Title: Quantum Fields and Strings Lecture - 230315

Speakers: Jaume Gomis

Collection: Quantum Fields and Strings (2022/2023)

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Summary

CFT data

1) $\{\Delta_I, S_I\}$ 2 pt. functions

2) C_{IJK} 3pt. functions

Constraint CFT data using associativity of OPE $O_1(O_2O_3) = (O_1O_2)O_3$

$$O_i(x_1)\, O_j(x_2) = \sum_k C_{ijk}(x_{12}, \partial_2)\, O_k(x_3)$$

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Constraint CFT data using associativity of OPE ①

② associativity

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completely fixed.

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↑ completely fixed.

② associativity

$$O_1(O_2 O_3) = (O_1 O_2) O_3$$

radius of convergence is

min $|x_2 - x_4|$

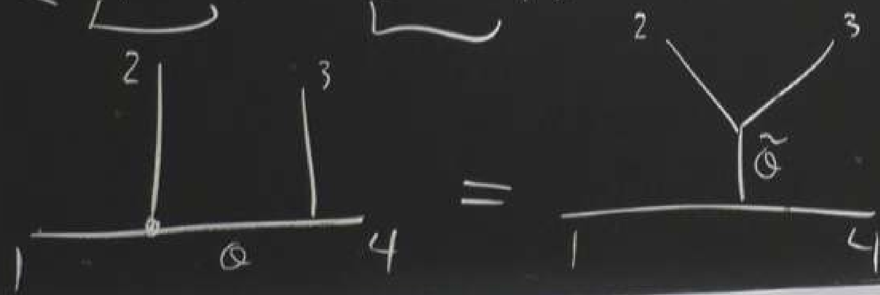
Main Focus: $\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \dfrac{g(u,v)}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}}$

$$u = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2}$$

$x_1 \leftrightarrow x_3$

$$v = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2}$$

$$\langle O_1 \; O_2 \; O_3 \; O_4 \rangle$$



for identical ϕ's $\{O\} = \{O'\}$

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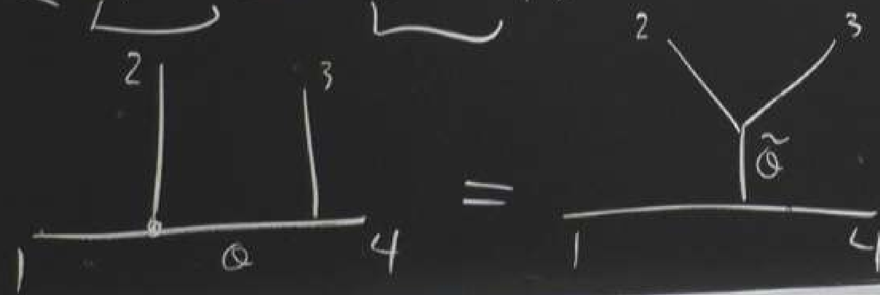
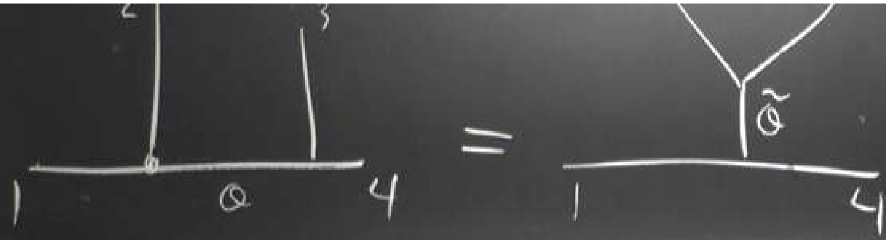
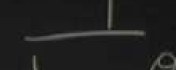
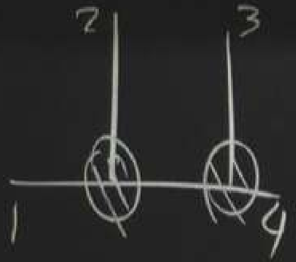




$$\phi(x_1)\phi(x_2) = \sum_{\mathcal{O}} C_{\phi\phi\mathcal{O}}\, C_a(x_{12},\partial_2)\, \underset{\substack{\uparrow \\ \text{spin}}}{\mathcal{O}^a(x_2)}$$





$$= \sum_{\mathcal{O},\mathcal{O}'} C_{\phi\phi\mathcal{O}}\, C_{\phi\phi\mathcal{O}'}\, C_a(x_{12},\partial_2)\, C_b(x_{34},\partial_4) \left\langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4) \right\rangle$$

$$\left\langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4) \right\rangle = \delta_{\mathcal{O}\mathcal{O}'} \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta_\mathcal{O}}}$$

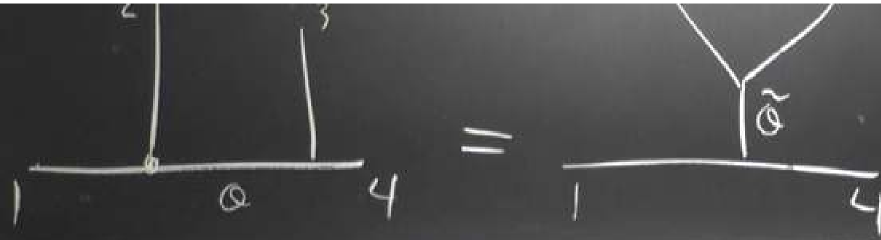
$$= \frac{1}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\, g_{\Delta_\mathcal{O},\ell_\mathcal{O}}(x_i)$$

conformal block:

$$g_{\Delta,\ell} = X_{12}^{2\Delta_\phi} X_{34}^{2\Delta_\phi} C_a(X_{12},\partial_2)\, C_b(X_{34},\partial_4)\, \frac{I^{ab}(X_{24})}{X_{24}^{2\Delta_O}}$$

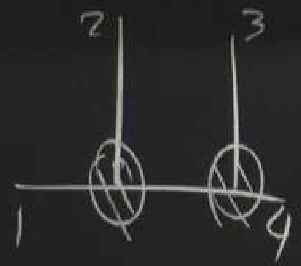
$$g(u,v) = \sum_O C^2_{\phi\phi O}\, g_{\Delta_O,\ell_O}(u,v)$$

- $g_{\Delta_O,\ell_O}(u,v)$ are harmonics of $SO(1,D+1)$



$$\phi(x_1)\phi(x_2) = \sum_{\mathcal{O}} C_{\phi\phi\mathcal{O}}\; C_a(x_{12},\partial_2)\; \mathcal{O}^a(x_2)$$

spin



$$= \sum_{\mathcal{O},\mathcal{O}'} C_{\phi\phi\mathcal{O}}\, C_{\phi\phi\mathcal{O}'}\, C_a(x_{12},\partial_2)\, C_b(x_{34},\partial_4)\, \langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4)\rangle$$

$$\langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4)\rangle = \delta_{\mathcal{O},\mathcal{O}'}\, \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta_\mathcal{O}}}$$

$$= \frac{1}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}} \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\, g_{\Delta_\mathcal{O},\ell_\mathcal{O}}(x_i)$$

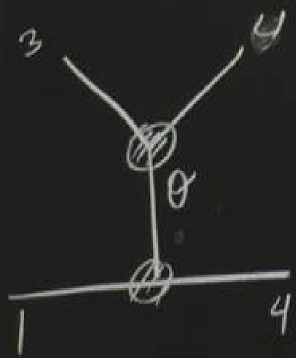
Conformal block:

$$g_{\Delta,\ell} = x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}\, C_a(x_{12},\partial_2)\, C_b(x_{34},\partial_4)\, \frac{I^{ab}(x_{24})}{x_{24}^{2\Delta_O}}$$

$$g(u,v) = \sum_{O} C_{\phi\phi O}^{2}\, g_{\Delta_O,\ell_O}(u,v)$$

– $g_{\Delta_O,\ell_O}(u,v)$ are harmonics of $SO(1,D+1)$

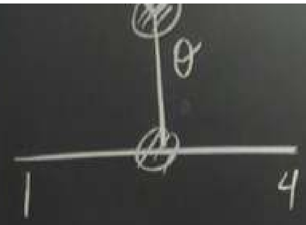
$\Longrightarrow$ associativity



$-$

$1 \leftrightarrow 3$

$$= \frac{g(v,u)}{x_{23}^{2\Delta_\phi}\, x_{14}^{2\Delta_\phi}}$$



$$= \frac{g(v,u)}{X_{23}^{2\Delta_\phi}\, X_{14}^{2\Delta_\phi}}$$

$$\boxed{g(u,v) - \left(\frac{u}{v}\right)^{\Delta_\phi} g(v,u) = 0}$$

$\Longrightarrow$ 1) Solve for all critical phenomena in $D=2$

2) Carve out disallowed CFT data domains.

$\Delta_\sigma , \Delta_\varepsilon$ critical exponents of the $D=3$ Ising model

$v = (1-z)(1-\bar{z})$

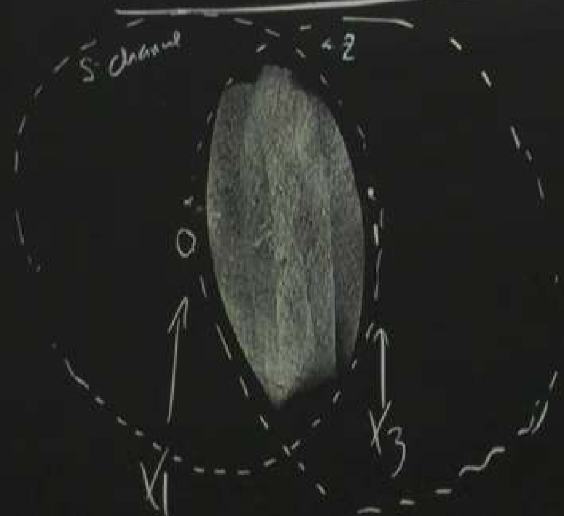
$$\boxed{g(u,v) - \left(\frac{u}{v}\right)^{\Delta_\phi} g(v,u) = 0}$$

$x_2 = z$

$\Longrightarrow$ 1) Solve for all critical phenomena in $D=2$

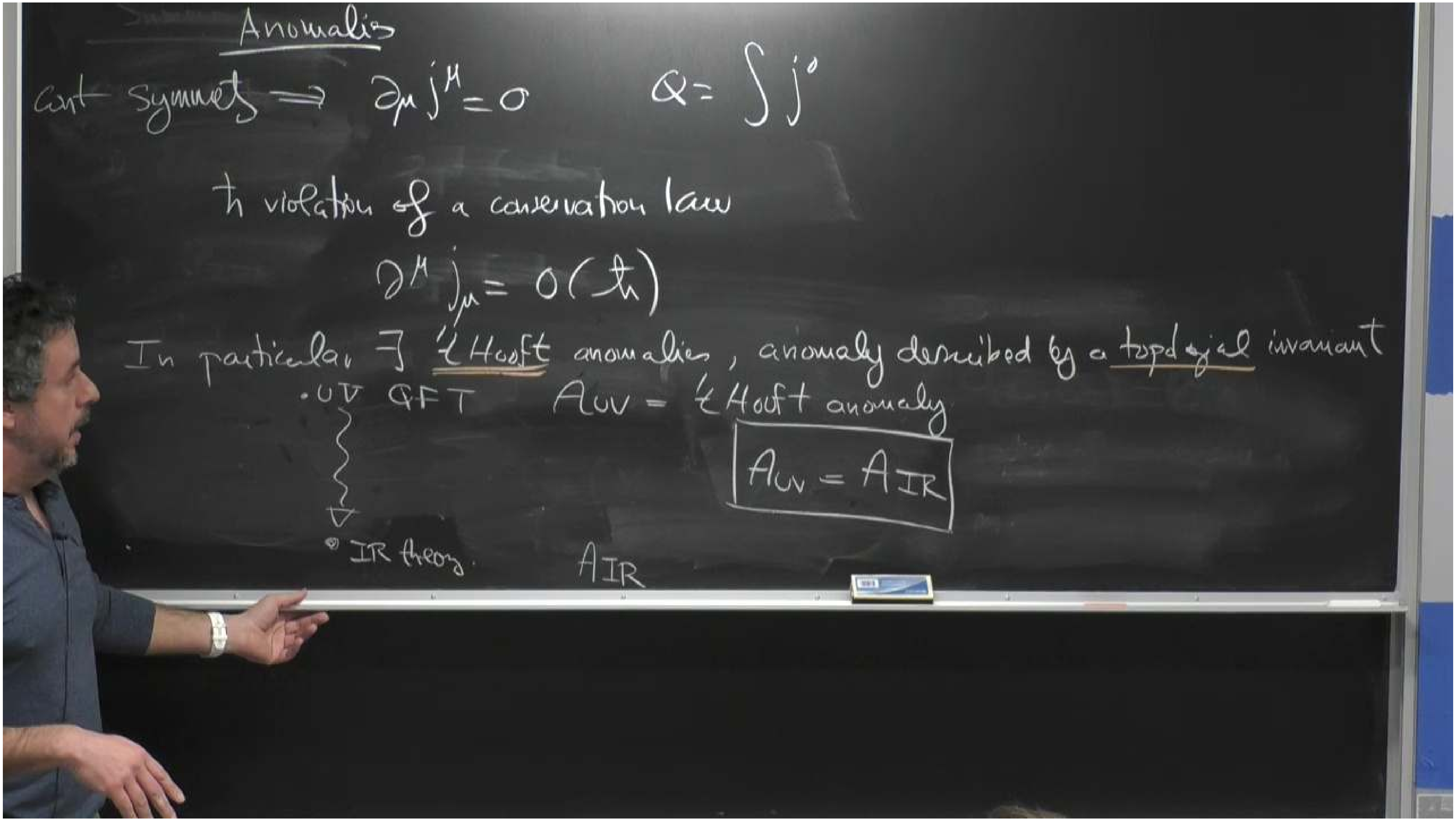
2) Carve out disallowed CFT data domains.

$\Delta_\sigma, \Delta_\varepsilon$ critical exponents of the $D=3$ Ising model



$\uparrow \infty$

x_4



Anomalies
cont symmetry $\Rightarrow \partial_\mu j^\mu = 0$
$Q = \int j^0$
$\hbar$ violation of a conservation law
$\partial^\mu j_\mu = O(\hbar)$
In particular $\exists$ 't Hooft anomalies, anomaly described by a topological invariant
• UV QFT
A_{UV} = 't Hooft anomaly
$\boxed{A_{UV} = A_{IR}}$
• IR theory
A_{IR}

In particular $\exists$ 't Hooft anomalies, anomaly described by a topological invariant

- UV QFT $\quad A_{UV} =$ 't Hooft anomaly

$\boxed{A_{UV} = A_{IR}}$

- IR theory $\quad A_{IR}$

Conformal anomaly in $D=2$. $\quad T^{\mu}_{\mu} = 0$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = p_\mu p_\nu p_\rho p_\sigma F_1(p^2) + 4 \text{ terms}$$

$$(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})\, G(p^2)$$

translational inv $\quad p^\mu T_{\mu\nu} = 0$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \frac{1}{p^2}\left[\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right] F(p^2)$$

- symmetric
- Transverse

$$\Pi_{\mu\nu} = p_\mu p_\nu - \eta_{\mu\nu} p^2$$

In $D=2$ $\quad \Pi_{\mu\nu} = \tilde{p}_\mu \tilde{p}_\nu \qquad \tilde{p}_\mu = \epsilon_{\mu\alpha} p^\alpha$

$D=2$

$$= c\,\frac{\tilde{p}_\mu \tilde{p}_\nu \tilde{p}_\rho \tilde{p}_\sigma}{p^2}$$

- Dimensional analysis

$$\langle T^\mu_\mu(p)\, T_{\rho\sigma}(-p)\rangle = c\, \tilde{p}_\rho \tilde{p}_\sigma$$

X_{12} $\quad X_{34}$ $\quad X_{24}$

$$\langle T_{\mu\nu}(p)\, T_{\rho\sigma}(-p)\rangle = \frac{1}{p^2}\left[\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \frac{2}{D-1}\Pi_{\mu\nu}\Pi_{\rho\sigma}\right]F(p^2)$$

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In $D=2$ $\quad \Pi_{\mu\nu} = \widehat{P}_\mu \widetilde{P}_\nu \qquad \widetilde{P}_\mu = \epsilon_{\mu\alpha} P^\alpha$

$D=2$

$$= c\,\frac{\widetilde{P}_\mu \widetilde{P}_\nu \widehat{P}_\rho \widetilde{P}_\sigma}{p^2}$$

- Dimensional analysis

$$\langle T^\mu_\mu(p)\, T_{\rho\sigma}(-p)\rangle = c\,\widetilde{P}_\rho \widetilde{P}_\sigma \Longrightarrow \text{encountered an irreducible anomaly.}$$

$X_{12} \quad X_{34} \quad X_{24}$

$$\langle T^{\mu}_{\mu}(p)\, T_{\rho\sigma}(-p)\rangle = c\, \tilde{P}_\rho \tilde{P}_\sigma \Longrightarrow$$ encountered an irreducible anomaly

$$\langle T^{\mu}_{\mu}(x)\, T_{\rho\sigma}(0)\rangle = c\, (\partial_\rho \partial_\sigma - \eta_{\rho\sigma} \Box)\, \delta^2(x)$$

couple the CFT to a background

$$\langle T^{\mu}_{\mu}(0)\rangle_h = \langle T^{\mu}_{\mu}(0)\, e^{\int h_{\rho\sigma} T^{\rho\sigma}} \rangle$$

$$= c \int d^2x\, \langle T^{\mu}_{\mu}(0)\, T_{\rho\sigma}(x)\rangle\, h_{\rho\sigma}(x) + \dots$$

$$\langle T^{\mu}_{\mu}(0)\rangle_h = c\,\underbrace{(\partial_\rho\partial_\sigma - \Box\,\eta_{\rho\sigma})\,h_{\rho\sigma}}_{R\big|_{\eta_{\mu\nu}+h_{\mu\nu}}}$$

$\boxed{D=2}$ $\boxed{\langle T^{\mu}_{\mu}\rangle = C\cdot R.}$

D

X_4

X_3

X_1

$$\langle T^\mu_\mu(0)\rangle_h = \underbrace{c\,(\partial_\rho\partial_\sigma - \Box\,\eta_{\rho\sigma})\,h_{\rho\sigma}}_{R\big|_{\eta_{\mu\nu}+h_{\mu\nu}}}$$

$$\boxed{\begin{array}{ll} D=2 & c_{UV} > c_{IR} \\ D=4 & a_{UV} > a_{IR} \end{array}}$$

CFT_{UV}

CFT_{IR}

$\boxed{D=2}$ $\boxed{\langle T^\mu_\mu\rangle = c\cdot R}$ $\Rightarrow$ c-theorems

$\boxed{D=3}$ $T^\mu_\mu \overset{?}{=} 0 \Rightarrow \nexists$ no conformal anomalies in odd D.

$$\delta_\sigma S_{c.t.} = \int d^4x\,\sqrt{g}\,R^2$$

$\boxed{D=4}$ $T^\mu_\mu = \underset{=}{a}\,\mathrm{Weyl}^2 - \underset{=}{c}\,\mathrm{Euler} + \cancel{\Box R}$

$\Delta_\sigma, \Delta_\varepsilon$ critical exponents of the $D=3$ Ising model

$$\langle T^\mu_\mu(0)\rangle_h = \underbrace{c\,(\partial_\rho\partial_\sigma - \Box\eta_{\rho\sigma})h_{\rho\sigma}}$$

$$R\Big|_{\eta_{\mu\nu}+h_{\mu\nu}}$$

CFT_{UV}

CFT_{IR}

$D=2$ $\quad c_{UV} > c_{IR}$

$D=4$ $\quad a_{UV} > a_{IR}$

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$\boxed{D=4}$ $\quad T^\mu_\mu = a\,\mathrm{Weyl}^2 - c\,\mathrm{Euler} + \cancel{\Box R}$

$$R^2_{\mu\nu\rho\sigma} + \alpha R^2_{\mu\nu} + \beta R^2$$

$\Delta_\sigma, \Delta_\varepsilon$ critical exponents of the $D=3$ Ising model