

Title: Quantum Fields and Strings Lecture - 230315

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Collection: Quantum Fields and Strings (2022/2023)

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Summary

CFT data

1) $\{\Delta_I, S_I\}$ 2 pt. functions

2) C_{IJK} 3pt. functions

Constraint CFT data using associativity of OPE

$$O_1(O_2 O_3) = (O_1 O_2) O_3$$

$$\left(\begin{array}{c} O_i(x_1) \\ \cdot \\ O_j(y_2) \end{array} \right) = \sum_K C_{ijk}(x_{12}, \alpha_2) \left(\begin{array}{c} \cdot \\ O_k(x_3) \end{array} \right)$$

CFT data

1) $\{\Delta_I, S_I\}$ 2 pt. functions

2) C_{IJK} 3 pt. functions

Constraint CFT data using associativity of OPE ^①

② associativity

$$O_1(O_2 O_3) = (O_1 O_2) O_3$$

$$\begin{array}{c} \circlearrowleft \\ \begin{array}{c} \cdot O_1(x_1) \\ \cdot O_j(y_2) \end{array} \\ \circlearrowright \end{array} = \sum_K C_{ijk}(x_{12}, z_2) \begin{array}{c} \circlearrowleft \\ \cdot O_k(x_3) \\ \circlearrowright \end{array}$$

↑ completely fixed.

CFT data

1) $\{\Delta_I, S_I\}$ 2 pt. functions

2) C_{IJK} 3 pt. functions

Constraint CFT data using associativity of OPE ①

$$\begin{array}{c} \circlearrowleft \\ \cdot O_i(x_1) \\ \cdot O_j(x_2) \\ \circlearrowright \end{array} = \sum_K C_{ijk}(x_{12}, \alpha_2) \cdot \begin{array}{c} \circlearrowleft \\ \cdot O_k(x_3) \\ \circlearrowright \end{array}$$

↑
completely fixed.

② associativity

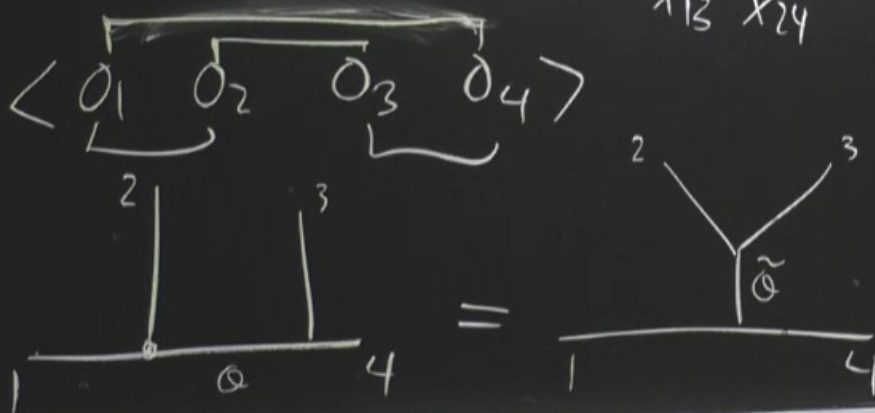
$$O_1(O_2 O_3) = (O_1 O_2) O_3$$

radius of convergence is
 $\min |x_2 - x_1|$

Main Focus: $\langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle = \frac{g(u, v)}{X_2^{2\Delta\phi} X_{34}^{2\Delta\phi}}$

$x_1 \leftrightarrow x_3$ $u = \frac{X_{12}^2 X_{34}^2}{X_{13}^2 X_{24}^2}$

$v = \frac{X_{14}^2 X_{23}^2}{X_{13}^2 X_{24}^2}$

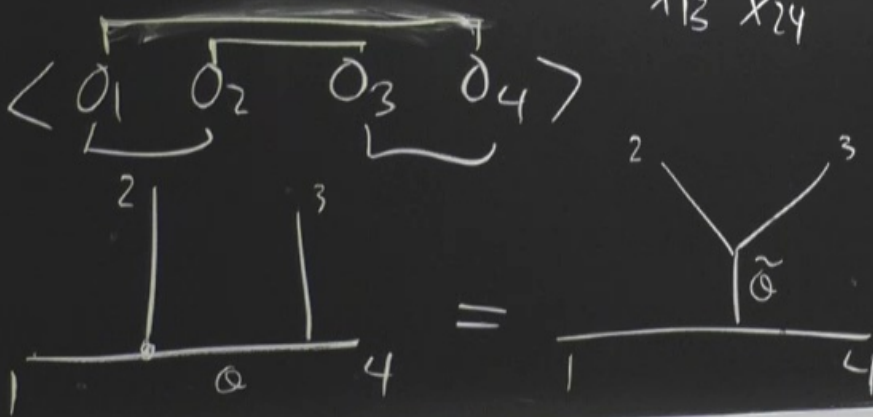


for identical ϕ 's $\{O = \phi\}$

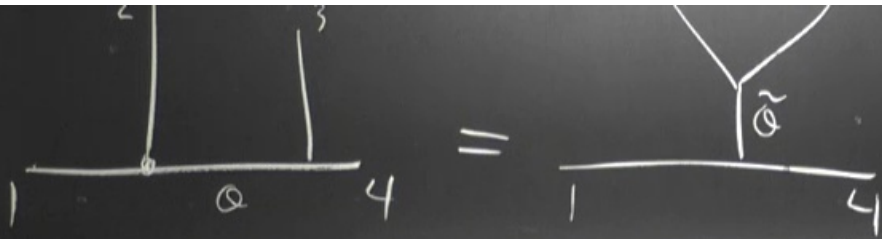
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$v = \frac{X_{14}^2 X_{23}^2}{X_{13}^2 X_{24}^2}$



for identical ϕ 's $\{O = SO\}$



$$\phi(x_1)\phi(x_2) = \frac{1}{\sigma} \langle \phi\phi\sigma \ C_a(x_{12}, \rho_2) \ \sigma^a(v_2) \ \frac{1}{\sigma} \rangle$$

$$= \sum_{\theta, \theta'} \langle \phi\phi\sigma \ C_{\phi\phi\theta'} \ C_a(x_{12}, \rho_2) \ C_b(x_{34}, \rho_4) \ \langle \theta^a(x_2) \ \theta'^b(x_4) \rangle \rangle$$

spin

$$= \frac{1}{\int_{X_{12}}^{2\Delta\phi} \int_{X_{34}}^{2\Delta\phi}} \sum_{\sigma} C_{\phi\phi\sigma}^2 \mathcal{G}_{\Delta\sigma, \rho_i}(X_i)$$

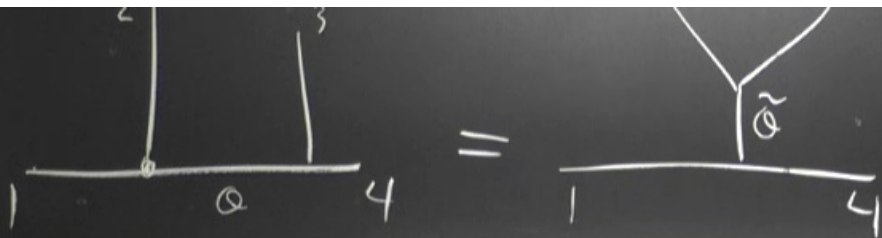
$$\int_{\theta, \theta'} \frac{I^{ab}(X_{24})}{X_{24}^{2\Delta\sigma}}$$

conformal block:

$$g_{\Delta, \ell} = X_{12}^{2\Delta} X_{34}^{2\Delta} C_a(X_{12}, \partial_2) C_b(X_{34}, \partial_4) \frac{\Gamma^{ab}(X_{24})}{X_{24}^{2\Delta}}$$

$$g(u, v) = \sum_{\Delta, \ell} C_{\Delta, \ell}^2 g_{\Delta, \ell}(u, v)$$

- $g_{\Delta, \ell}(u, v)$ are harmonics of $SO(1, D+1)$



$$\phi(x_1)\phi(x_2) = \frac{1}{\sigma} \langle \phi\phi\sigma \ C_a(x_{12}, \sigma_2) \ \sigma^a(v_2) \ \frac{1}{\sigma} \rangle$$

$$= \sum_{\sigma, \sigma'} C_{\phi\phi\sigma} C_{\phi\phi\sigma'} C_a(x_{12}, \sigma_2) C_b(x_{34}, \sigma_4) \langle \sigma^a(x_{12}) \ \sigma'^b(x_{34}) \rangle$$

$$= \frac{1}{\int_{X_{12}}^{2\Delta\phi} \int_{X_{34}}^{2\Delta\phi}} \sum_{\sigma} C_{\phi\phi\sigma}^2 \int_{\Delta\sigma, l_0}^g(X_i)$$

$\int_{\sigma, \sigma'} \frac{I^{ab}(X_{24})}{X_{24}^{2\Delta\sigma}}$

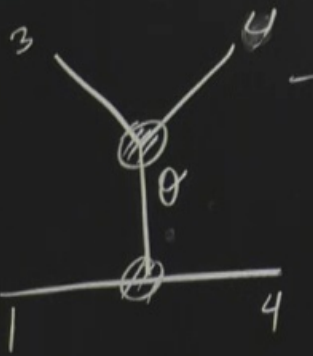
Correspondence

$$g_{\Delta, \ell} = X_{12}^{2\Delta\phi} X_{34}^{2\Delta\phi} C_a(X_{12}, \partial_2) C_b(X_{34}, \partial_4) \frac{\Gamma^{ab}(X_{24})}{X_{24}^{2\Delta\theta}}$$

$$g(u, v) = \sum_{\alpha} C_{\phi\theta}^2 g_{\Delta\theta, \ell\alpha}(u, v)$$

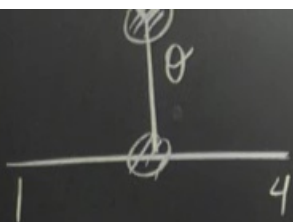
- $g_{\Delta\theta, \ell\alpha}(u, v)$ are harmonics of $SO(1, D+1)$

\Rightarrow associativity



$1 \leftrightarrow 3$

$$= \frac{g(\delta, u)}{X_{23}^{2\Delta\phi} X_{14}^{2\Delta\phi}}$$



$$= \frac{g(\sigma, u)}{X_{23}^{2\Delta\phi} X_{14}^{2\Delta\phi}}$$

$$g(u, \sigma) - \left(\frac{u}{\nu}\right)^{\Delta\phi} g(\sigma, u) = 0$$

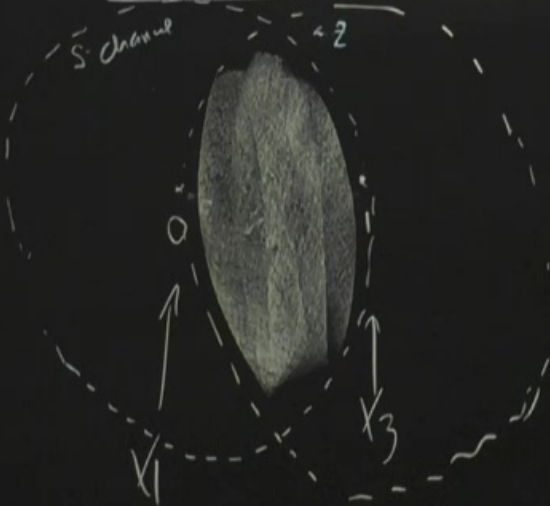
⇒ 1) Solve for all critical phenomena in $D=2$

2) Carry out disallowed CFT data domains.

Δ_r, Δ_E critical exponents of the $D=3$ Ising model

$x_2 = z$

$$g(u, v) - \left(\frac{u}{v}\right)^{\Delta\phi} g(\sigma_1 u) = 0$$



\Rightarrow 1) Solve for all critical phenomena in $D=2$

2) Carve out disallowed CFT data domains.

Δ_r, Δ_E critical exponents of the $D=3$ Ising model

\uparrow
 x_4

Anomalies

Cont symmetry $\Rightarrow \partial_\mu j^\mu = 0 \quad Q = \int j^0$

In violation of a conservation law

$$\partial^\mu j_\mu = 0(\hbar)$$

In particular \exists Hooft anomalies, anomaly described by a topological invariant

UV QFT $A_{UV} = \text{Hooft anomaly}$



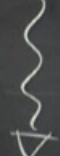
IR theory

AIR

$$A_{UV} = A_{IR}$$

In particular \rightarrow 2 Hooft anomalies, anomaly described by a topological invariant

• UV QFT



• IR theory

$A_{UV} = \text{2 Hooft anomaly}$

$$A_{UV} = A_{IR}$$

AIR

Conformal anomaly in $D=2$.

$$T^M_M = 0$$

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle = P_{\mu\rho} P_{\nu\sigma} F_1(p^2) + 4 \text{ terms}$$

translational inv $P^\mu T_{\mu\nu} = 0$

$$(\eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\rho}) G(p^2)$$

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle = \frac{1}{p^2} \left[\Pi_{\mu\rho} \Pi_{\nu\sigma} + \Pi_{\mu\sigma} \Pi_{\nu\rho} - \frac{2}{D-1} \Pi_{\mu\nu} \Pi_{\rho\sigma} \right] F(p^2)$$

- symmetric
- transverse

$$\Pi_{\mu\nu} = P_\mu P_\nu - \eta_{\mu\nu} P^2$$

$$\text{In } D=2 \quad \Pi_{\mu\nu} = \hat{P}_\mu \tilde{P}_\nu \quad \tilde{P}_\mu = \epsilon_{\mu\alpha} P^\alpha$$

$D=2$

$$= c \frac{\hat{P}_\mu \hat{P}_\nu \hat{P}_\rho \hat{P}_\sigma}{p^2}$$

- Dimensional analysis

$$\langle T_{\mu\nu}^M(p) T_{\rho\sigma}(-p) \rangle = c \tilde{P}_\mu \tilde{P}_\sigma$$

X12

X34

X24

$$\langle T_{\mu\nu}(p) T_{\rho\sigma}(-p) \rangle = \frac{1}{p^2} \left[\Pi_{\mu\rho} \Pi_{\nu\sigma} + \Pi_{\mu\sigma} \Pi_{\nu\rho} - \frac{2}{D-1} \Pi_{\mu\nu} \Pi_{\rho\sigma} \right] F(p^2)$$

- symmetric
- transverse

$$\Pi_{\mu\nu} = P_\mu P_\nu - \eta_{\mu\nu} P^2$$

In $D=2$ $\Pi_{\mu\nu} = \hat{P}_\mu \tilde{P}_\nu$ $\hat{P}_\mu = \epsilon_{\mu\alpha} P^\alpha$

$D=2$

$$= c \frac{\hat{P}_\mu \hat{P}_\nu \hat{P}_\rho \hat{P}_\sigma}{p^2}$$

- Dimensional analysis

$$\langle T_{\mu\nu}^M(p) T_{\rho\sigma}(-p) \rangle = c \hat{P}_\mu \hat{P}_\sigma \Rightarrow \text{encountered an irreducible anomaly}$$

X12

X34

Q

X24

$$\langle T_{\mu}^{\nu}(p) T_{\rho\sigma}(-p) \rangle = c \tilde{P}_{\rho} \tilde{P}_{\sigma} \Rightarrow \text{encountered an irreducible anomaly}$$

$$\langle T_{\mu}^{\nu}(x) T_{\rho\sigma}(0) \rangle = c (\partial_{\rho} \partial_{\sigma} - \eta_{\rho\sigma} \square) \delta^2(x)$$

couple the CFT to a background

$$\langle T_{\mu}^{\nu}(0) \rangle_h = \langle T_{\mu}^{\nu}(0) e^{\int h_{\rho\sigma} T^{\rho\sigma}} \rangle$$

$$= c \int d^2x \langle T_{\mu}^{\nu}(0) T_{\rho\sigma}(x) \rangle h^{\rho\sigma}(x) + \dots$$

$$\langle T_{\mu}^{\mu}(0) \rangle_h = c \underbrace{(2\partial\partial - \square \eta_{\rho\sigma}) h_{\rho\sigma}}$$

$$R \Big|_{\eta_{\mu\nu} + h_{\mu\nu}}$$

$D=2$

$$\langle T_{\mu}^{\mu} \rangle = c \cdot R.$$

D

mschl

x4

$$\langle T_{\mu}^{\mu}(0) \rangle_h = c \underbrace{(2\partial\partial_0 - \square \eta_{\rho\sigma}) h_{\rho\sigma}}$$

$D=2$	$c_W > c_{IR}$
$D=4$	$a_{UV} > a_{IR}$

$$R \left| \begin{matrix} \nearrow \\ \eta_{\mu\nu} + h_{\mu\nu} \end{matrix} \right.$$

CFT_{UV}
CFT_{IR}

$D=2$

$$\langle T_{\mu}^{\mu} \rangle = c \cdot R$$

\Rightarrow c-theorems

$D=3$

$T_{\mu}^{\mu} = 0 \Rightarrow$ ~~\neq~~ no conformal anomaly in odd D .

$$\int_{\mathcal{S}} \text{Scal} = \int d^4x \sqrt{g} R^2$$

$D=4$

$$T_{\mu}^{\mu} = a \text{ Weyl}^2 - c \text{ Euler} + \cancel{DR}$$

$\Delta_{\sigma}, \Delta_{\epsilon}$ critical exponents of the $D=3$ Ising model

$$\langle T_{\mu}^{\mu}(0) \rangle_h = c \underbrace{(\partial\rho\partial\sigma - \square\eta_{\rho\sigma}) h_{\rho\sigma}}$$

$$R \Big|_{\eta_{\mu\nu} + h_{\mu\nu}}$$

CFT_{UV}
CFT_{IR}

D=2	$c_W > c_{IR}$
D=4	$a_{UV} > a_{IR}$

$D=2$ $\langle T_{\mu}^{\mu} \rangle = c \cdot R \Rightarrow c$ -theorems

$D=3$ $T_{\mu}^{\mu} = 0 \Rightarrow \nexists$ no conformal anomaly in odd D.

$$\int_{\mathcal{S}} S_{\text{ct}} = \int d^4x \sqrt{g} R^2$$

$D=4$ $T_{\mu}^{\mu} = a \text{ Weyl}^2 - c \text{ Euler} + \frac{DR}{2} + \alpha R_{\mu\nu\rho\sigma}^2 + \beta R_{\mu\nu}^2$

$\Delta_{\sigma}, \Delta_{\epsilon}$ critical exponents of the D=3 Ising model