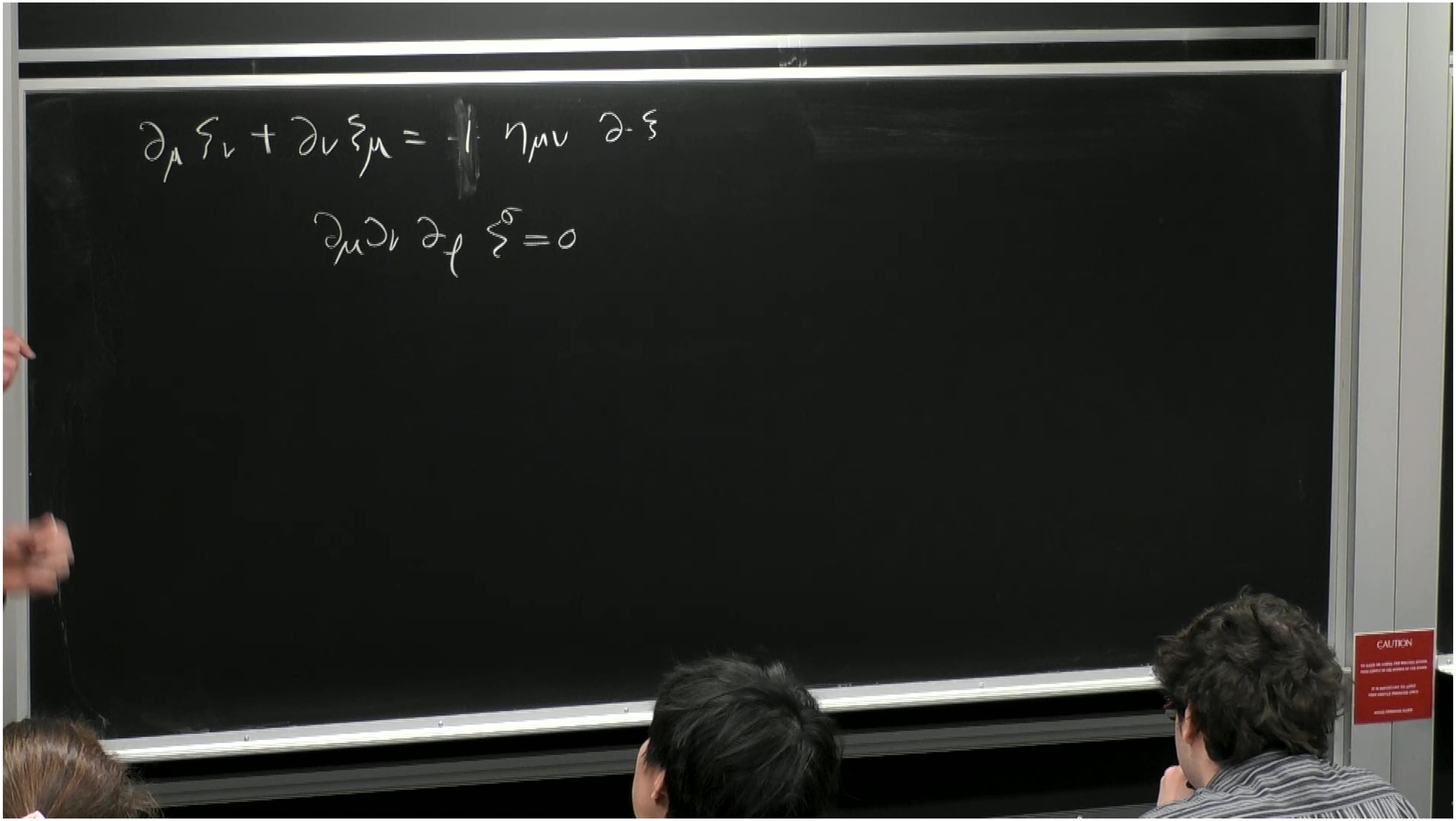
Title: Quantum Fields and Strings Lecture - 230310

Speakers: Jaume Gomis

Collection: Quantum Fields and Strings (2022/2023)

Date: March 10, 2023 - 10:15 AM

URL: https://pirsa.org/23030019


$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \eta_{\mu\nu}\, \partial\cdot\xi$
$\partial_\mu \partial_\nu \partial_\rho \xi^\rho = 0$
CAUTION

1) Translations $\Rightarrow T_{\mu\nu}$ $\qquad \partial_\mu T_{\mu\nu} = 0$

2) Lorentz symmetry: $\quad T_{\nu\mu} = T_{\mu\nu}$

3) $\displaystyle\int T_{\mu\nu}\,\delta h^{\mu\nu}$ $\qquad$ - Diffeo: $\delta h^{\mu\nu} = \partial^\mu \xi^\nu + \partial^\nu \xi^\mu$

$$\int T_{\mu\nu}\left(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu\right)$$

2) Lorentz symmetry: $T_{\nu\mu} = T_{\mu\nu}$

3) $\int T_{\mu\nu}\, \delta h^{\mu\nu}$ — **Diffeo:** $\delta h^{\mu\nu} = \partial^\mu \xi^\nu + \partial^\nu \xi^\mu$

$$\int T_{\mu\nu}\left(\partial^\mu \xi^\nu + \partial^\nu \xi^\mu\right) = \int T_\mu{}^\mu \,(\partial \cdot \xi)$$

$$\partial^\mu \xi^\nu + \partial^\nu \xi^\mu = \eta^{\mu\nu}(\partial\cdot\xi)$$

– **Dilations** $\xi \sim x$

- Dilations $\xi \sim x$

$$\Rightarrow \; T^{\mu}_{\mu} = \partial_\mu \boxed{L^{\mu}} \qquad \eta^{\mu\nu}(\partial\cdot\xi) \qquad \sim x$$

- Special conformal $\Rightarrow \; \xi \sim x^2$

$$\boxed{T^{\mu}_{\mu} = \partial_\mu \partial_\nu L^{\mu\nu}}$$

Improve $T_{\mu\nu}$ s.t. $\hat{T}_{\mu\nu} = T_{\mu\nu} + \Theta_{\mu\nu}$

$$T_{\mu\nu} = \partial_\mu \partial_\nu L^{\mu\nu}$$

s.t. $\boxed{\hat{T}^{\mu}_{\mu} = 0}$

$$\tilde{\mathcal{L}}(\phi) = \mathcal{L}(\phi, g) + R_{\mu\nu\rho\sigma}\,\Theta^{\mu\nu\rho\sigma}$$

$$\Downarrow$$

$$T_{\mu\nu}$$

$$\hat{T}_{\mu\nu}$$

Dilatations $\xi \sim x$ $\qquad \sim x$

$$\Rightarrow T^\mu_\mu = \partial_\mu \boxed{L^\mu} \qquad \eta^{\mu\nu}(\partial\cdot\xi)$$

- Special conformal $\Rightarrow \xi \sim x^2$

$$\boxed{T^\mu_\mu = \partial_\mu \partial_\nu L^{\mu\nu}}$$

Improve $T_{\mu\nu}$ s.t $\hat{T}_{\mu\nu} = T_{\mu\nu} + \Theta_{\mu\nu}$

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2$$

s.t $\boxed{\hat{T}^\mu_\mu = 0}$ $\qquad T_{\mu\nu} = \partial_\mu\partial_\nu L^{\mu\nu}$

$$\tilde{\mathcal{L}}(\phi) = \mathcal{L}(\phi, g) + R_{\mu\nu\rho\sigma}\,\Theta^{\mu\nu\rho\sigma}$$

$$T^\mu_\mu = \frac{1}{2}m^2\phi^2$$

$$\Downarrow$$

$$T_{\mu\nu}$$

$$T^\mu_\mu = \beta(g)\,F^2_{\mu\nu}$$

$$\underbrace{\phantom{T_{\mu\nu} + R_{\mu\nu\rho\sigma}\Theta^{\mu\nu\rho\sigma}}}_{\hat{T}_{\mu\nu}}$$

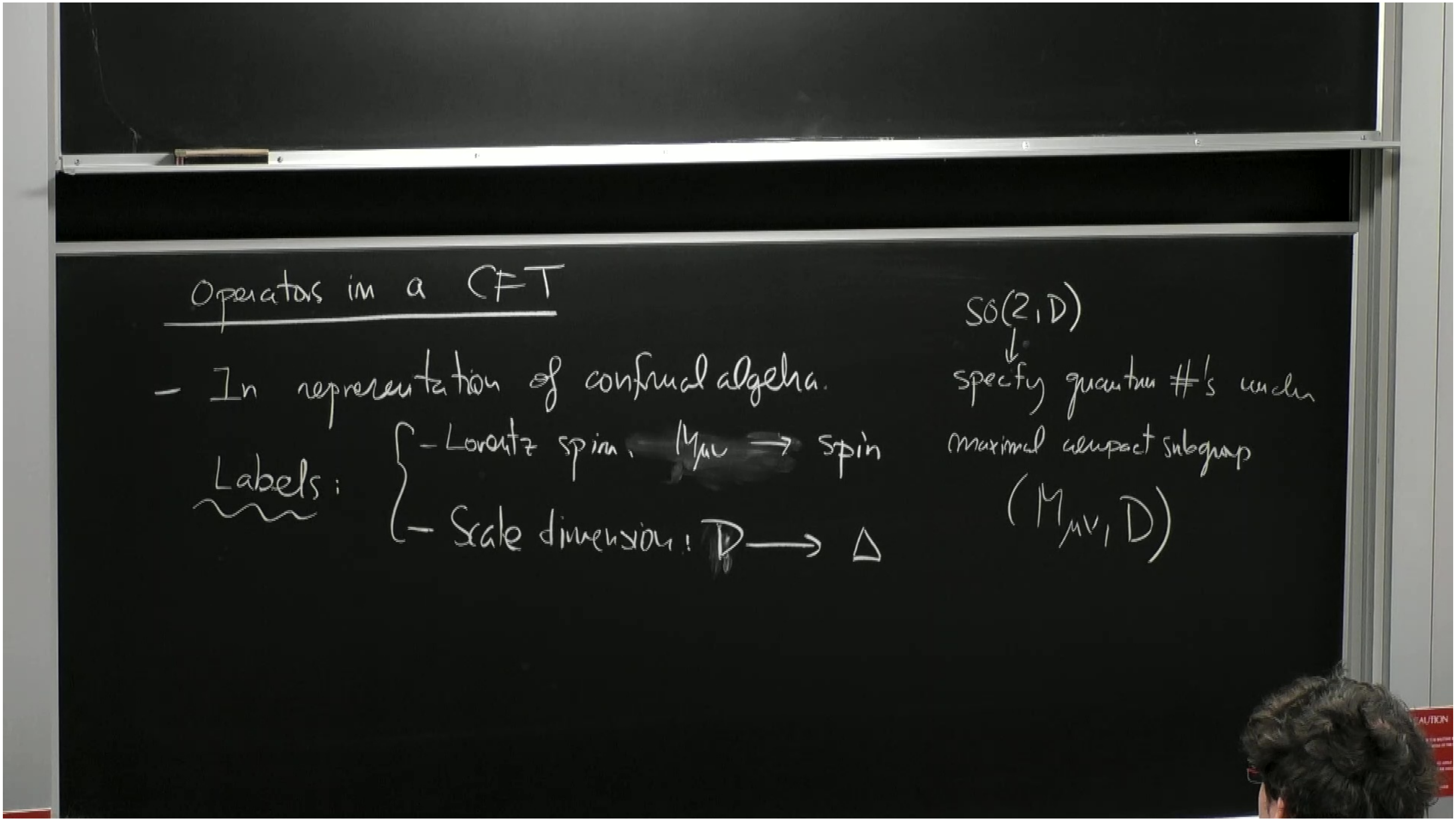
$x^\mu \to x^\mu + \xi^\mu$

$$\xi^\mu = \underbrace{a^\mu}_{P_\mu} + \underbrace{\omega^\mu{}_\nu x^\nu}_{M_{\mu\nu}} + \underbrace{\lambda x^\mu}_{D} + \underbrace{b^\mu x^2 - 2x^\mu b\cdot x}_{K_\mu}$$

Lie algebra $SO(2,D)$

- Inversion, $\mathbb{Z}_2$

- conformal group is $SO(2,D)$


Operators in a CFT
- In representation of conformal algebra.
Labels:
- Lorentz spin: $M_{\mu\nu} \to$ spin
- Scale dimension: $D \longrightarrow \Delta$
$SO(2,D)$
specify quantum #'s under maximal compact subgroup
$(M_{\mu\nu}, D)$

Operators in a CFT

- In representation of conformal algebra.

$SO(2,D)$ $\downarrow$ specify quantum #'s under maximal compact subgroup $(M_{\mu\nu}, D)$

Labels:
- Lorentz spin: $M_{\mu\nu} \longrightarrow$ spin
- Scale dimension: $D \longrightarrow \boxed{\Delta}$ $\leftarrow$ critical exponents

$M_{\mu\nu}, D$ preserve the origin.

$\mathcal{O}$ that has scale dim Δ.

$$[D, \mathcal{O}] = \Delta\, \mathcal{O}$$

$$[M_{\mu\nu}, \mathcal{O}] = S_{\mu\nu} \cdot \mathcal{O}$$

Operators split:

– Primary ops. (transform simply under)

– Descendants ops.

$$[K_\mu, \mathcal{O}] = 0 \iff \text{primary}$$

conformal family

$$\left\{ \begin{array}{l} \mathcal{O} \\ P_\mu \mathcal{O} \\ P_{\mu_1} P_{\mu_2} \mathcal{O} \\ \vdots \end{array} \right. \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{descendants}$$

$$[D, K_\mu] = -i K_\mu$$

$$[D, P_\mu] = i P_\mu$$

Primary Operators transform under conformal transformation:

$$x^\mu \longrightarrow \tilde{x}^\mu = (g\,x)^\mu \qquad g(a, \omega, \lambda, b)$$

$$\frac{\partial \tilde{x}}{\partial x} \frac{\partial \tilde{x}}{\partial x} \eta = e^{2\sigma} \eta .$$

$$R^T \eta R = \eta \qquad R = \frac{\partial \tilde{x}}{\partial x} e^{-\sigma}$$

$$\mathcal{O}_{A,\Delta}(x) \longrightarrow \tilde{\mathcal{O}}_{A,\Delta} = \left|\frac{\partial x}{\partial \tilde{x}}\right|^{\Delta/D} L_A^{\ B}(R(x))\, \mathcal{O}_B(g^{-1}x)$$

Lorentz spin. — Scaling dimension — spin transf. — orbital transf.

Primary operators transform under conformal transformation:

$$x^\mu \longrightarrow \tilde{x}^\mu = (g\,x)^\mu \qquad g(a,\omega,\lambda,b)$$

$$L(g_1)L(g_2) = L(g_1 g_2)$$

representation of Lorentz

$$\frac{\partial\tilde{x}}{\partial x}\frac{\partial\tilde{x}}{\partial x}\eta = e^{2\sigma}\eta.$$

$$R^T \eta R = \eta \qquad R = \frac{\partial\tilde{x}}{\partial x} e^{-\sigma}$$

$$\mathcal{O}_{A,\Delta}(x) \longrightarrow \tilde{\mathcal{O}}_{A,\Delta} = \left|\frac{\partial x}{\partial\tilde{x}}\right|^{\Delta/D} L_A^{\ B}(R(x))\, \mathcal{O}_B(g^{-1}x)$$

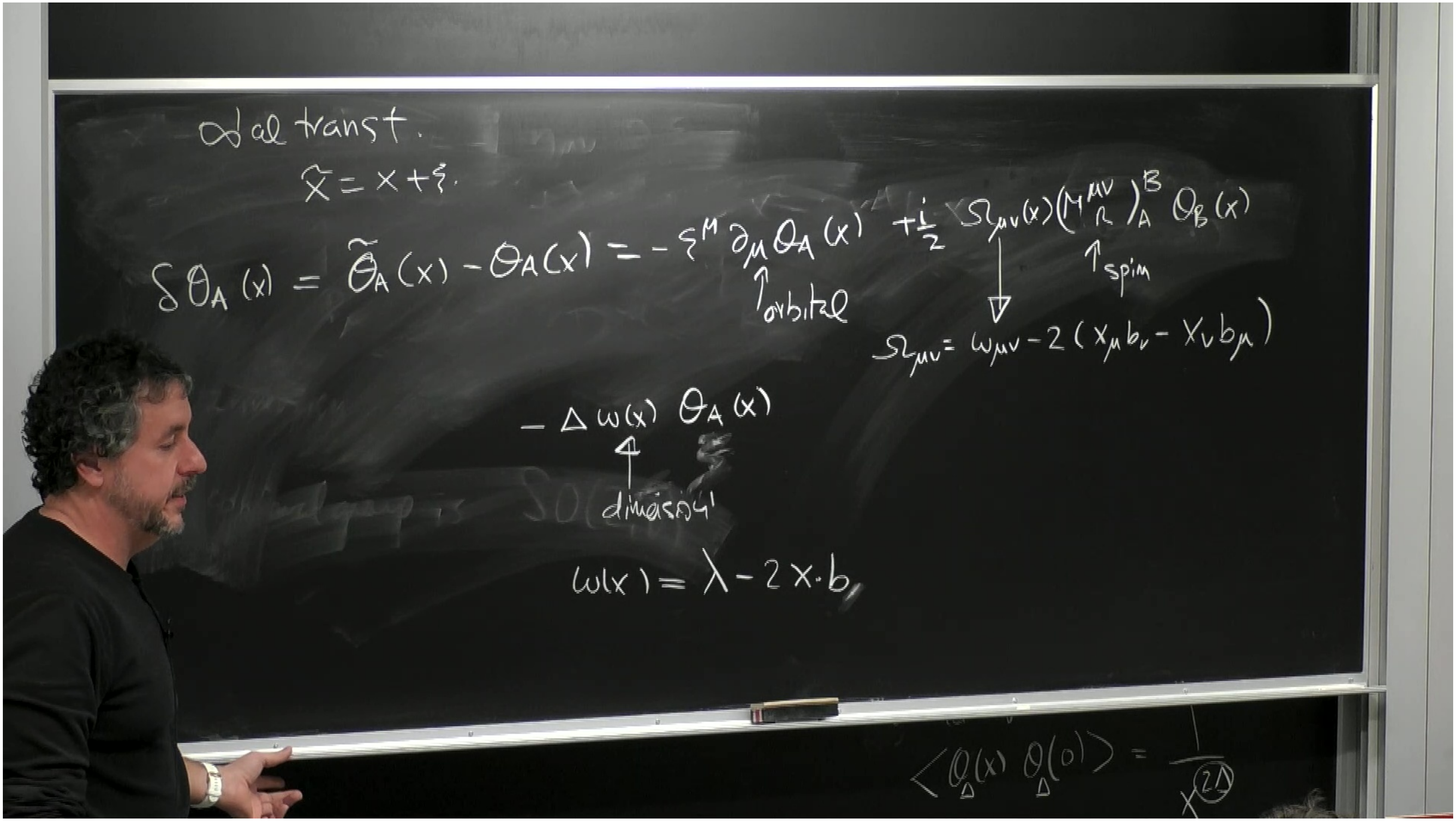
Lorentz spin

scaling dimension

spin transf.

orbital transf.

$T_{\mu\nu}$


∞'al transf.
x̃ = x + ξ.
δθ_A(x) = θ̃_A(x) − θ_A(x) = −ξ^μ ∂_μ θ_A(x) + i/2 Ω_μν(x)(M^μν)_A^B θ_B(x)
orbital
spin
Ω_μν = ω_μν − 2(x_μ b_ν − x_ν b_μ)
− Δ ω(x) θ_A(x)
dimension
ω(x) = λ − 2 x·b
<O_Δ(x) O_Δ(0)> = 1/x^(2Δ)

$$\omega(x) = \lambda - 2x \cdot b,$$

want is to constraint observables: correlation functions

Wand identity

$\langle \mathcal{O}(x_1)\, \mathcal{O}(x_2) \rangle =$

$\langle \mathcal{O}(x_1)\, \mathcal{O}(x_2)\, \mathcal{O}(x_3) \rangle$

the position dependence is completely fixed by conformal symmetries

∞dal transf.

$$\tilde{x} = x + \xi .$$

$$\delta\Theta_A(x) = \tilde{\Theta}_A(x) - \Theta_A(x) = -\xi^\mu \underset{\text{orbital}}{\partial_\mu} \Theta_A(x) + \frac{i}{2}\Omega_{\mu\nu}(x)\underset{\text{spin}}{(M^{\mu\nu}{}_R)_A{}^B}\, \Theta_B(x)$$

$$-\underset{\text{dimension}}{\Delta}\,\omega(x)\,\Theta_A(x)$$

$$\Omega_{\mu\nu} = \omega_{\mu\nu} - 2(x_\mu b_\nu - x_\nu b_\mu)$$

$$\omega(x) = \lambda - 2x\cdot b$$

$$\Theta(x) = e^{iP^\mu x_\mu}\,\Theta(0)\,e^{-iP^\mu x_\mu}$$