Title: Quantum Fields and Strings Lecture - 230313

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Collection: Quantum Fields and Strings (2022/2023)

Date: March 13, 2023 - 10:15 AM

URL: https://pirsa.org/23030020

Conformal symmetry completely fixes position dependence of 2 and 3pt functions.

e.g. scalar operators

$$x_{ij} = |x_i - x_j|$$

$$\langle \mathcal{O}_I(x_1)\, \mathcal{O}_J(x_2) \rangle = \frac{\delta_{IJ}}{x_{12}^{\Delta_I}}$$

$$\langle \mathcal{O}_I(x_1)\, \mathcal{O}_J(x_2)\, \mathcal{O}_K(x_3) \rangle = \frac{C_{IJK}}{x_{12}^{\Delta_1+\Delta_2-\Delta_3}\, x_{23}^{\Delta_2+\Delta_3-\Delta_1}\, x_{13}^{\Delta_1+\Delta_3-\Delta_2}}$$

$\Rightarrow$ CFT data $\{\Delta_I\}, \{C_{IJK}\}$

Δ_I: scaling dimensions

C_{IJK}: structure constants

$x^\mu = 0$ is a fixed point of D and $M_{\mu\nu}$

$\theta(x)$ primary

$\downarrow P_\mu$

$\partial_\mu \theta(x)$, this is not annihilated by k_μ at $x^\mu = 0$

Conformal symmetry completely fixes position dependence of 2 and 3pt functions.
e.g. scalar operators

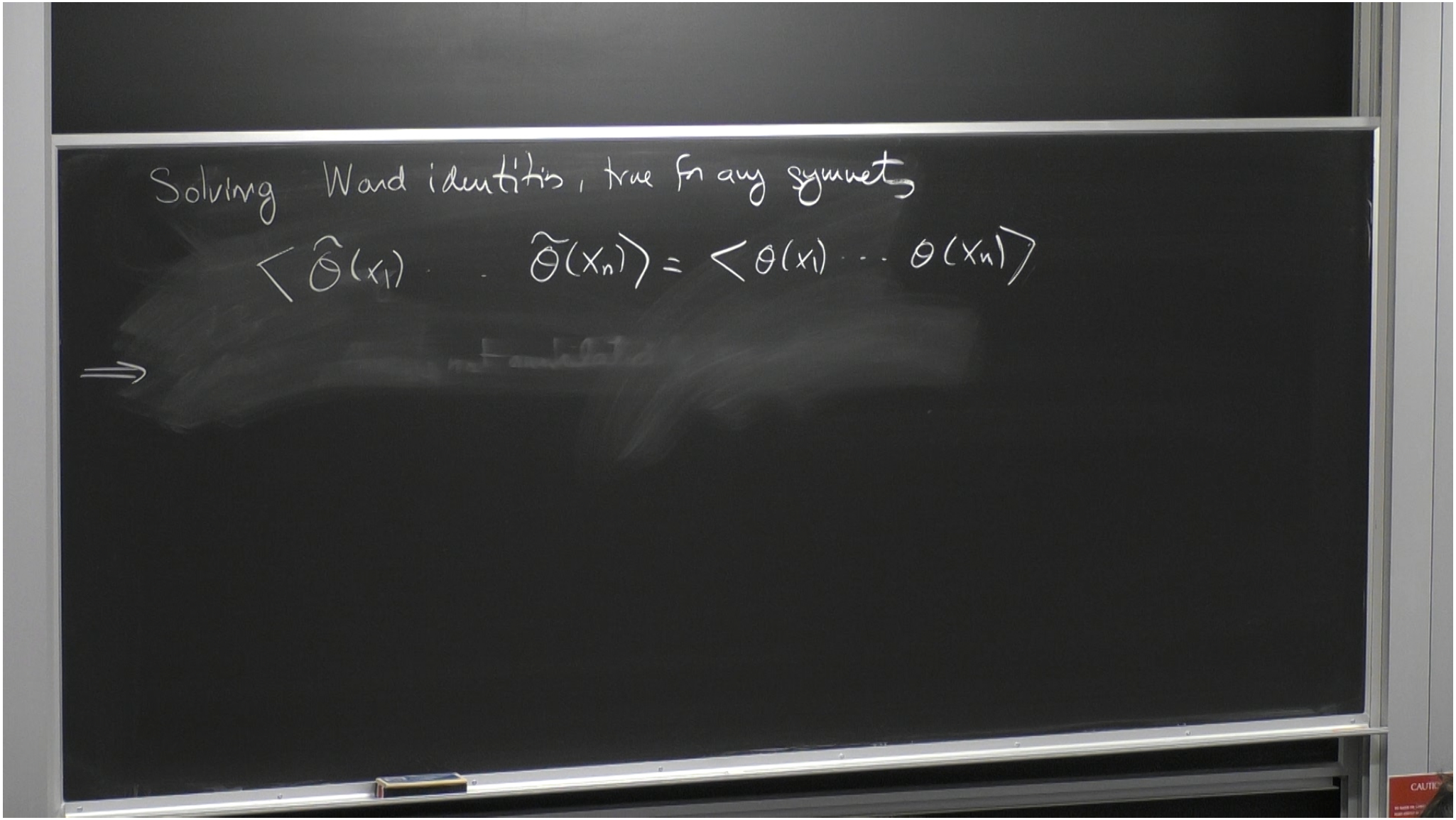
$$\langle \mathcal{O}_I(x_1)\mathcal{O}_J(x_2)\rangle = \frac{\delta_{IJ}}{x_{12}^{\Delta_I}} \qquad x_{ij} = |x_i - x_j|$$

$$\widetilde{\mathcal{O}}(x) = \lambda^{-\Delta}\mathcal{O}(\lambda^{-1}x)$$

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Solving Ward identities, true for any symmetry
$\langle \widehat{\theta}(x_1) \cdots \widetilde{\theta}(x_n) \rangle = \langle \theta(x_1) \cdots \theta(x_n) \rangle$
$\Rightarrow$

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e.g. scalar operators

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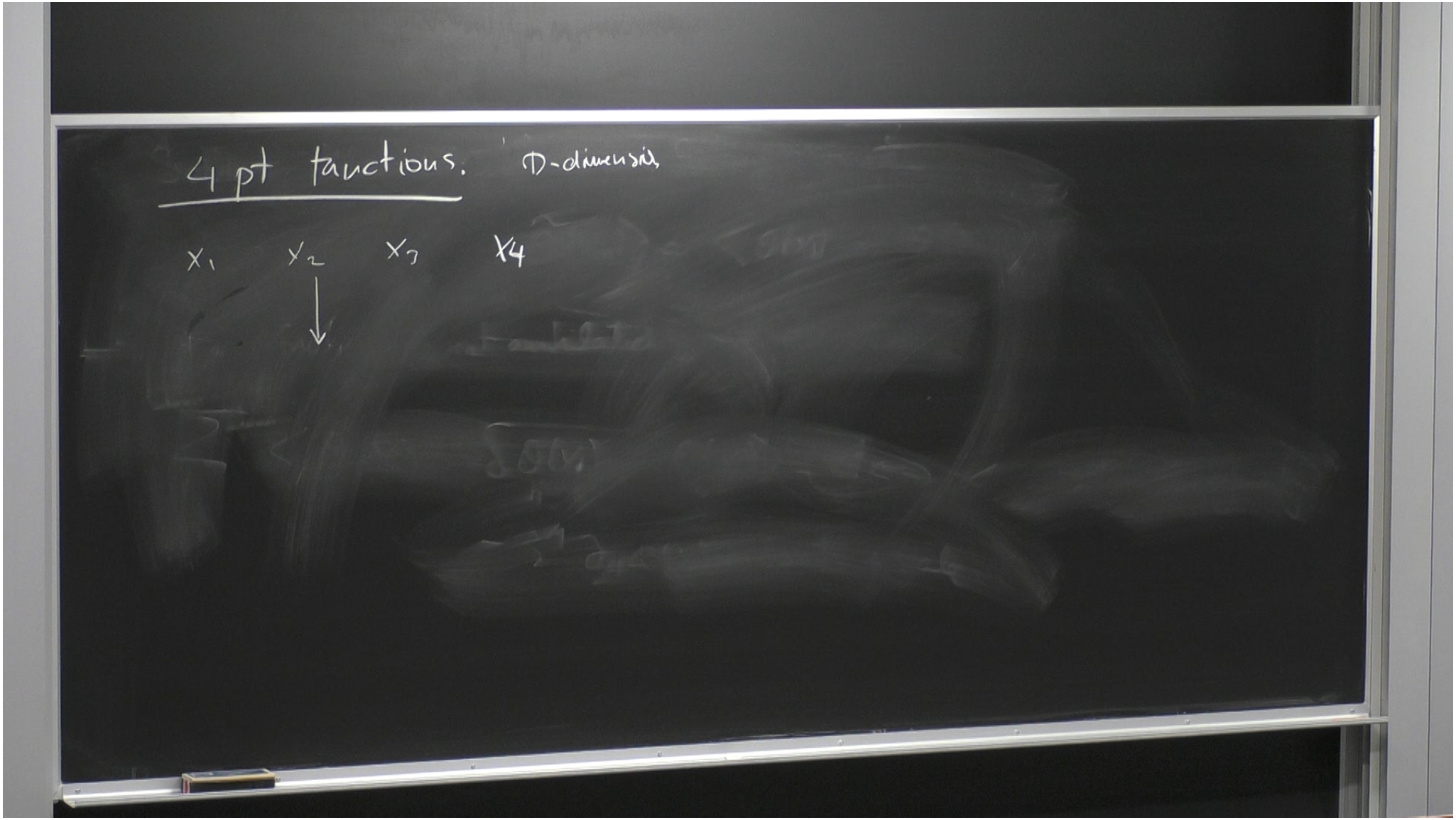
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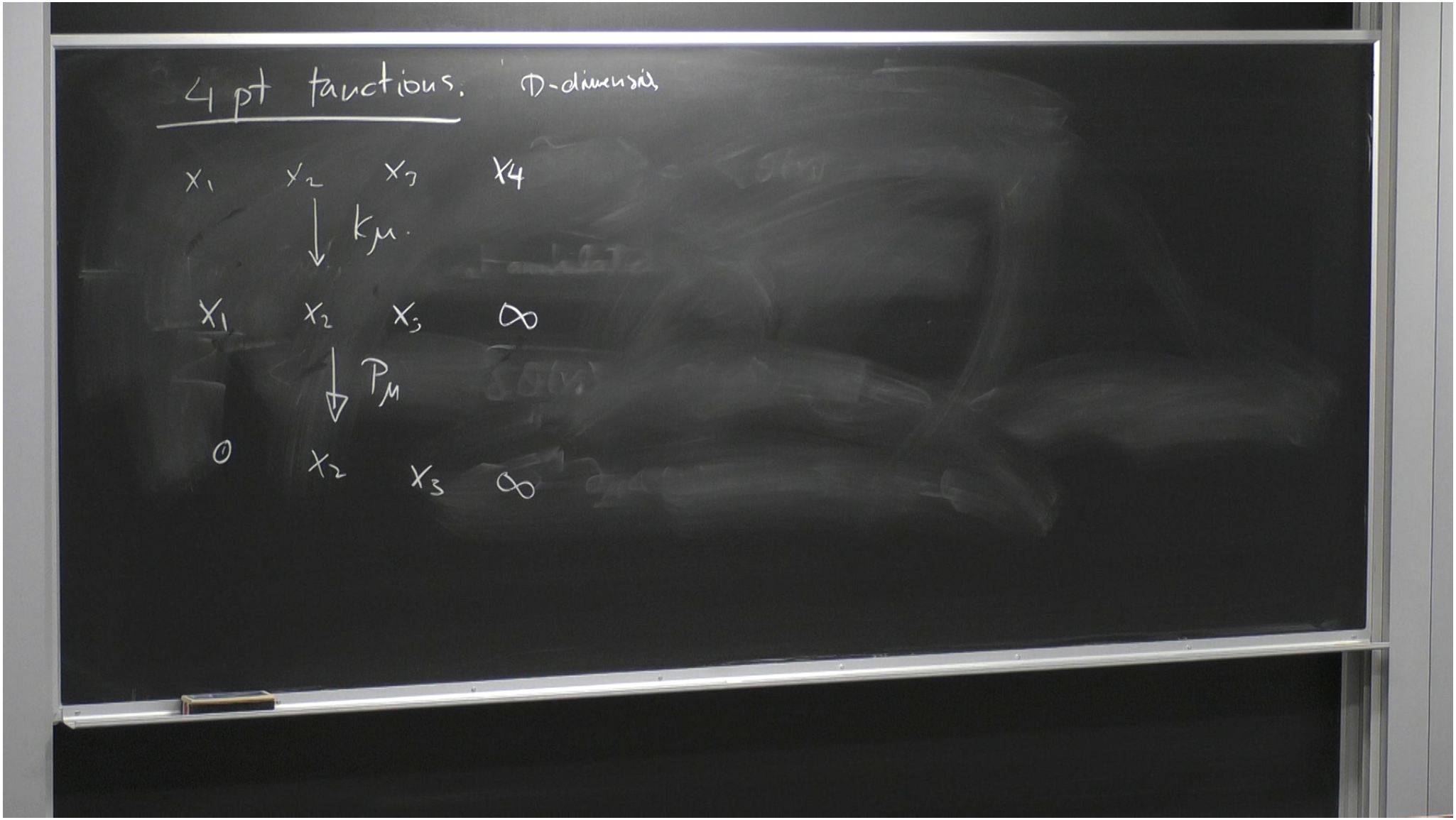
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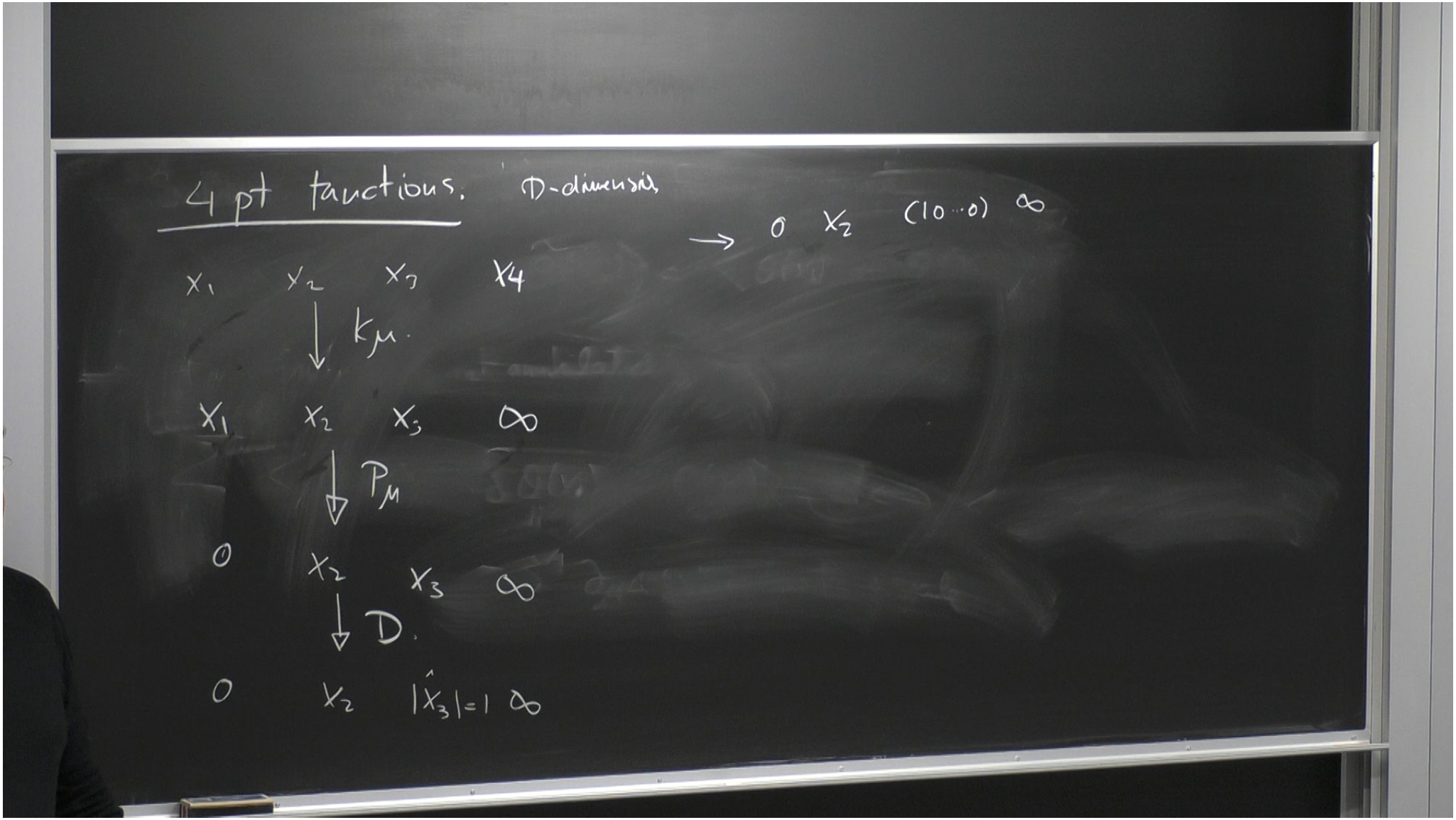
Δ_I: scaling dimensions

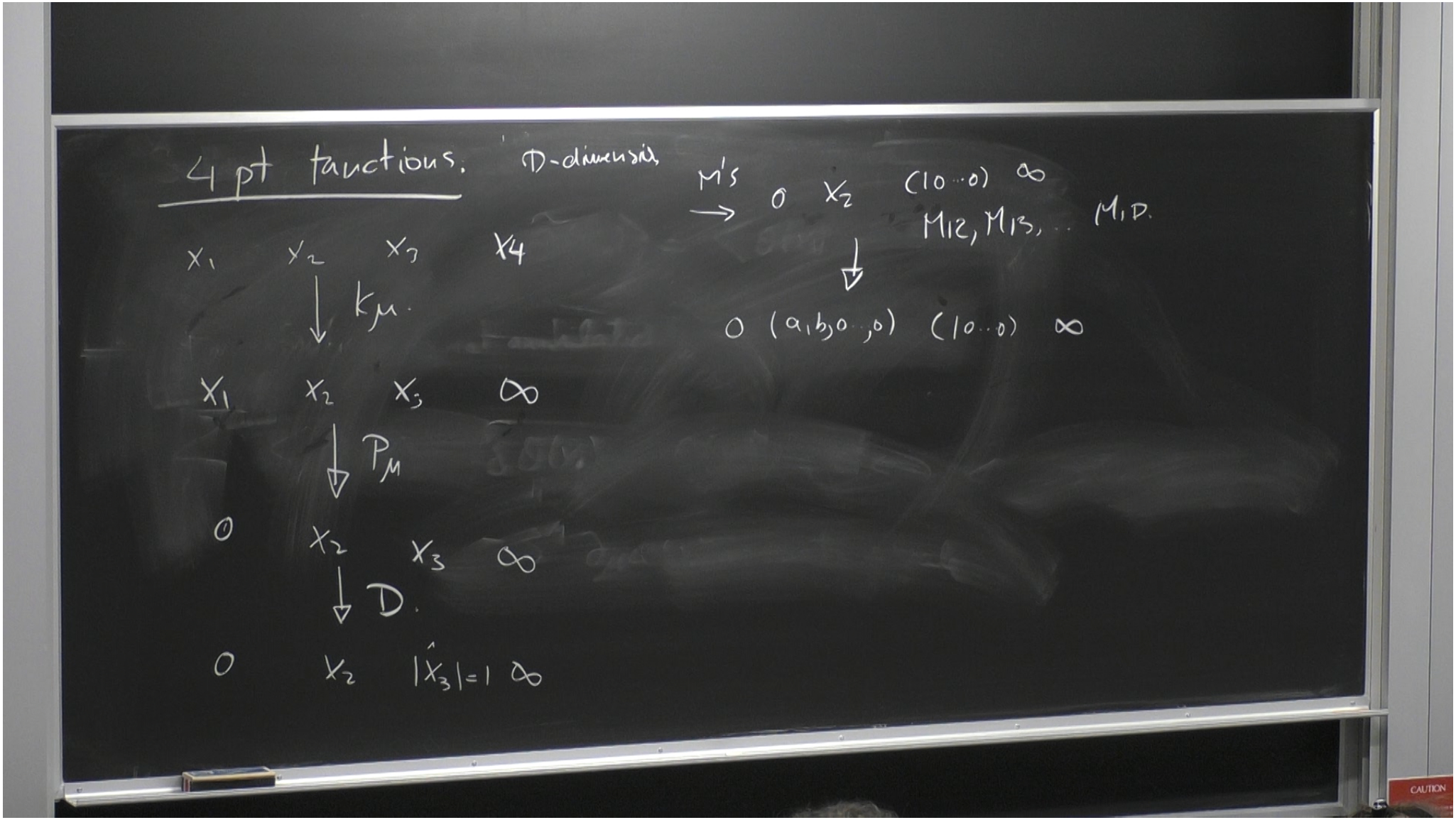
C_{ijk}: structure constants

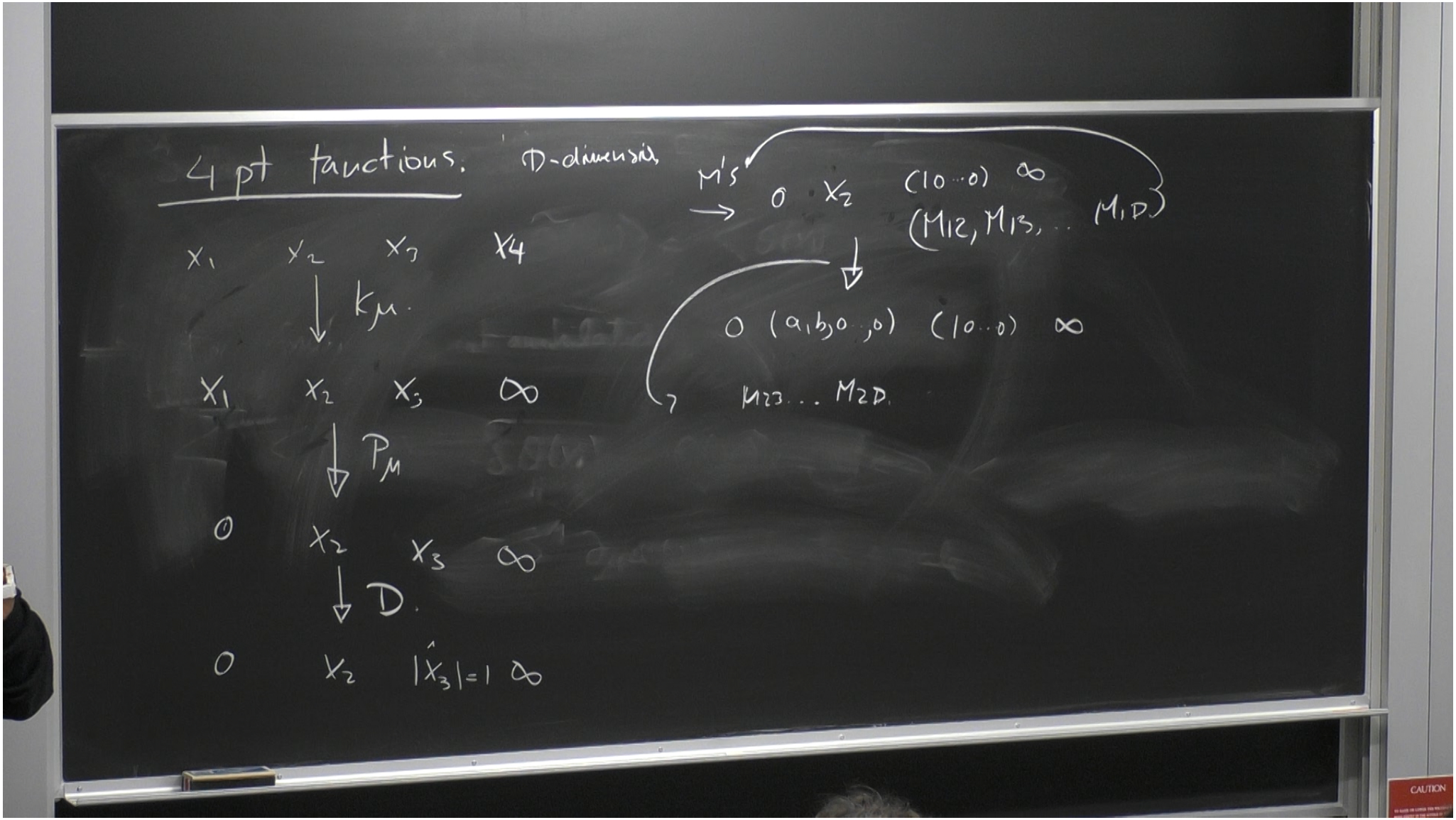

4 pt functions. D-dimensi
x_1 x_2 x_3 x_4

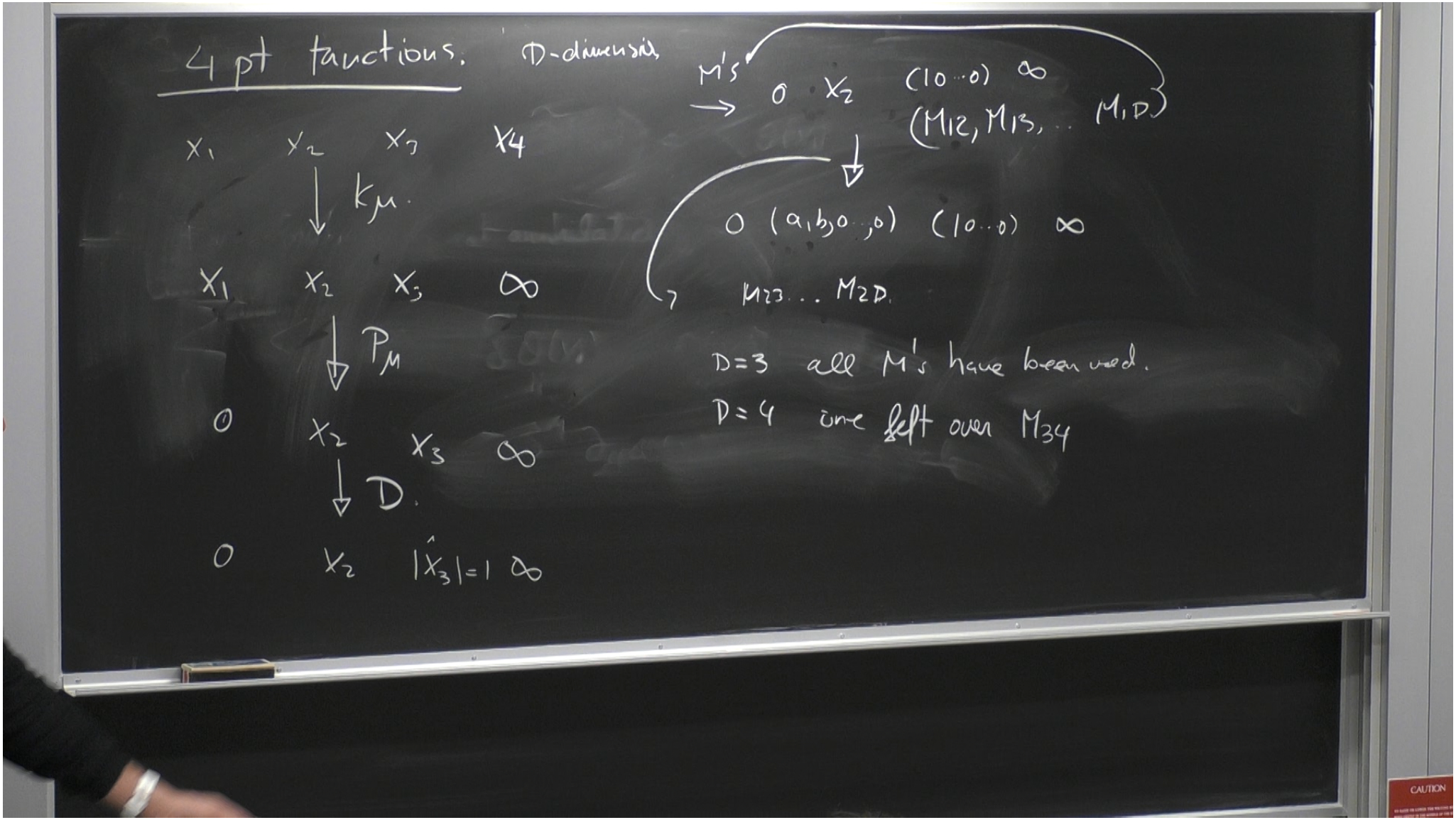

4 pt functions. D-dimensio
x_1 x_2 x_3 x_4
k_μ.
x_1 x_2 x_3 ∞

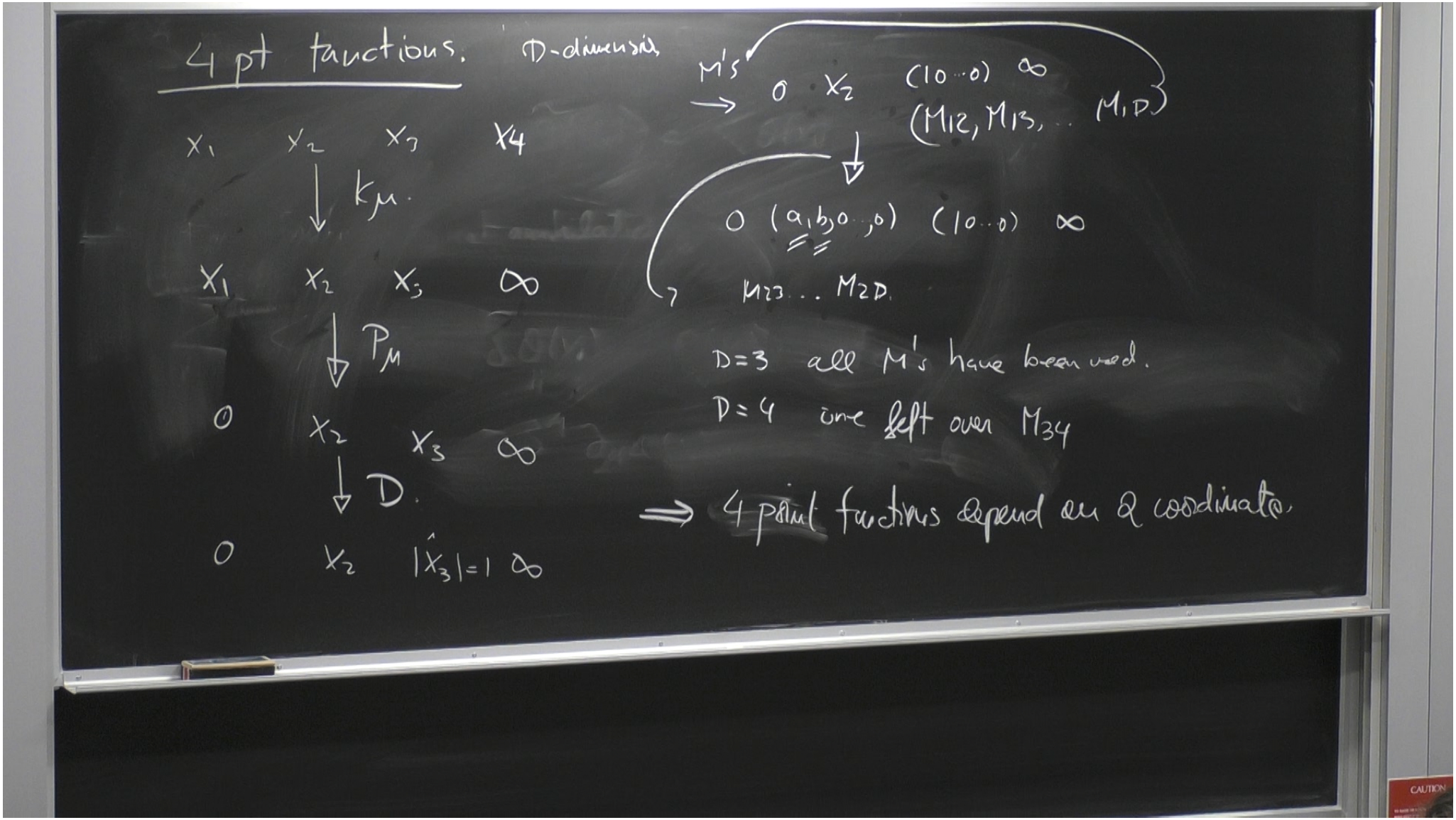

4 pt functions. D-dimensional
x_1 x_2 x_3 x_4
k_μ
x_1 x_2 x_3 ∞
P_μ
0 x_2 x_3 ∞

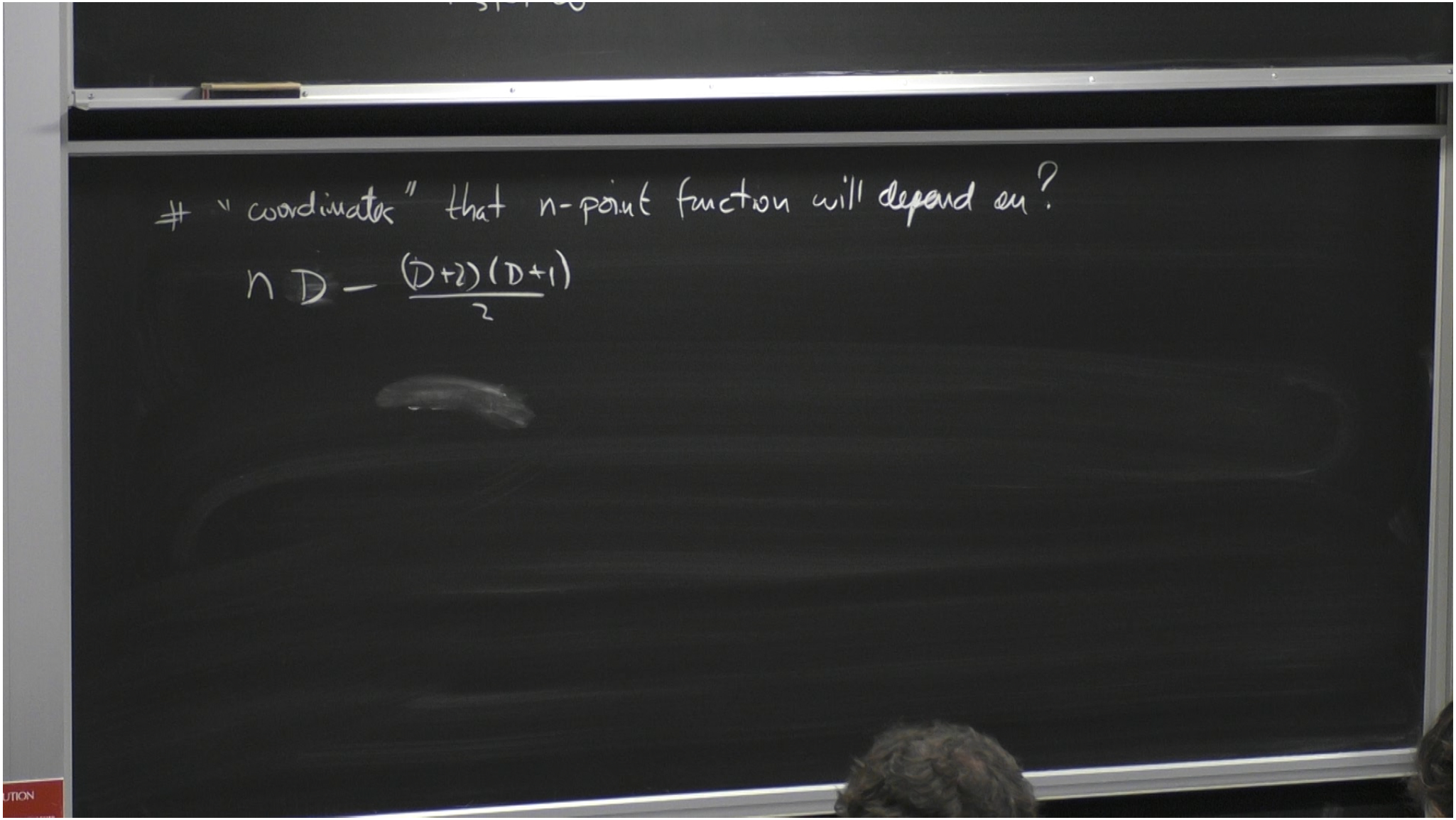

4 pt functions. D-dimensional
→ 0 x_2 (10⋯0) ∞
x_1 x_2 x_3 x_4
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x_1 x_2 x_3 ∞
P_μ
0 x_2 x_3 ∞
D.
0 x_2 $|\hat{x}_3|=1$ ∞


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M's
0 x_2 (10...0) ∞
$M_{12}, M_{13}, \ldots M_{1D}$.
0 (a,b,0...,0) (10...0) ∞


4 pt functions. D-dimensional
M's
x_1 x_2 x_3 x_4
k_μ.
x_1 x_2 x_3 ∞
P_μ
0 x_2 x_3 ∞
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0 x_2 $|\hat{x}_3|=1$ ∞
0 x_2 $(10\cdots0)$ ∞
$(M_{12}, M_{13}, \ldots M_{1D})$
0 $(a,b,0\ldots,0)$ $(10\cdots0)$ ∞
$M_{23}\ldots M_{2D}$


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$M_{23}\ldots M_{2D}$
D=3 all M's have been used.
D=4 one left over M_{34}


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0 $(a,b,0\ldots,0)$ $(10\cdots0)$ ∞
$M_{23}\ldots M_{2D}$
D=3 all M's have been used.
D=4 one left over M_{34}
$\Rightarrow$ 4 point functions depend on 2 coordinates.


"coordinates" that n-point function will depend on?
n D − (D+2)(D+1)/2

"coordinates" that n-point function will depend on?

$$n\,D - \frac{(D+2)(D+1)}{2}$$

Define cross ratios or conformal ratios

$$u = \frac{X_{12}^2 X_{34}^2}{X_{13}^2 X_{24}^2} \qquad v = \frac{X_{14}^2 X_{23}^2}{X_{13}^2 X_{24}^2}$$

Most general 4 pt function of identical scalar operator.

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}}$$

- $g(u,v)$ is not an independent function.

Most general 4 pt function of identical scalar operator:

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta_\phi}\, x_{34}^{2\Delta_\phi}}$$

- $g(u,v)$ is not an independent function. (determined by Δ_I, C_{IJK}).

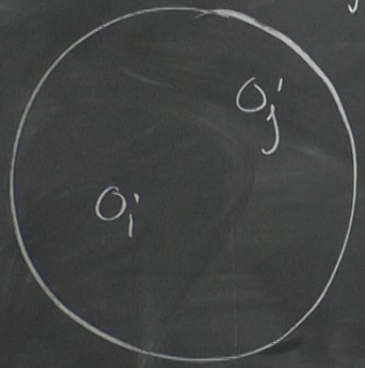
- $\langle \cdots \rangle$ constraints Δ_I, C_{IJK}.

scaling dimensions

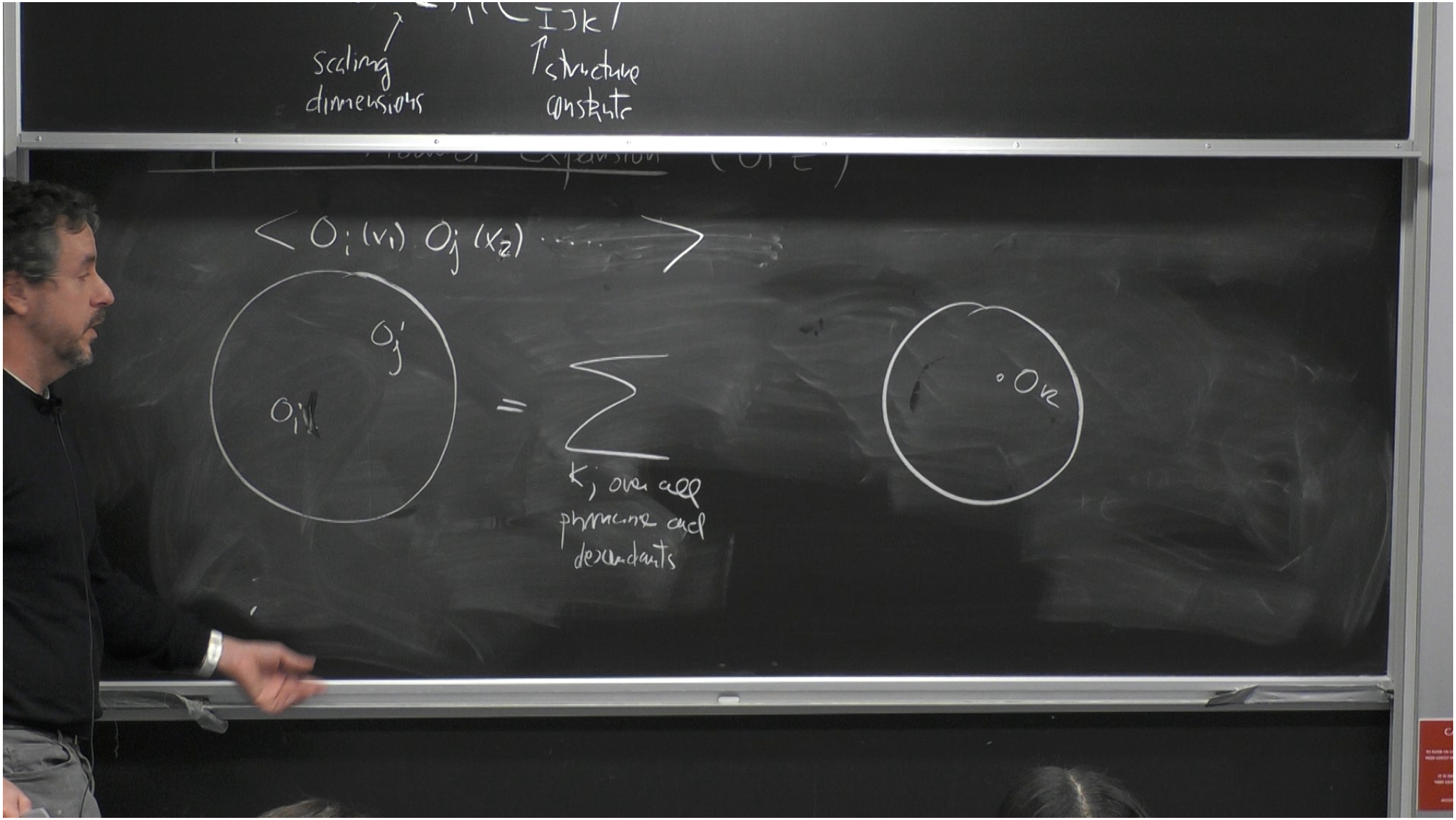
C_{IJK} ← structure constants

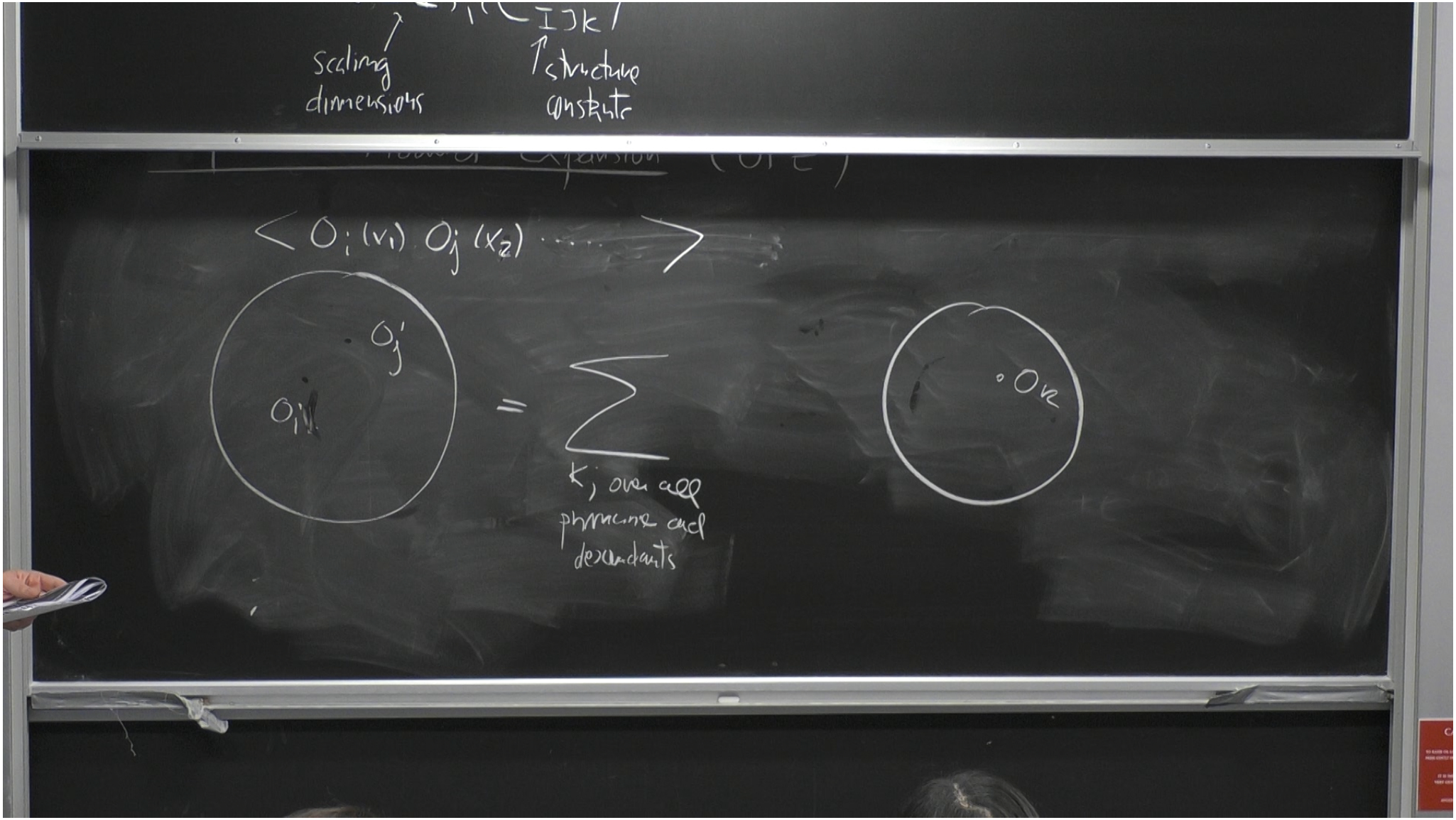
Operator Product Expansion (OPE)

$\langle O_i(x_1)\, O_j(x_2) \cdots \rangle$



$=$


scaling dimensions
structure constants
C_{IJK}
$\langle O_i(x_1)\, O_j(x_2) \cdots \rangle$
O_i
O_j
O_k
$=$
$\sum$
k, over all primaries and descendants


scaling dimensions
structure constants
$\langle O_i(x_1)\, O_j(x_2) \cdots \rangle$
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$\Rightarrow$ CFT data $\{\Delta_I\}, \{C_{IJK}\}$

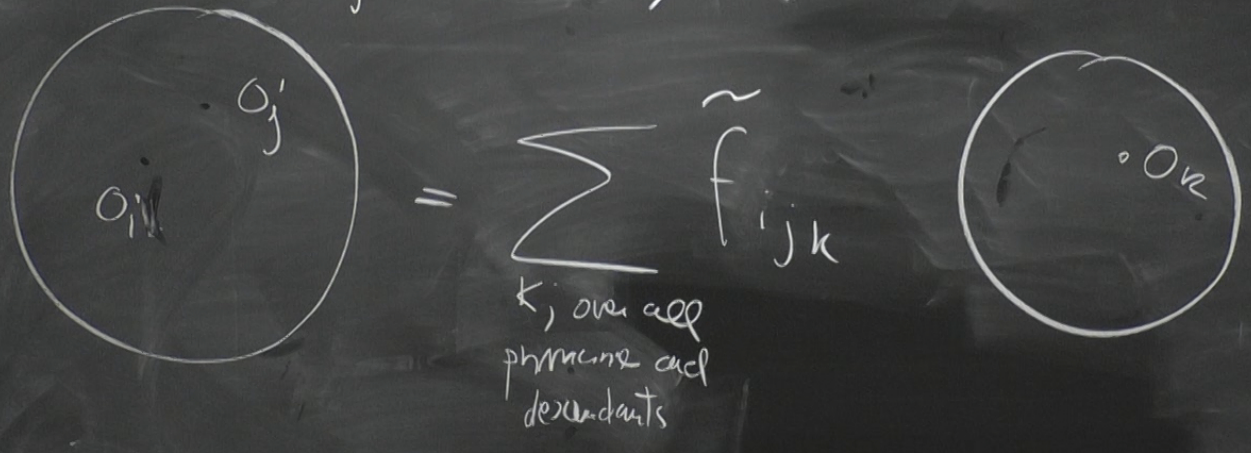
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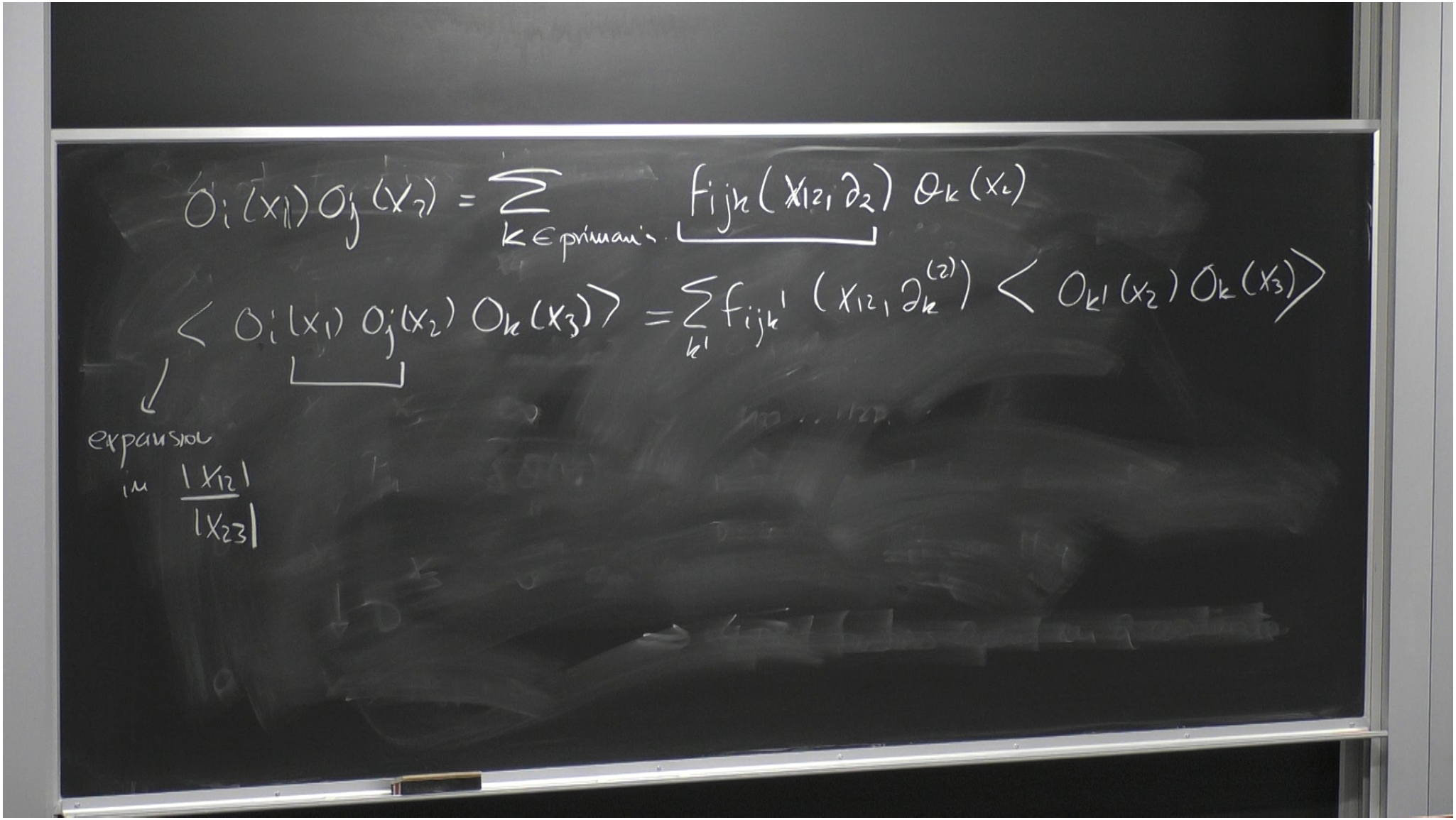
structure constants

x_{12} x_{23} x_{13}

$\Delta_1+\Delta_2-\Delta_3$

$\langle O_i(x_1)\, O_j(x_2) \cdots \rangle$




$O_i(x_1) O_j(x_2) = \sum_{k \in \text{primaries}} f_{ijk}(x_{12}, \partial_2) O_k(x_2)$
$\langle O_i(x_1) O_j(x_2) O_k(x_3) \rangle = \sum_{k'} f_{ijk'}(x_{12}, \partial_k^{(2)}) \langle O_{k'}(x_2) O_k(x_3) \rangle$
expansion in $\frac{|x_{12}|}{|x_{23}|}$

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$$= f_{ijk}(x_{12}, \partial_k^{(2)}) \frac{1}{x_{23}^{2\Delta_k}}$$

expansion in $\frac{|x_{12}|}{|x_{23}|}$

$$f_{ijk} = C_{ijk} \cdot (1 +$$

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expansion in $\frac{|x_{12}|}{|x_{23}|}$

$$f_{ijk} = C_{ijk} \cdot (1 + \alpha x^\mu \partial_\mu + \beta\, x^\mu x^\nu \partial_\mu \partial_\nu + \dots)$$

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