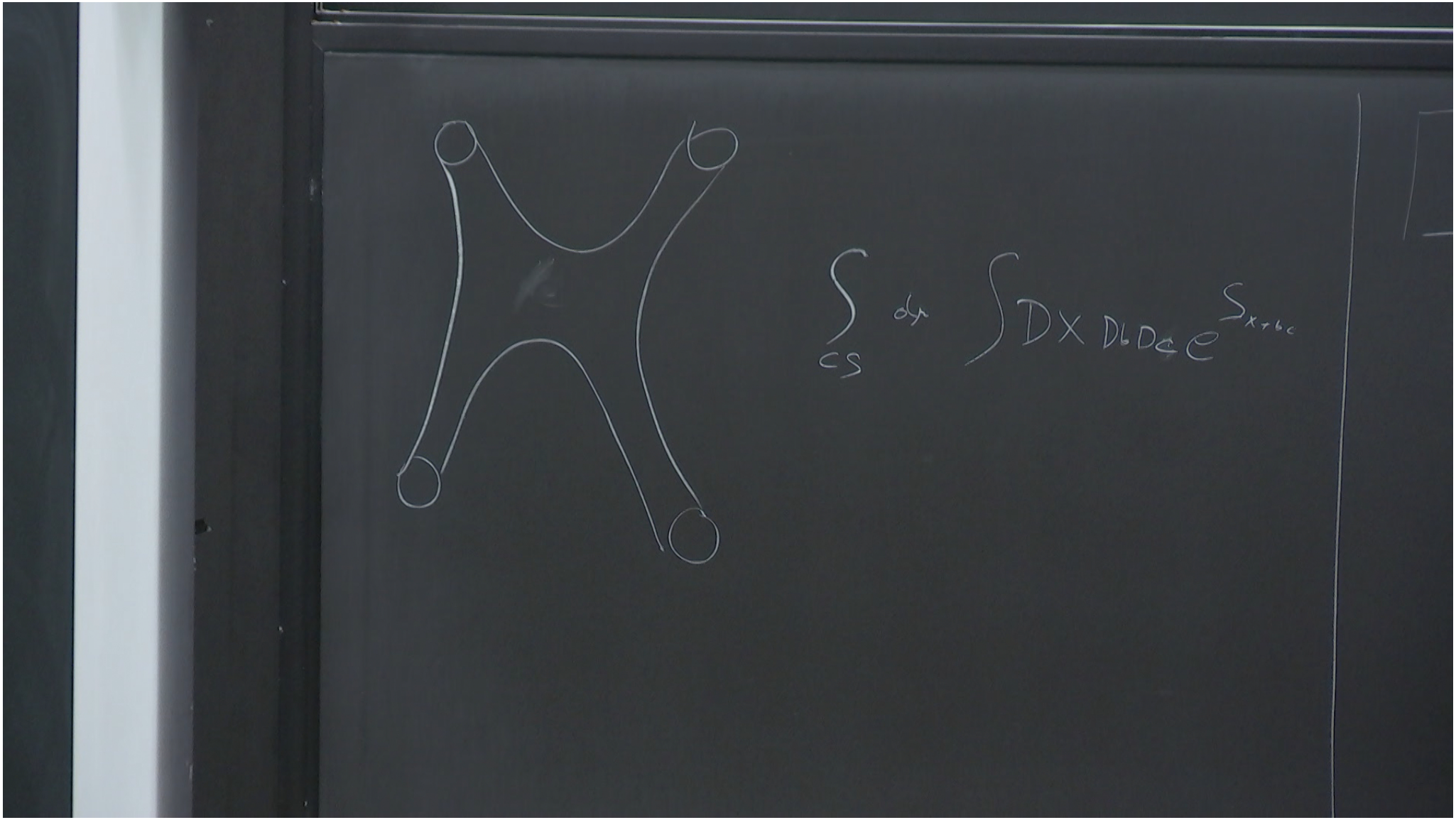
Title: Quantum Fields and Strings Lecture - 230329

Speakers: Davide Gaiotto

Collection: Quantum Fields and Strings (2022/2023)

Date: March 29, 2023 - 10:15 AM

URL: https://pirsa.org/23030027


$\int_{CS} d\tau \int DX\, D\mathfrak{b}\, D\mathfrak{c}\, e^{S_{X+\mathfrak{b}\mathfrak{c}}}$

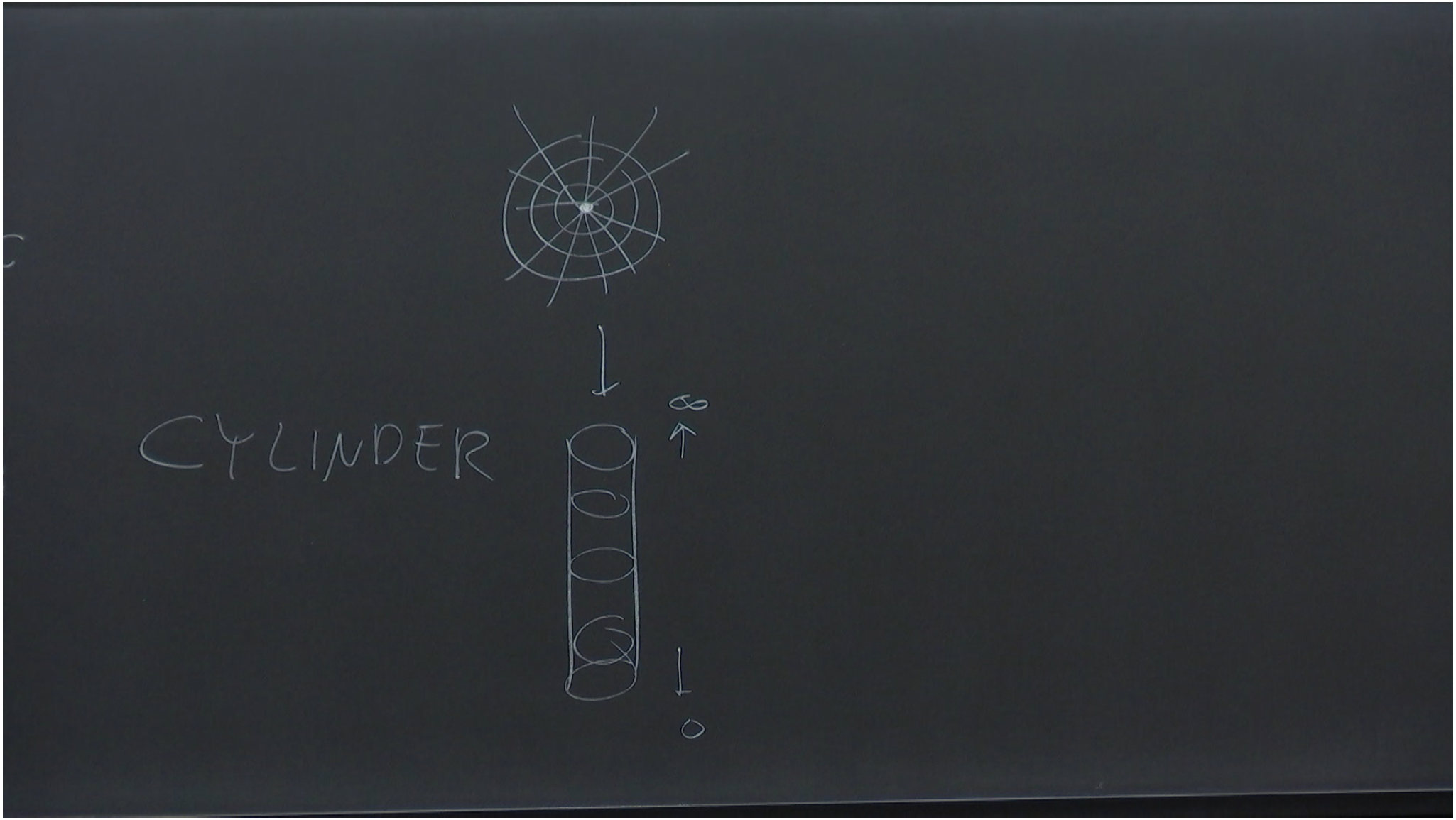
$z, \bar{z}$

FLAT PLANE $\quad ds^2 = |dz|^2 = dz\,d\bar{z} \quad$ ON $\mathbb{C}$

dr^2

$\frac{|dz|^2}{z^2} + d\theta^2 = \frac{dz^2 + z^2 d\theta^2}{z^2} \leftarrow ds^2 = \frac{|dz|^2}{|z|^2}$ OU $\mathbb{C}/\{0\}$

$R = e^T$


CYLINDER
1
∞
1
0

$= dz\,d\bar{z}$ ON $\mathbb{C}$

$dr^2 = dz^2 + dz^2 d\theta^2 = \frac{dz^2 + z^2 d\theta^2}{z^2}$ $\leftarrow$ $ds^2 = \frac{|dz|^2}{|z|^2}$ OU $\mathbb{C}/_{\mathbb{Z}}$ CYL

$R = e^r$

$$ds^2 = \frac{|dz|^2}{1+|z|^2}$$

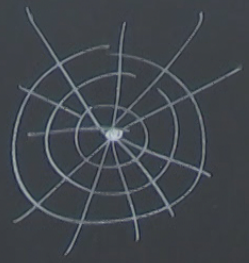


PLANE $ds^2 = |\partial_z z|^2 \, dz\, d\bar{z}$ ON $\mathbb{C}$

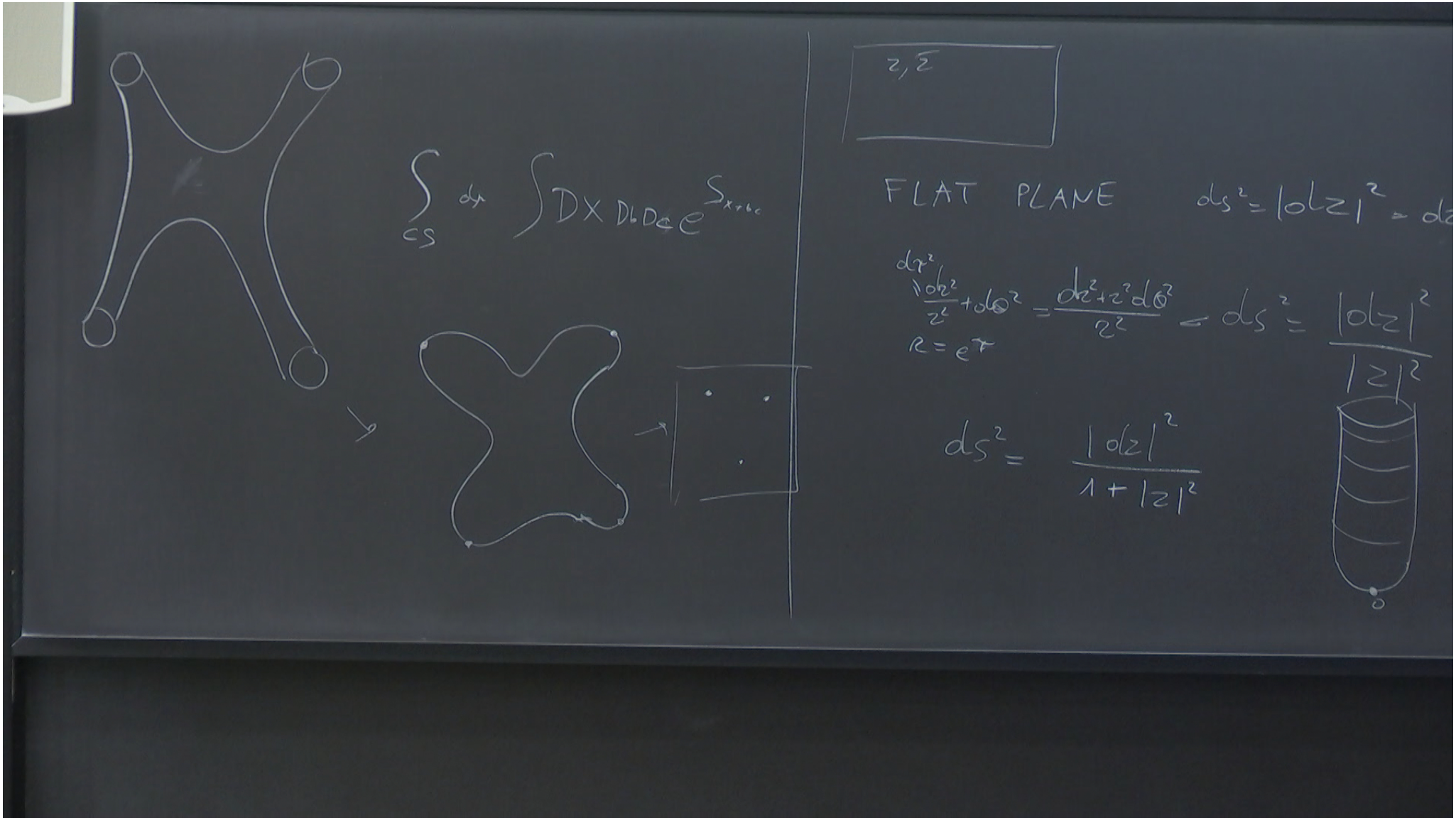
$\rho^2 = \frac{dz^2 + z^2 d\theta^2}{z^2} \leadsto ds^2 = \frac{|\partial_z z|^2}{|z|^2}$ OU $\mathbb{C}/\mathbb{Z}_3$

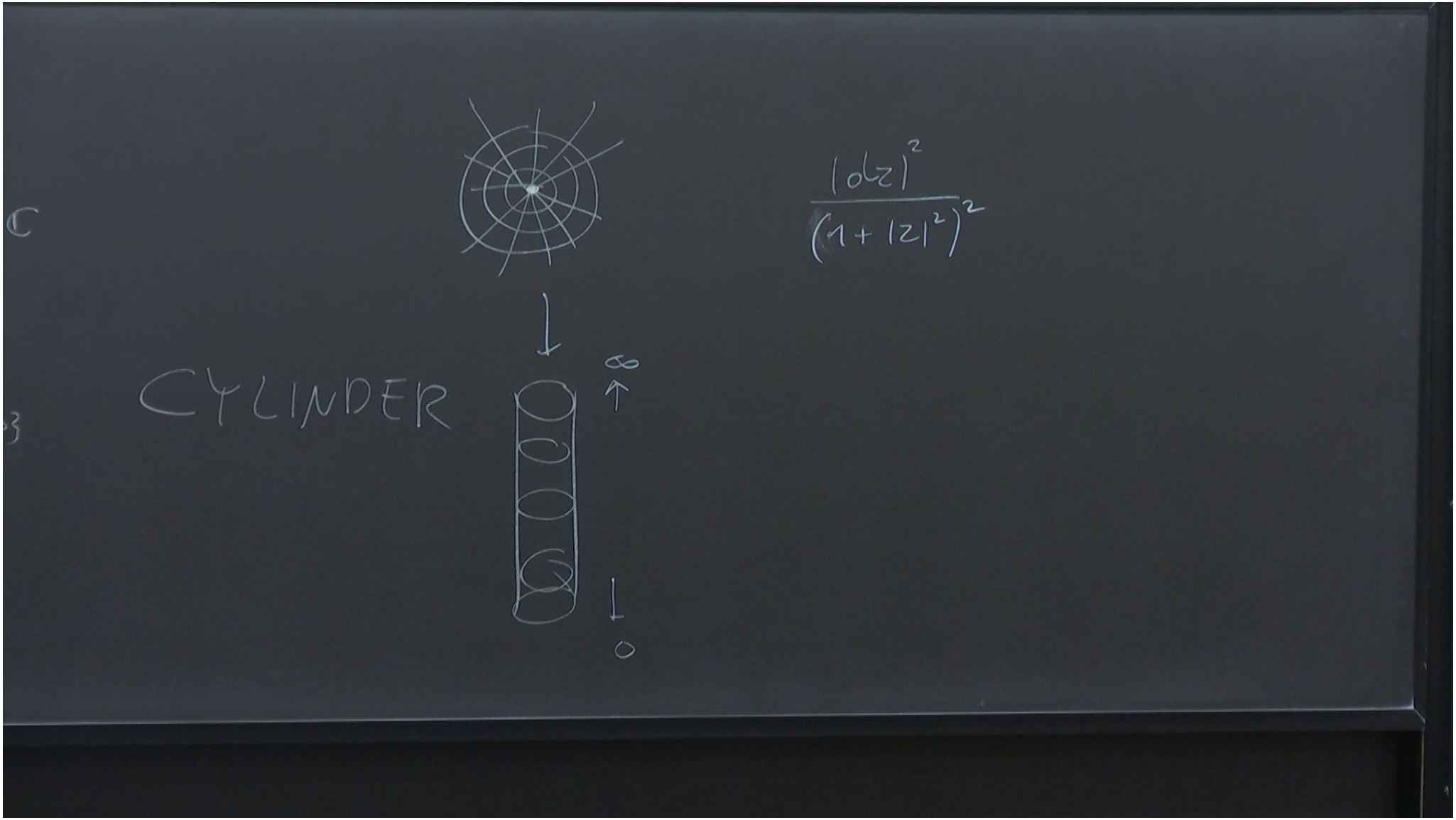
$= \frac{|\partial_z z|^2}{1 + |z|^2}$








$\int_{CS} d\mu \int DX\, Db\, Dc\, e^{S_{X+b,c}}$
z, z̄
FLAT PLANE
$ds^2 = |\partial z|^2 = dz$
$\frac{dr^2}{r^2} + d\theta^2 = \frac{dx^2 + r^2 d\theta^2}{r^2}$
$R = e^T$
$ds^2 = \frac{|\partial z|^2}{|z|^2}$
$ds^2 = \frac{|\partial z|^2}{1 + |z|^2}$


$\frac{|dz|^2}{(1+|z|^2)^2}$
CYLINDER
∞
0

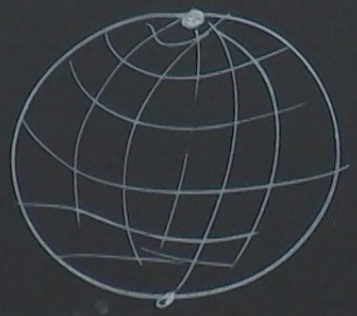
$$\frac{|dz|^2}{(1+|z|^2)^2} = \frac{dx^2}{(1+z^2)^2} + \frac{z^2}{(1+z^2)^2} d\theta^2$$

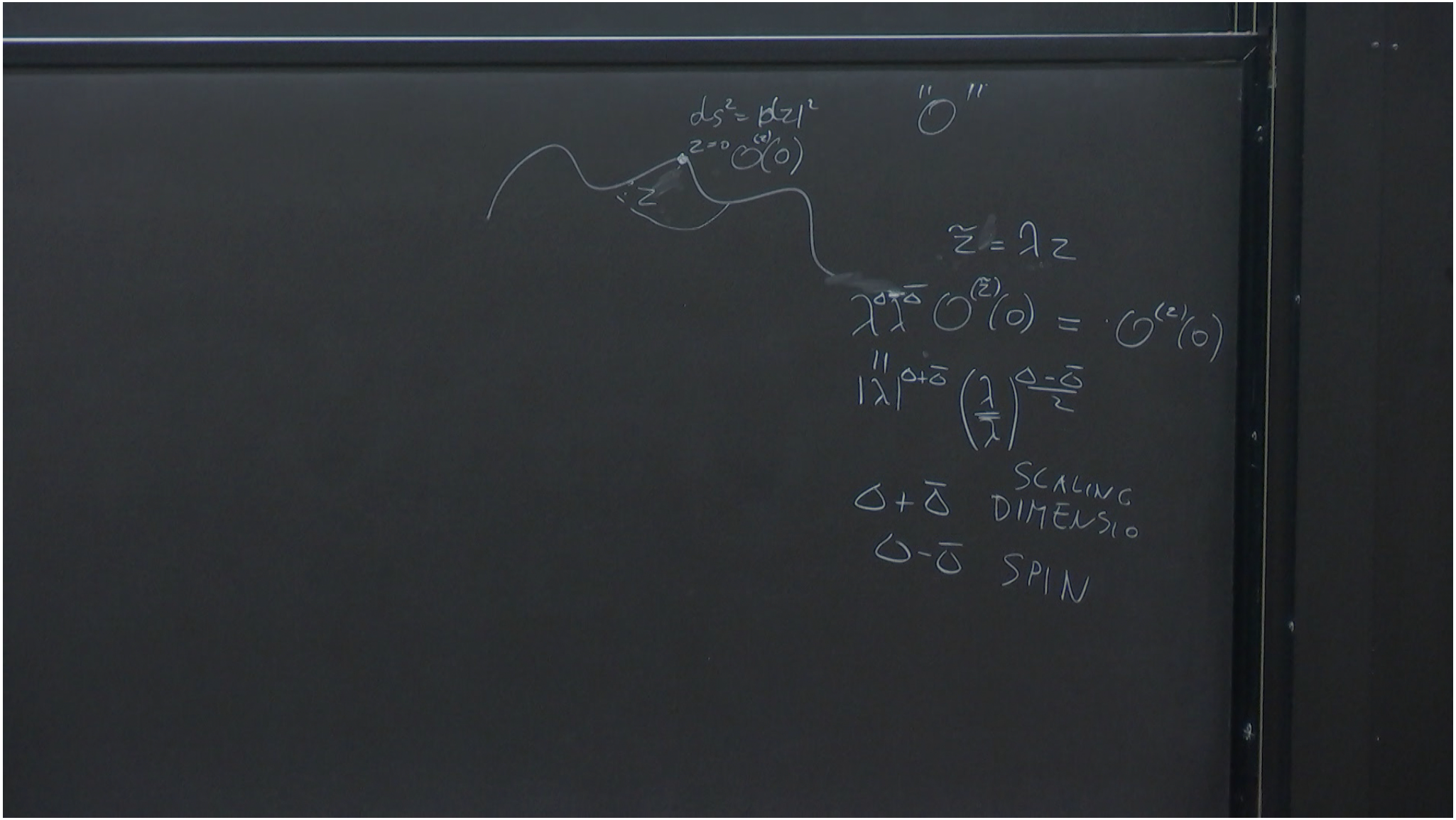
$$\| \qquad d\alpha = \frac{dz}{1+z^2}$$

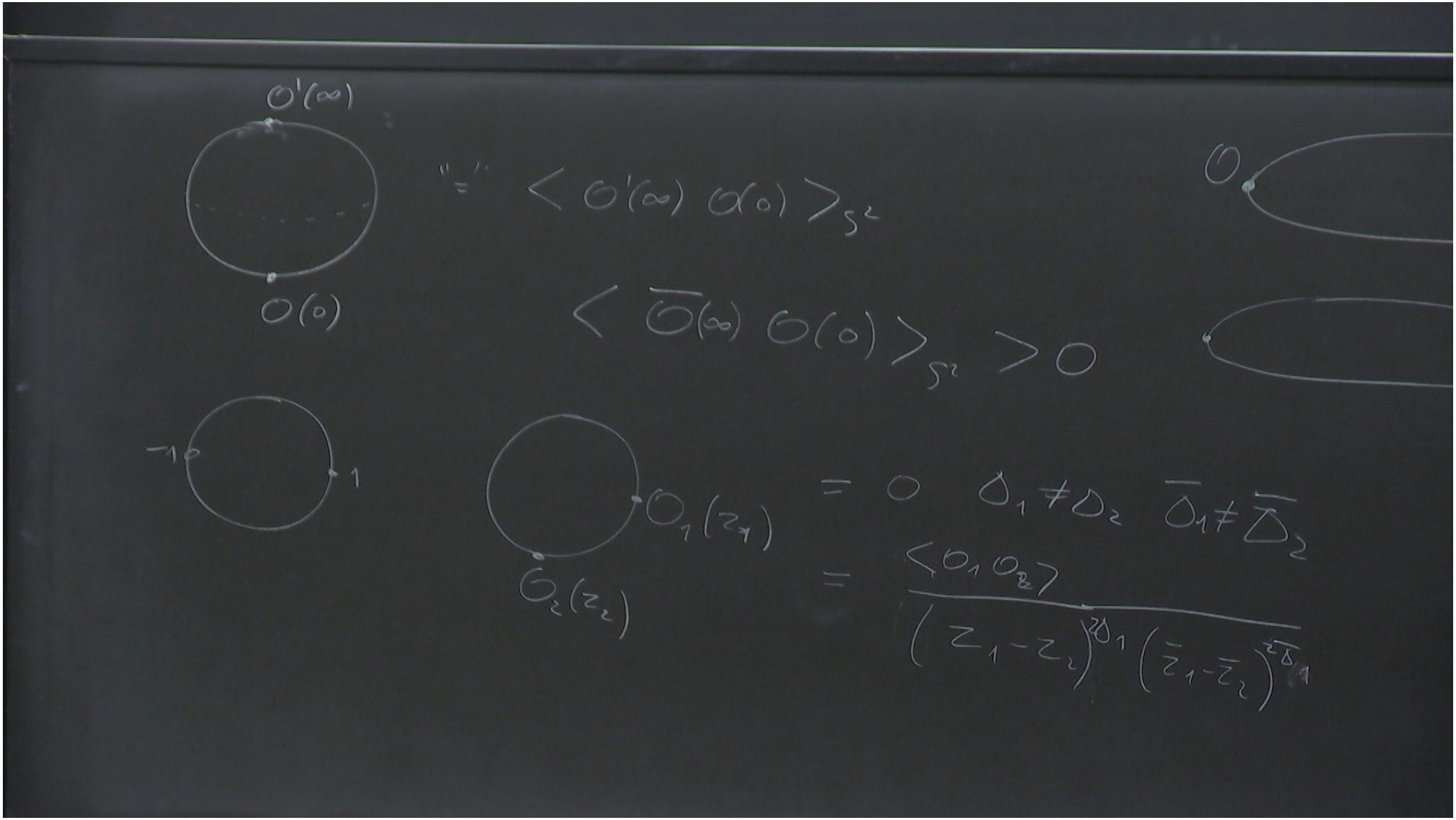
$$z = \tan\alpha$$

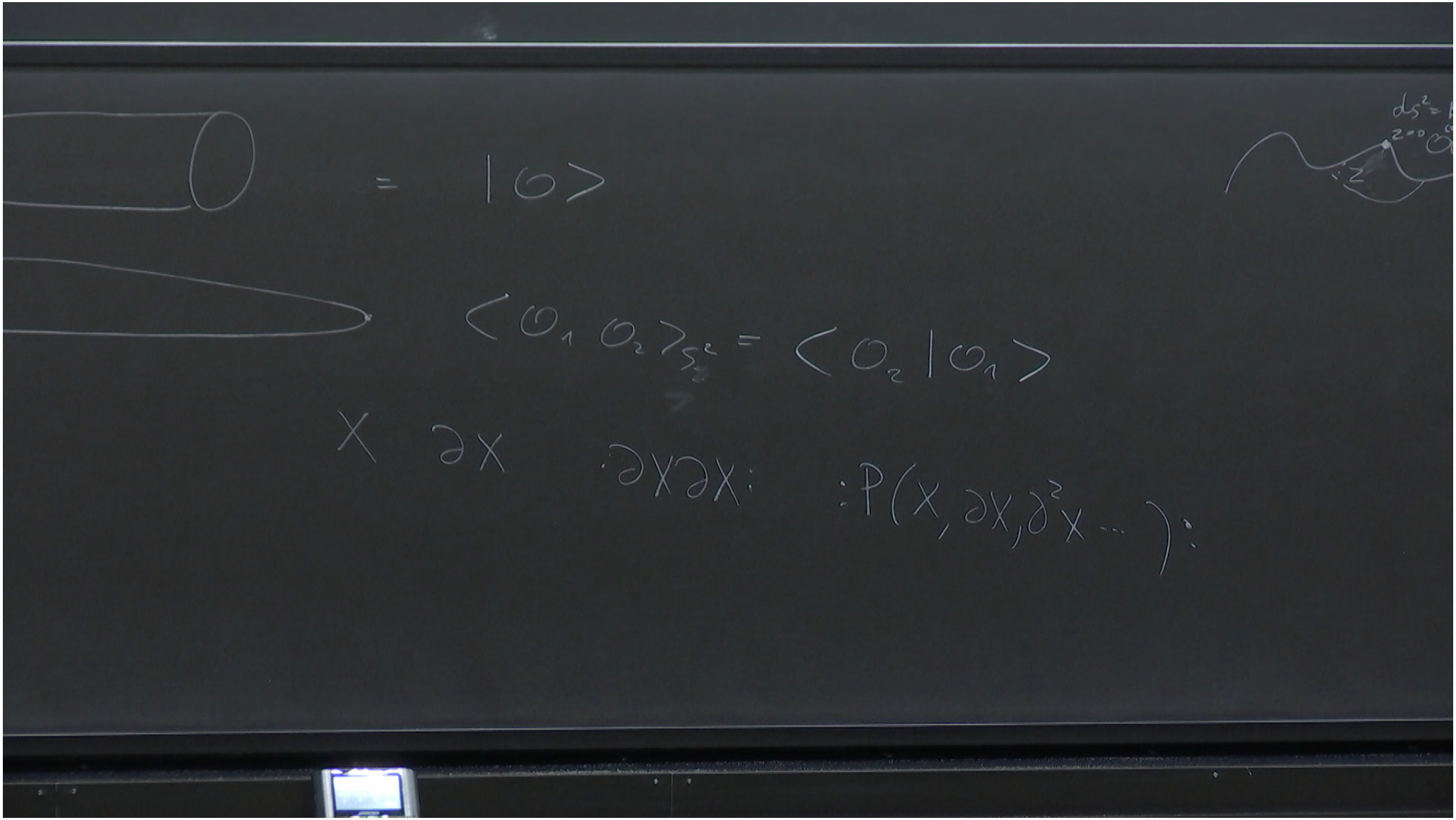
$$\frac{z}{1+z^2} = \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$$

$$\frac{1}{4}\left(d(2\alpha)^2 + \sin^2 2\alpha\, d\theta^2\right)$$



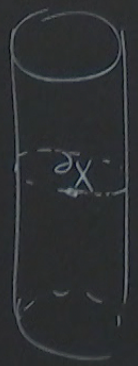

$ds^2 = |dz|^2$
$z=0$ $\mathcal{O}^{(z)}(0)$
z
"$\mathcal{O}$"
$\tilde{z} = \lambda z$
$\lambda^{\Delta}\bar{\lambda}^{\bar{\Delta}}\mathcal{O}^{(\tilde{z})}(0) = \mathcal{O}^{(z)}(0)$
$|\lambda|^{\Delta+\bar{\Delta}}\left(\frac{\lambda}{\bar{\lambda}}\right)^{\frac{\Delta-\bar{\Delta}}{2}}$
$\Delta+\bar{\Delta}$ SCALING DIMENSION
$\Delta-\bar{\Delta}$ SPIN

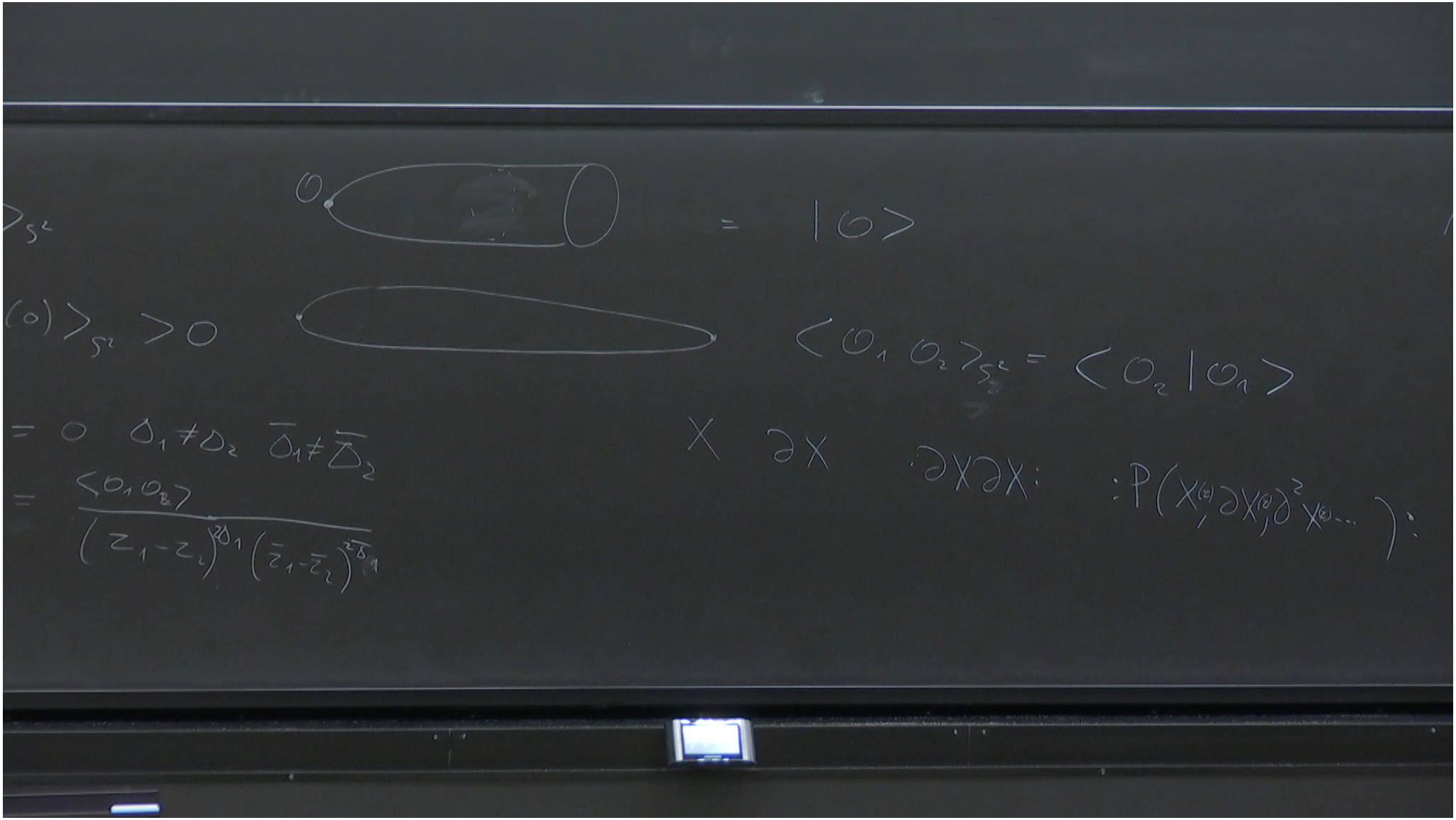

O'(∞)
O(0)
"="
< O'(∞) O(0) >_{S²}
< Ō(∞) O(0) >_{S²} > 0
-1
1
O₁(z₁)
O₂(z₂)
= 0 Δ₁ ≠ Δ₂ Δ̄₁ ≠ Δ̄₂
= < O₁ O₂ > / (z₁ - z₂)^{2Δ₁} (z̄₁ - z̄₂)^{2Δ̄₁}

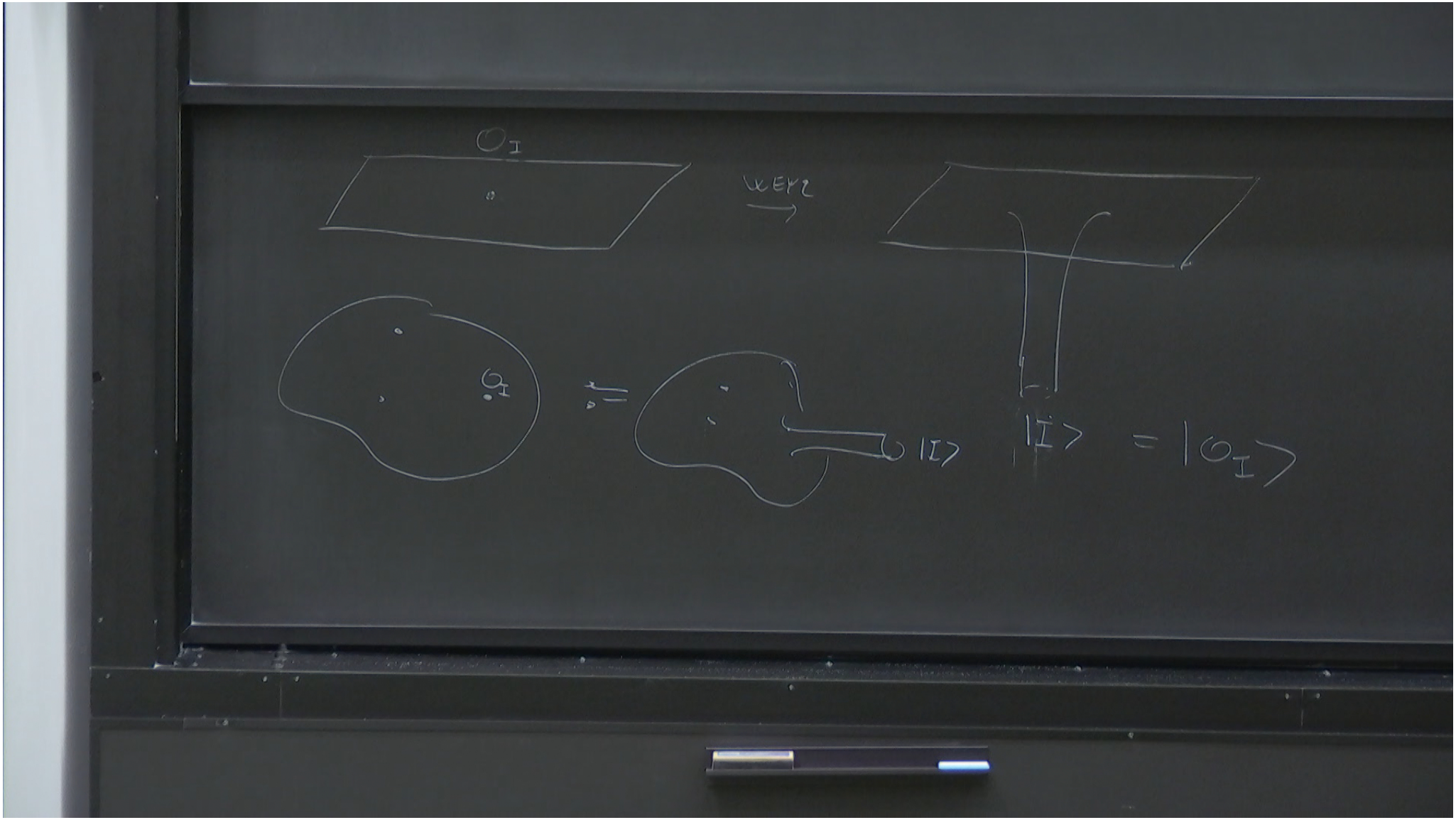

= |O>
<O1 O2 ...>_{S^2} = <O2|O1>
X ∂X :∂X∂X: :P(X, ∂X, ∂²X ...):

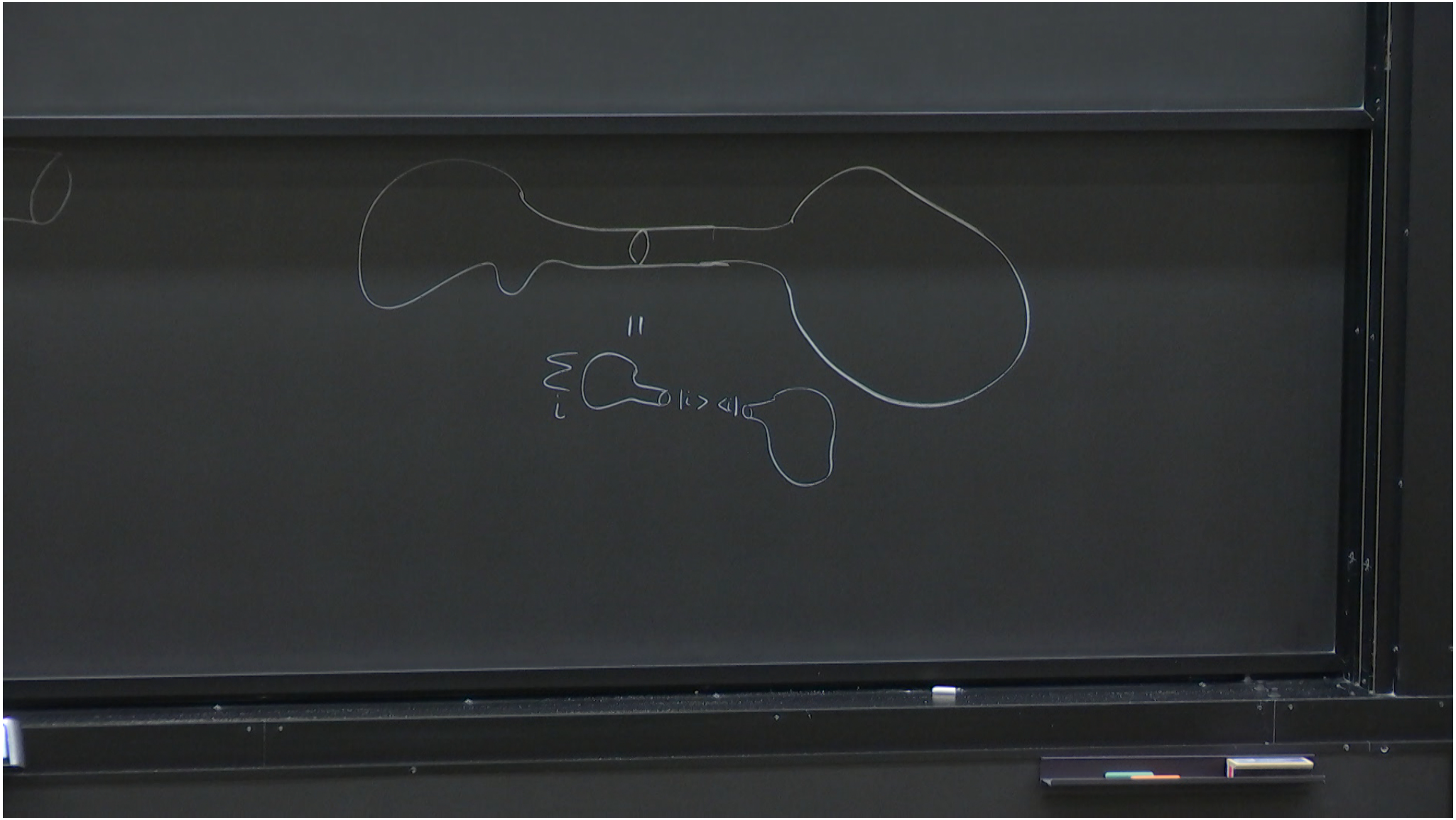
$$\partial X = \sum_n a_n e^{ns}$$

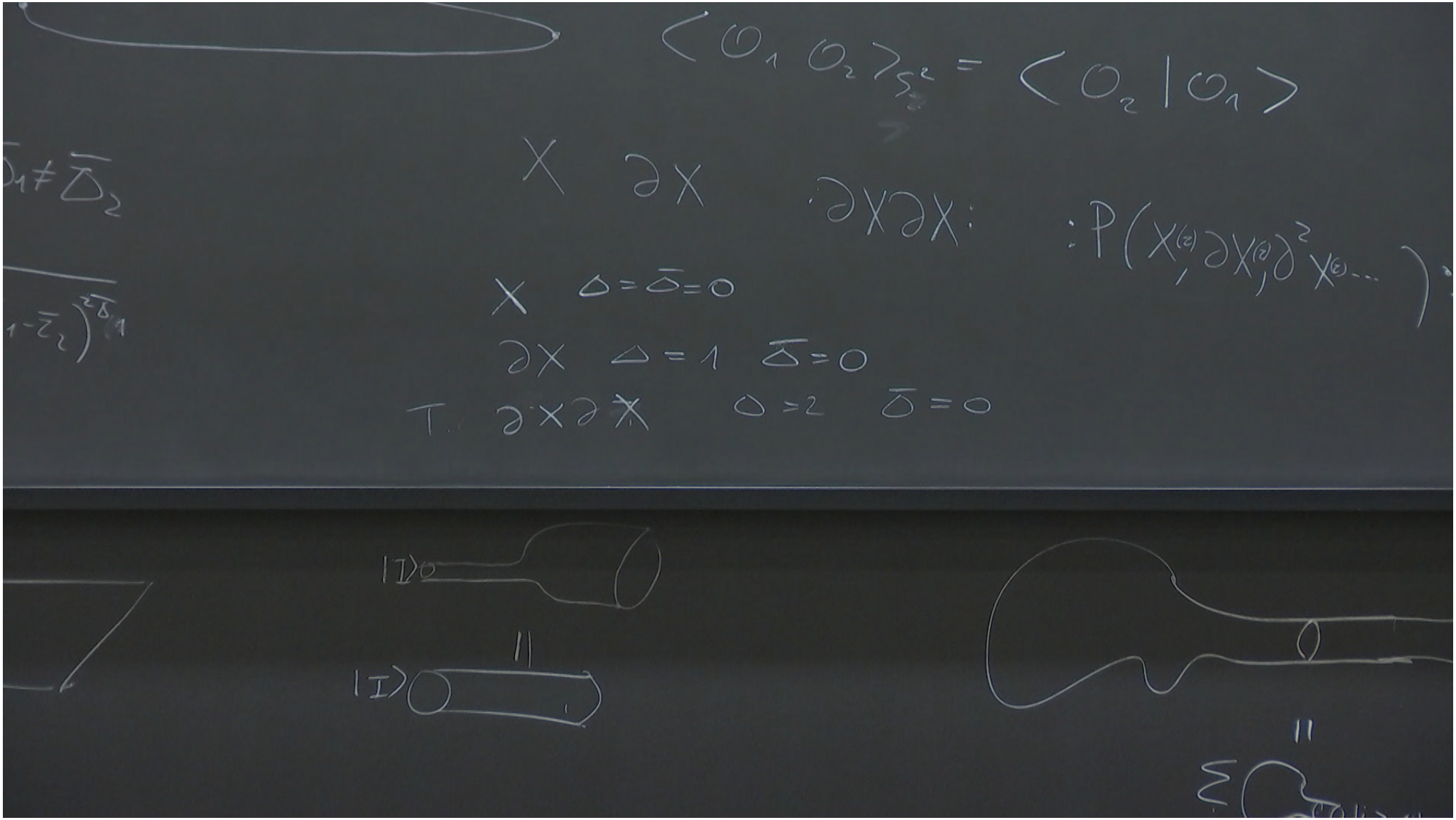
$$a_n = \int_{\sigma=0}^{2\pi} \partial X(s') \, e^{ns} \, ds$$

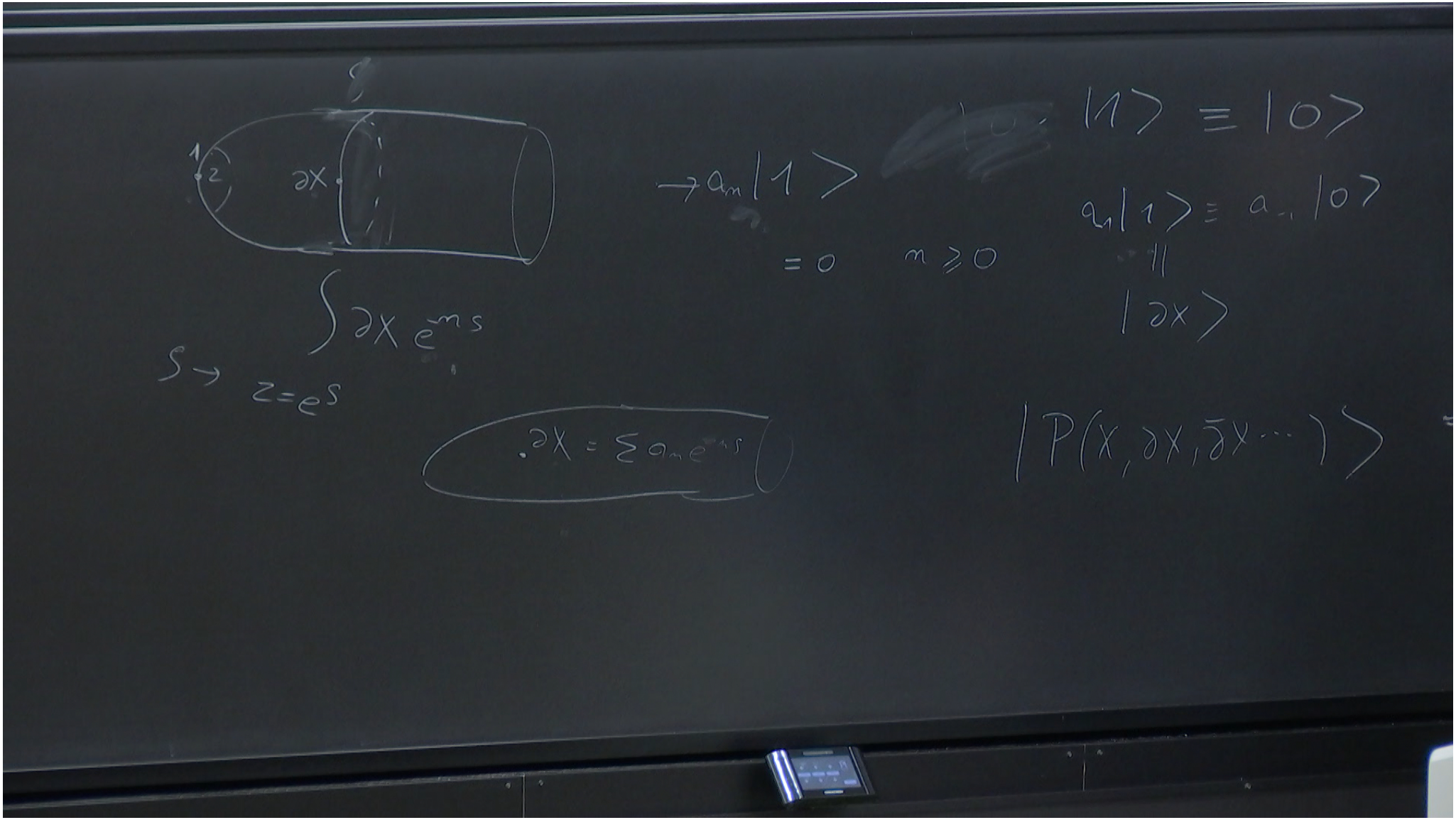


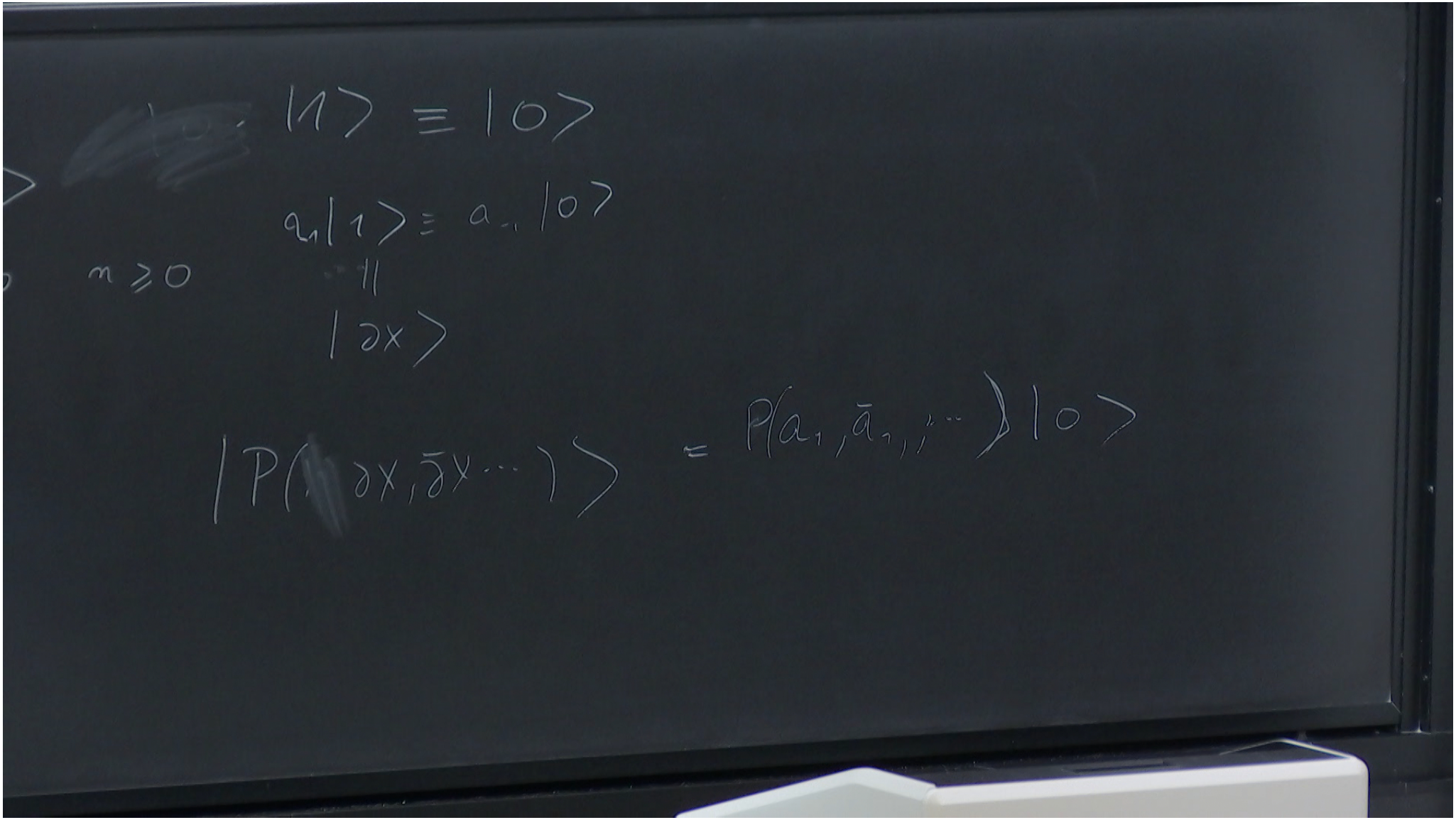

$\rangle_{S^2}$
$= |0\rangle$
$\langle 0 \rangle_{S^2} > 0$
$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle_{S^2} = \langle \mathcal{O}_2 | \mathcal{O}_1 \rangle$
$= 0 \quad \Delta_1 \neq \Delta_2 \quad \bar{\Delta}_1 \neq \bar{\Delta}_2$
$= \frac{\langle \mathcal{O}_1 \mathcal{O}_2 \rangle}{(z_1 - z_2)^{2\Delta_1} (\bar{z}_1 - \bar{z}_2)^{2\bar{\Delta}_1}}$
$X \quad \partial X \quad \partial X \partial X : \quad : P(X^{(a)}, \partial X^{(a)}, \partial^2 X^{(a)} \ldots) :$


O_I
$|I\rangle$
$|I\rangle = |O_I\rangle$


=
i
|i><i|


$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle_{S^2} = \langle \mathcal{O}_2 | \mathcal{O}_1 \rangle$
$\Delta_1 \neq \bar{\Delta}_2$
$X \quad \partial X \quad :\partial X \partial X: \quad :P(X^{(z)}, \partial X^{(z)}, \partial^2 X^{(z)} \ldots)$
$X \quad \Delta = \bar{\Delta} = 0$
$\partial X \quad \Delta = 1 \quad \bar{\Delta} = 0$
$T \quad \partial X \partial X \quad \Delta = 2 \quad \bar{\Delta} = 0$
$|I\rangle$
$|I\rangle$
Σ


1
2
∂X
$\int \partial X e^{ns}$
$s \to z = e^s$
$\partial X = \sum a_n e^{ns}$
$a_n|1\rangle = 0 \quad n \geq 0$
$|1\rangle \equiv |0\rangle$
$a_{-1}|1\rangle \equiv a_{-1}|0\rangle$
$\| \quad |\partial X\rangle$
$|P(X, \partial X, \bar{\partial} X \cdots)\rangle$


$|1\rangle \equiv |0\rangle$
$a_1|1\rangle \equiv a_1|0\rangle$
$n \geq 0$
$||$
$|\partial x\rangle$
$|P(\partial x, \bar{\partial} x \cdots)\rangle = P(a_1, \bar{a}_1, \cdots)|0\rangle$

$|p\rangle \equiv |\mathcal{O}_p\rangle$

$|p\rangle = e^{ipx}|0\rangle$

$\mathcal{O}_p = "e^{ipX}"$

$\int DX\, e^{-S}\, e^{ipX(z_1)} :=$

$$\int DX\, e^{S}\, e^{ipX(z_1)} := \int DX\, e^{S + ipX(z_1)}$$

$$\langle X \rangle = \lim_{f \to \delta^2(z_1)} e^{ip \int f(z,\bar{z}) \langle X(z,\bar{z}) \rangle - \frac{p^2}{2} \int f(z) f(z') \log |z - z'|^2}$$

$$f = \int_{\Delta}$$

$|p\rangle \equiv |\mathcal{O}_p\rangle$

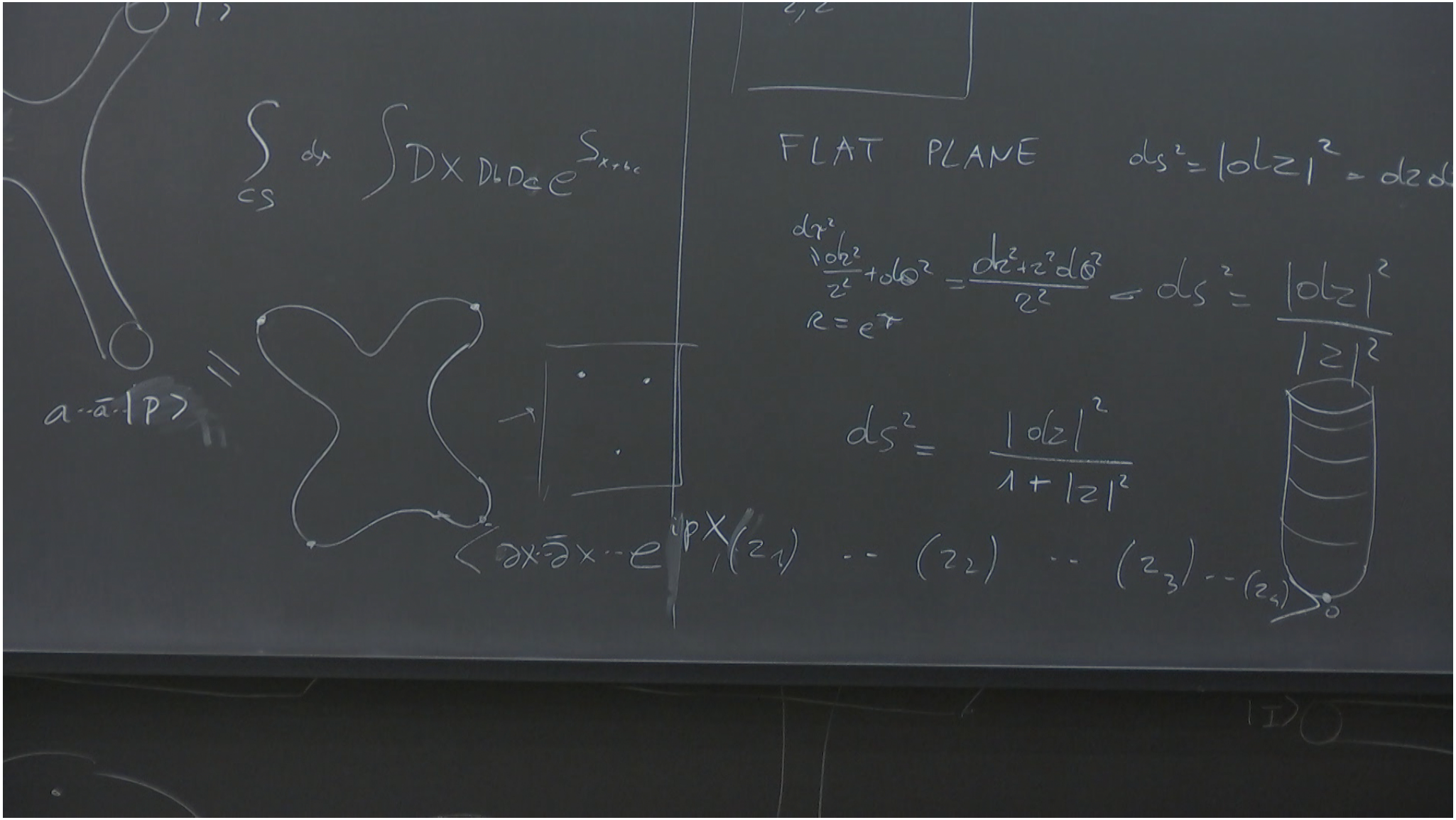
$\int DX\, e^{S}\, e^{ipX(z_1)}$

$|p\rangle = e^{ip\alpha}|0\rangle$

$\boxed{\mathcal{O}_p = "e^{ipX}"}$

$\lim_{f \to \delta^2(z_1)} e^{ip\int f(z,\bar{z})X(z,\bar{z})\rangle} - \frac{p^2}{2}\int f(z)f(z'$

$f =$ 


FLAT PLANE
$ds^2 = |\partial_z|^2$
$ds^2 = \frac{|\partial_z|^2}{|z|^2}$
$ds^2 = \frac{|\partial_z|^2}{1+|z|^2}$
$a_{-1}\bar{a}_{-1}|p\rangle$
$\langle \partial X \bar{\partial} X \cdots e^{ipX}(z_1) \cdots (z_2) \cdots (z_3) \cdots (z_4) \rangle$

$$\int DX\, e^{S}\, e^{ipX(z_1)} := \int DX\, e^{S + ipX(z_1)}$$

$$X(z,\bar{z})X(z,\bar{z}) - \frac{p^2}{2}\int f(z)f(z')\log|z-z'|^2$$

$q_1|1\rangle \equiv a_{-1}|0\rangle$

$n \geq 0$

$\|$

$|\partial x\rangle$

$|P(\partial x, \bar{\partial} x \cdots)\rangle = P(a_1, \bar{a}_1, \cdots)|0\rangle$

$|\partial^n x\rangle \to a_{-n}|0\rangle$