Title: Quantum Fields and Strings Lecture - 230322

Speakers: Davide Gaiotto

Collection: Quantum Fields and Strings (2022/2023)

Date: March 22, 2023 - 10:15 AM

URL: https://pirsa.org/23030024

$x^\mu)$

$$S(h_{ab} + \delta h_{ab}, x^\mu) = S(h_{ab}, x^\mu) + \int d\sigma^2 \sqrt{h}\, \delta h_{ab} T^{ab}$$

DIFF : $h_{ab} \to h_{ab} + \nabla_a v_b + \nabla_b v_a$

$$\int d\sigma^2 \sqrt{h}\, \nabla_a v_b T^{ab} = 0$$

$$\Downarrow$$

$$\boxed{\nabla_a T^{ab} = 0}$$

WEYL : $h_{ab} \to h_{ab} + \delta\phi\, h_{ab}$

$$\Downarrow$$

$$\boxed{T^{ab} h_{ab} = 0}$$

$$\boxed{\nabla_a T^{ab} = 0}$$

$$\ldots \rangle_{h_{ab} + \delta h_{ab}} = \langle \ldots \rangle_{h_{ab}} + \int d^2 u \, \delta h_{ab}(u) \langle T^{ab}(u) \ldots \rangle$$

$$\langle \cdots\cdots \rangle_{h_{ab}+\delta h_{ab}} = \langle \cdots \rangle_{h_{ab}} + \int d^2 u \sqrt{h}\, \delta\phi(u) \langle h_{ab} T^{ab}(u) \cdots \rangle$$

$$\sum c_i \mathcal{O}^i$$

$\delta\phi$ CONST h_{ab}

$$= \sum_i c_i \frac{\delta}{\delta g_i} \langle \quad \rangle$$

$$S = \cdots + \sum_i g_i \int \mathcal{O}^i$$

$$T^{ab} h_{ab} = \sum c_i \mathcal{O}^i$$

$c_i = \beta_i$ BETA FUNCTION

$S(h_{ab}, x^\mu)$

$$S(h_{ab} + \delta h_{ab}, x^\mu) = S(h_{ab}, x^\mu) + \int d\sigma^2 \sqrt{h}\, \delta h_{ab} T^{ab}$$

$$T_{ab} = \frac{\delta S}{\delta h_{ab}} = 0$$

DIFF : $h_{ab} \to h_{ab} + \nabla_a v_b + \nabla_b v_a$

$$\int d\sigma^2 \sqrt{h}\, \nabla_a v_b T^{ab} = 0$$

$$\Downarrow$$

$$\boxed{\nabla_a T^{ab} = 0}$$

$$\frac{\delta S}{\delta x^\mu} = 0$$

WEYL : $h_{ab} \to h_{ab} + \delta\phi\, h_{ab}$

$$\Downarrow$$

$$\boxed{T^{ab} h_{ab} = 0}$$

$$h_{ab} + \delta\phi\, h_{ab} \qquad h_{ab} + \int d^2u \sqrt{h}\, \delta\phi_3(u) \langle h_{ab} T^{ab}(u) \dots \rangle$$

$$T^{ab} h_{ab} = \sum c_i \mathcal{O}^i$$

$c_i = \beta_i$ BETA FUNCTION

$$T^{ab} h_{ab} = c\,\mathbb{1}$$

$$(D-26)$$

$\delta\phi$ const $\parallel$

h_{ab} "

$$\sum_i c_i \frac{\delta}{\delta g_i} \langle \quad \rangle$$

$$S = \dots + \sum_i g_i \int \mathcal{O}^i$$

$$T_1 \times T_2$$

$$T_{ab} = T^{(1)}_{ab} + T^{(2)}_{ab}$$

$$T^{ab} l_{ab} = \left(T^{ab} l_{ab}\right)^{(1)} + \left(T^{ab} l_{ab}\right)^{(2)}$$

DRYWALL

GAUGE-FIX

$h_{ab} \to \delta_{ab}$

$$S = \sum_\mu \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X_\mu}{\partial \sigma_a}$$

$$\Downarrow$$

$$\boxed{T^{ab} h_{ab} = 0}$$

$$\boxed{\quad = 0}$$

$||$

β_i BETA FUNCTION

$$T^{ab} h_{ab} = c \mathbb{1}$$

$$c \propto (\underbrace{D}_{D \times \text{SCALAR}} - \underbrace{26}_{\text{GHOSTS}})$$

$$\sum_i c_i \frac{\delta}{\delta g_i} \langle \quad \rangle$$

$$S = \cdots + \sum_i g_i \int \mathcal{O}^i$$

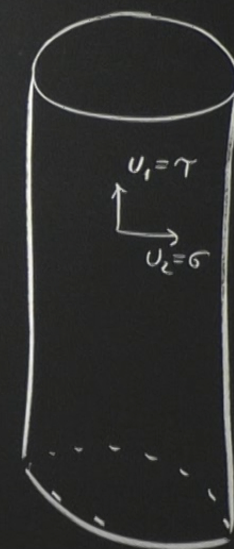
$$S[X] = \frac{1}{2} \int \frac{\partial X}{\partial U_a} \frac{\partial X}{\partial U^a} \, d^2U$$

EOM

$$\partial \bar{\partial} X = 0$$

$$X = f(\sigma + i\tau) + \bar{f}(\sigma - i\tau)$$

$$\sigma \to \sigma + 2\pi L$$



HILBERT SPACE ?

$$X(\sigma, \tau) = x_0(\tau) + \sum_{n \neq 0} e^{i \frac{n}{L} \sigma} x_n(\tau)$$

$$\mathcal{H} = L^2(\mathbb{R}) \otimes \text{FOCK SPACE}$$

$$|p\rangle, \; a_{-1}|p\rangle, \; \bar{a}_{-1}|p\rangle, \; a_{-2}|p\rangle, \; a_{-1}^2|p\rangle, \; a_{-1}\bar{a}_{-1}|p\rangle \; \ldots$$

$\ldots, a_n$ DESTRUCTION

$$\langle p | p' \rangle = \delta(p-p')$$

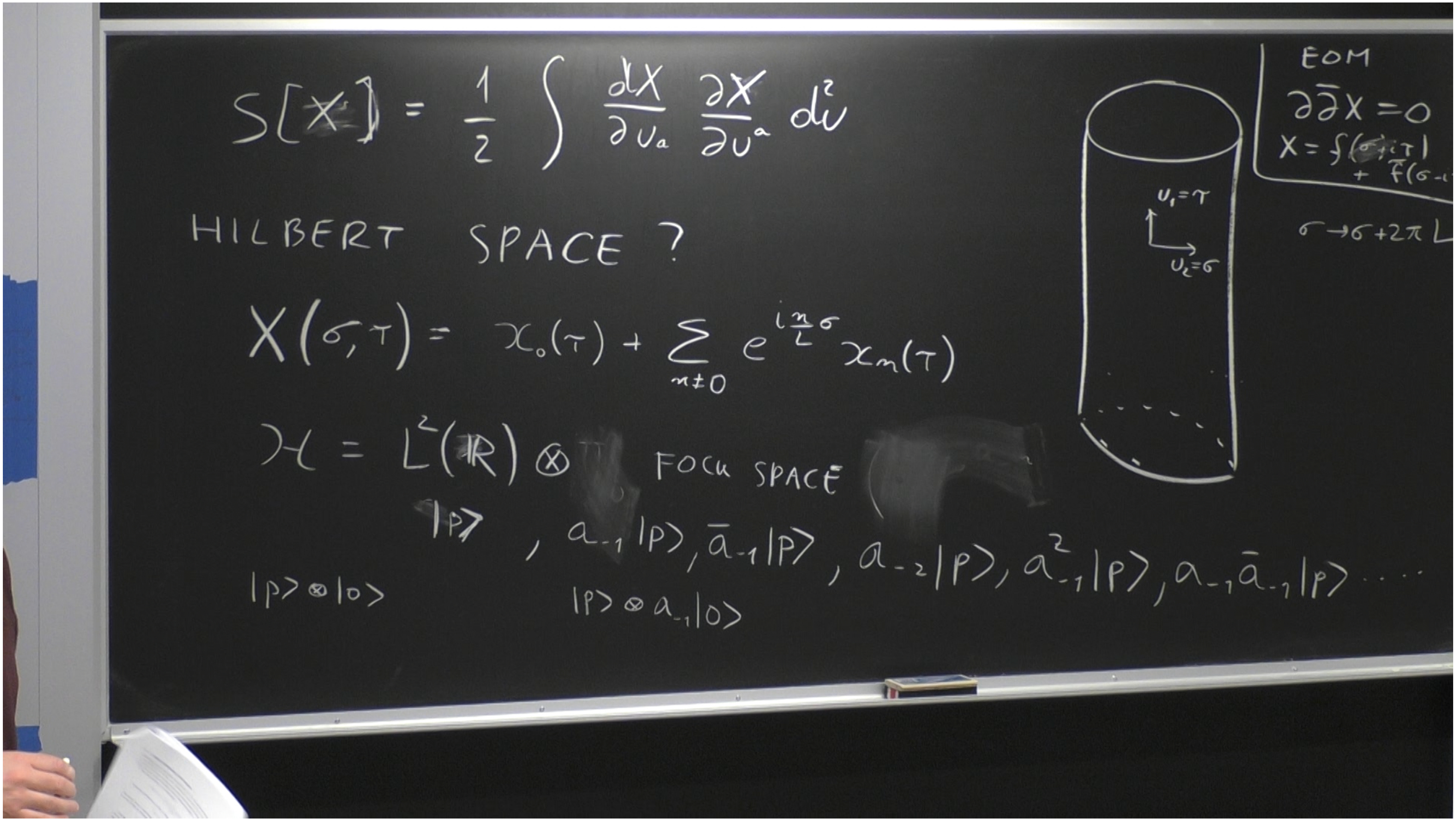
$$\langle p | a_2 \, a_{-1}^2 | p' \rangle = 0$$

$$\big(\langle p | a_1\big)\big(a_{-1} | p' \rangle\big) = \langle p | p' \rangle + \cancel{\langle p | a_{-1} a_1 | p' \rangle}$$

$$a_1 a_{-1} = 1 + a_{-1} a_1$$

$$\langle p | a_1^2 \, a_{-1}^2 | p' \rangle = 2 \langle p | p' \rangle$$

$$\langle p | a_1^n \, a_{-1}^n | p' \rangle = n! \langle p | p' \rangle$$


$S[X] = \frac{1}{2}\int \frac{\partial X}{\partial v_a}\frac{\partial X}{\partial v^a} d^2v$
HILBERT SPACE ?
$X(\sigma,\tau) = x_0(\tau) + \sum_{n\neq 0} e^{i\frac{n}{L}\sigma} x_n(\tau)$
$\mathcal{H} = L^2(\mathbb{R}) \otimes$ FOCK SPACE
$|p\rangle$, $a_{-1}|p\rangle$, $\bar{a}_{-1}|p\rangle$, $a_{-2}|p\rangle$, $a_{-1}^2|p\rangle$, $a_{-1}\bar{a}_{-1}|p\rangle$ . . .
$|p\rangle \otimes |0\rangle$
$|p\rangle \otimes a_{-1}|0\rangle$
$v_1 = \tau$
$v_2 = \sigma$
EOM
$\partial\bar{\partial}X = 0$
$X = f(\sigma + i\tau)$
$+ \bar{f}(\sigma - i$
$\sigma \to \sigma + 2\pi L$

$$\langle p | a_1^n a_{-1}^n | p \rangle = n! \langle p | p' \rangle$$

$$\langle p | \prod_i a_i^{k_i} \prod_i a_{-i}^{k_i} | p' \rangle = \prod_i k_i! \, i^{k_i} \langle p | p' \rangle$$

a_{-m} CREATES $E = m$, $P = m$

$\bar{a}_{-m}$ '' $E = m$, $P = -m$

$$P = \int_0^{2\pi L} T_{\sigma\tau} \, d\sigma$$

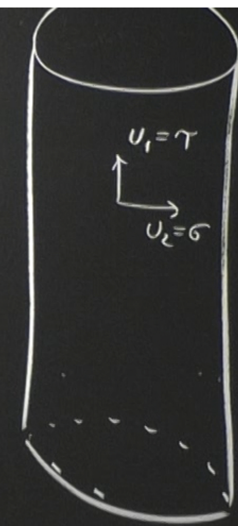
$$E = \int_0^{2\pi L} T^{\tau\tau} \, d\sigma$$

$$S[X] = \frac{\;}{2}\int \frac{\partial X}{\partial U_a}\frac{\partial X}{\partial U^a}\, dU$$

$$\partial\bar\partial X = 0$$

$$X = f(\sigma + i\tau) + \bar f(\sigma - i\tau)$$

$$\sigma \to \sigma + 2\pi L$$



HILBERT SPACE ?

$$X(\sigma,\tau) = x_0(\tau) + \sum_{n\neq 0} e^{i\frac{n}{L}\sigma} x_n(\tau)$$

$$\mathcal{H} = L^2(\mathbb{R}) \otimes \text{FOCK SPACE}$$

$$|p\rangle,\ a_{-1}|p\rangle,\ \bar a_{-1}|p\rangle,\ a_{-2}|p\rangle,\ a_{-1}^2|p\rangle,\ a_{-1}\bar a_{-1}|p\rangle \ldots$$

$|p\rangle \otimes |0\rangle$

$|p\rangle \otimes a_{-1}|0\rangle$

$|p\rangle :\ E = p^2/2$

$P = 0$

$$S[X] = \frac{1}{2}\int \frac{\partial X}{\partial v_a}\frac{\partial X}{\partial v^a}\, d^2v$$

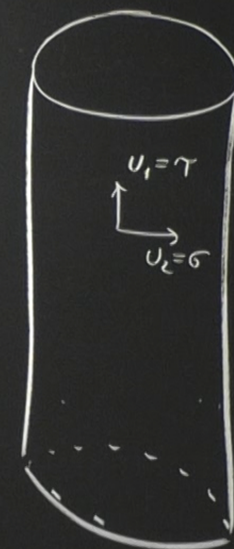
EOM

$$\partial\bar\partial X = 0$$

$$X = f(\sigma + i\tau) + \bar{f}(\sigma - i\tau)$$

$$\sigma \to \sigma + 2\pi L$$

HILBERT SPACE ?



$$X(\sigma,\tau) = x_0(\tau) + \sum_{n\neq 0} e^{i\frac{n}{L}\sigma} x_n(\tau)$$

$\mathcal{H} = L^2(\mathbb{R}) \otimes$ FOCK SPACE

$|p\rangle$, $a_{-1}|p\rangle$, $\bar{a}_{-1}|p\rangle$, $a_{-2}|p\rangle$, $a_{-1}^2|p\rangle$, $a_{-1}\bar{a}_{-1}|p\rangle$ ....

$|p\rangle \otimes |0\rangle$

$|p\rangle \otimes a_{-1}|0\rangle$

$|p\rangle$: $E = p^2/2$

$P = 0$

$$\frac{\delta S}{\delta x^\mu} = 0$$

GAUGE-FIX

$h_{ab} \to \delta_{ab}$

$$\boxed{S = \sum_\mu \frac{\partial x^\mu}{\partial \upsilon^a} \frac{\partial x_\mu}{\partial \upsilon_a}}$$

WEYL: $h_{ab} \longrightarrow h_{ab} + \delta\phi\, h_{ab}$

$\Downarrow$

$$\boxed{T^{ab} h_{ab} = 0}$$

$$\int d\upsilon^2 \sqrt{h}\, \nabla_a \upsilon_b T^{ab} = 0$$

$\Downarrow$

$$\boxed{\nabla_a T^{ab} = 0}$$

$$-i \frac{\partial}{\partial t} = H \qquad e^{itH}$$

$$\frac{\partial}{\partial \tau} = -H \qquad e^{-\tau H}$$

$$T^{ab} h_{ab} = \sum c_i \mathcal{O}^i$$

$\parallel$

β_i BETA FUNCTION

$$T^{ab} h_{ab} = c \mathbb{1}$$

$$(D-26)$$

$\uparrow$ D x SCALAR $\uparrow$ GHOSTS

$\delta\phi$ const $\parallel$

h_{ab} "

$$\sum_i c_i \frac{\delta}{\delta g_i}$$

$$S = \cdots + \sum_i g_i \int \mathcal{O}^i$$