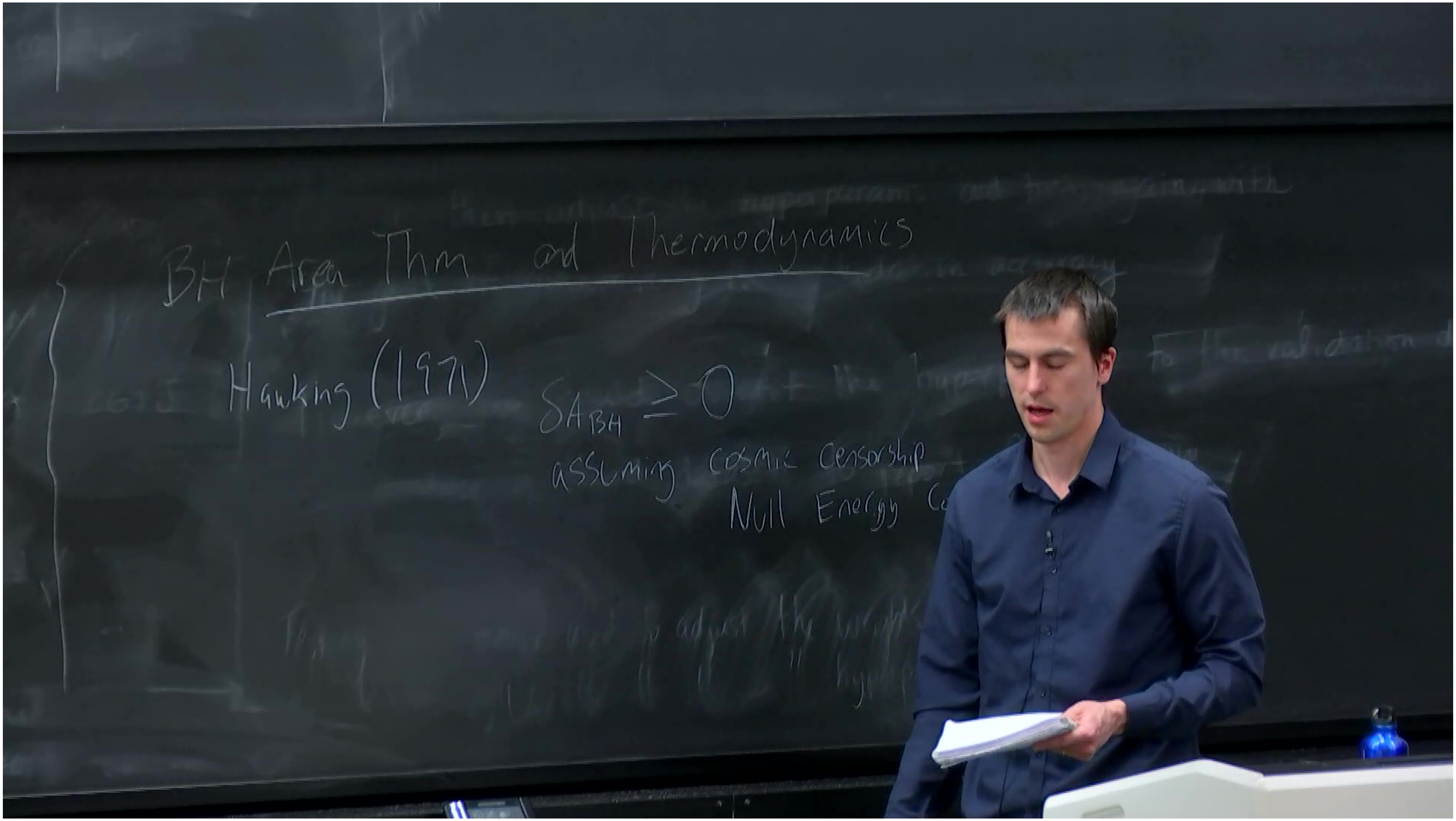
$$= \left(\frac{g_{tt} + g_{t\phi}\Omega_S}{g_{tt} + g_{t\phi}\Omega} \right) \left(\frac{\Omega - \Omega_R}{\Omega_S - \Omega_R} \right)$$

$$\eta = \frac{E_S - E}{E} = \frac{E_S}{E} - 1$$

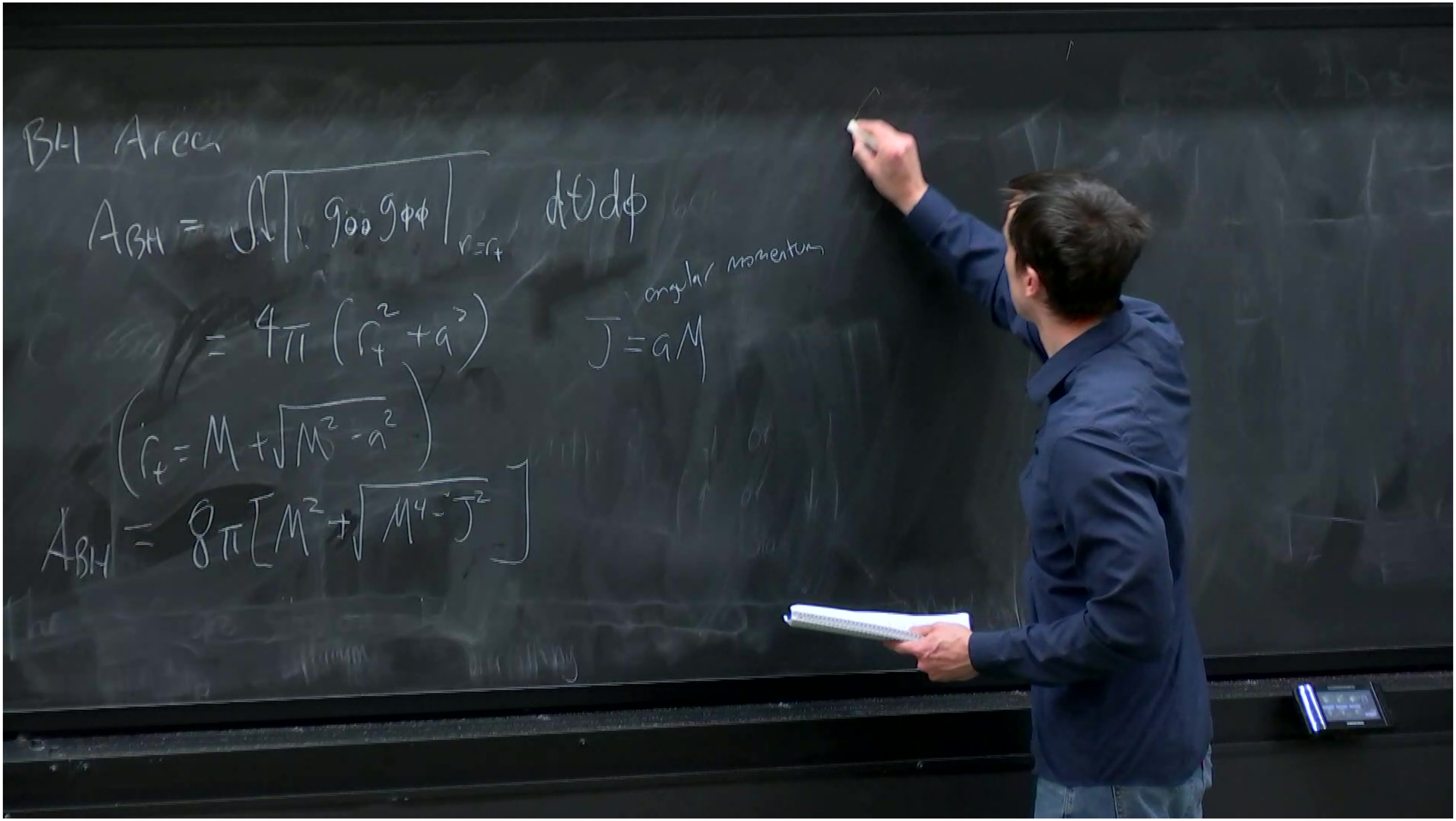

ā = 1
phases
Labels
ā → 1

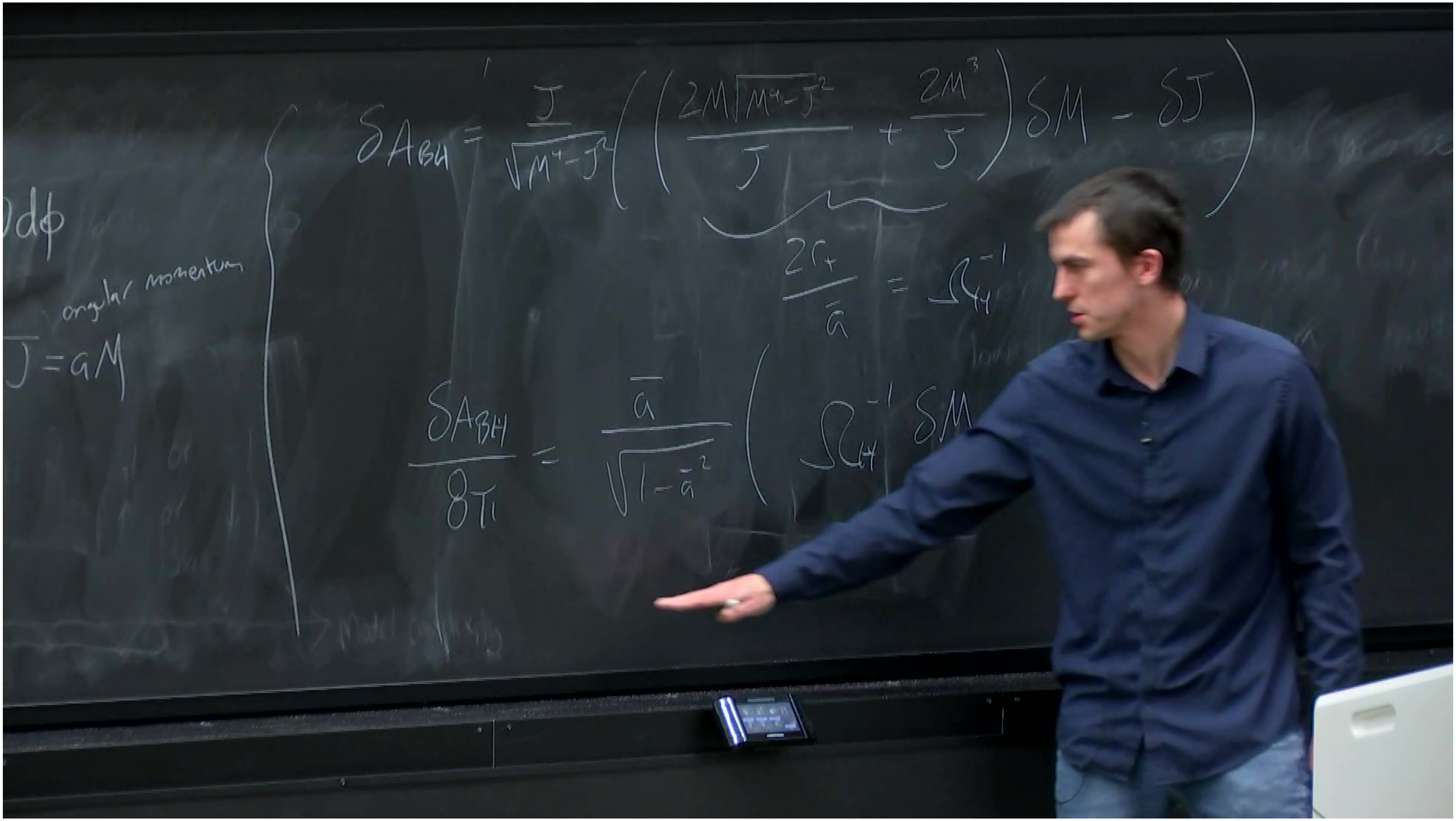

ā = 1
Ω_H = 1/2M
Ω_+
Ω
Ω_-
r_+
r=2M
turning radius
0.21
r_+
2M

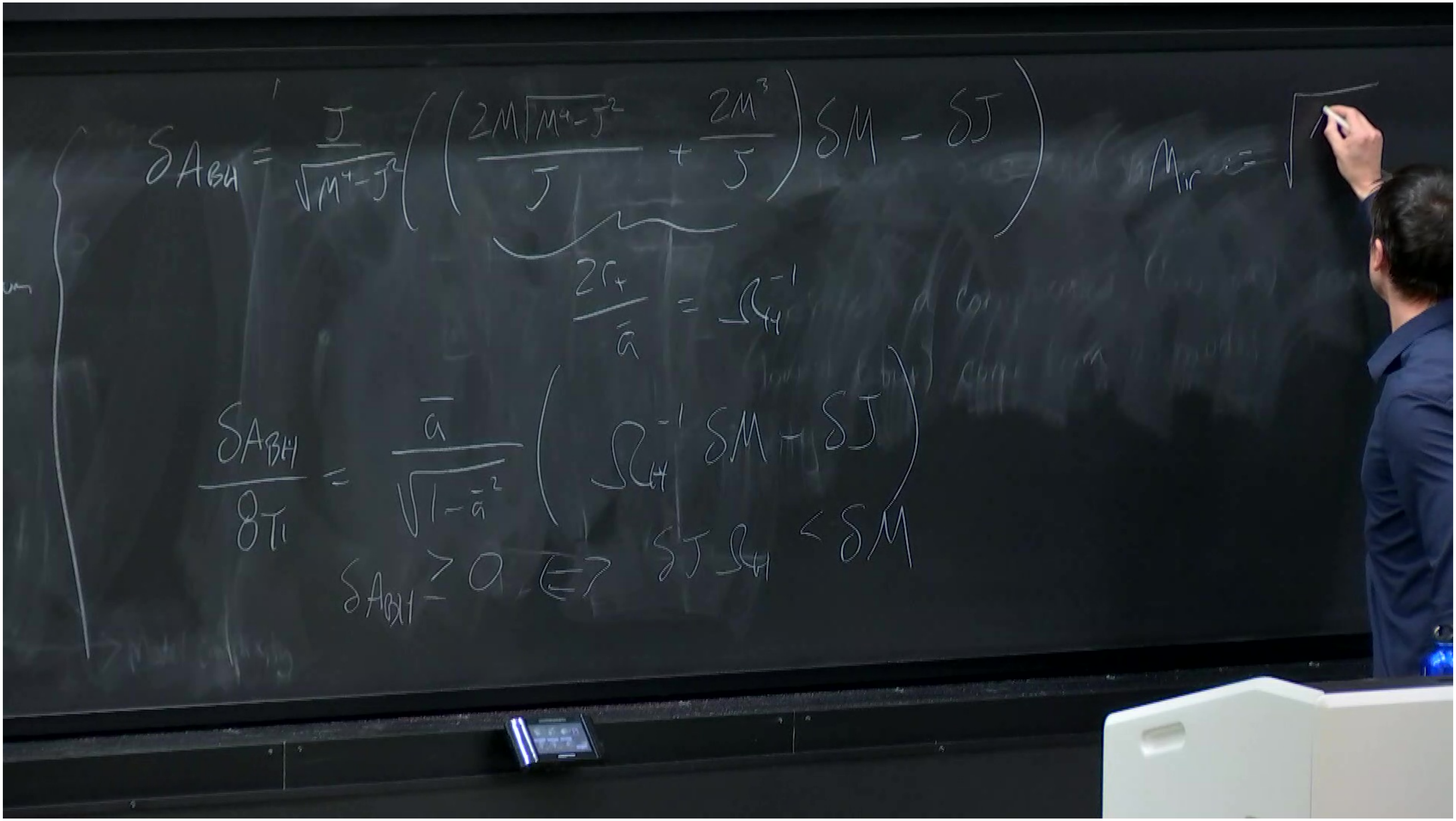

χ^a = t^a + Ω_H φ^a
p_a χ^a < 0
−δE + Ω_H δJ < 0
δJ Ω_H < δE



BH Area Thm and Thermodynamics
Hawking (1971)
$\delta A_{BH} \geq 0$
assuming cosmic censorship
Null Energy


BH Area
$A_{BH} = \iint \sqrt{g_{\theta\theta} g_{\phi\phi}}\Big|_{r=r_+} d\theta\, d\phi$
$= 4\pi (r_+^2 + a^2)$
$\left(r_+ = M + \sqrt{M^2 - a^2}\right)$
$A_{BH} = 8\pi\left[M^2 + \sqrt{M^4 - J^2}\right]$
angular momentum
$J = aM$


$\delta A_{BH} = \frac{J}{\sqrt{M^4 - J^2}}\left(\left(\frac{2M\sqrt{M^4-J^2}}{J} + \frac{2M^3}{J}\right)\delta M - \delta J\right)$
$\frac{2r_+}{\bar{a}} = \Omega_H^{-1}$
$\frac{\delta A_{BH}}{8\pi} = \frac{\bar{a}}{\sqrt{1-\bar{a}^2}}\left(\Omega_H^{-1}\,\delta M\right.$
angular momentum
$J = aM$


$\delta A_{BH} = \frac{J}{\sqrt{M^4-J^2}}\left(\left(\frac{2M\sqrt{M^4-J^2}}{J} + \frac{2M^3}{J}\right)\delta M - \delta J\right)$
$\frac{2r_+}{\bar{a}} = \Omega_H^{-1}$
$\frac{\delta A_{BH}}{8\pi} = \frac{\bar{a}}{\sqrt{1-\bar{a}^2}}\left(\Omega_H^{-1}\delta M - \delta J\right)$
$\delta A_{BH} \geq 0 \iff \delta J\,\Omega_H < \delta M$

$$\delta A_{BH} = \frac{J}{\sqrt{M^4-J^2}}\left(\left(\underbrace{\frac{2M\sqrt{M^4-J^2}}{J} + \frac{2M^3}{J}}_{\frac{2r_+}{\bar{a}} = \Omega_H^{-1}}\right)\delta M - \delta J\right)$$

$$M_{ir} = \sqrt{\frac{A_{BH}}{16\pi}}$$

$\bar{a}=0$ $\quad M_{ir} = M_{BH}$

$\bar{a}=1,$ $\quad M_{ir} = \frac{1}{\sqrt{2}} M_{BH}$

$$\frac{\delta A_{BH}}{8\pi} = \frac{\bar{a}}{\sqrt{1-\bar{a}^2}}\left(\Omega_H^{-1}\delta M - \delta J\right)$$

$E_{rot} = M_{BH} - M_{ir} \,\hat{=}\, 0.29\, M_{BH}$ for $\bar{a}=1$

$$\delta A_{BH} \geq 0 \iff \delta J\, \Omega_H < \delta M$$