Title: Quantum Fields and Strings Lecture - 230320

Speakers: Davide Gaiotto

Collection: Quantum Fields and Strings (2022/2023)

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$$S_{PARTICLE}[x^\mu(\tau)] = m\int \sqrt{\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\; d\tau$$

$x^\mu(\tau)$

$\mathbb{R} \to \mathbb{R}^{D-1,1}$

$$\tilde{\tau} = f(\tau) \qquad \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\tilde{\tau}}\frac{d\tilde{\tau}}{d\tau}$$

$$\int Dx^\mu \, e^{S_P(x^\mu)}$$

RELATIVISTIC STRING

$$x^\mu = e v \tau + x_0$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\cancel{\frac{1}{\sqrt{\cdots}}}\frac{dx^\mu}{d\tau}\right) = 0 \qquad \cancel{\frac{1}{\sqrt{\cdot}}}\,\frac{1}{e}\frac{dx^\mu}{d\tau} = e v^\mu \qquad \eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = e^2$$

$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[\underset{\sqrt{h}\,h^{\tau\tau}}{e^{-1}}\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} + m^2 \underset{\sqrt{h}}{e}\right] d\tau$$

e^2 = METRIC

$$e^2 = h_{\tau\tau}$$

$$e^2 = \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau}$$

$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[e^{-1} \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} + m^2 e \right] d\tau$$

e^2 = METRIC

$$e^2 = h_{\tau\tau}$$

$$e(\tau)\, d\tau = \tilde{e}(\tilde{\tau})\, d\tilde{\tau}$$

$$e^2 = \frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau}$$

GAUGE-FIXING : $e(\tau) = k$

$$x^\mu = e\, v^\mu \tau + x_0$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\,\cancel{\frac{1}{\sqrt{\cdots}}}\,\frac{dx^\mu}{d\tau}\right) = 0 \qquad \cancel{\frac{1}{\sqrt{\cdots}}}\,\frac{dx^\mu}{d\tau} = e\,v^\mu$$

$$\eta_{\mu\nu}\,\frac{dx^\mu}{d\tau}\,\frac{dx^\nu}{d\tau} = e^2$$

$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[e^{-1}\,\frac{dx^\mu}{d\tau}\,\frac{dx_\mu}{d\tau} + m^2 e\right] d\tau$$

e^2 = METRIC

$$e^2 = h_{\tau\tau}$$

$$e^2 = \frac{dx^\mu}{d\tau}\,\frac{dx_\mu}{d\tau}$$

$$e(\tau)\,d\tau = \tilde{e}(\tilde{\tau})\,d\tilde{\tau}$$

$$x^\mu(\tau) = \tilde{x}^\mu(\tilde{\tau})$$

GAUGE-FIXING : $e(\tau) = k$

$-t$

$$S_{\text{PARTICLE}}[x^\mu(\tau)] = m\int \sqrt{\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\,d\tau$$

$$\sqrt{1+\left(\frac{dx^i}{dx^0}\right)^2} = (\ )^2 + (\ )^4 + \dots$$

$$x^\mu(\tau)$$

$$\mathbb{R} \to \mathbb{R}^{D-1,1}$$

$$[0,1] \to \mathbb{R}^{D-1,1}$$

$$\text{DIFF}_{1d}: \ \tilde{\tau} = f(\tau)$$

$$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\tilde{\tau}}\frac{d\tilde{\tau}}{d\tau}$$

$$\int Dx^\mu \, e^{S_P(x^\mu)}$$

$$x^\mu = e v^\mu \tau + x_0$$

$$\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = e^2$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\cancel{\frac{1}{\sqrt{\dots}}}\frac{dx^\mu}{d\tau}\right) = 0$$

$$\cancel{\frac{1}{\sqrt{\cdot}}}\ \frac{dx^\mu}{d\tau} = e v^\mu$$

$$\int_0^\infty dL \int \mathcal{D}x \; e^{\frac{1}{i}\int \left[\left(\frac{dx}{d\tau}\right)^2 L^{-1} + m^2 L\right]} = \int \frac{\mathcal{D}x\,\mathcal{D}e}{\mathrm{DIFF}_{1d}}$$

$$\int_0^\infty dL \; e^{-\frac{(x_i - x_f)^2}{4L} - Lm^2} = \int dp\, e^{2ip(x_i - x_f)} \int_0^\infty dL \; e^{-Lp^2 - Lm^2}$$

$$\int d \qquad \frac{1}{p^2 + m^2}$$

$$\frac{\cdots}{4L} - Lm^2$$

$$= \int dp\, e^{2ip(x_i - x_f)} \int_0 dL\, e^{-Lp^2 - Lm^2} = G(x_i, x_f)$$

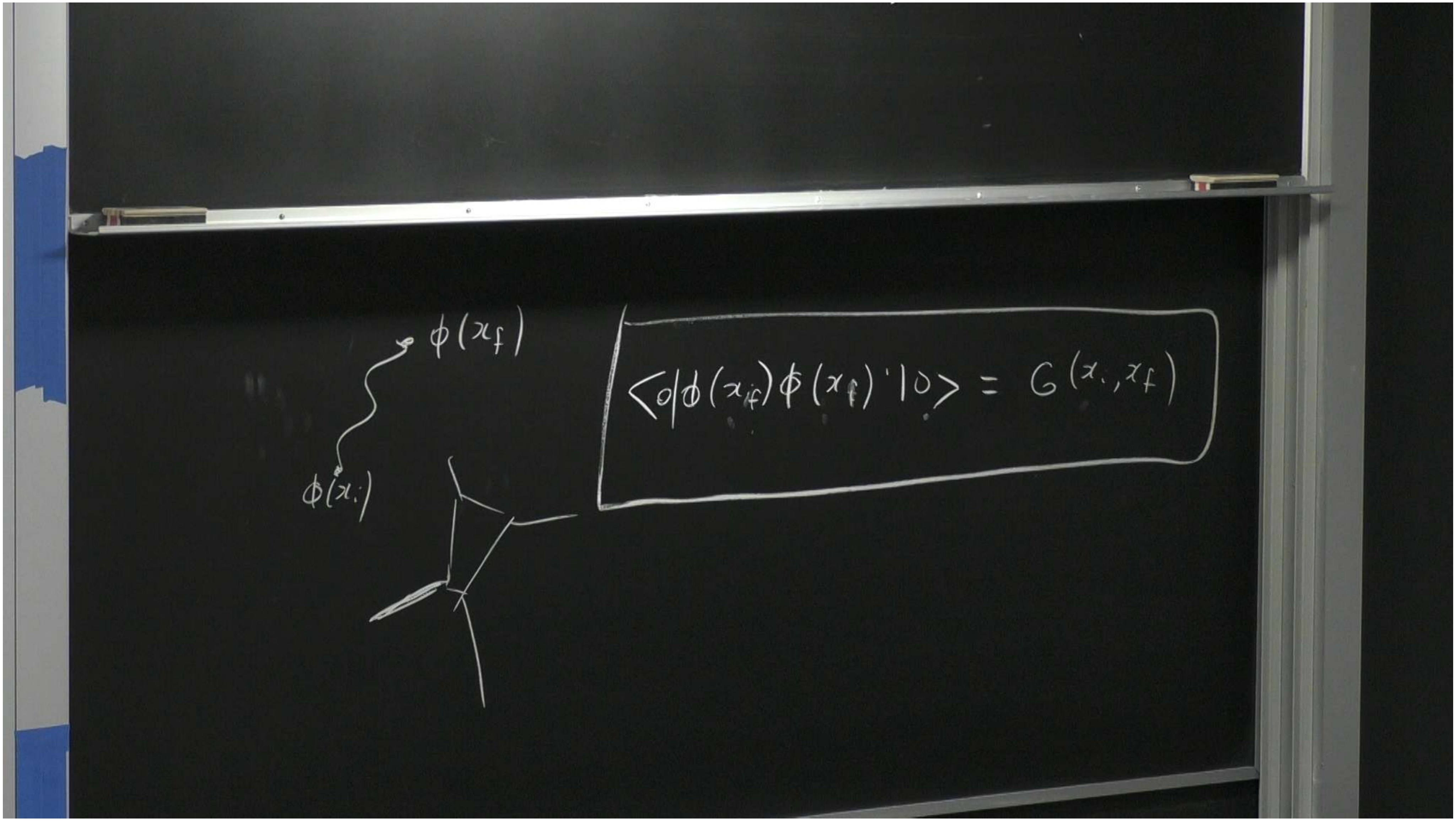
$$\parallel$$

$$\frac{1}{p^2 + m^2}$$

$$\int_0^\infty dL \int \mathcal{D}x \; e^{\frac{1}{2}\int \left[\left(\frac{dx}{d\tau}\right)^2 L^{-1} + m^2 L\right]} = \int \frac{\mathcal{D}x\,\mathcal{D}e}{\mathrm{DIFF}_{1d}}\, e^{S[x,e]}$$

$$\int_0^\infty dL \; e^{-\frac{(x_i - x_f)^2}{4L} - Lm^2} = \int dp\, e^{2ip(x_i - x_f)} \int_0^\infty dL\, e^{-Lp^2 - Lm^2} = G(x_i, x_f)$$

$$\int d \qquad \frac{1}{p^2+m^2}$$


φ(x_f)
φ(x_i)
<0|φ(x_if)φ(x_f)'|0> = G(x_i, x_f)

$= f(\tau)$

$$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\tilde{\tau}} \frac{d\tilde{\tau}}{d\tau}$$

(x^μ)

$$e^{2\phi(\tau)} d\tau^2 \xrightarrow{\text{DFF}} L^2 d\tilde{\tau}^2$$

$$x^\mu = e u^\mu \tau + x_0$$

$$\frac{dx^\mu}{d\tau} = e U^\mu$$

$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} =$$

$$S_{\text{PARTICLE}}[x^\mu(\tau)] = m\int \sqrt{\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\; d\tau$$

$$\sqrt{1+\left(\frac{dx^i}{dx^0}\right)^2} = (\;)^2 + (\;)^4, \dots$$

τ

$x^\mu(\tau)$

$\mathbb{R} \to \mathbb{R}^{D-1,1}$

$[0,1] \to \mathbb{R}^{D-1,1}$

$\text{DIFF}_{1d}: \; \tilde{\tau} = f(\tau)$

$$\int Dx^\mu \, e^{S_P(x^\mu)}$$

$$\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\tilde{\tau}}\frac{d\tilde{\tau}}{d\tau}$$

$$e^2(\tau)\, d\tau^2 \xrightarrow{\text{DFF}} l^2 \, d\tilde{\tau}^2$$

$$x^\mu = e v^\mu \tau + x_0$$

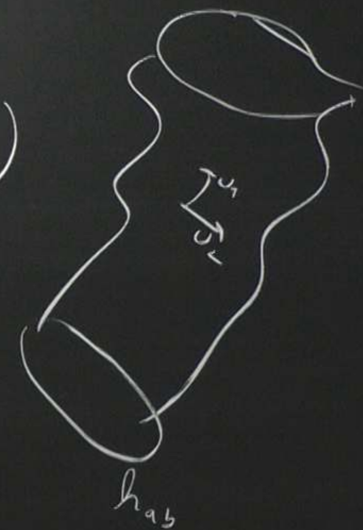
$$\frac{d}{d\tau}\left(\frac{1}{e}\frac{1}{\sqrt{\cdots}}\frac{dx^\mu}{d\tau}\right) = 0 \qquad \cancel{\frac{1}{\sqrt{\cdot}}}\,\frac{dx^\mu}{d\tau} = e v^\mu$$

$$\frac{d}{d\tau}\left(\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right) = 0$$

RELATIVISTIC STRING

$$S = T \int \sqrt{\det_{2\times 2}\left[\eta_{\mu\nu} \frac{\partial x^\mu}{\partial u^a} \frac{\partial x^\nu}{\partial u^b}\right]}\, du^1 du^2$$

$x^\mu(u^1, u^2)$

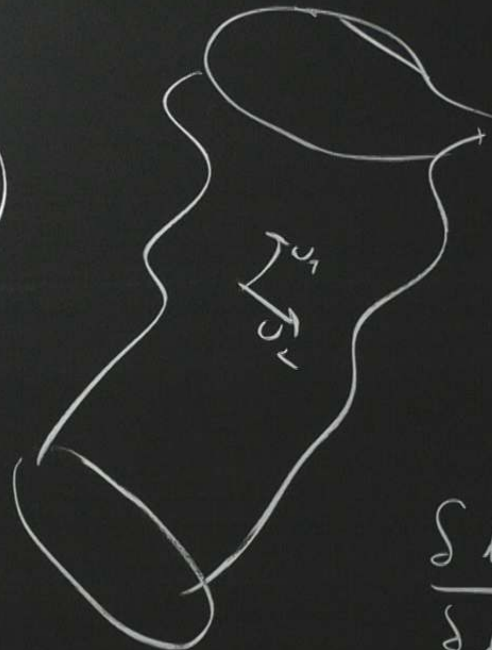


$$S_{\text{POLYAKOV}} = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial x^\nu}{\partial u^b} \eta_{\mu\nu}\, du^1 du^2$$

STRING

$$x^\mu(u^1, u^2)$$

$$\left[\frac{\partial x^\mu}{\partial u^a}\frac{dx^\nu}{\partial u^b}\right] du^1 du^2$$



h_{ab}

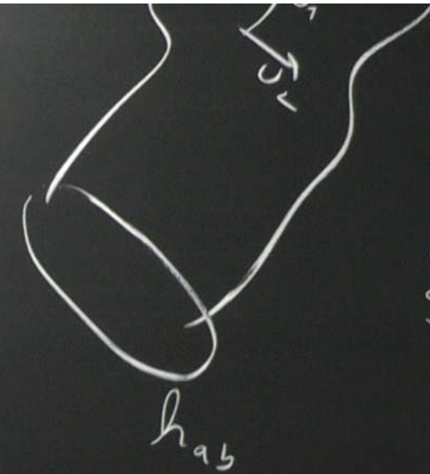
$$h\, h^{ab}\frac{d\cancel{X}^{\mu}}{du^a}\frac{dx^\nu}{\partial u^b}\eta_{\mu\nu}\, du^1 du^2$$

$$\frac{\delta h^{ab}}{\delta h_{cd}} = h^{ac} h^{bd} + (\leftrightarrow)$$

$$\frac{\delta h}{\delta h_{cd}}$$

$$\int \sqrt{\ldots_{2\times 2}\left[\eta_{\mu\nu}\,\frac{\ldots}{\partial u^a}\,\frac{\ldots}{\partial u^b}\right]}\; du^1 du^2$$

$$S_{\text{POLYAKOV}} = \frac{T}{2}\int \sqrt{h}\; h^{ab}\,\frac{dX^\mu}{du^a}\,\frac{dx^\nu}{du^b}\,\eta_{\mu\nu}\; du^1 du^2$$



$$h_{ab} \longrightarrow e^\phi \delta_{ab}$$

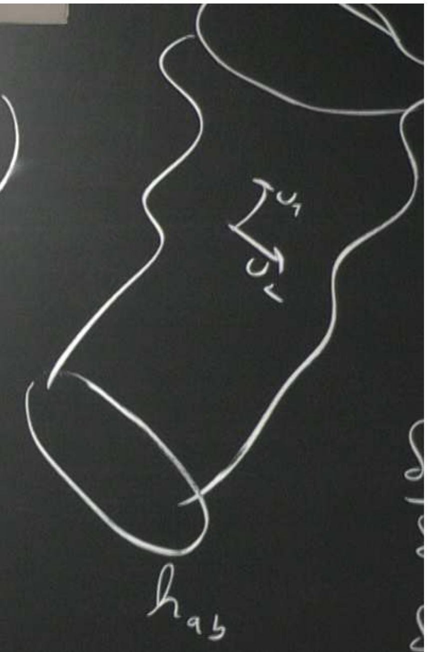
$$\frac{\partial x^\mu}{\partial u^a}\,\frac{\partial x^\nu}{\partial u^b} - \delta_{ab}\cdots = 0$$

$$\frac{T}{2}\int \frac{dx^\mu}{du^a}\,\frac{dx_\mu}{du_a}\, d^2u \quad = \quad \frac{\partial^2 x^\mu}{\partial u^a \partial u_a} = 0$$

$$\frac{\delta}{\delta}\ldots \qquad \frac{\delta}{\delta h}\ldots$$

$$S = T\int \sqrt{\det_{2\times 2}\left[\eta_{\mu\nu}\frac{\partial x^\mu}{\partial u^a}\frac{dx^\nu}{\partial u^b}\right]}\, du^1 du^2$$

$$x^\mu(u^1, u^2)$$



$$S_{\text{POLYAKOV}} = \frac{T}{2}\int \sqrt{h}\, h^{ab}\frac{dx^\mu}{du^a}\frac{dx^\nu}{\partial u^b}\eta_{\mu\nu}\, du^1 du^2$$

$$h_{ab} \longrightarrow e^\phi \delta_{ab}$$

$$\frac{dx^\mu}{\partial u^a}\frac{\partial x^\nu}{\partial u^b} - \delta_{ab}\cdots = 0$$

$$\frac{T}{2}\int \frac{dx^\mu}{du^a}\frac{dx_\mu}{du_a}\, du^2 \Rightarrow \frac{\partial^2 x^\mu}{\partial u^a \partial u_a} = 0$$

$$\int \sqrt{\det_{2\times 2}\left[\eta_{\mu\nu}\frac{\partial x}{\partial u^a}\frac{\partial x}{\partial u^b}\right]}\, du^1 du$$

$$S_{\text{POLYAKOV}} = \frac{T}{2}\int \sqrt{h}\; h^{ab}\,\frac{dx^\mu}{du^a}\,\frac{dx^\nu}{\partial u^b}\,\eta_{\mu\nu}$$

$$h_{ab} \longrightarrow e^{\phi}\,\delta_{ab}$$

$$\frac{dx^\mu}{\partial u^a}\,\frac{\partial x^\nu}{\partial u^b} - \delta_{ab}\cdots = 0$$

$$\frac{T}{2}\int \frac{dx^\mu}{du^a}\,\frac{d}{\partial}$$

$$x^\mu = e U^\tau + x_0$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\frac{\cancel{1}}{\cancel{\sqrt{\cdots}}}\frac{dx^\mu}{d\tau}\right) = 0 \qquad \frac{\cancel{1}}{\cancel{\sigma}}\frac{dx^\mu}{d\tau} = eU^\mu \qquad \frac{d}{d\tau}\left(\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right) = 0$$

$$S[x(\tau), e(\tau)] = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^\mu}{d\tau}\frac{dx_\mu}{d\tau} + m^2 e\right] d\tau$$

e^2 = METRIC

$e^2 = h_{\tau\tau}$

$$e^2 = \frac{dx^\mu}{d\tau}\frac{dx^\mu}{d\tau}$$

$$e(\tau)\,d\tau = \tilde{e}(\tilde{\tau})\,d\tilde{\tau}$$

$$x^\mu(\tau) = \tilde{x}^\mu(\tilde{\tau})$$

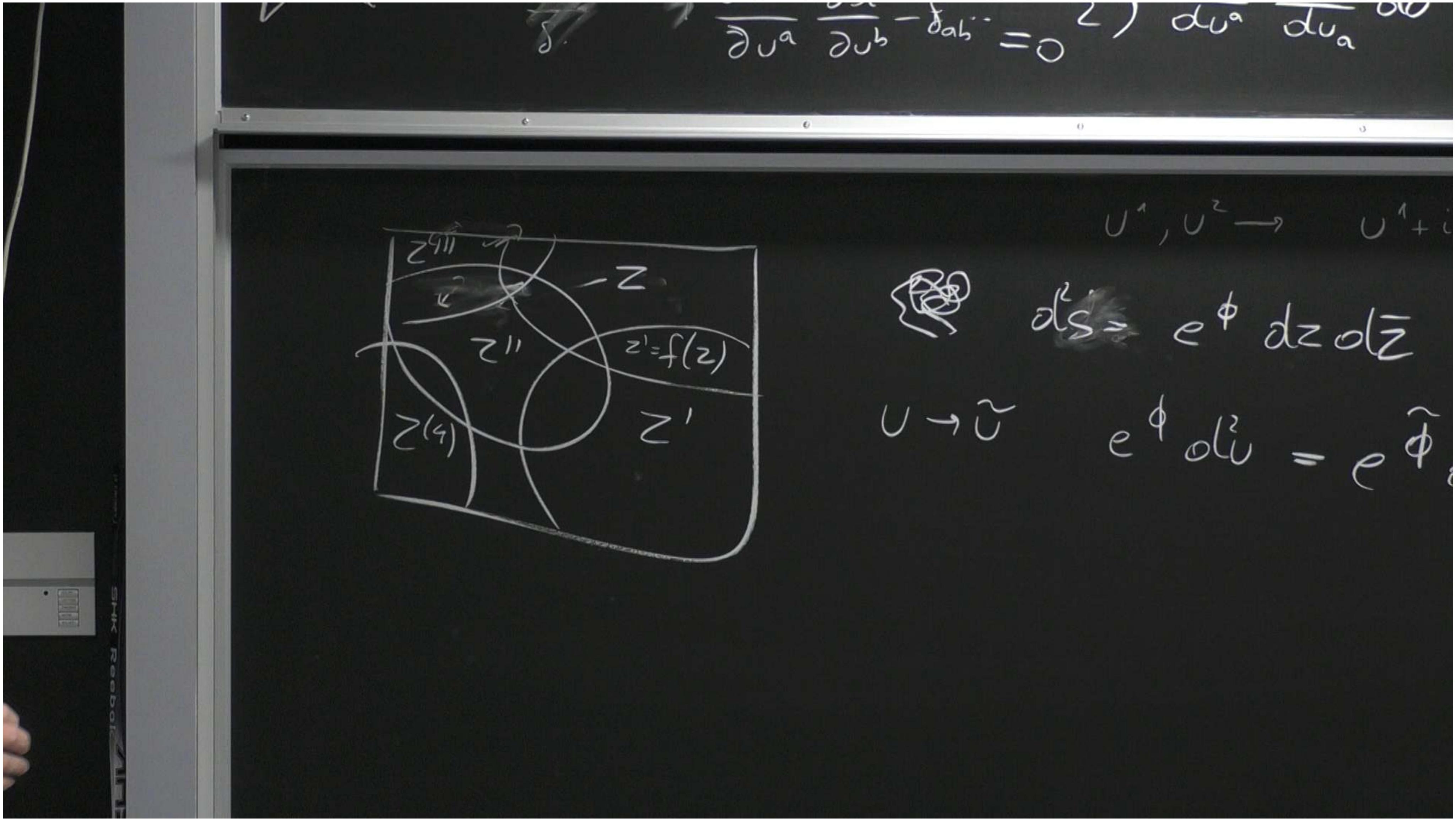
GAUGE-FIXING : $e(\tau) = k$

$$\partial u^a \; \partial u^b \; g_{ab} = 0$$

$$u^1, u^2 \longrightarrow \quad u^1 + i u^2 = z$$

$$ds^2 = e^{\phi}\, dz\, d\bar{z} \qquad\qquad z = f(\tilde{z})$$

$$u \to \tilde{u} \qquad e^{\phi}\, d^2u = e^{\tilde{\phi}}\, d^2\tilde{u} \qquad e^{\phi}\, dz\, d\bar{z} = e^{\phi} \left|\frac{dz}{d\tilde{z}}\right|^2 d\tilde{z}\, d\bar{\tilde{z}}$$


z'''
z
z''
z' = f(z)
z(4)
z'
u¹, u² →
ds² = e^φ dz dz̄
u → ũ
e^φ dũ² = e^φ̂
=0

h_{ab} $\delta h / \delta h_{cd}$

$h_{ab} \longrightarrow e^{\phi} \delta_{ab}$

$\frac{\partial x^\mu}{\partial u^a} \frac{\partial x^\nu}{\partial u^b} - \delta_{ab} \cdots = 0$

$\frac{T}{2} \int \frac{dx^\mu}{du^a} \frac{dx_\mu}{du_a} d^2u \Rightarrow \frac{\partial^2 x^\mu}{\partial u^a \partial u_a} = 0$

$\partial_{\bar{z}} z' = 0$

$e^{\phi} dz d\bar{z} = e^{\phi} \left| \frac{dz}{d\tilde{z}} \right|^2 d\tilde{z} d\bar{\tilde{z}}$

$\int \frac{Dh}{\text{DIFF} \times \text{WEYL}}$

$\int D? \int Dx \, e^{\frac{T}{2} \int \left(\frac{dx}{du}\right)^2 d^2u}$

COMPLEX STRUCTURES ON WORLD SHEET

WEYL : $h_{ab} \to e^{\lambda} h_{ab}$

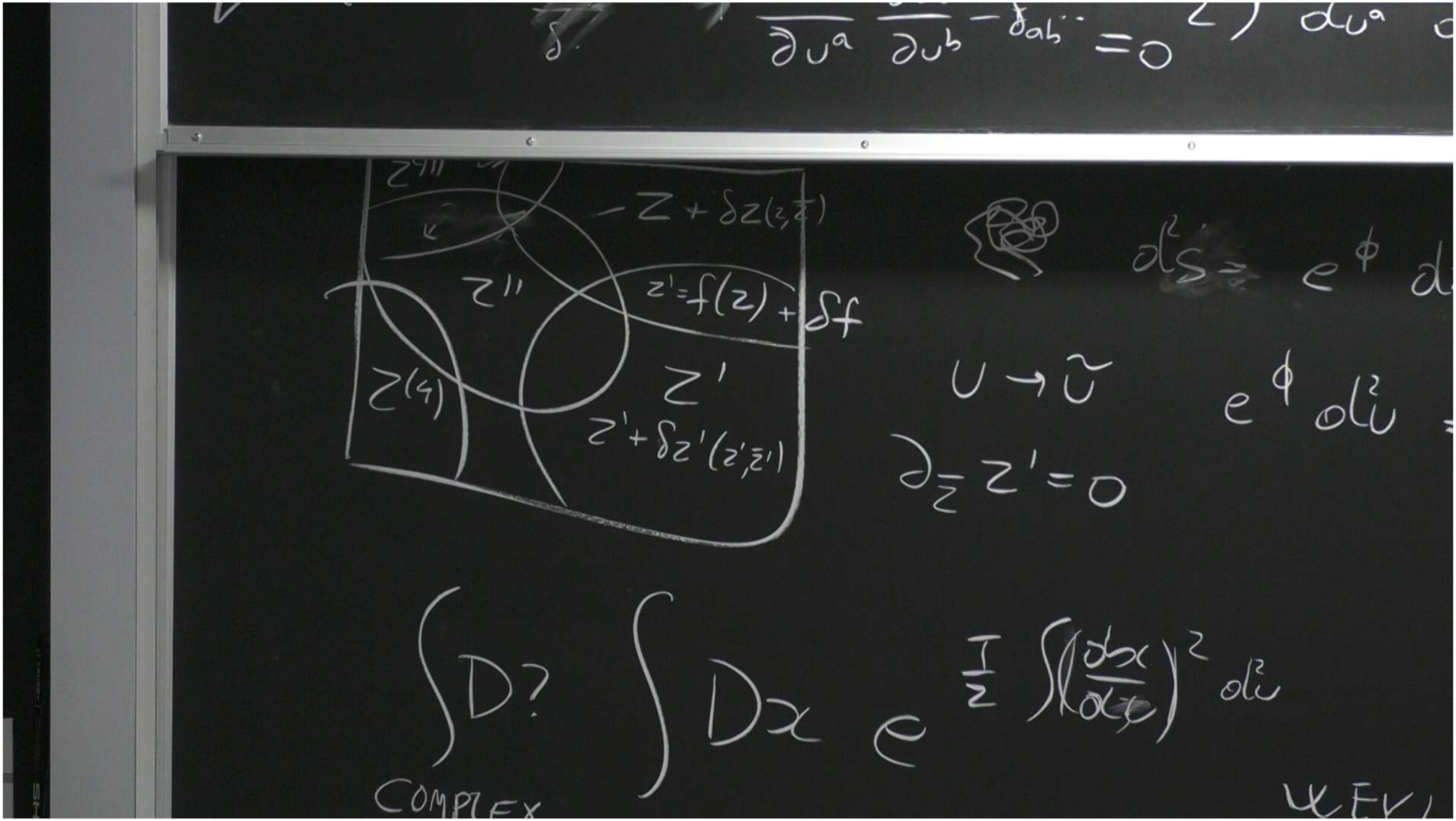
$$= e^{\phi}\, dz\, d\bar{z} \qquad z = f(\tilde{z})$$

$$e^{\phi}\, ds^2 = e^{\tilde{\phi}}\, d\tilde{s}^2 \qquad e^{\phi}\, dz\, d\bar{z} = e^{\phi} \left|\frac{dz}{d\tilde{z}}\right|^2 d\tilde{z}\, d\bar{\tilde{z}}$$

$$ds^2$$

$$\int \frac{Dh}{\mathrm{DIFF} \times \mathrm{WEYL}} \qquad \frac{\mathrm{METRIC}}{\mathrm{DIFF} \times \mathrm{WEYL}} \longleftrightarrow \text{COMPLEX STRUCTURES}$$

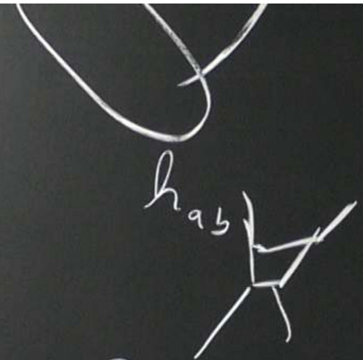
$$\mathrm{WEYL}: \; h_{ab} \to e^{\lambda} h_{ab}$$


$-Z + \delta Z(z,\bar{z})$
Z''
$z' = f(z) + \delta f$
$Z^{(4)}$
Z'
$z' + \delta z'(z',\bar{z}')$
$ds^2 = e^{\phi} d$
$u \to \tilde{u}$
$e^{\phi} du^2$
$\partial_{\bar{z}} z' = 0$
$\int D?$
$\int Dx \, e^{\frac{T}{2}\int\left(\frac{dx}{du}\right)^2 du^2}$
COMPLEX

$$\frac{dx^\nu}{\partial u^b}\eta_{\mu\nu}\; du^1 du^2$$

$$h_{ab}$$

$$\frac{\delta h^{ab}}{\delta h_{cd}} = h^{ac} h^{bd} + (\leftrightarrow)$$



$$\frac{\delta h}{\delta h_{cd}}$$

$$\frac{dx^\mu}{du^a}\frac{dx_\mu}{du_a}\, d\hat{u}^2 \Rightarrow \frac{\partial^2 x^\mu}{\partial u^a \partial u_a} = 0$$

$$\frac{d}{d\tau}\left(\frac{1}{e}\frac{\cancel{1}}{\sqrt{\cancel{\;}\cdots}}\right.$$

$$e^\phi\, dz\, d\bar z \qquad z = f(\tilde z)$$

$$d\hat{u}^2 = e^{\hat\Phi} d\tilde{u}^2 \qquad e^\phi dz\, d\bar z = e^\phi \left|\frac{dz}{d\tilde z}\right|^2 d\tilde z\, d\bar{\tilde z}$$

$$\int Dh$$

METRIC

$$S\,[x(\tau), e$$

$$e^2 =$$

$$S_{\text{POLYAKOV}} = \frac{T}{2}\int \sqrt{h}\, h^{ab} \frac{dX^{\mu}}{du^{a}} \frac{dx}{\partial u}$$

$$h_{ab} \longrightarrow e^{\phi}\,\delta_{ab}$$

$$\frac{\delta S}{\delta h_{ab}} = T_{ab} \qquad T_{ab} = \frac{dx^{\mu}}{\partial u^{a}} \frac{\partial x_{\mu}}{\partial u^{b}} - \delta_{ab} \cdots = 0$$

$$\frac{T}{2}\int \frac{dx}{du}$$

$$e^{\phi}\, d\ell^2 = e^{\tilde{\Phi}}\, d\tilde{\ell}^2$$

$$e^{\phi}\, dz\, d\bar{z} = e^{\phi} \left|\frac{dz}{d\tilde{z}}\right|^2 d\tilde{z}\, d\bar{\tilde{z}}$$

$$\int \frac{Dx \int Dh}{\mathrm{DIFF} \times \mathrm{WEYL}}$$

$$\frac{\text{METRIC}}{\text{DIFF} \times \text{WEYL}} \longleftrightarrow \text{COMPLEX STRUCTURES}$$

$$\mathrm{WEYL}: \; h_{ab} \to e^{\lambda} h_{ab} \qquad (D-26) \cdots$$