Title: Machine Learning Lecture - 230309

Speakers: Lauren Hayward

Collection: Machine Learning for Many-Body Physics (2022/2023)

Date: March 09, 2023 - 9:00 AM

URL: https://pirsa.org/23030031

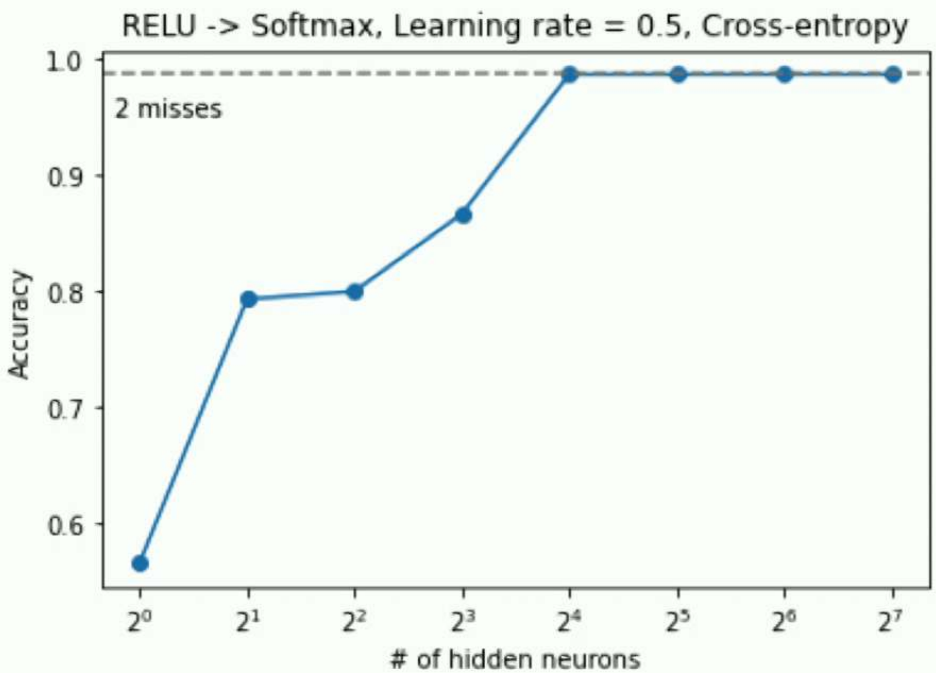
Machine Learning for Many-Body Physics

Lecture 6

PERIMETER
SCHOLARS
INTERNATIONAL

Tutorial 2, Problem 2

Accuracy vs. the number of neurons in the hidden layer

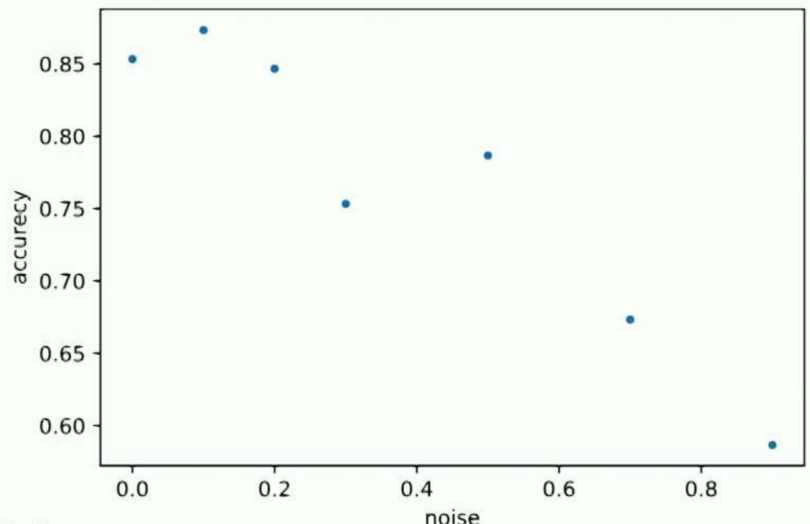


PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Results from 2022 students

Accuracy vs. the magnitude of noise in the data



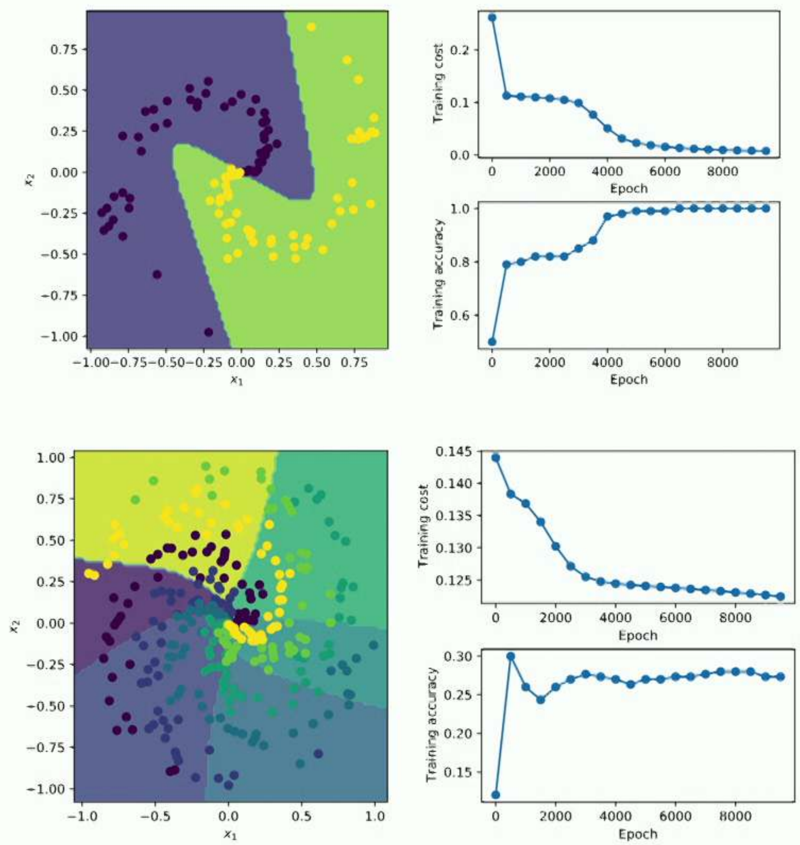

```
- learning_rate = 0.1
- cost_func = torch.nn.MSELoss()
- activation_function = sigmoid
- number_of_layer and neurons: 1, 4
- N_epochs = 100000
```

PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Results from 2022 students

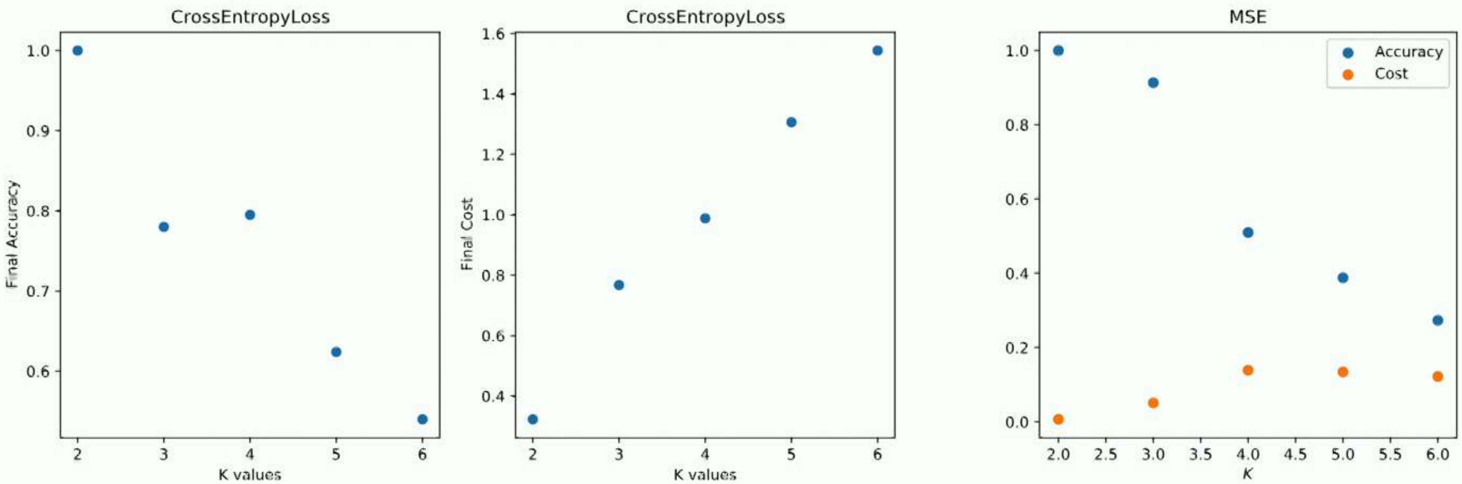
Varying the number of different labels (branches)



Tutorial 2, Problem 2

Results from 2022 students

Accuracy vs. the number of different labels (branches)

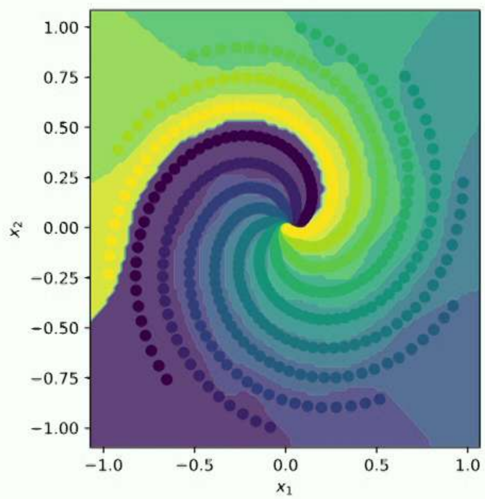


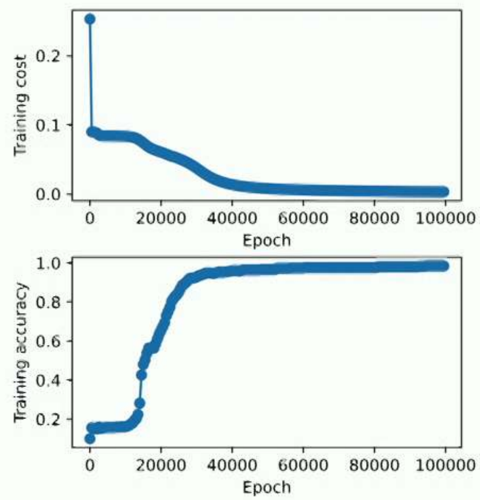
- learning rate = 1
- 4 neurons in the hidden layer
- first layer uses sigmoid
- outer layer uses softmax

PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Group 3



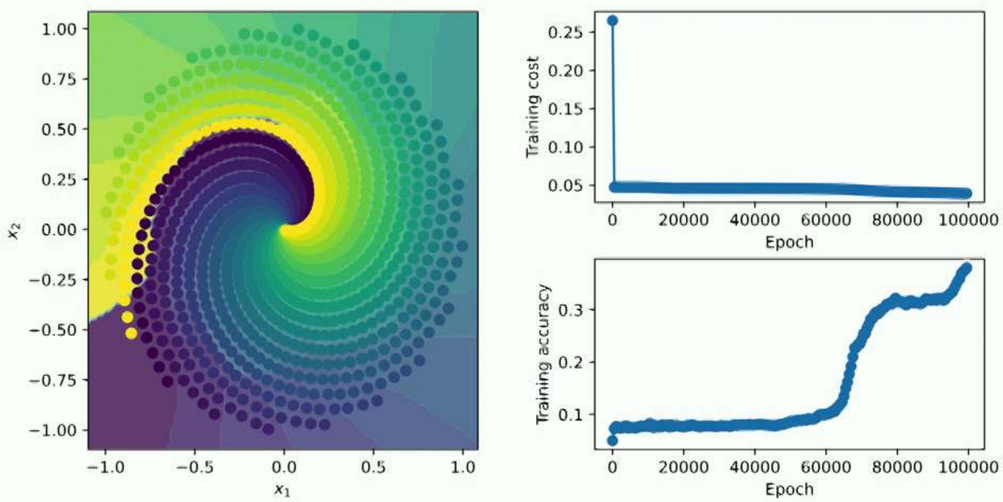


- 4 layers with 2, 20, 40, and k neurons
- Adam optimization wasn't working!
- Sigmoid activation function
- MSELoss

PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Group 3

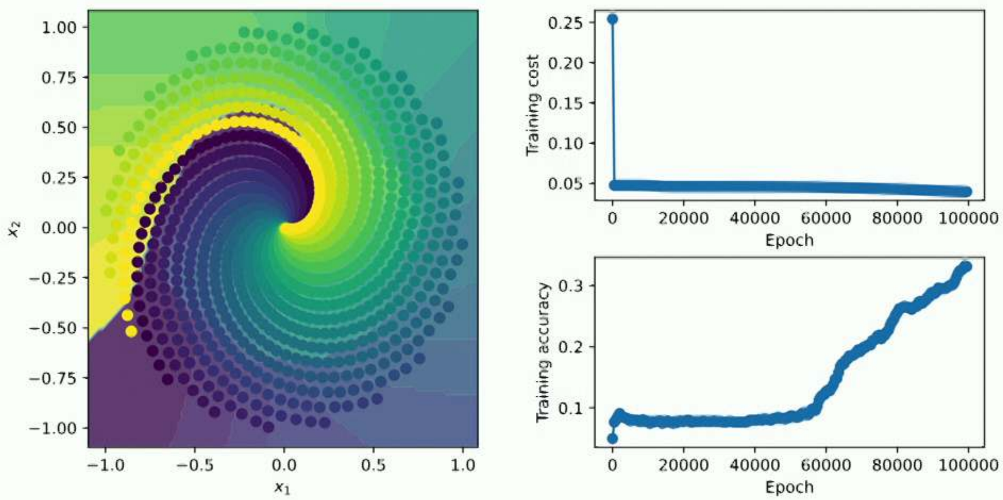


- 4 layers with 2, 20, 40, and k neurons
- Adam optimization wasn't working!
- Sigmoid activation function
- MSELoss

PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Group 3

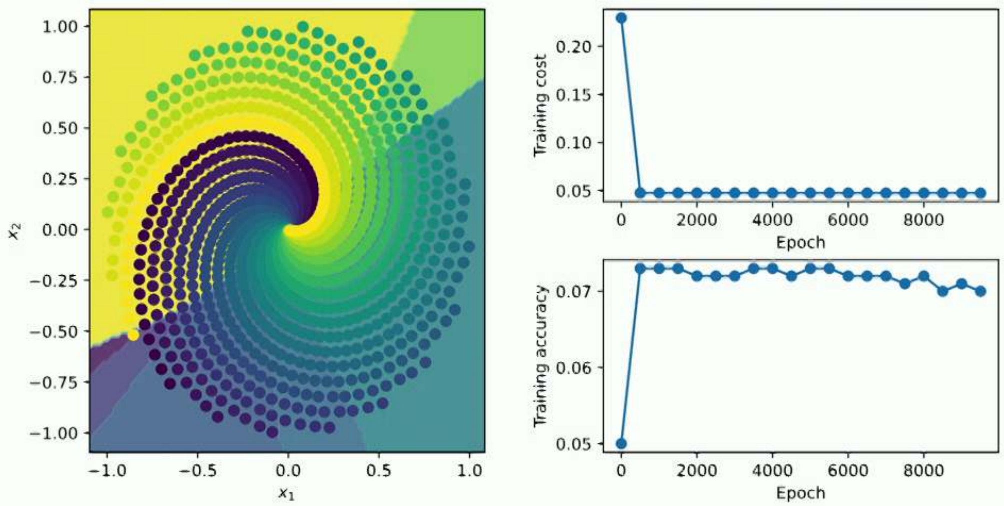


- 4 layers with 2, 20, 40, and k neurons
- Adam optimization wasn't working!
- Sigmoid activation function
- MSELoss

PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Group 3

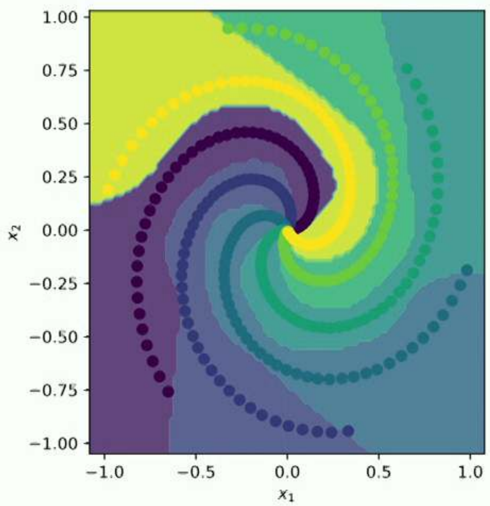


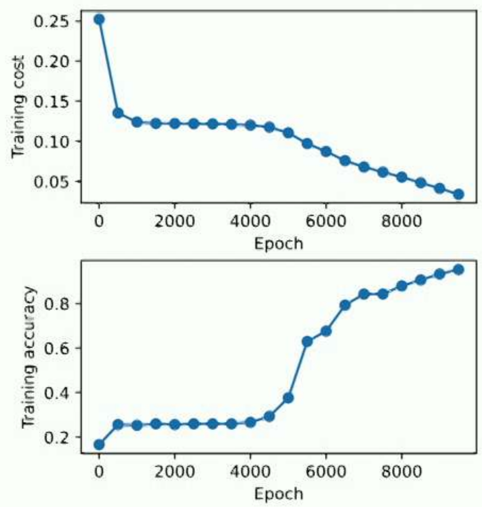
- 4 layers with 2, 20, 40, and k neurons
- Adam optimization wasn't working!
- Sigmoid activation function
- MSELoss

PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Group 3



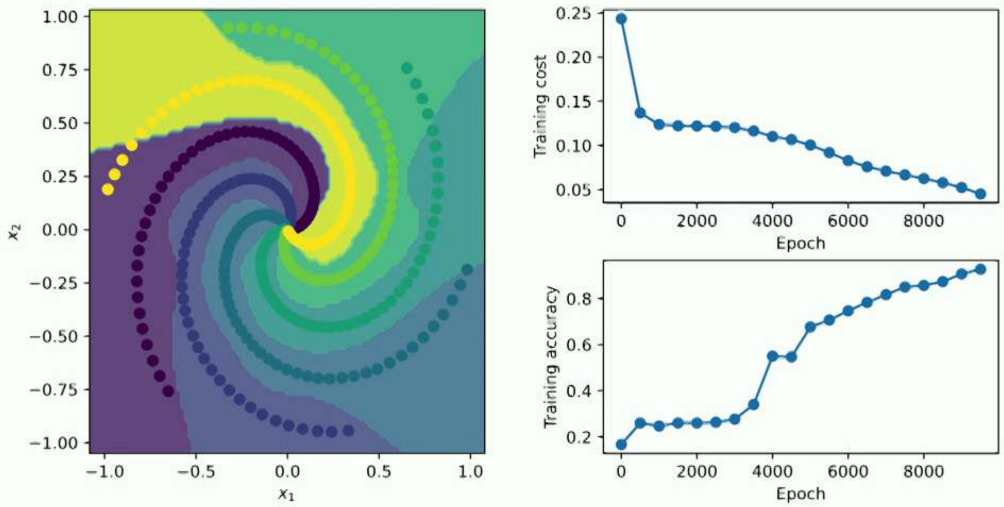


- 4 layers with 2, 20, 40, and k neurons
- Adam optimization wasn't working!
- Sigmoid activation function
- MSELoss

PERIMETER SCHOLARS INTERNATIONAL

Tutorial 2, Problem 2

Group 3



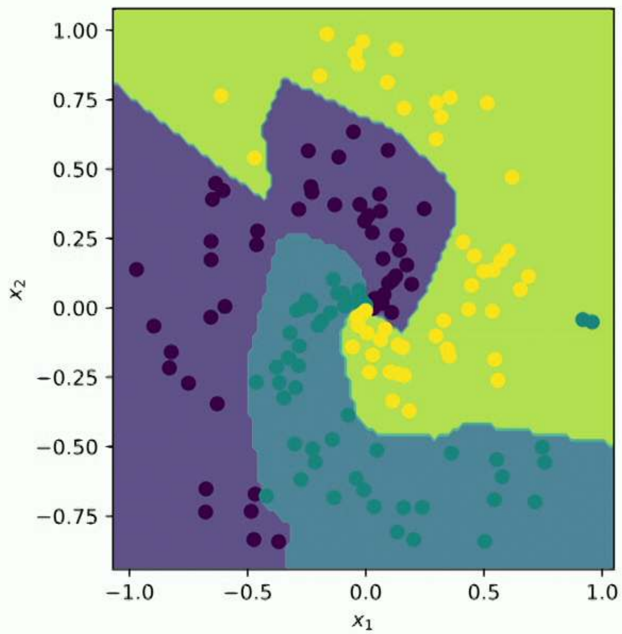
- 4 layers with 2, 20, 40, and k neurons
- Adam optimization wasn't working!
- Sigmoid activation function
- MSELoss

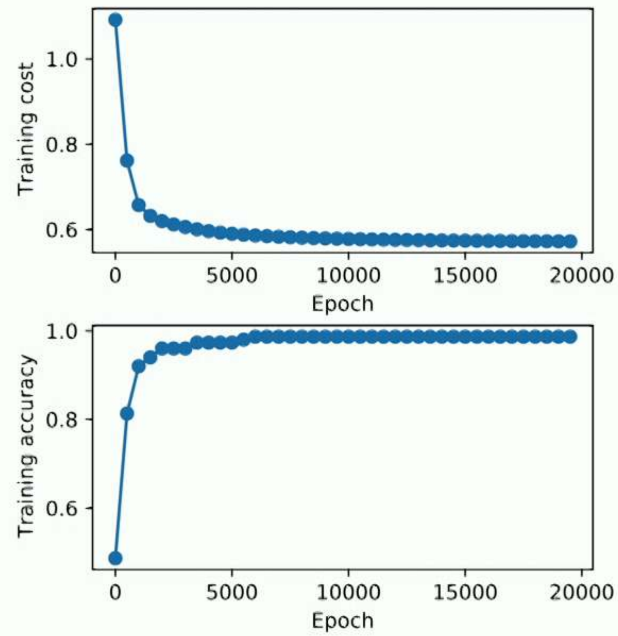
PERIMETER SCHOLARS INTERNATIONAL

Overfitting

Results from 2022 students

```
mag_noise   = 0.5
hidden_size = 100
```

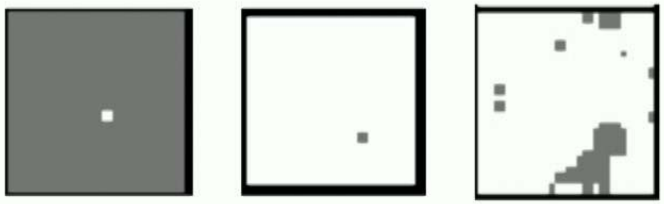




PERIMETER SCHOLARS INTERNATIONAL

Ising model classification

Ferromagnetic phase:



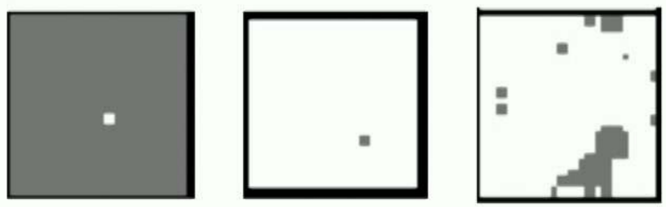
Paramagnetic phase:

Carrasquilla and Melko, arXiv:1605.01735

PERIMETER SCHOLARS INTERNATIONAL

Ising model classification

Ferromagnetic phase:



?

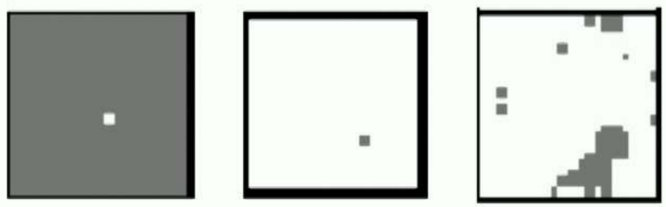
Paramagnetic phase:

Carrasquilla and Melko, arXiv:1605.01735

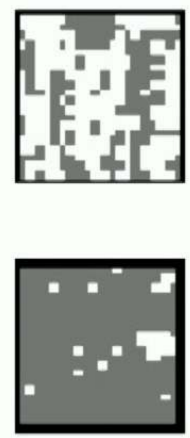
PERIMETER SCHOLARS INTERNATIONAL

Ising model classification

Ferromagnetic phase:



Paramagnetic phase:



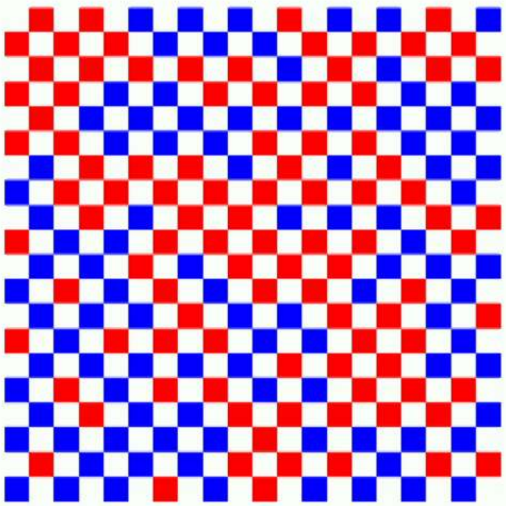
?

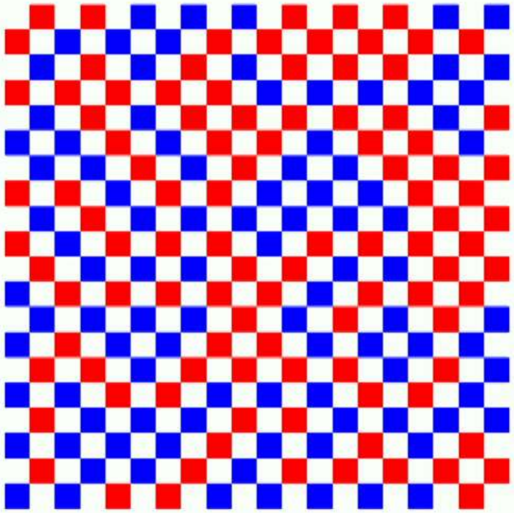
?

Carrasquilla and Melko, arXiv:1605.01735

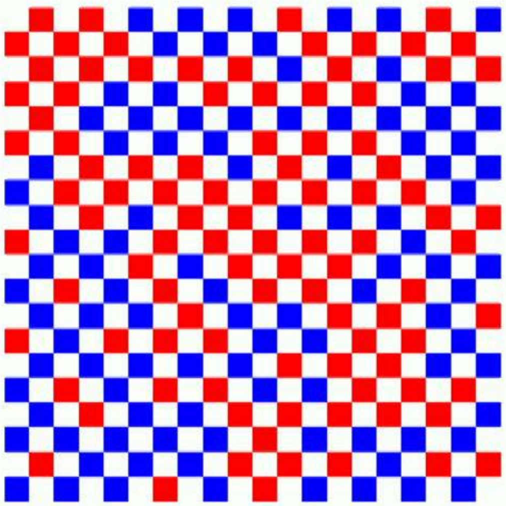
PERIMETER SCHOLARS INTERNATIONAL

Ising gauge theory classification

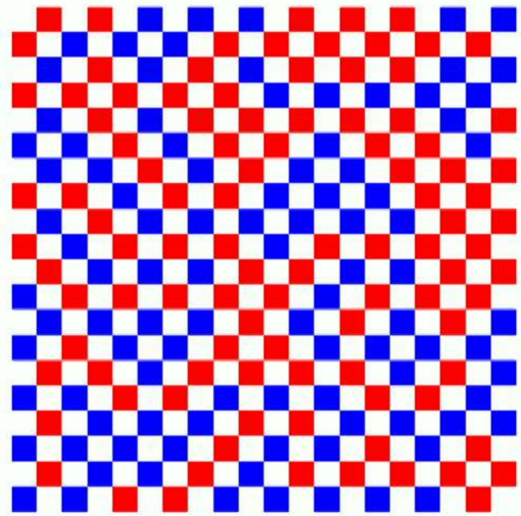




Ising gauge theory classification



Random state



Ordered state

PERIMETER SCHOLARS INTERNATIONAL

Outline for today:

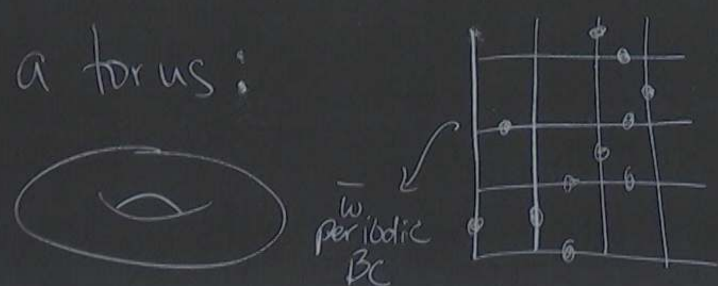
- Using supervised neural networks (NNs) to learn about phases of matter
  - ↳ Ising $\mathbb{Z}_2$ gauge theory
- Recap of hyperparams. in NNs
- Monte Carlo sampling

Distinguishing phases of the classical Ising $\mathbb{Z}_2$ gauge theory

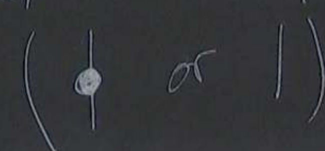
$$H = -J \sum_{\substack{\text{plaquettes} \\ p}} \left(\prod_{i \in p} s_i \right) \quad \text{with } s_i = +1 \text{ or } -1$$

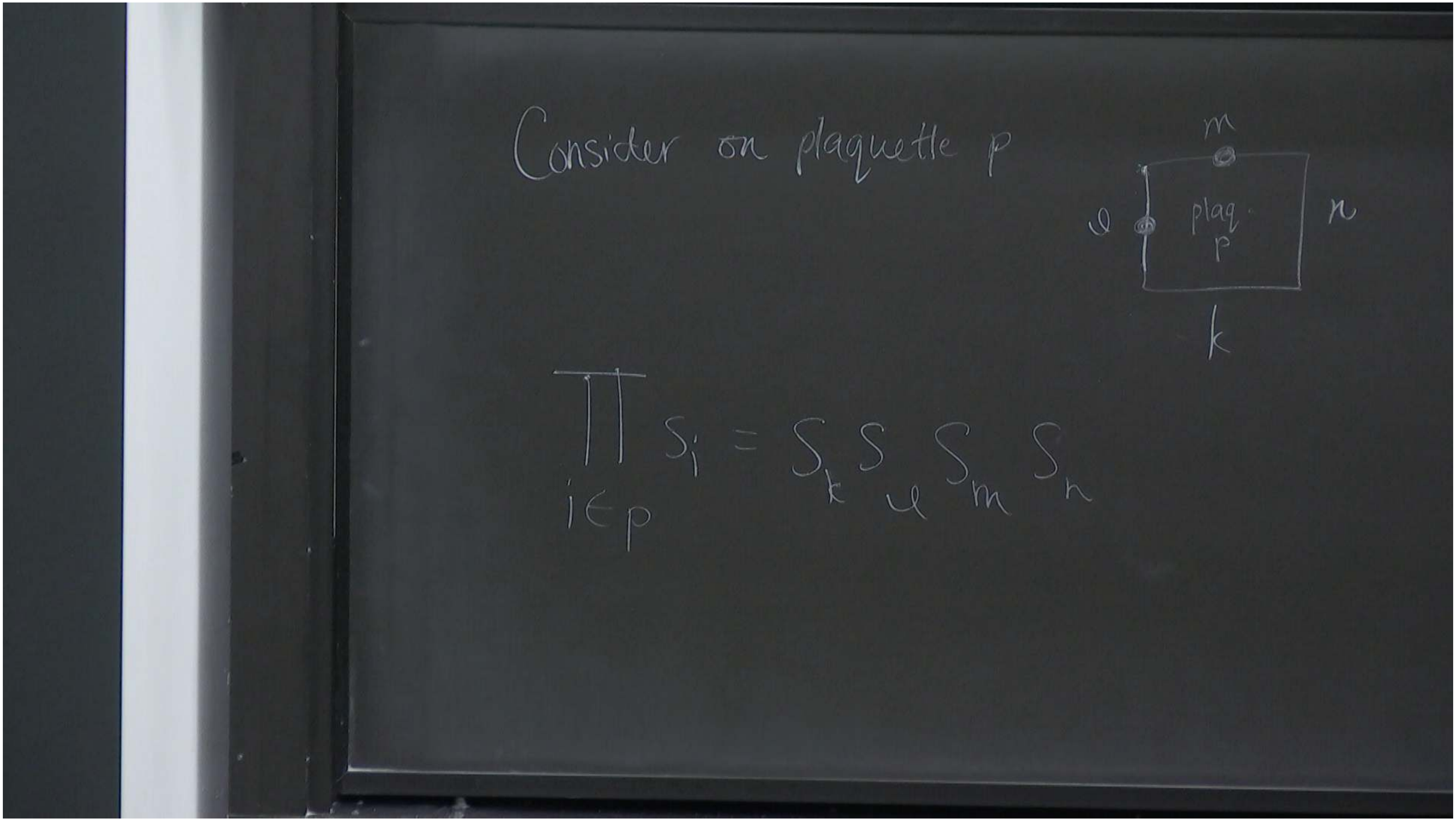
The d.o.f. are on the bonds of the lattice

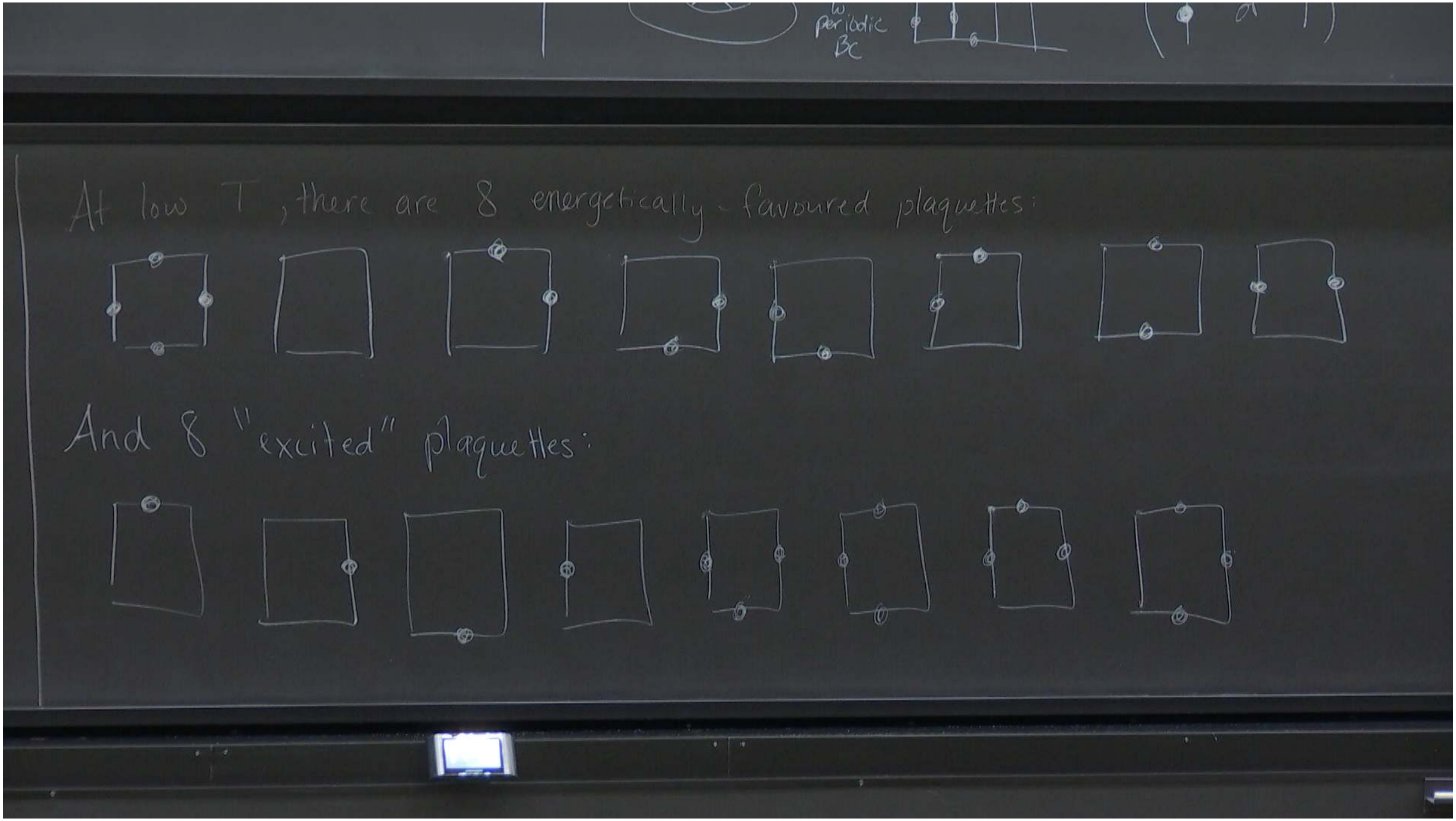
On a torus:



$s_i = +1$ or -1



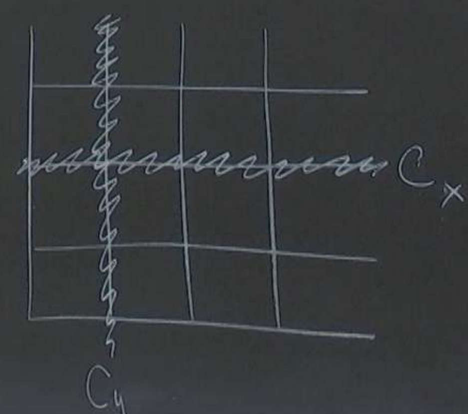

Consider on plaquette p
m
n
l
k
plaq.
p
$\prod_{i \in p} S_i = S_k S_l S_m S_n$


periodic
BC
At low T, there are 8 energetically-favoured plaquettes:
And 8 "excited" plaquettes:

There is topological order exactly at $T=0$ in the thermodynamic limit and no order for $T>0$

We can use the topological Wilson loop W_c as a measure of this order

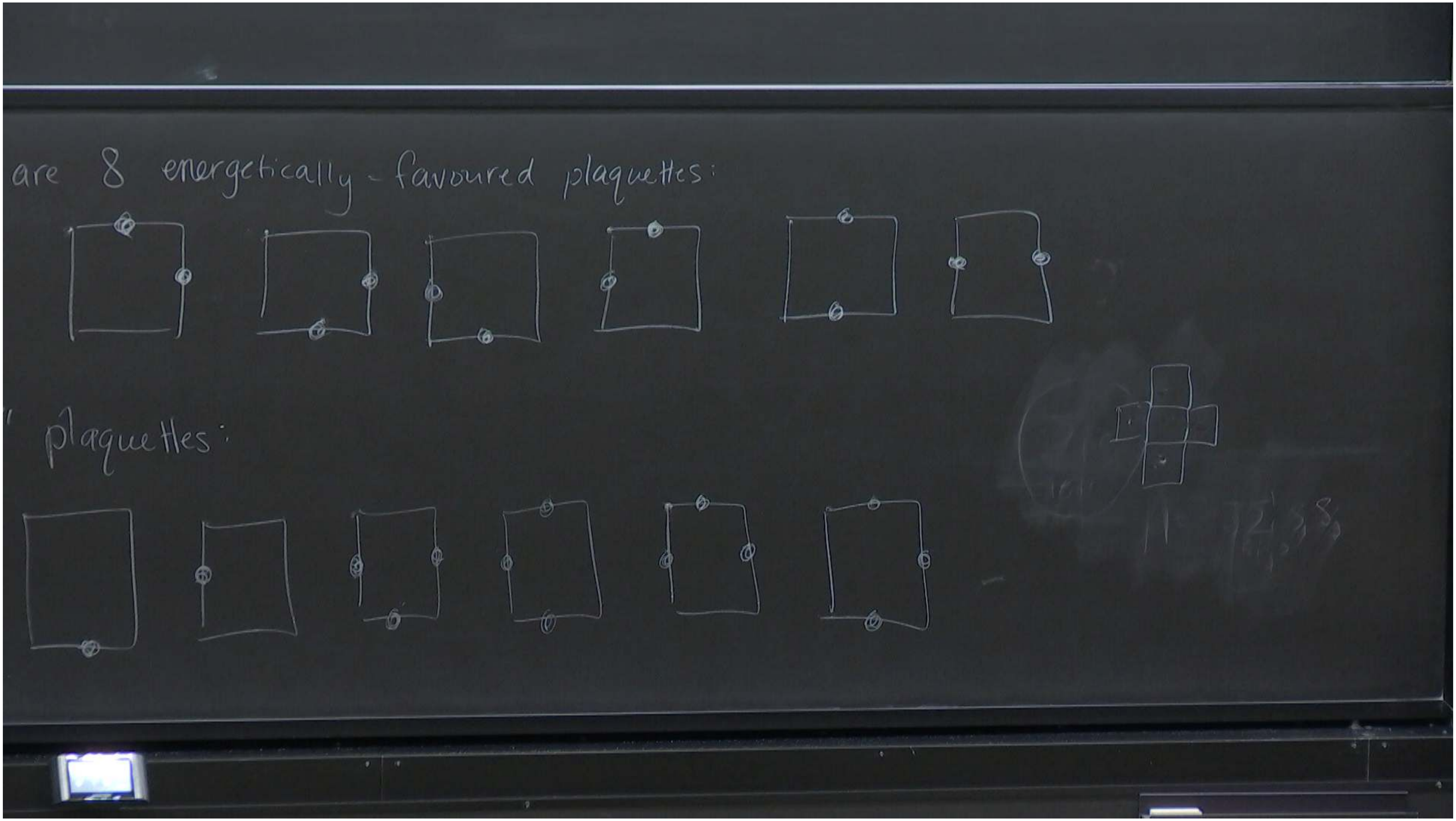
$$W_c = \prod_{i \in C} S_i = \pm 1$$



In the topologically-ordered groundstate, W_{C_x} and W_{C_y} are constant for all choices of C_x and C_y

Four possibilities:

W_{C_x}	W_{C_y}
+1	+1
+1	−1
−1	+1
−1	−1


are 8 energetically-favoured plaquettes:
plaquettes:

Hyperparams for feedforward NNs

- # of layers (how "deep" the NN is)
- # of neurons in each hidden layer
- learning rate value, whether it decays with time
- activation function
- cost func.
- training algorithm (GD, SGD, Adam, ...)
- hyperparams. within training alg.: momentum γ, mini-batch size $|B|$, learning rate

- how much regularization is added to the cost (param. λ) and what kind (L1, L2, ...)
- masking prob. if using dropout
- how we partition into training, validation, testing
- whether or not we add noise to our data, what kind and how much
- how the weights and biases are initialized

⋮

Monte Carlo (MC) Sampling

(Reference: Newman & Barkema)

(Used to generate data samples for HW1)

Consider a classical system where we want to calculate expectation values of quantities Q

$$\langle Q \rangle = \sum_{\mu} Q_{\mu} p_{\mu}$$

For Boltzmann dist.:

$$p_{\mu} = \frac{1}{Z} e^{-\beta E_{\mu}}, \quad Z = \sum_{\mu} e^{-\beta E_{\mu}}$$

- E_μ is the energy of config μ
- $\beta = \frac{1}{k_B T}$

- The sums $\sum_\mu$ are over huge # of terms

(2^N terms for classical Ising). Analytically intractable

Idea of MC: estimate $\langle Q \rangle$ from M

States $\{\mu_1, \mu_2, \ldots, \mu_M\}$, $M \ll 2^N$

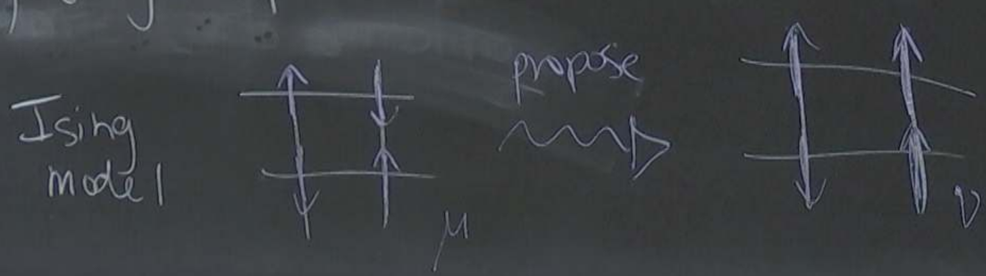
In particular, if each state μ_i is selected from the set of all states according to the desired dist. p_{μ_i}, then

$$\langle Q \rangle \simeq Q_M = \frac{1}{M} \sum_{i=1}^{M} Q_{\mu_i} \quad \left(\text{makes use of "importance sampling"} \right)$$

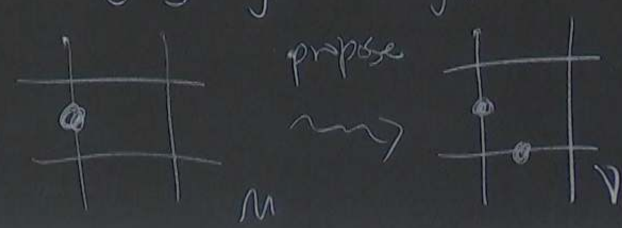
To implement this sampling, we can use Markov chain MC (MCMC)

↳ use the current state μ to propose a move new related state ν, and then decide to accept or reject proposal

eg) single spin flip:



Ising gauge theory:



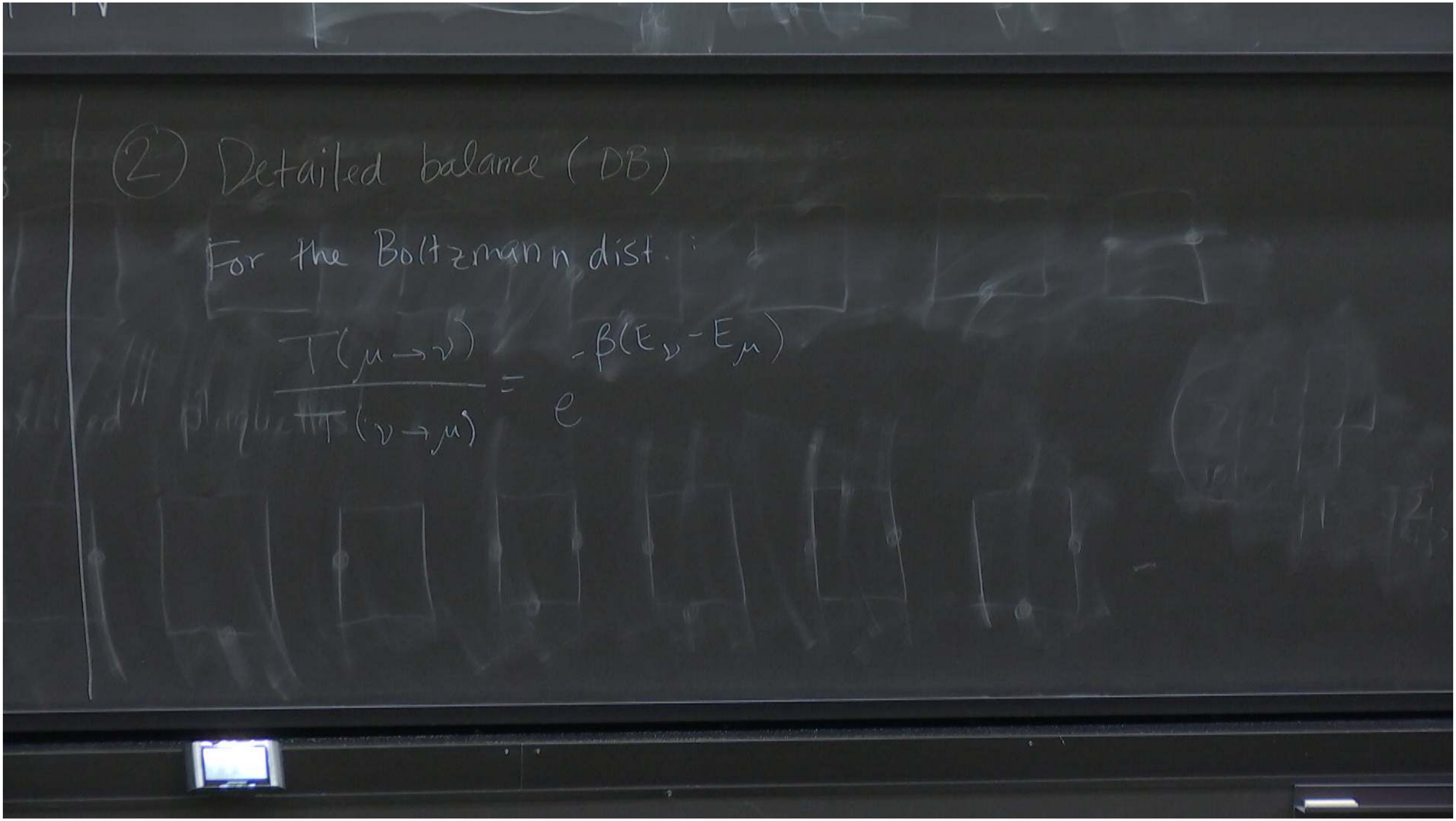
In priciple, any proposal is allowed

The prob. of moving from μ to ν is the transition prob. $T(\mu \to \nu)$, with $\sum_{\nu} T(\mu \to \nu) = 1$

We can choose any set of transition probs. $\{T(\mu \to \nu)\}$
as long as we satisfy two conditions:

(1) Ergodicity: there is a non-zero prob. of transitioning (eventually) from μ to ν

$$\mu \longrightarrow \ldots \longrightarrow \nu$$


(2) Detailed balance (DB)
For the Boltzmann dist.:
$\frac{T(\mu \to \nu)}{T(\nu \to \mu)} = e^{-\beta(E_\nu - E_\mu)}$