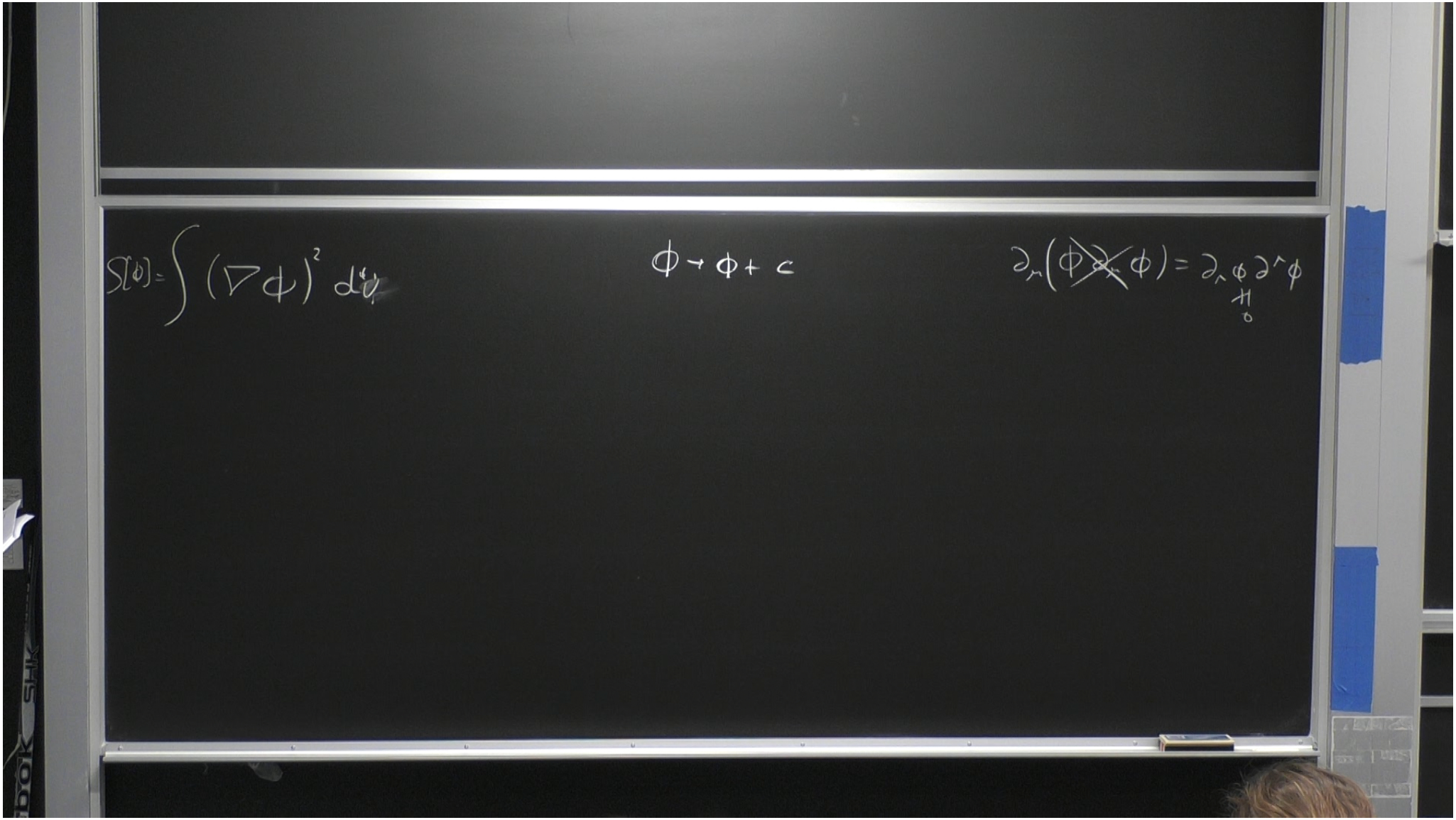
Title: Quantum Fields and Strings Lecture - 230324

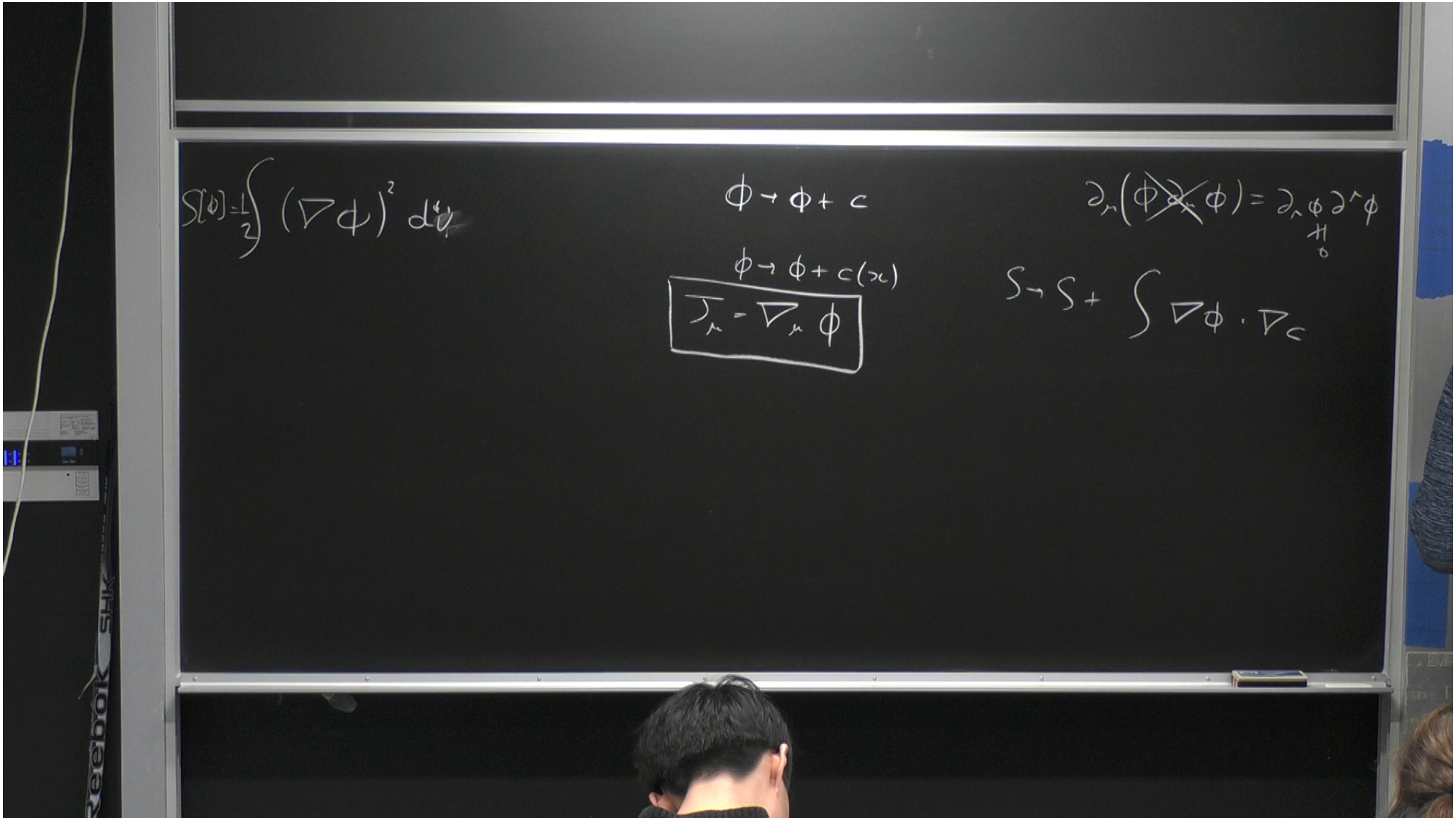
Speakers: Davide Gaiotto

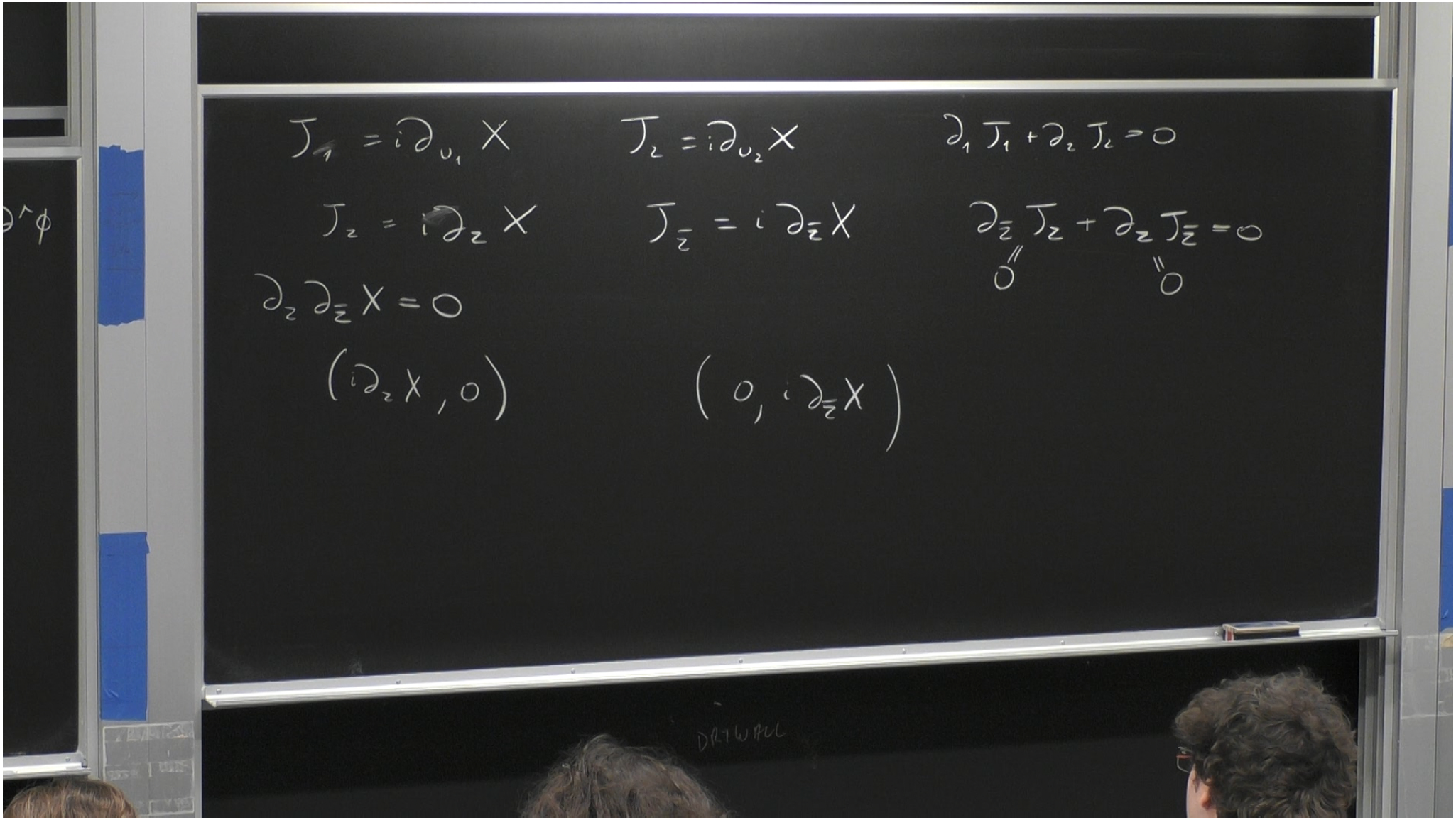
Collection: Quantum Fields and Strings (2022/2023)

Date: March 24, 2023 - 10:15 AM

URL: https://pirsa.org/23030025

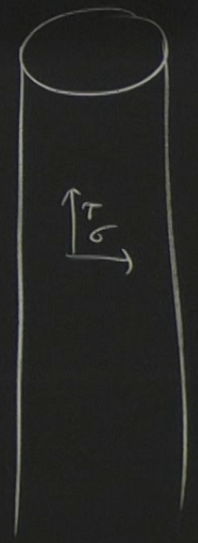

$S[\phi] = \int (\nabla \phi)^2 \, d^4x$
$\phi \to \phi + c$
$\partial_\mu(\phi \partial^\mu \phi) = \partial_\mu \phi \partial^\mu \phi + \phi \Box \phi$
$= 0$


$S[\phi] = \frac{1}{2}\int (\nabla\phi)^2 \, d^4x$
$\phi \to \phi + c$
$\partial_\mu(\phi\partial^\mu\phi) = \partial_\mu\phi\,\partial^\mu\phi + \phi\,\partial_\mu\partial^\mu\phi$
$\phi \to \phi + c(x)$
$J_\mu = \nabla_\mu\phi$
$S \to S + \int \nabla\phi \cdot \nabla c$


$J_1 = i\partial_{v_1} X$
$J_2 = i\partial_{v_2} X$
$\partial_1 J_1 + \partial_2 J_2 = 0$
$J_z = i\partial_z X$
$J_{\bar{z}} = i\partial_{\bar{z}} X$
$\partial_{\bar{z}} J_z + \partial_z J_{\bar{z}} = 0$
0
0
$\partial_z \partial_{\bar{z}} X = 0$
$(i\partial_z X, 0)$
$(0, i\partial_{\bar{z}} X)$

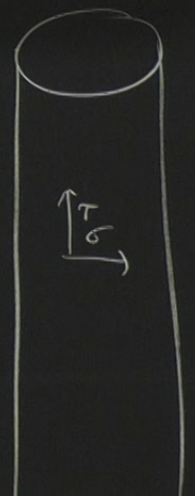
$$a_n \propto \int_0^{2\pi L} e^{i\frac{n}{L}s}\, i\partial_s X \, d\sigma \qquad \bar{a}_n \propto \int_0^{2\pi L} e^{i\frac{n}{L}\bar{s}}\, i\partial_{\bar{s}} X \, d\sigma$$

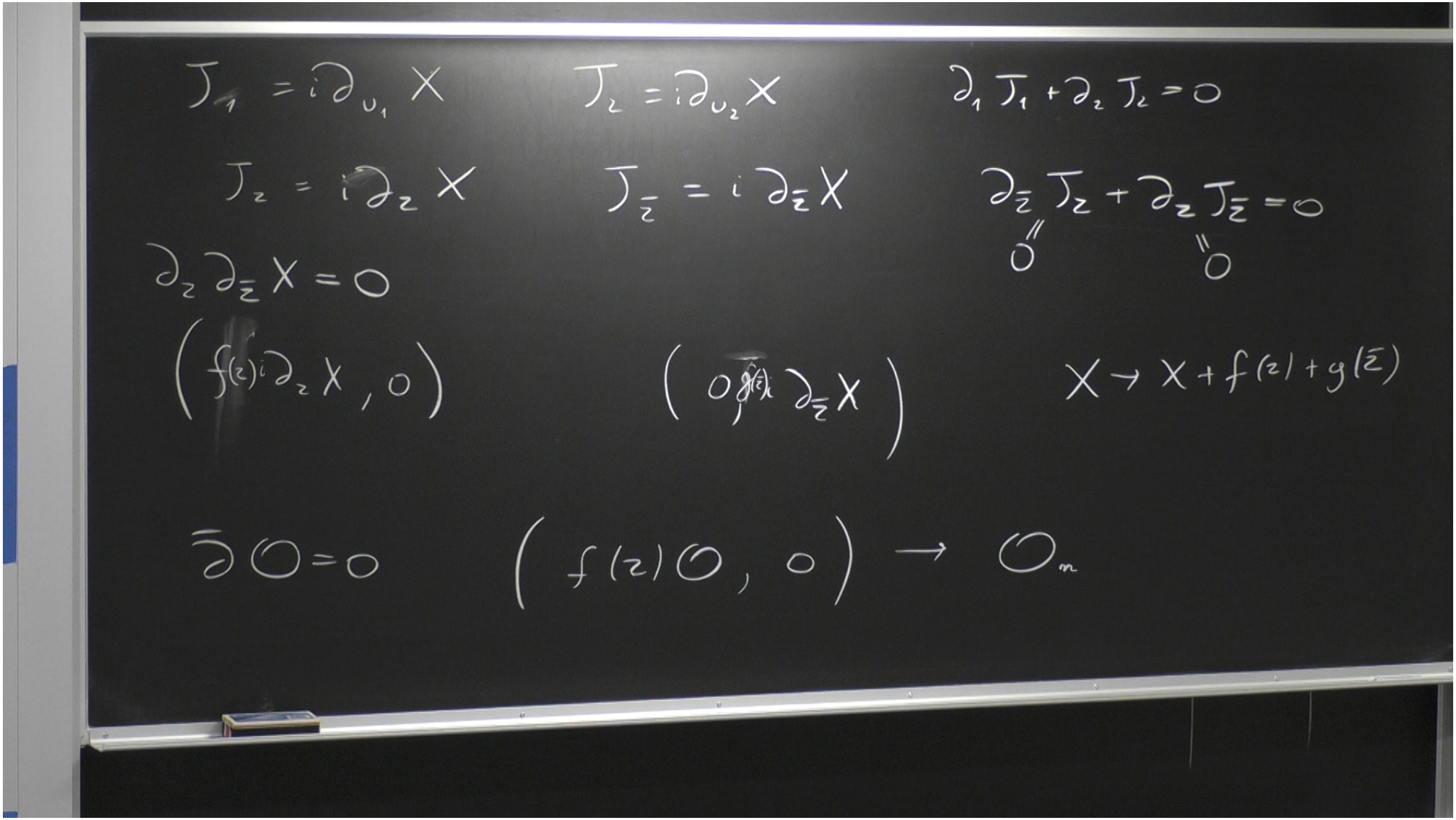
$s = \tau - i\sigma$

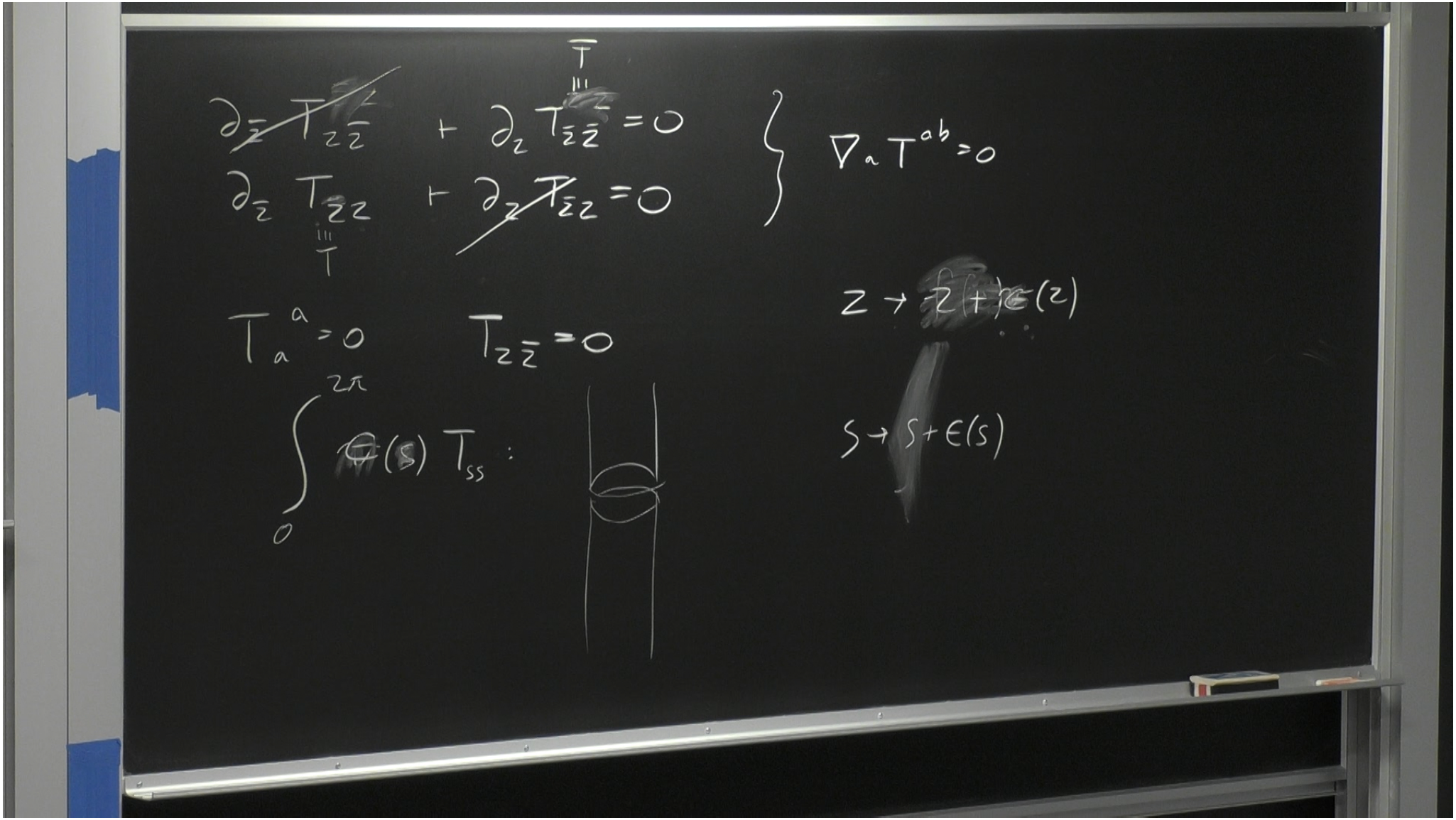


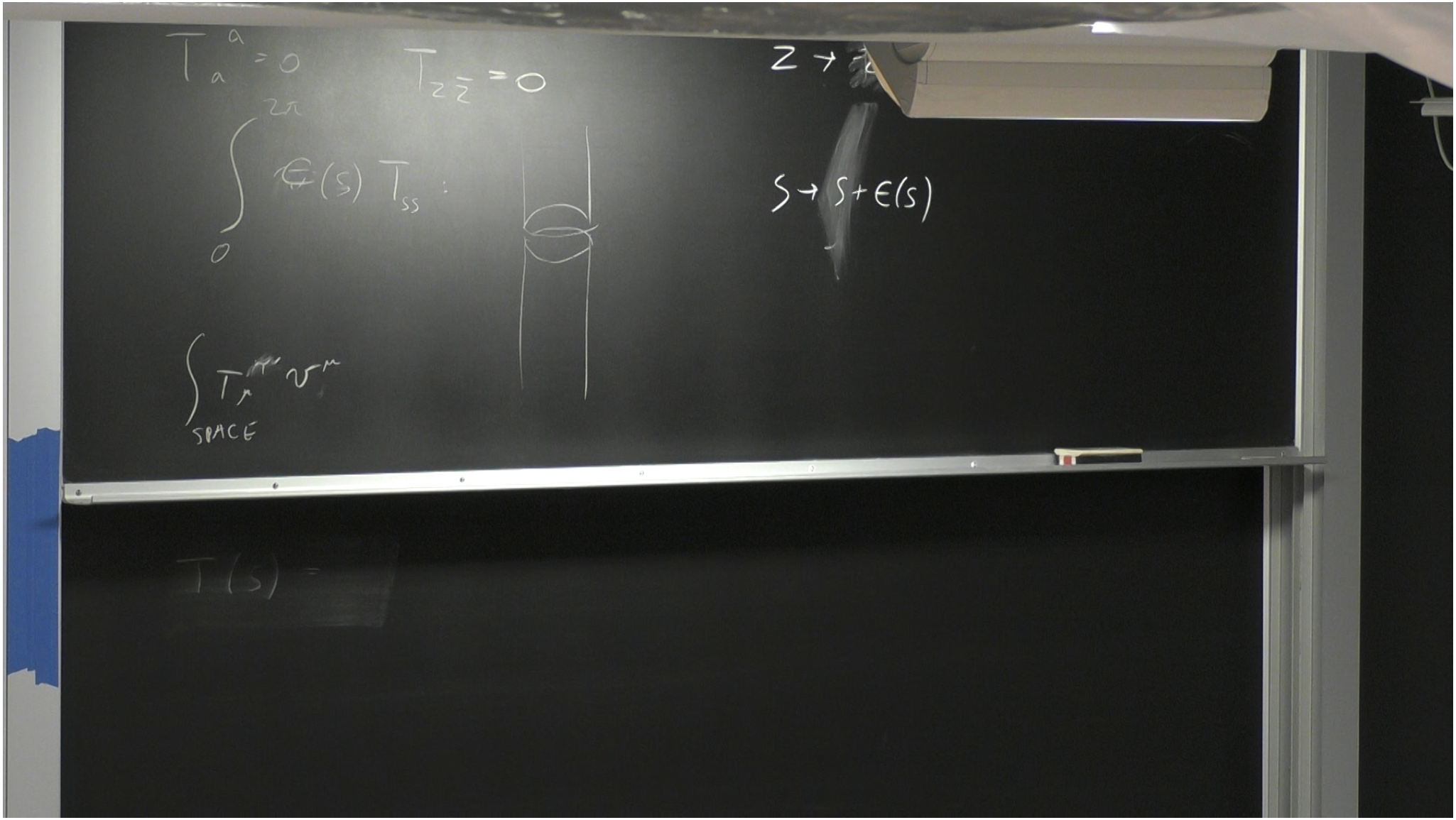
$\left(f(z) \partial_z X , 0 \right)$ $\left(0 , \bar{g}(\bar{z}) \partial_{\bar{z}} X \right)$ $X \to X + f(z) + g(\bar{z})$

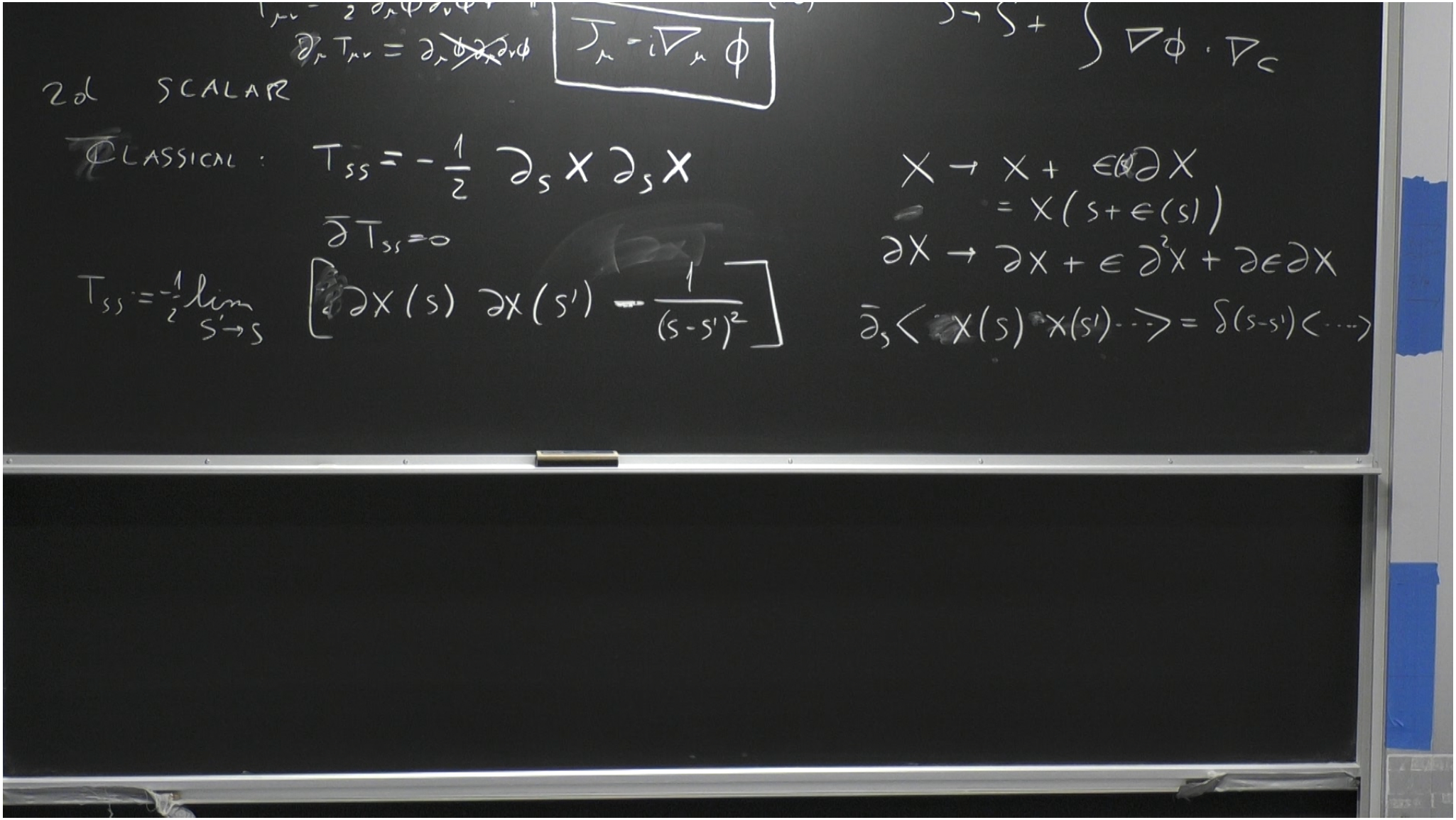
$a_m \propto \int_0^{2\pi L} e^{i \frac{m}{L} s} \, i \partial_s X \, d\sigma$ $\bar{a}_m \propto \int_0^{2\pi L} e^{i \frac{m}{L} \bar{s}} \, i \partial_{\bar{s}} X \, d\sigma$ $s = \tau - i\sigma$

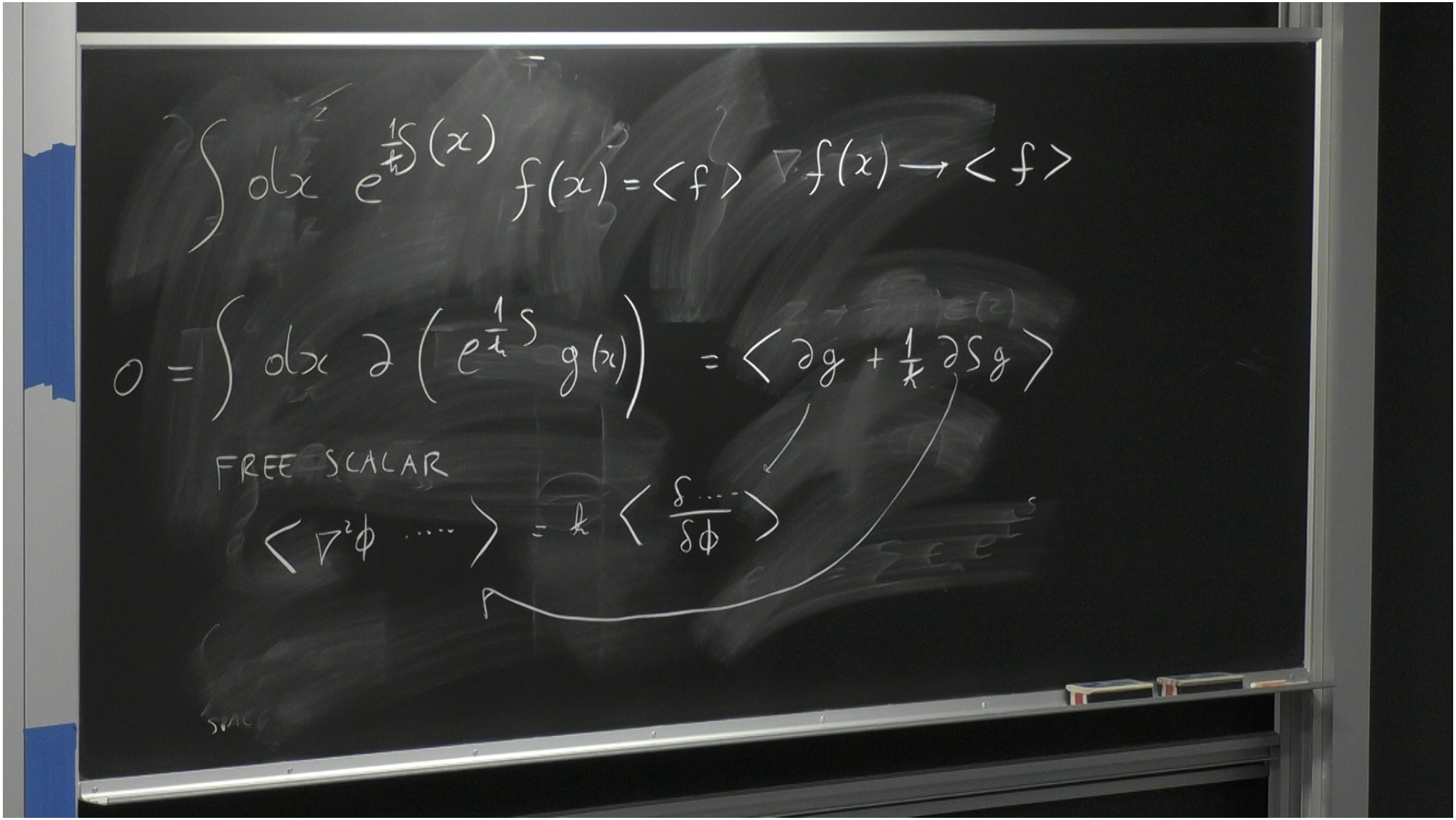


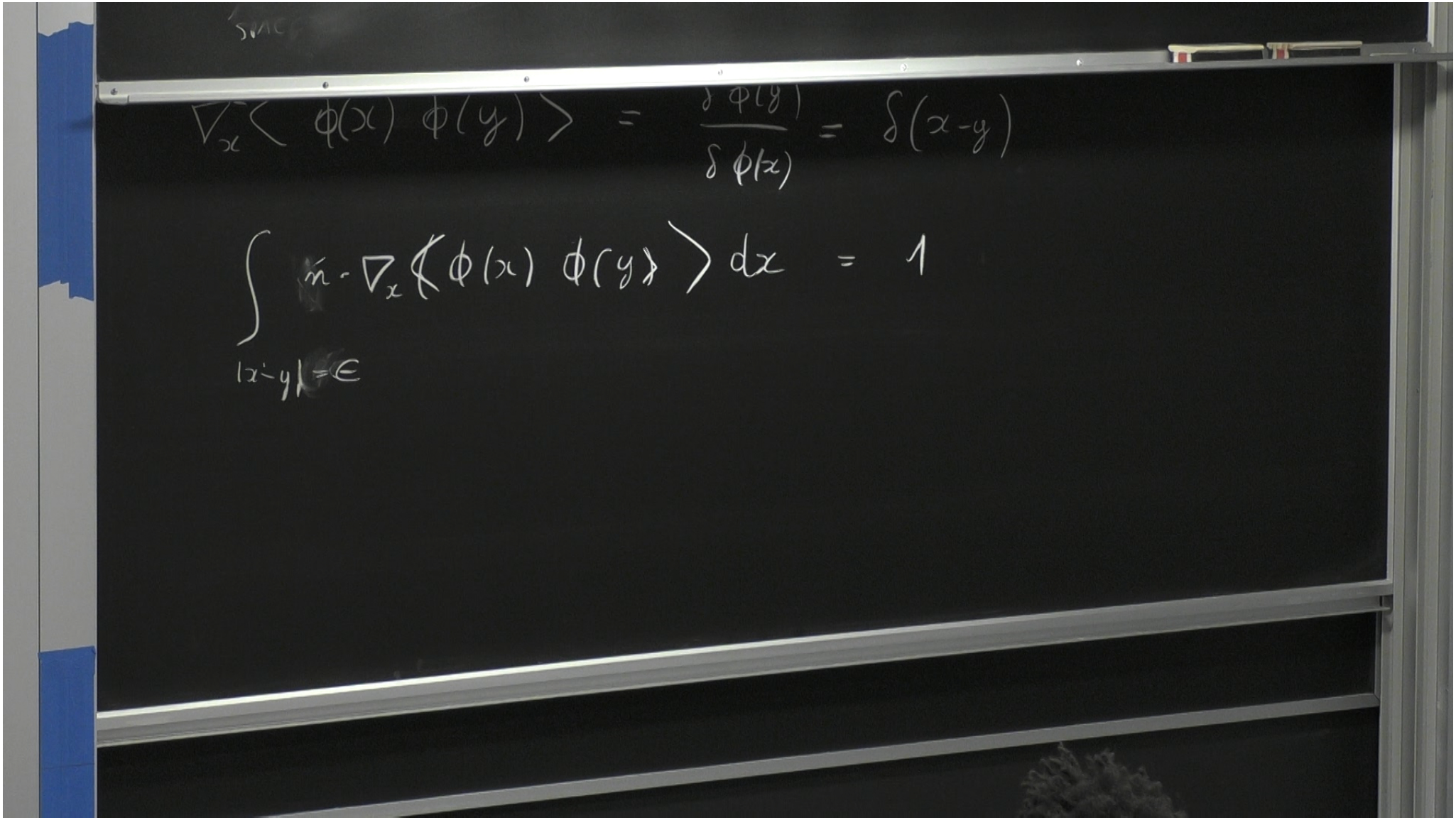

$J_1 = i\partial_{v_1} X$
$J_2 = i\partial_{v_2} X$
$\partial_1 J_1 + \partial_2 J_2 = 0$
$J_z = i\partial_z X$
$J_{\bar{z}} = i\partial_{\bar{z}} X$
$\partial_{\bar{z}} J_z + \partial_z J_{\bar{z}} = 0$
0
0
$\partial_z \partial_{\bar{z}} X = 0$
$(f(z) i\partial_z X, 0)$
$(0, \bar{f}(\bar{z}) \partial_{\bar{z}} X)$
$X \to X + f(z) + g(\bar{z})$
$\bar{\partial} \mathcal{O} = 0$
$(f(z) \mathcal{O}, 0) \to \mathcal{O}_m$

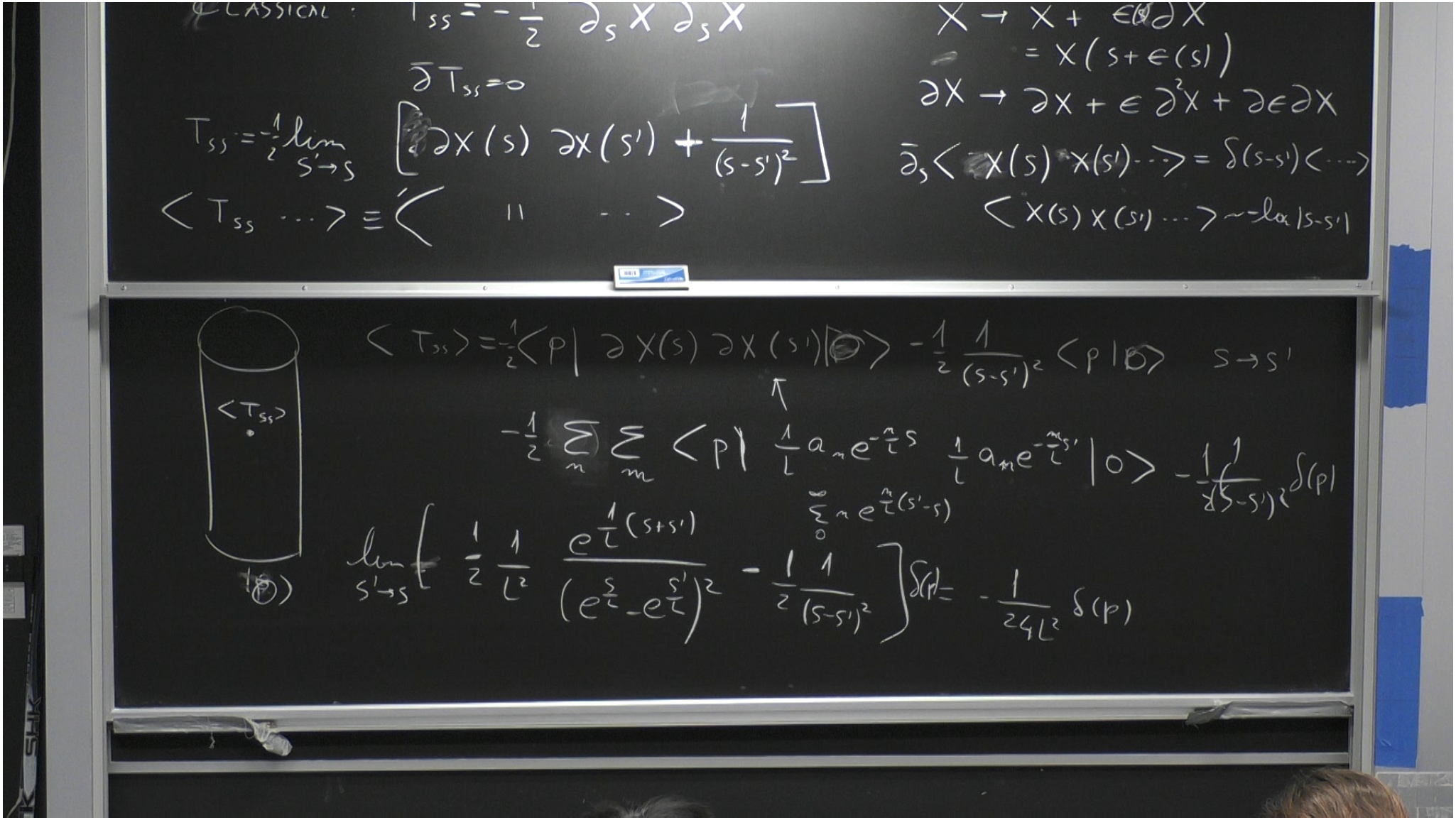

$\partial_{\bar{z}} T_{zz} + \partial_z T_{\bar{z}\bar{z}} = 0$
$\partial_z T_{\bar{z}z} + \partial_{\bar{z}} T_{zz} = 0$
$\nabla_a T^{ab} = 0$
$T_a^{\ a} = 0$
$T_{z\bar{z}} = 0$
$z \to z + \epsilon(z)$
$\int_0^{2\pi} \epsilon(s)\, T_{ss}$:
$s \to s + \epsilon(s)$

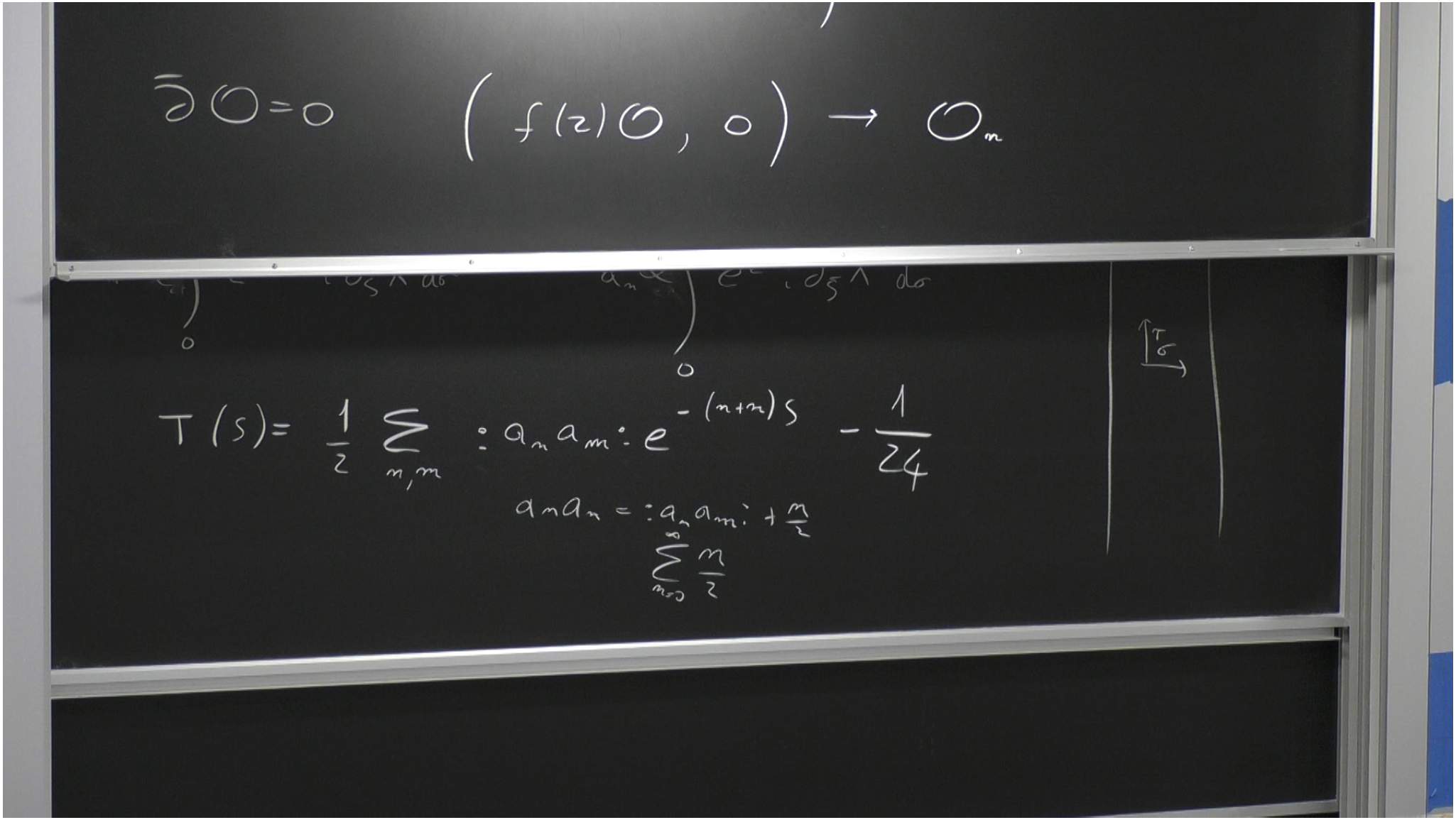

$T_a^a = 0$
$T_{z\bar{z}} = 0$
$z \to$
$\int_0^{2\pi} \epsilon(s)\, T_{ss}$:
$s \to s + \epsilon(s)$
$\int T^{\mu\nu} v^{\mu}$
SPACE
$T(s) =$

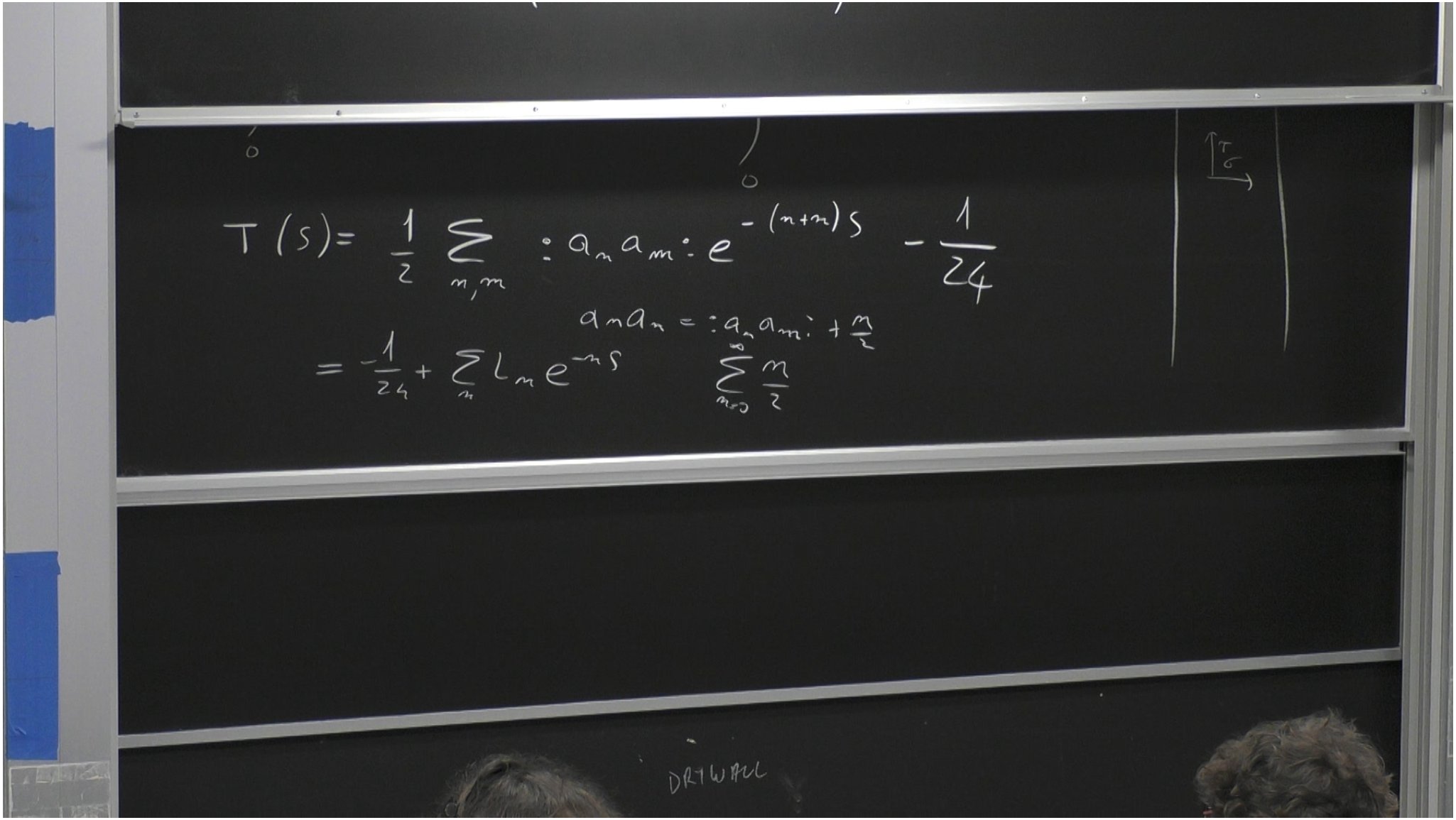

$\partial_\mu T_{\mu\nu} = \partial_\mu \partial_\nu \phi$
$J_\mu = i \nabla_\mu \phi$
$S \to S + \int \nabla\phi \cdot \nabla c$
2d SCALAR
CLASSICAL : $T_{ss} = -\frac{1}{2} \partial_s X \partial_s X$
$\bar{\partial} T_{ss} = 0$
$T_{ss} = -\frac{1}{2} \lim_{s' \to s} \left[\partial X(s) \partial X(s') - \frac{1}{(s-s')^2} \right]$
$X \to X + \epsilon \partial X$
$= X(s + \epsilon(s))$
$\partial X \to \partial X + \epsilon \partial^2 X + \partial\epsilon \partial X$
$\bar{\partial}_s \langle X(s) X(s') \cdots \rangle = \delta(s-s') \langle \cdots \rangle$

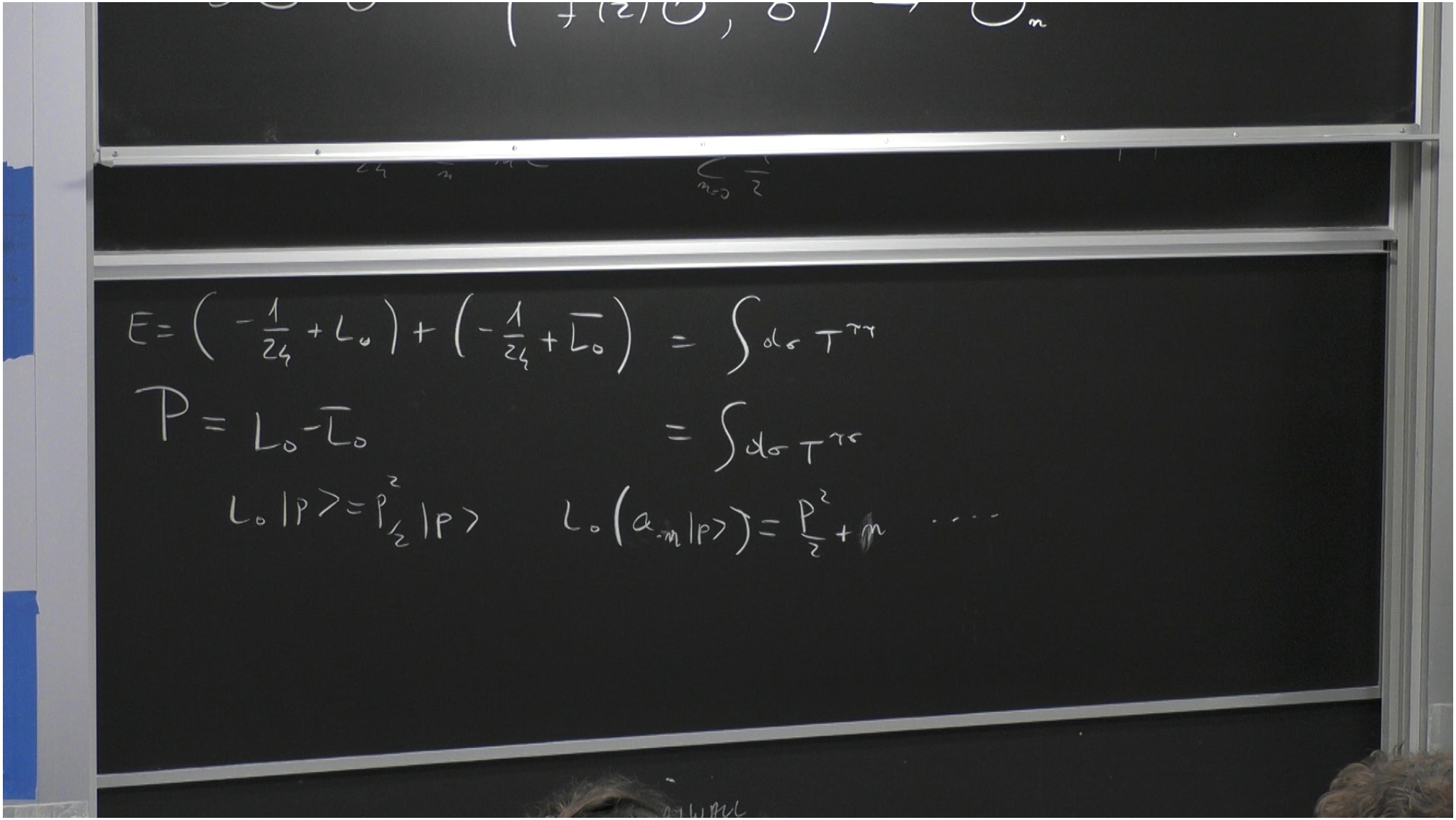

$\int dx\, e^{\frac{1}{\hbar}S(x)} f(x) = \langle f \rangle \quad f(x) \to \langle f \rangle$
$0 = \int dx\, \partial \left(e^{\frac{1}{\hbar}S} g(x) \right) = \langle \partial g + \frac{1}{\hbar} \partial S g \rangle$
FREE SCALAR
$\langle \nabla^2 \phi \ldots \rangle = \hbar \left\langle \frac{\delta \ldots}{\delta \phi} \right\rangle$

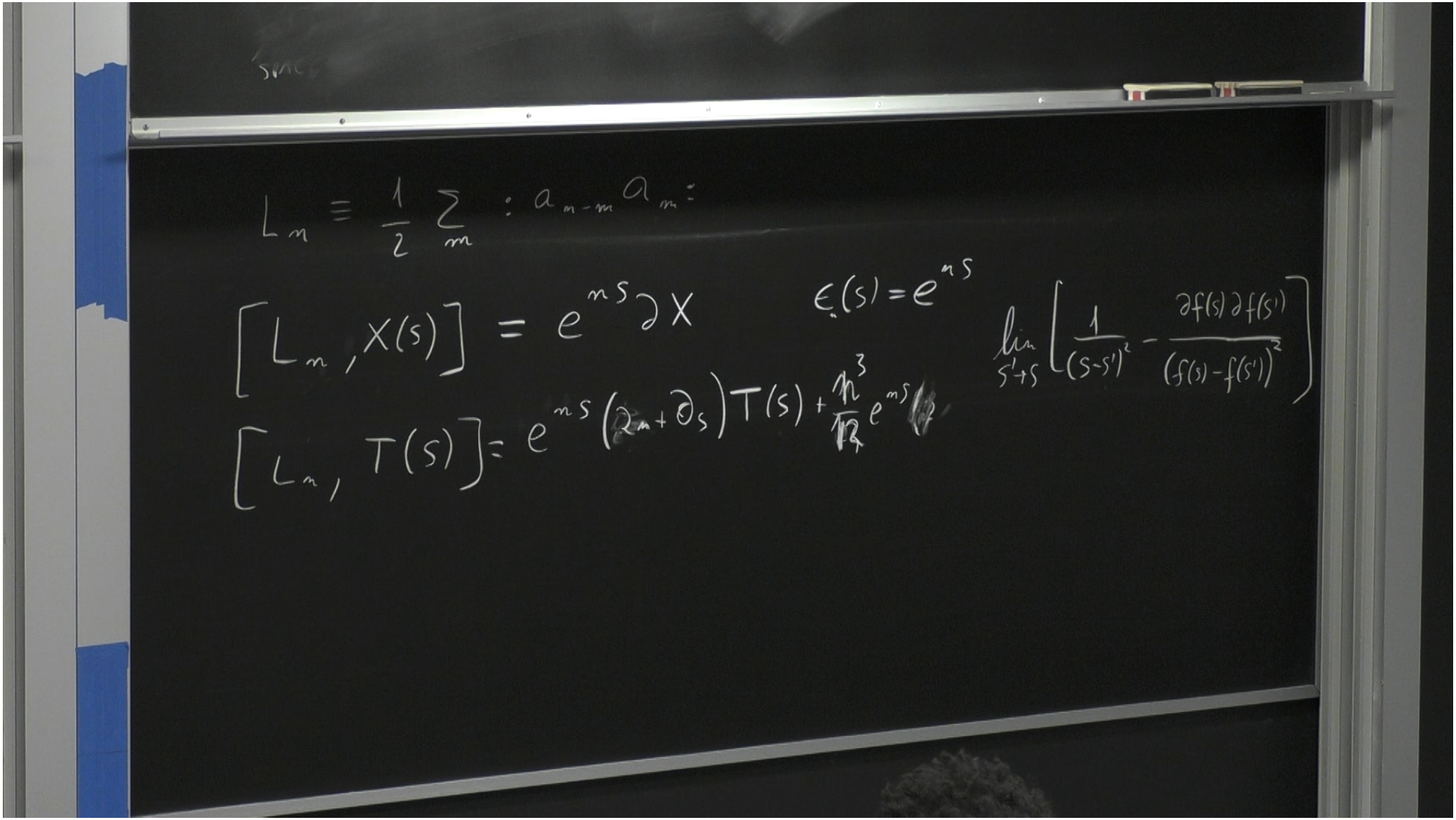

since
$\nabla_x \langle \phi(x)\, \phi(y) \rangle = \frac{\delta \phi(y)}{\delta \phi(x)} = \delta(x-y)$
$\int_{|x-y|=\epsilon} \hat{n} \cdot \nabla_x \langle \phi(x)\, \phi(y) \rangle \, dx = 1$

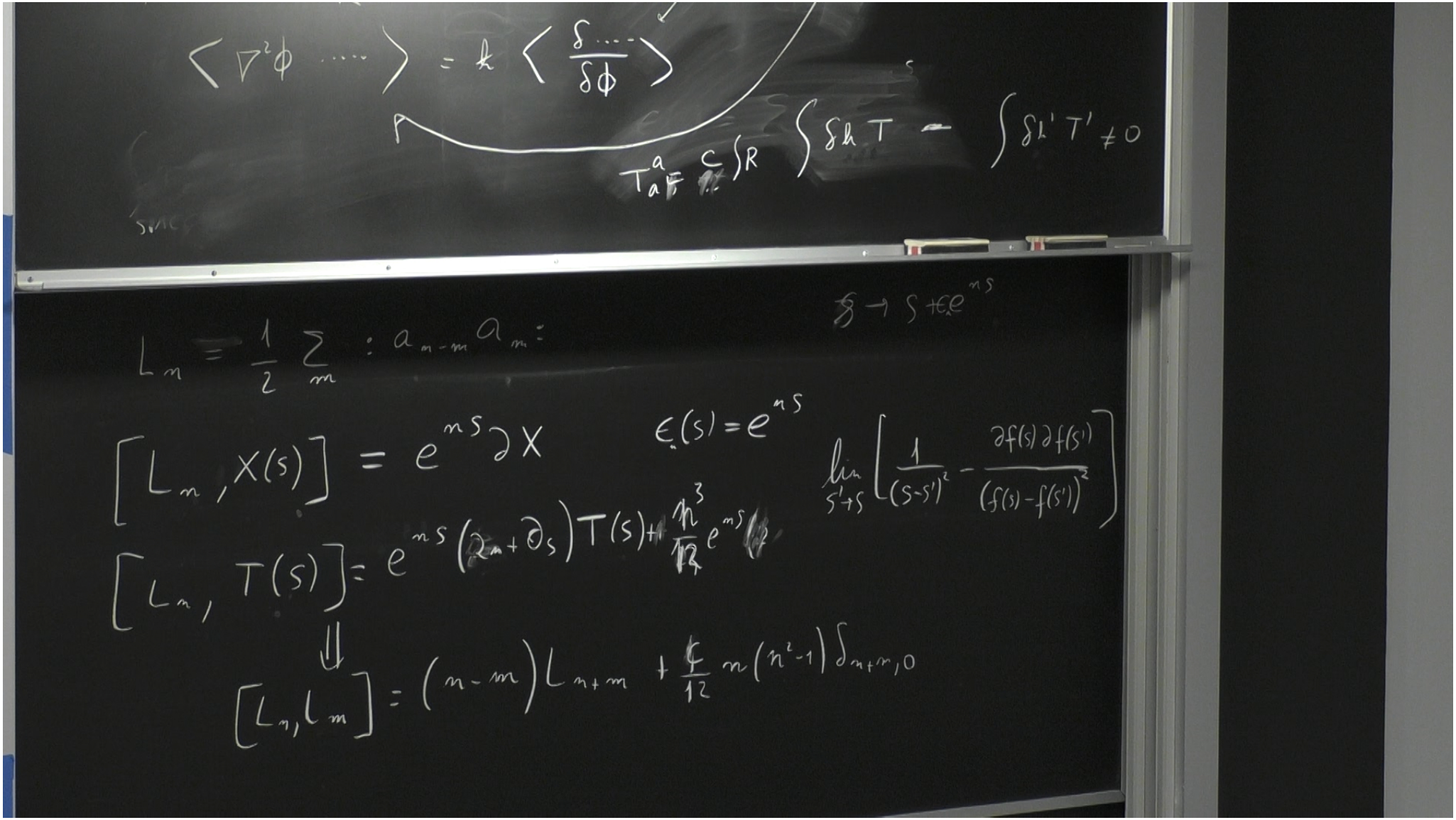

CLASSICAL : $T_{ss} = -\frac{1}{2}\partial_s X \partial_s X$
$\bar\partial T_{ss} = 0$
$T_{ss} = -\frac{1}{2}\lim_{s'\to s}\left[\partial X(s)\,\partial X(s') + \frac{1}{(s-s')^2}\right]$
$\langle T_{ss} \cdots \rangle \equiv$
$X \to X + \epsilon\partial X$
$= X(s+\epsilon(s))$
$\partial X \to \partial X + \epsilon\partial^2 X + \partial\epsilon\partial X$
$\bar\partial_s \langle X(s)\,X(s')\cdots\rangle = \delta(s-s')\langle\cdots\rangle$
$\langle X(s)\,X(s')\cdots\rangle \sim -\log|s-s'|$
$\langle T_{ss}\rangle$
$\langle T_{ss}\rangle = -\frac{1}{2}\langle p|\,\partial X(s)\,\partial X(s')|0\rangle - \frac{1}{2}\frac{1}{(s-s')^2}\langle p|0\rangle \quad s\to s'$
$-\frac{1}{2}\sum_n\sum_m \langle p|\,\frac{1}{L}a_n e^{-\frac{n}{L}s}\,\frac{1}{L}a_m e^{-\frac{m}{L}s'}|0\rangle - \frac{1}{2}\frac{1}{(s-s')^2}\delta(p)$
$\sum_0^\infty n\, e^{\frac{n}{L}(s'-s)}$
$\lim_{s'\to s}\left[\frac{1}{2}\frac{1}{L^2}\frac{e^{\frac{1}{L}(s+s')}}{\left(e^{\frac{s}{L}}-e^{\frac{s'}{L}}\right)^2} - \frac{1}{2}\frac{1}{(s-s')^2}\right]\delta(p) = -\frac{1}{24L^2}\delta(p)$


$\bar{\partial}\mathcal{O}=0$
$\left(f(z)\mathcal{O},\ 0\right) \to \mathcal{O}_m$
$T(s)= \frac{1}{2}\sum_{m,n} :a_n a_m: e^{-(n+m)s} - \frac{1}{24}$
$a_n a_m = :a_n a_m: + \frac{n}{2}$
$\sum_{n=0}^{\infty} \frac{n}{2}$


$T(s) = \frac{1}{2} \sum_{n,m} :a_n a_m: e^{-(n+m)s} - \frac{1}{24}$
$a_n a_m = :a_n a_m: + \frac{n}{2}$
$= -\frac{1}{24} + \sum_n L_n e^{-ns}$
$\sum_{n=0}^{\infty} \frac{n}{2}$
DRYWALL


$E = \left(-\frac{1}{24} + L_0\right) + \left(-\frac{1}{24} + \bar{L}_0\right) = \int d\sigma\, T^{\tau\tau}$
$P = L_0 - \bar{L}_0 = \int d\sigma\, T^{\tau\sigma}$
$L_0 |p\rangle = \frac{p^2}{2} |p\rangle$
$L_0 \left(a_{-m} |p\rangle\right) = \frac{p^2}{2} + m \ \ldots$


$L_n \equiv \frac{1}{2} \sum_m : a_{n-m} a_m :$
$[L_n, X(s)] = e^{ns} \partial X$
$\epsilon_n(s) = e^{ns}$
$\lim_{s' \to s} \left[\frac{1}{(s-s')^2} - \frac{\partial f(s) \partial f(s')}{(f(s) - f(s'))^2} \right]$
$[L_n, T(s)] = e^{ns} (2n + \partial_s) T(s) + \frac{n^3}{12} e^{ns}$


$\langle \nabla^2\phi \cdots \rangle = k \left\langle \frac{\delta \cdots}{\delta\phi} \right\rangle$
$T^a_{a\,F} = C \int_R \int \delta h\, T - \int \delta h' T' \neq 0$
$L_n = \frac{1}{2} \sum_m : a_{n-m} a_m :$
$\delta \to s + \epsilon e^{ns}$
$\epsilon(s) = e^{ns}$
$[L_n, X(s)] = e^{ns} \partial X$
$\lim_{s' \to s} \left[\frac{1}{(s-s')^2} - \frac{\partial f(s)\, \partial f(s')}{(f(s)-f(s'))^2} \right]$
$[L_n, T(s)] = e^{ns} (2n + \partial_s) T(s) + \frac{n^3}{12} e^{ns}$
$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12} n (n^2-1) \delta_{n+m,0}$