Title: Quantum Fields and Strings Lecture - 230320

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Collection: Quantum Fields and Strings (2022/2023)

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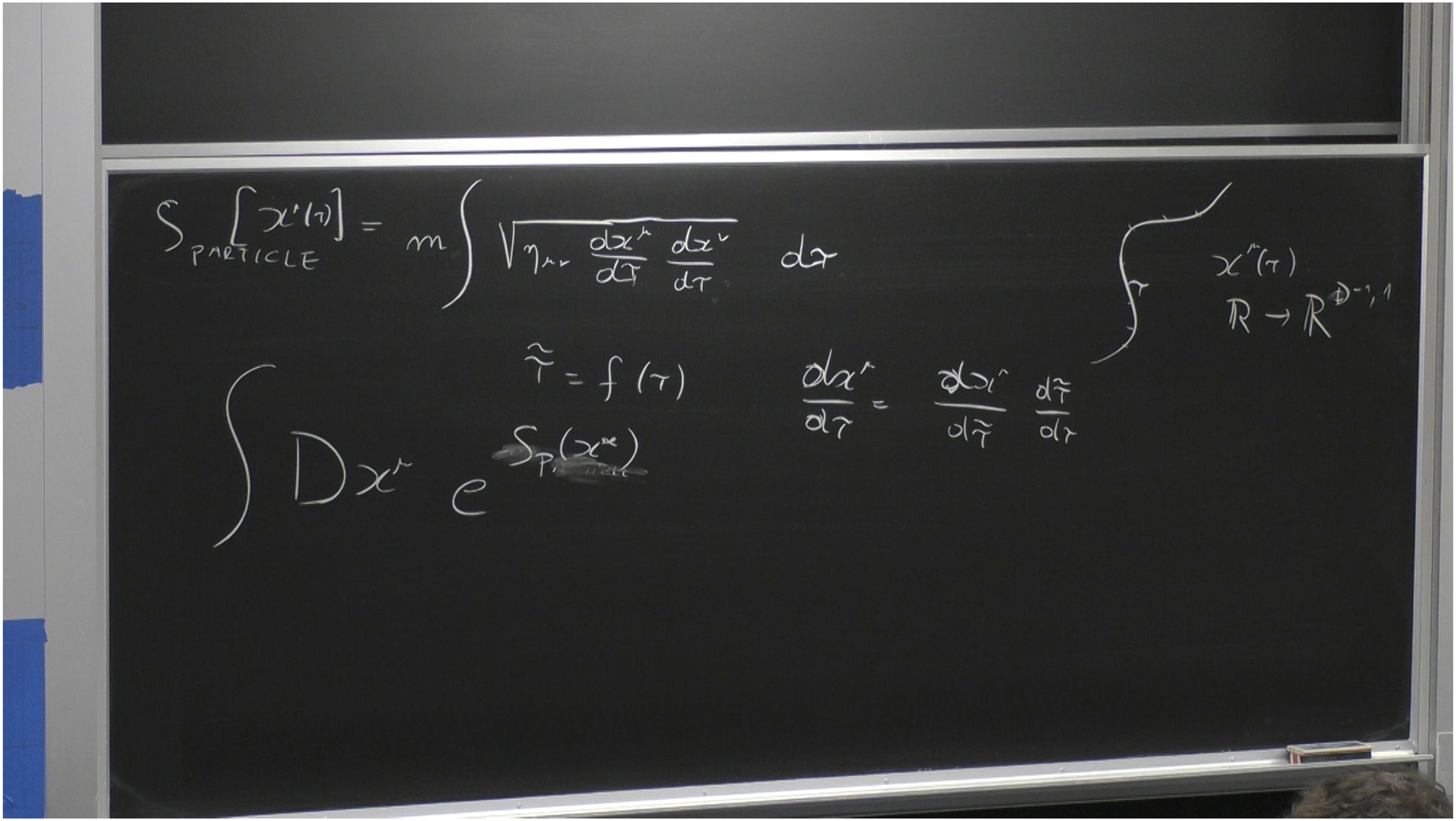

S_PARTICLE [x^μ(τ)] = m ∫ √(η_μν dx^μ/dτ dx^ν/dτ) dτ
x^μ(τ)
R → R^{D-1,1}
τ̃ = f(τ)
dx^μ/dτ = dx^μ/dτ̃ dτ̃/dτ
∫ Dx^μ e^{S_P(x^μ)}

RELATIVISTIC STRING

$$x^\mu = eU^\tau + x_0$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\frac{1}{\sqrt{\cdots}}\frac{dx^\mu}{d\tau}\right) = 0 \qquad \frac{1}{\sqrt{\cdot}}\,\frac{dx^\mu}{d\tau} = eU^\mu \qquad \eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = e^2$$

$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} + m^2 e\right] d\tau$$

$$\sqrt{h}\, h^{\tau\tau} \qquad \sqrt{h}$$

$e^2$ = METRIC

$$e^2 = h_{\tau\tau}$$

$$e^2 = \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau}$$

$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[ e^{-1} \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} + m^2 e \right] d\tau$$

$e^2$ = METRIC

$$e^2 = h_{\tau\tau}$$

$$e(\tau)\,d\tau = \tilde{e}(\tilde{\tau})\,d\tilde{\tau}$$

$$e^2 = \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau}$$

GAUGE-FIXING : $e(\tau) = k$

$$x^\mu = e v^\mu \tau + x_0$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\frac{1}{\sqrt{\cdots}}\frac{dx^\mu}{d\tau}\right) = 0 \qquad \frac{1}{\sqrt{\cdot}}\frac{dx^\mu}{d\tau} = e v^\mu \qquad \eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = e^2$$

$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} + m^2 e\right] d\tau$$

$e^2$ = METRIC

$$e^2 = h_{\tau\tau}$$

$$e^2 = \frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$$

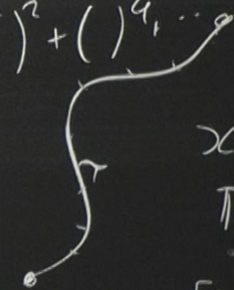
$$e(\tau)\,d\tau = \tilde{e}(\tilde{\tau})\,d\tilde{\tau}$$

$$x^\mu(\tau) = \tilde{x}^\mu(\tilde{\tau})$$

GAUGE-FIXING: $e(\tau) = k$

$$S_{\text{PARTICLE}}[x^\mu(\tau)] = m \int \sqrt{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}\, d\tau$$

$$\sqrt{1 + \left(\frac{dx^i}{dx^0}\right)^2} = (\ )^2 + (\ )^4 + \dots$$



$x^\mu(\tau)$

$\mathbb{R} \to \mathbb{R}^{D-1,1}$

$[0,1] \to \mathbb{R}^{D-1,1}$

DIFF: $\tilde{\tau} = f(\tau)$

$$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\tilde{\tau}} \frac{d\tilde{\tau}}{d\tau}$$

$$\int Dx^\mu \, e^{S_P(x^\mu)}$$

$$x^\mu = e v^\mu \tau + x_0$$

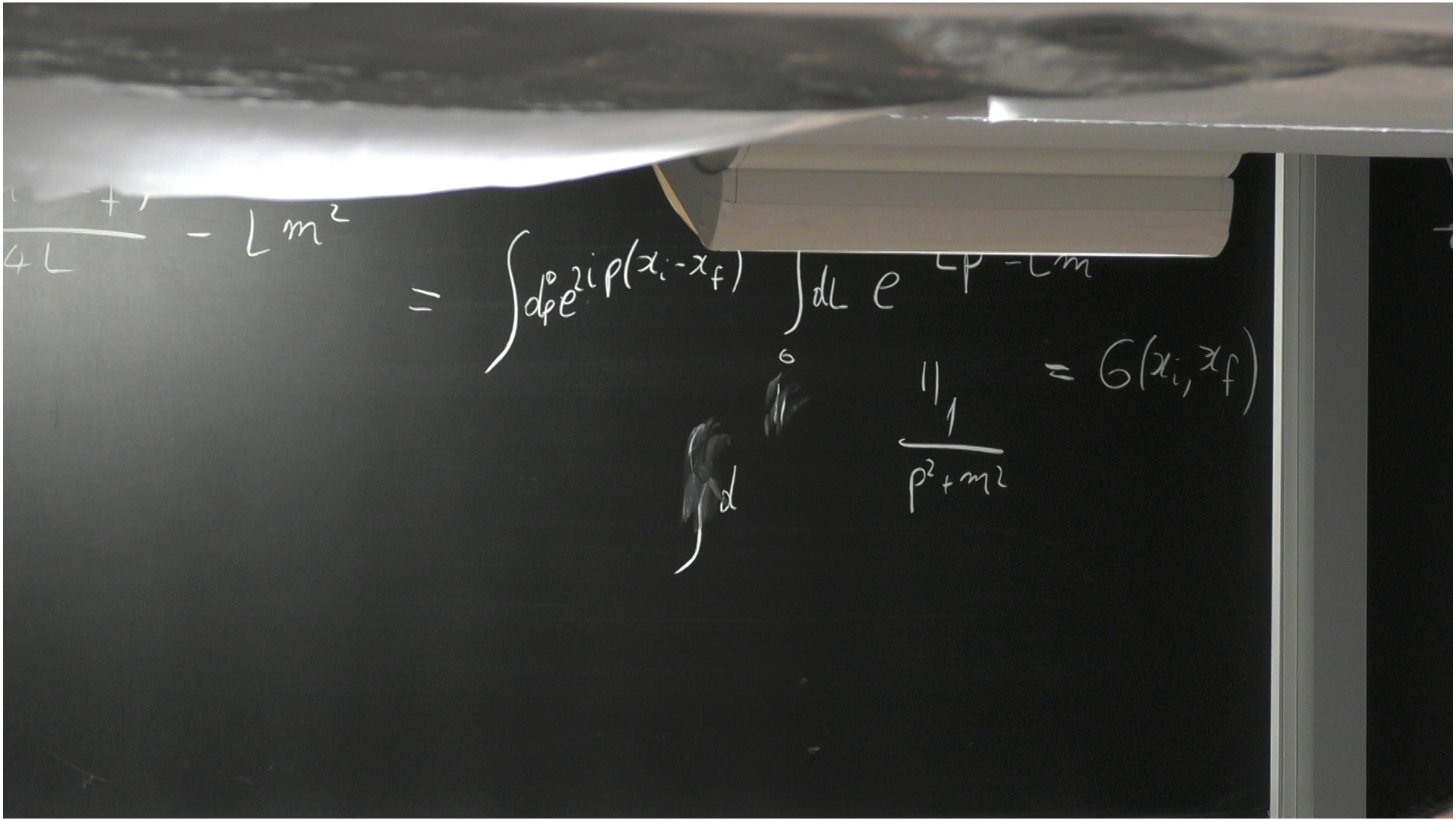
$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = e^2$$

$$\frac{d}{d\tau}\left(\frac{1}{e} \frac{1}{\sqrt{\dots}} \frac{dx^\mu}{d\tau}\right) = 0 \qquad \frac{1}{\sqrt{\cdot}} \frac{dx^\mu}{d\tau} = e v^\mu$$

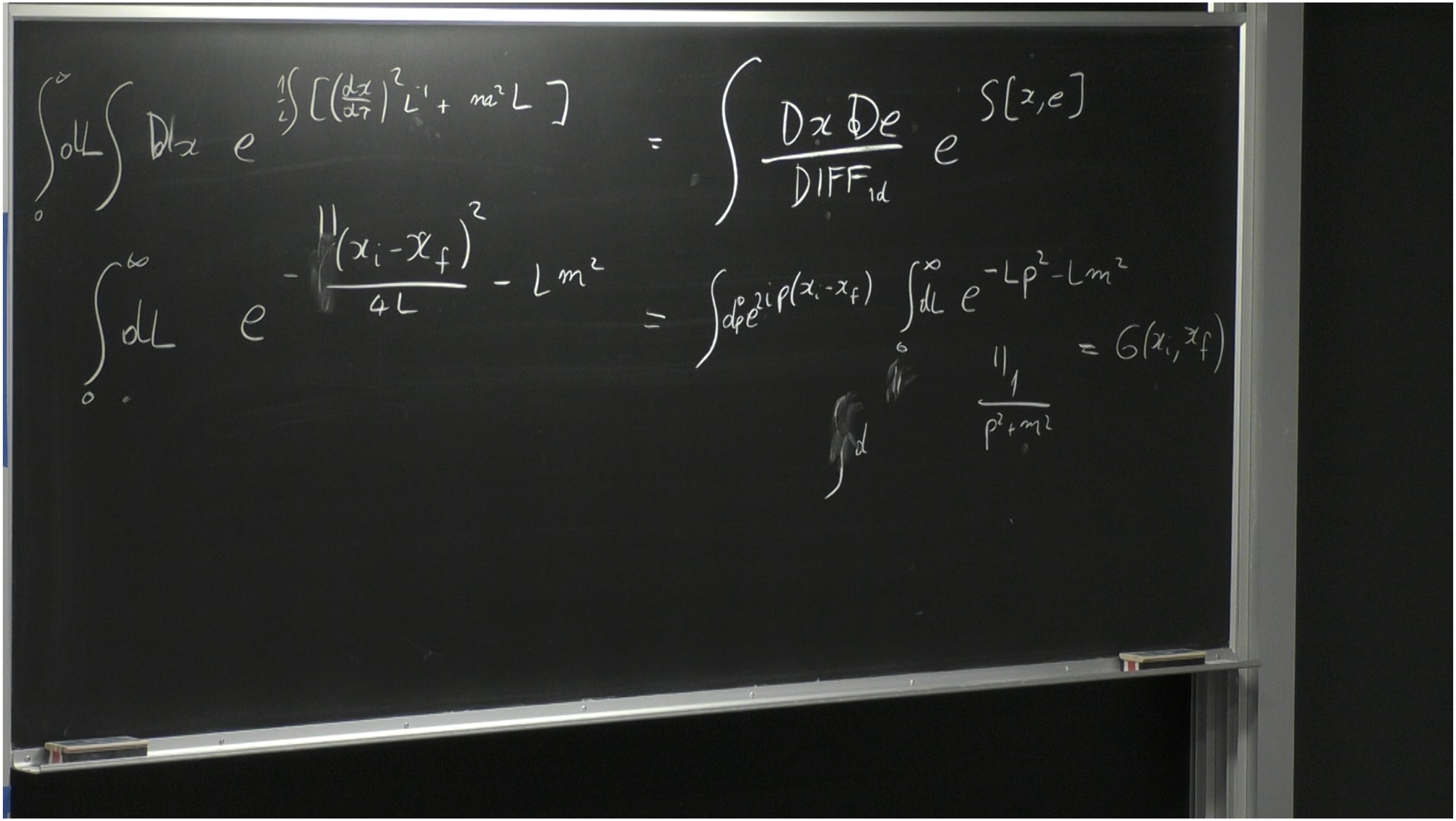
$$\int_0^\infty dL \int Dx \; e^{\frac{1}{i}\int \left[\left(\frac{dx}{d\tau}\right)^2 L^{-1} + m^2 L\right]} = \int \frac{Dx\, De}{DIFF_{1d}}$$

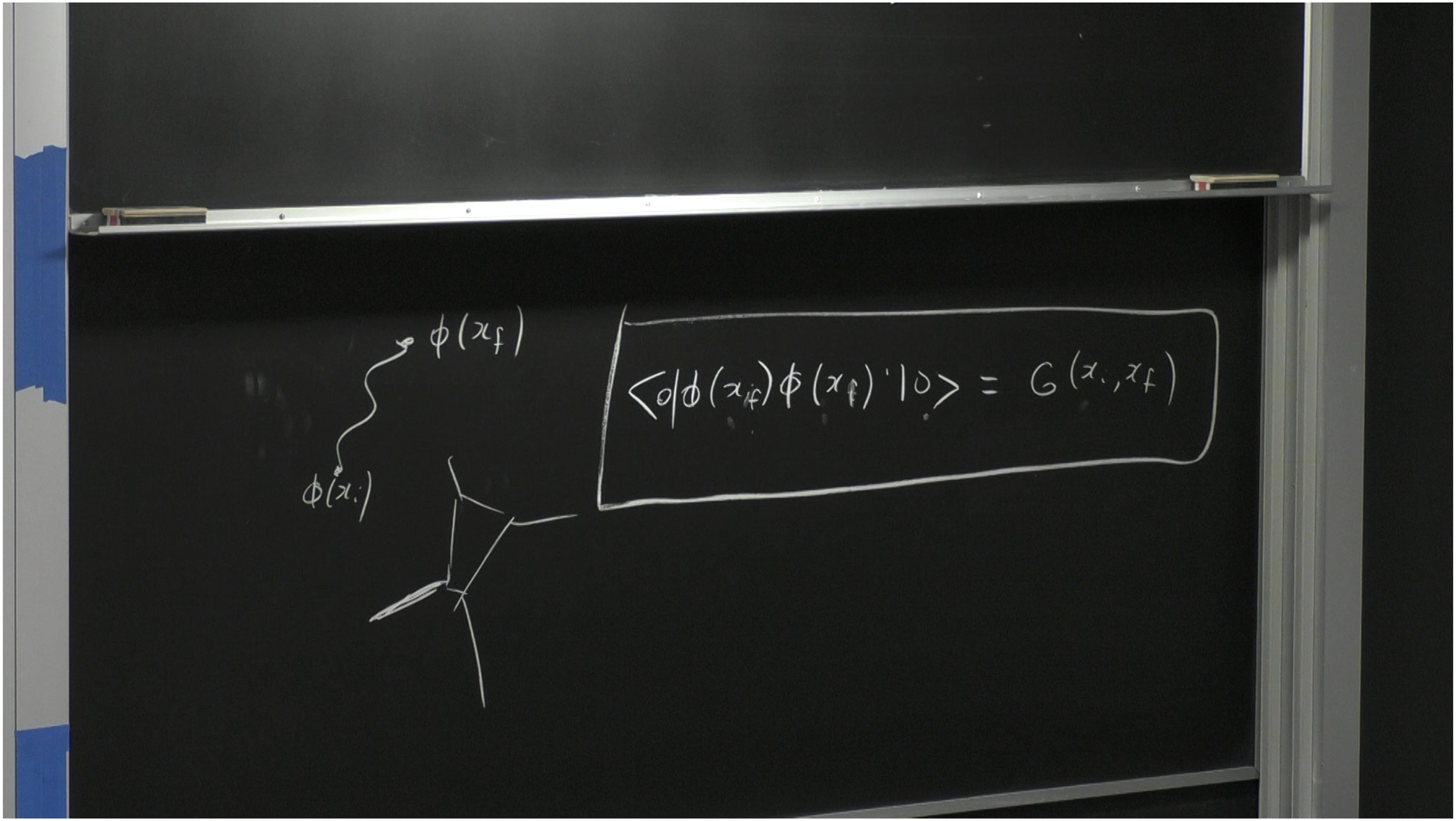
$$\int_0^\infty dL \; e^{-\frac{(x_i - x_f)^2}{4L} - Lm^2} = \int dp\, e^{2ip(x_i - x_f)} \int_0^\infty dL \; e^{-Lp^2 - Lm^2}$$

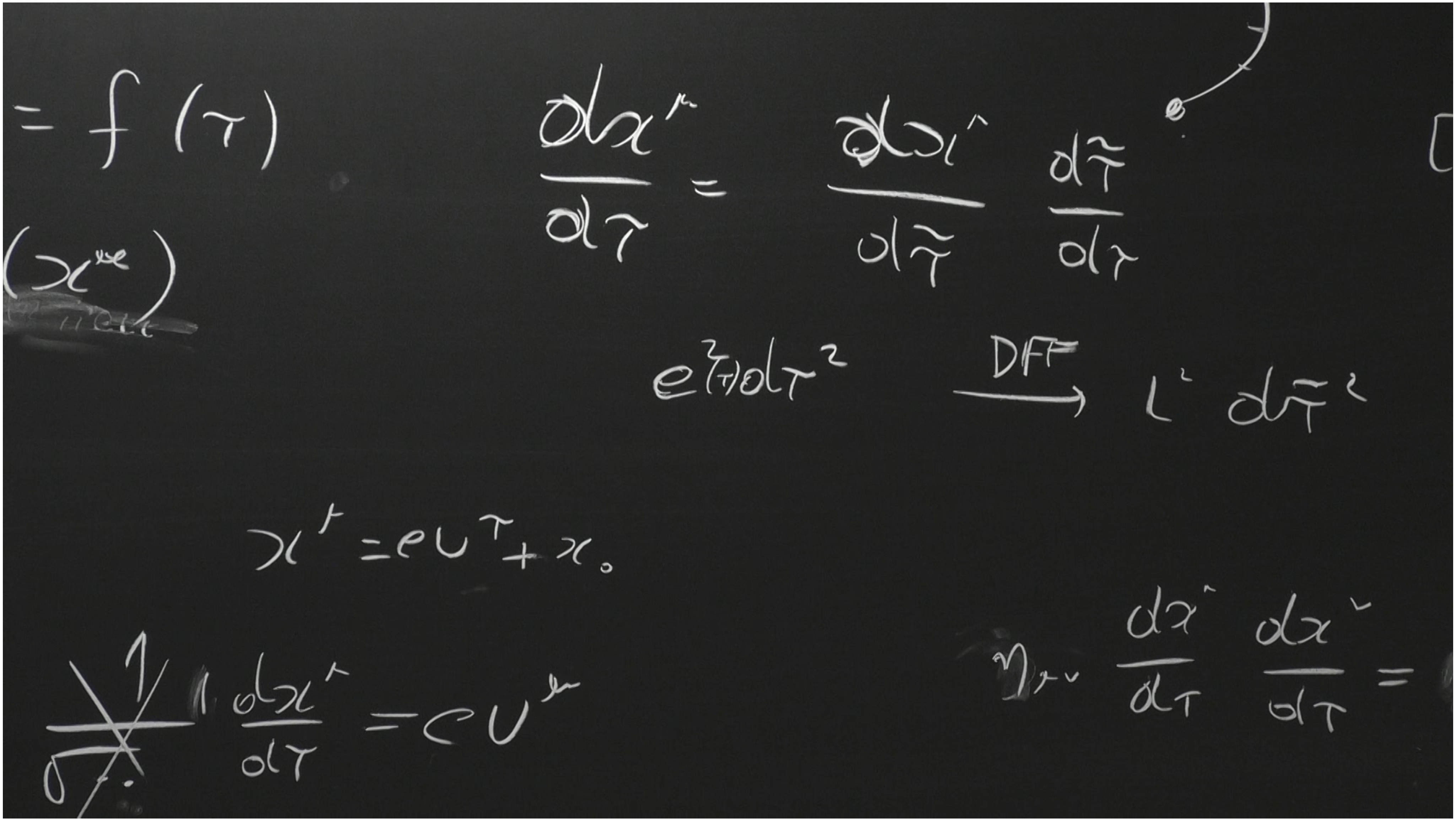
$$\int d \qquad \frac{1}{p^2 + m^2}$$

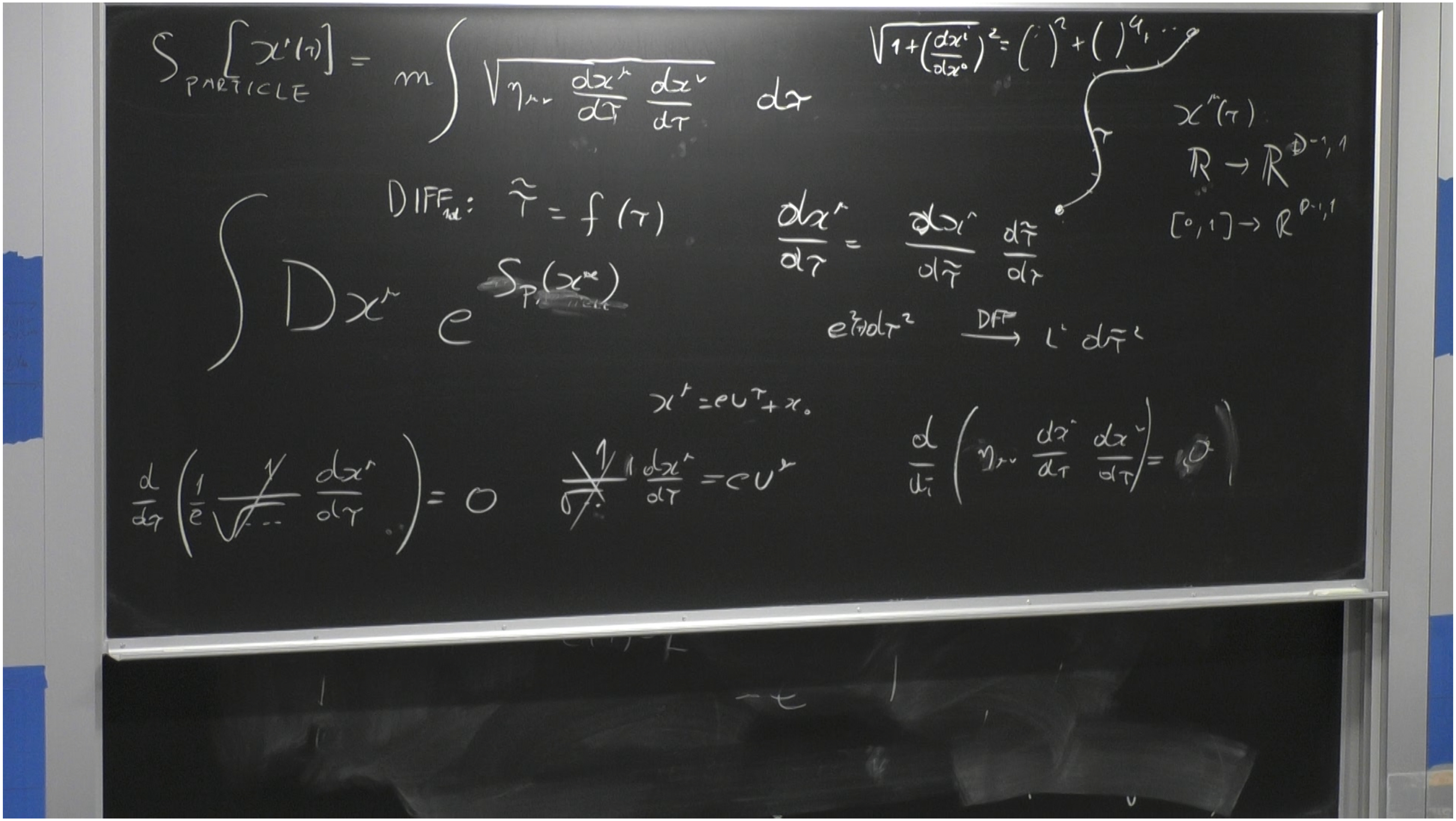


4L
- L m²
= ∫dp e^{2ip(x_i - x_f)} ∫dL e
= G(x_i, x_f)
1
p² + m²


$\int_0^\infty dL \int Dx \, e^{\frac{1}{i}\int [(\frac{dx}{d\tau})^2 L^{-1} + m^2 L]} = \int \frac{Dx\,De}{DIFF_{1d}} e^{S[x,e]}$
$\int_0^\infty dL \, e^{-\frac{(x_i - x_f)^2}{4L} - Lm^2} = \int dp\, e^{2ip(x_i - x_f)} \int_0^\infty dL \, e^{-Lp^2 - Lm^2} = G(x_i, x_f)$
$\frac{1}{p^2+m^2}$


ϕ(x_f)
ϕ(x_i)
⟨0|ϕ(x_if)ϕ(x_f)'|0⟩ = G(x_i, x_f)


$= f(\tau)$
$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\tilde{\tau}} \frac{d\tilde{\tau}}{d\tau}$
$(x^\mu)$
$e^{2\omega(\tau)} d\tau^2 \xrightarrow{DIFF} L^2 d\tilde{\tau}^2$
$x^\mu = eU^\tau + x_0$
$\frac{dx^\mu}{d\tau} = eU^\mu$
$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} =$

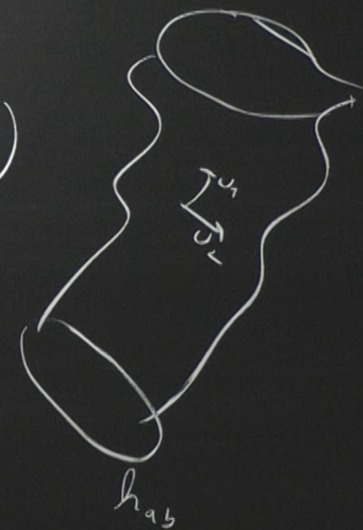


$S_{PARTICLE}[x^\mu(\tau)] = m \int \sqrt{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}\, d\tau$
$\sqrt{1+\left(\frac{dx^i}{dx^0}\right)^2} = (\ )^2 + (\ )^4, \dots$
$x^\mu(\tau)$
$\mathbb{R} \to \mathbb{R}^{D-1,1}$
$[0,1] \to \mathbb{R}^{D-1,1}$
DIFF: $\tilde{\tau} = f(\tau)$
$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\tilde{\tau}} \frac{d\tilde{\tau}}{d\tau}$
$\int Dx^\mu\, e^{S_P(x^\mu)}$
$e^2(\tau) d\tau^2 \xrightarrow{DIFF} 1^2 d\tilde{\tau}^2$
$x^\mu = e v^\mu \tau + x_0$
$\frac{d}{d\tau}\left(\frac{1}{e} \frac{1}{\sqrt{\dots}} \frac{dx^\mu}{d\tau}\right) = 0$
$\frac{dx^\mu}{d\tau} = e v^\mu$
$\frac{d}{d\tau}\left(\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}\right) = 0$

# RELATIVISTIC STRING

$$S = T \int \sqrt{\det_{2\times 2}\left[\eta_{\mu\nu} \frac{\partial x^\mu}{\partial u^a} \frac{\partial x^\nu}{\partial u^b}\right]}\, du^1 du^2$$

$$x^\mu(u^1, u^2)$$



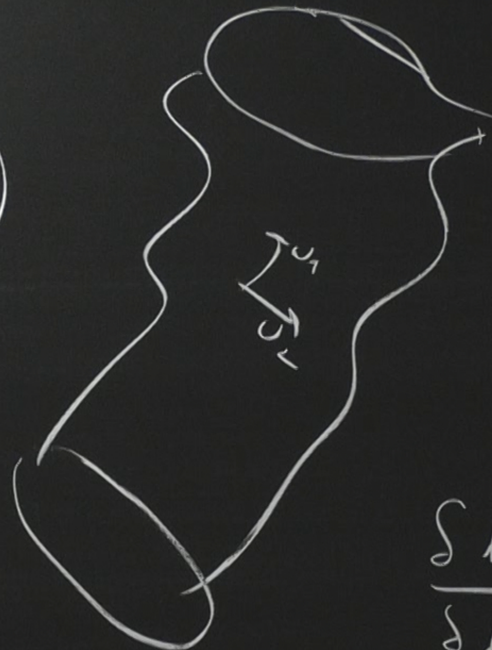

$$S_{POLYAKOV} = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial x^\mu}{\partial u^a} \frac{\partial x^\nu}{\partial u^b} \eta_{\mu\nu}\, du^1 du^2$$

STRING

$$x^\mu(u^1, u^2)$$

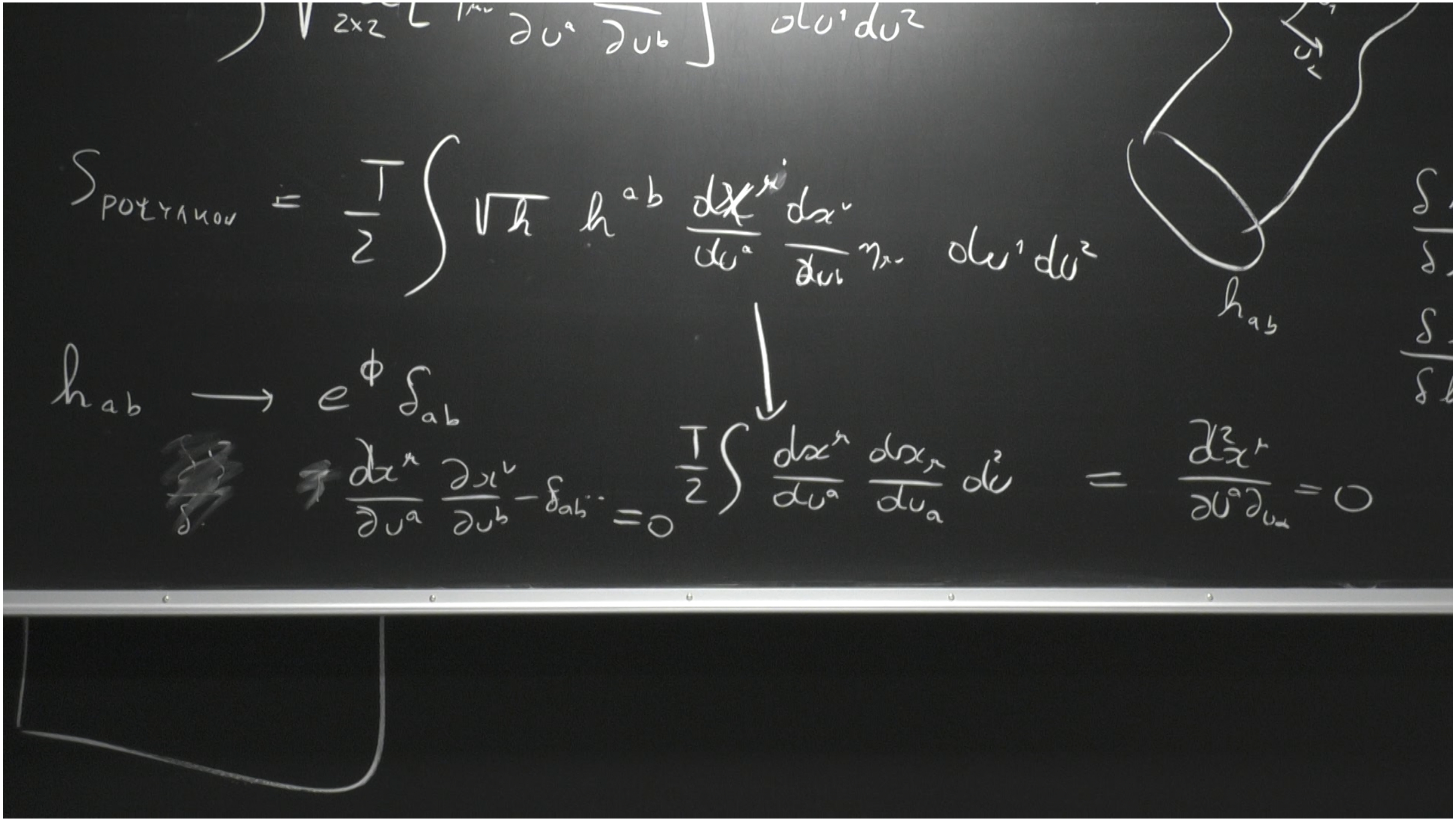
$$\left[\frac{\partial x^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b}\right] du^1 du^2$$

$$h\, h^{ab} \frac{\partial x^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b}\eta_{\mu\nu}\, du^1 du^2$$



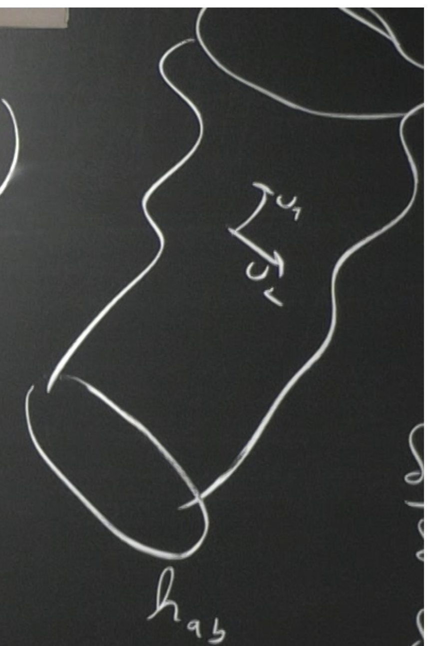

$$\frac{\delta h^{ab}}{\delta h_{cd}} = h^{ac} h^{bd} + (\leftrightarrow)$$

$$\frac{\delta h}{\delta h_{cd}}$$


$S_{POLYAKOV} = \frac{T}{2}\int \sqrt{h}\, h^{ab} \frac{dx^\mu}{du^a}\frac{dx^\nu}{du^b}\eta_{\mu\nu}\, du^1 du^2$
$h_{ab}$
$h_{ab} \longrightarrow e^\phi \delta_{ab}$
$\frac{\partial x^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b} - \delta_{ab} \cdots = 0$
$\frac{T}{2}\int \frac{dx^\mu}{du^a}\frac{dx_\mu}{du_a} d^2u = \frac{\partial^2 x^\mu}{\partial u^a \partial u_a} = 0$

$$S = T\int \sqrt{\det_{2\times 2}\left[\eta_{\mu\nu}\frac{\partial x^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b}\right]}\; du^1 du^2$$

$$x^\mu(u^1, u^2)$$




$$S_{\text{POLYAKOV}} = \frac{T}{2}\int \sqrt{h}\; h^{ab}\,\frac{\partial x^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b}\,\eta_{\mu\nu}\; du^1 du^2$$

$$h_{ab} \longrightarrow e^\phi \delta_{ab}$$

$$\frac{\partial x^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b} - \delta_{ab}\cdots = 0$$

$$\frac{T}{2}\int \frac{\partial x^\mu}{\partial u^a}\frac{\partial x_\mu}{\partial u_a}\, d^2u \;\Rightarrow\; \frac{\partial^2 x^\mu}{\partial u^a \partial u_a} = 0$$

$$\int \sqrt{\det_{2\times 2}\left[\eta_{\mu\nu}\frac{\partial x^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b}\right]}\, du^1 du$$

$$S_{\text{POLYAKOV}} = \frac{T}{2}\int \sqrt{h}\, h^{ab}\frac{dx^\mu}{du^a}\frac{dx^\nu}{\partial u^b}\eta_{\mu\nu}$$

$$h_{ab} \longrightarrow e^{\phi}\delta_{ab}$$

$$\frac{dx^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b} - \delta_{ab}\cdots = 0$$

$$\frac{T}{2}\int \frac{dx^\mu}{du^a}\frac{d}{\partial}$$

$$x^\mu = e v^\mu \tau + x_0$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\frac{1}{\sqrt{\cdots}}\frac{dx^\mu}{d\tau}\right) = 0$$

$$\frac{1}{\sqrt{\cdot}}\frac{dx^\mu}{d\tau} = e v^\mu$$

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$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} + m^2 e\right] d\tau$$

$e^2$ = METRIC

$$e^2 = h_{\tau\tau}$$

$$e^2 = \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau}$$

$$e(\tau)\,d\tau = \tilde{e}(\tilde{\tau})\,d\tilde{\tau}$$

$$x^\mu(\tau) = \tilde{x}^\mu(\tilde{\tau})$$

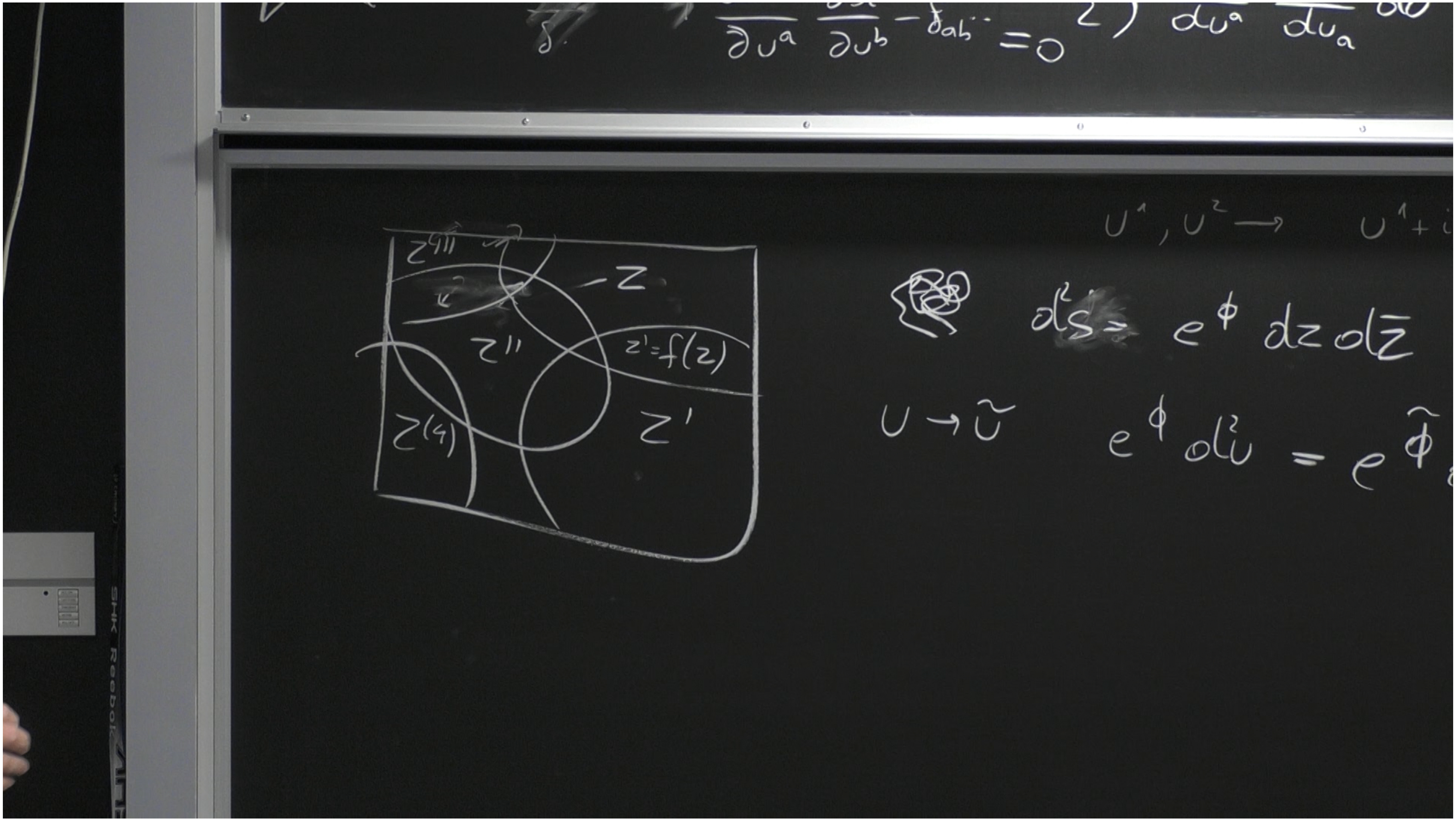
GAUGE-FIXING: $e(\tau) = k$

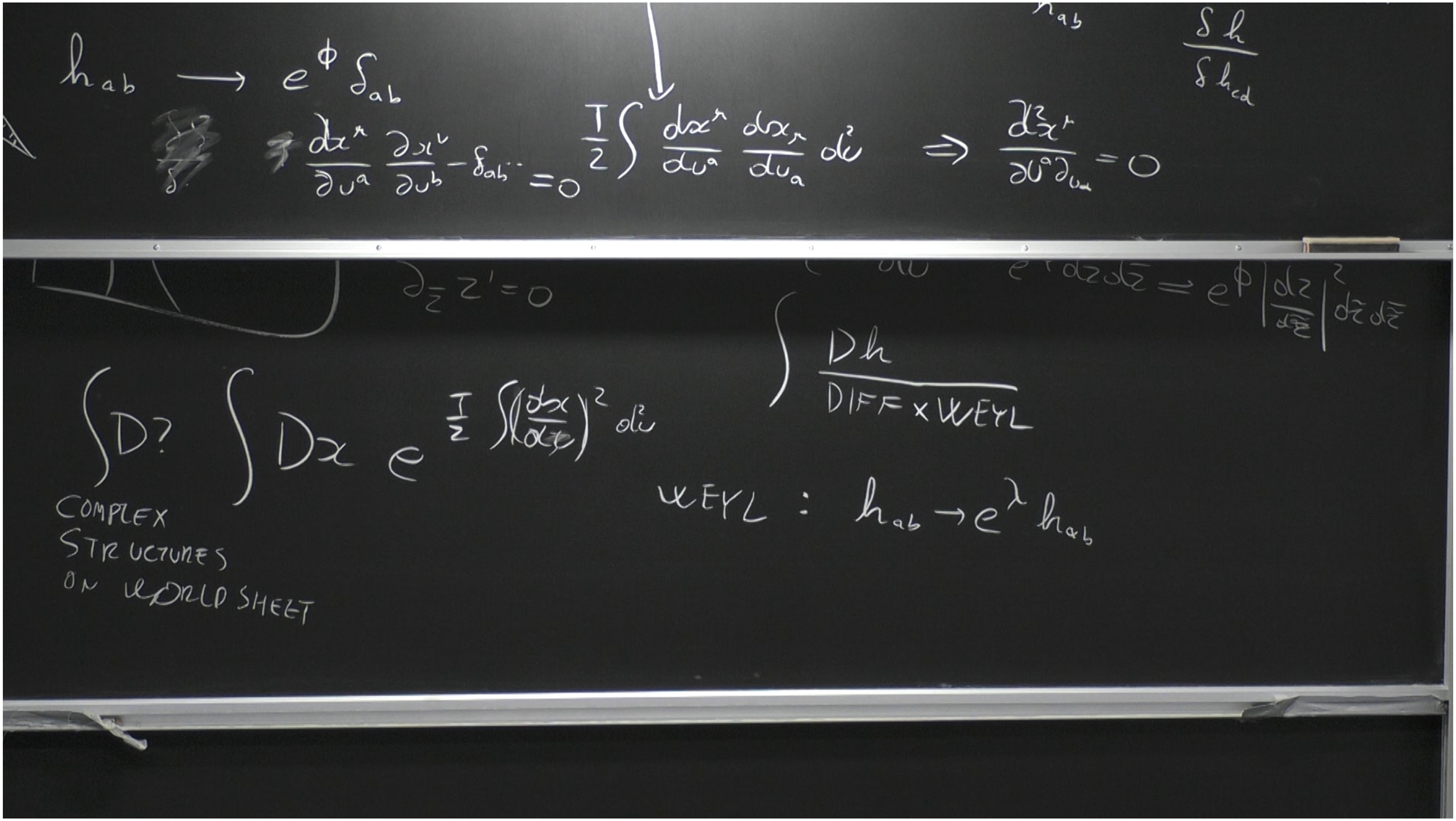
$\partial u^a \; \partial u^b \; g_{ab} = 0 \Rightarrow \partial u^a \; \partial u_a \qquad \partial u^a \partial u_a \; 0$

$u^1, u^2 \longrightarrow \quad u^1 + i u^2 = z$

$ds^2 = e^{\phi} \, dz \, d\bar{z} \qquad z = f(\tilde{z})$

$u \to \tilde{u} \qquad e^{\phi} d^2 u = e^{\tilde{\phi}} d^2 \tilde{u} \qquad e^{\phi} dz\, d\bar{z} = e^{\phi} \left| \frac{dz}{d\tilde{z}} \right|^2 d\tilde{z}\, d\bar{\tilde{z}}$


∂U^a ∂U^b − δ_ab = 0
2)
U^1, U^2 →
Z
Z''
z' = f(z)
Z^(4)
Z'
ds² = e^φ dz dz̄
U → Ũ
e^φ dU² = e^φ̃



h_ab → e^Φ δ_ab
(T/2) ∫ (dx^μ/du^a)(dx_μ/du_a) d²u ⇒ ∂²x^μ/∂u^a∂u_a = 0
δh/δh_cd
∂_z̄ z' = 0
e^Φ |dz/dz̃|² dz̃ dz̄̃
∫ Dh / DIFF × WEYL
∫D? ∫Dx e^{(T/2)∫(dx/du)² d²u}
COMPLEX
STRUCTURES
ON WORLD SHEET
WEYL : h_ab → e^λ h_ab

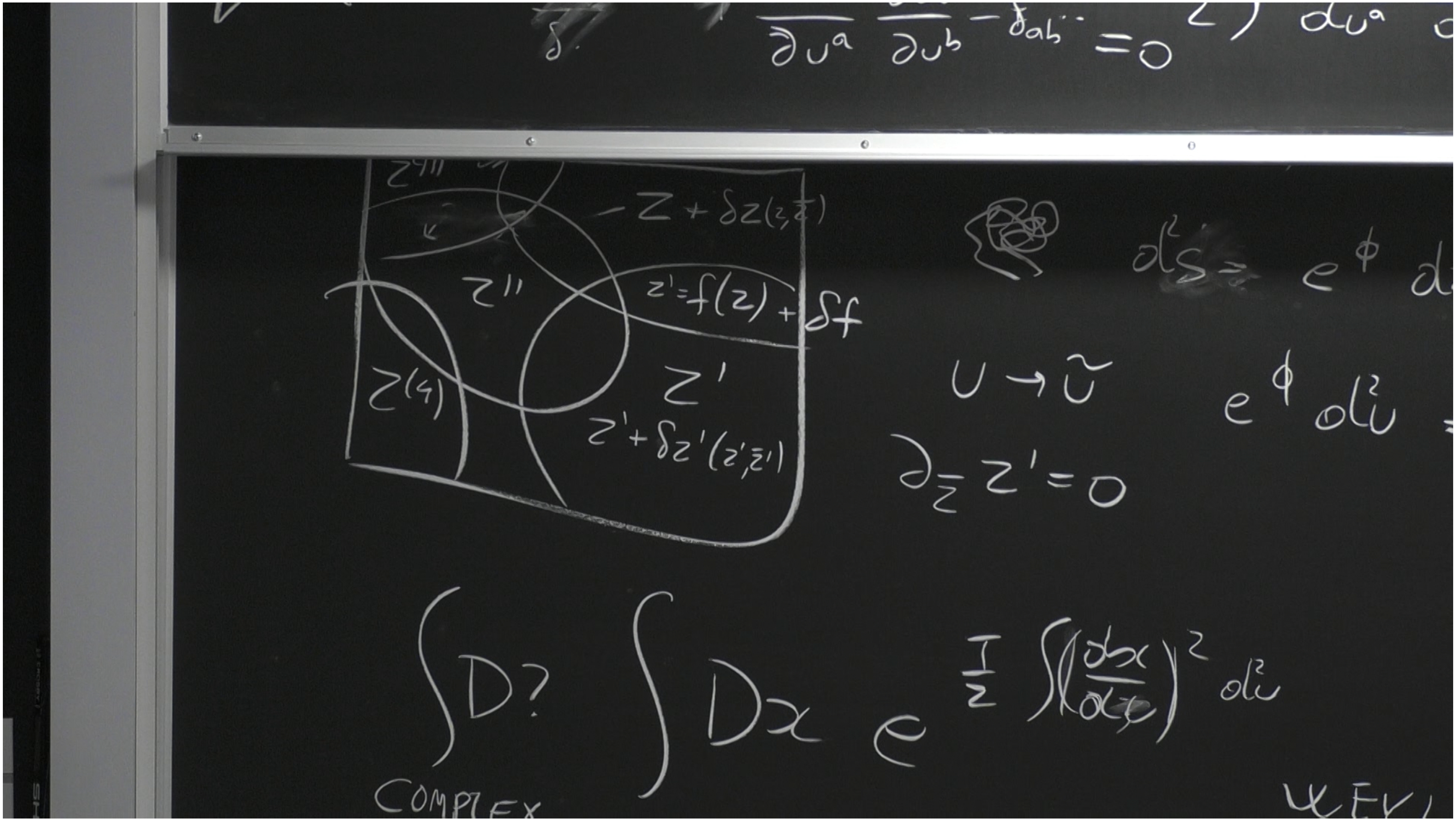
$$e^{\phi}\, dz\, d\bar{z} \qquad z = f(\tilde{z})$$

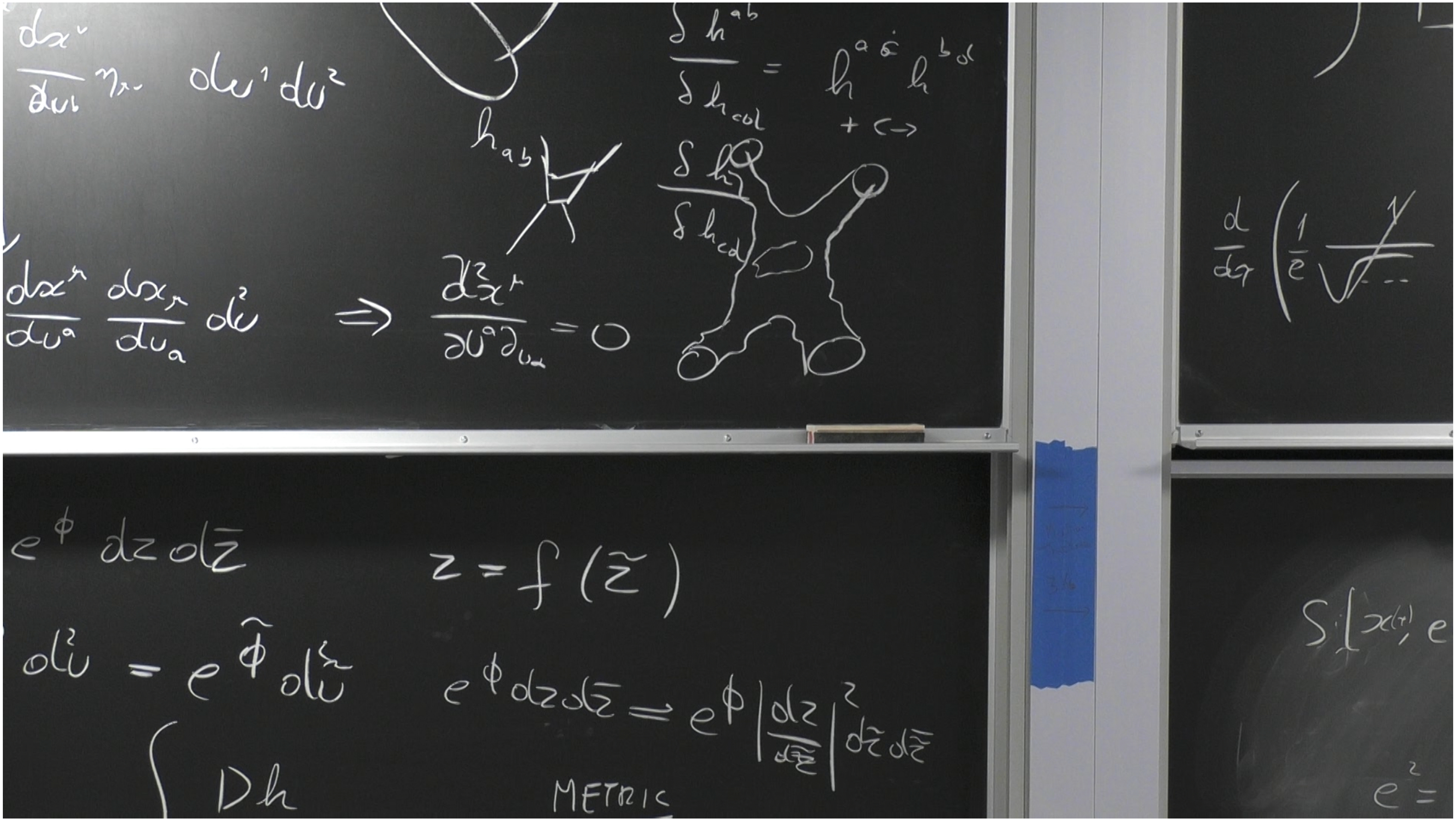
$$e^{\phi}\, dl^2 = e^{\hat{\phi}}\, d\tilde{l}^2 \qquad e^{\phi}\, dz\, d\bar{z} = e^{\phi} \left| \frac{dz}{d\tilde{z}} \right|^2 d\tilde{z}\, d\bar{\tilde{z}}$$

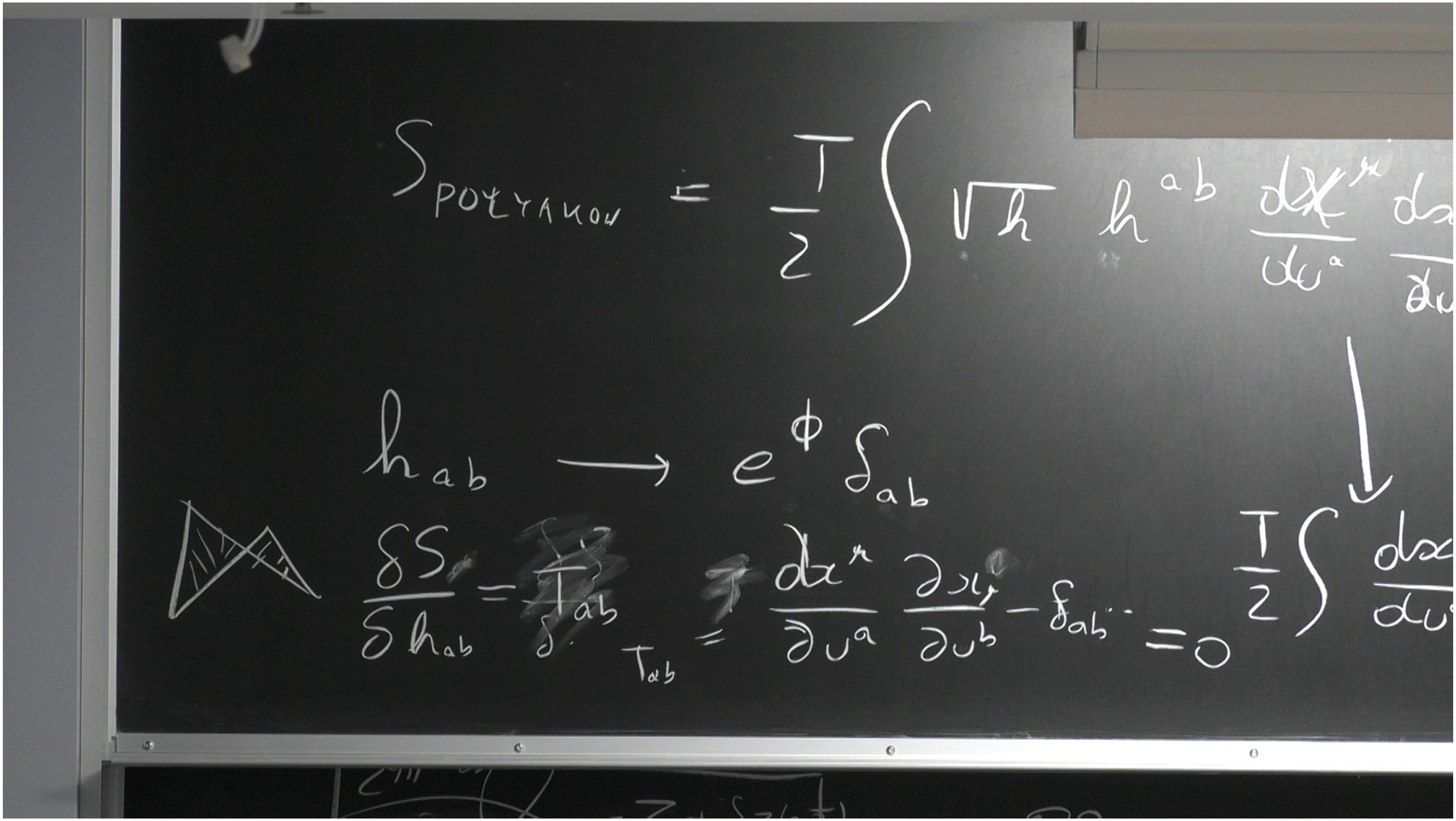
$$\int \frac{Dh}{\text{DIFF} \times \text{WEYL}} \qquad \frac{\text{METRIC}}{\text{DIFF} \times \text{WEYL}} \longleftrightarrow \text{COMPLEX STRUCTURES}$$

$dl^2$

$$\text{WEYL}: \; h_{ab} \to e^{\lambda} h_{ab}$$


-Z + δZ(z, z̄)
z' = f(z) + δf
Z''
Z(4)
Z'
Z' + δZ'(z', z̄')
u → ũ
∂_z̄ Z' = 0
ds² = e^φ d
e^φ du²
∫D?
COMPLEX
∫Dx e^{I/2 ∫(dx/dz)² du²}


z = f(z̃)
e^φ dz dz̄
METRIC
∫ Dh


S POLYAKOV = T/2 ∫ √h h^ab dx^μ/dσ^a
h_ab ⟶ e^φ δ_ab
δS/δh_ab = T_ab = dx^μ/∂σ^a ∂x_μ/∂σ^b − δ_ab = 0
T/2 ∫

$$e^{\phi} dw^2 = e^{\tilde{\phi}} d\tilde{w}^2$$

$$e^{\phi} dz\, d\bar{z} = e^{\phi} \left| \frac{dz}{d\tilde{z}} \right|^2 d\tilde{z}\, d\bar{\tilde{z}}$$

$$\int \frac{Dx \int Dh}{\text{DIFF} \times \text{WEYL}}$$

$$\frac{\text{METRIC}}{\text{DIFF} \times \text{WEYL}} \longleftrightarrow \text{COMPLEX STRUCTURES}$$

$$\text{WEYL}: \; h_{ab} \to e^{\lambda} h_{ab}$$

$$(D-26) \ldots$$