Title: Quantum Fields and Strings Lecture - 230308

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Collection: Quantum Fields and Strings (2022/2023)

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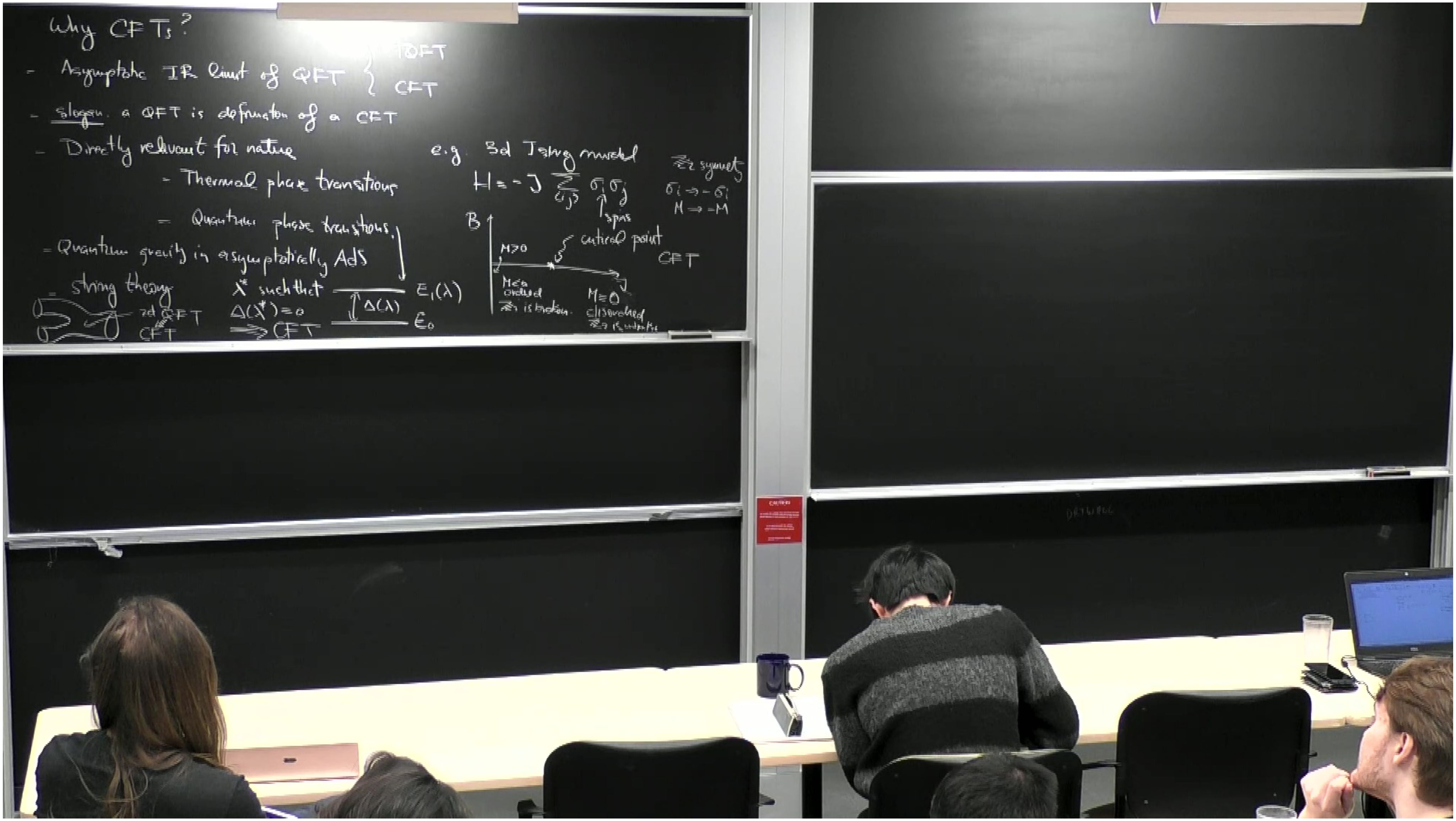
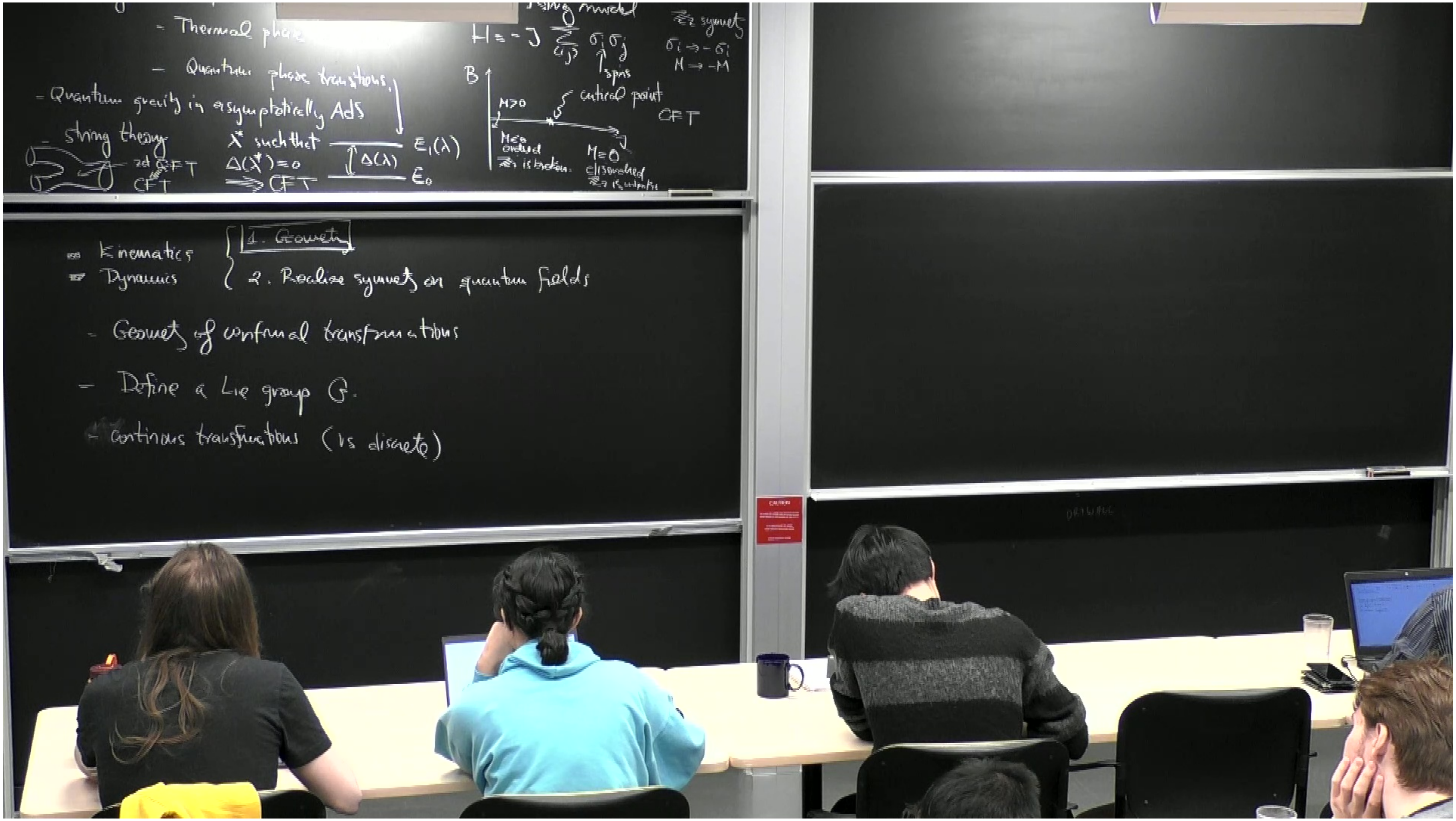
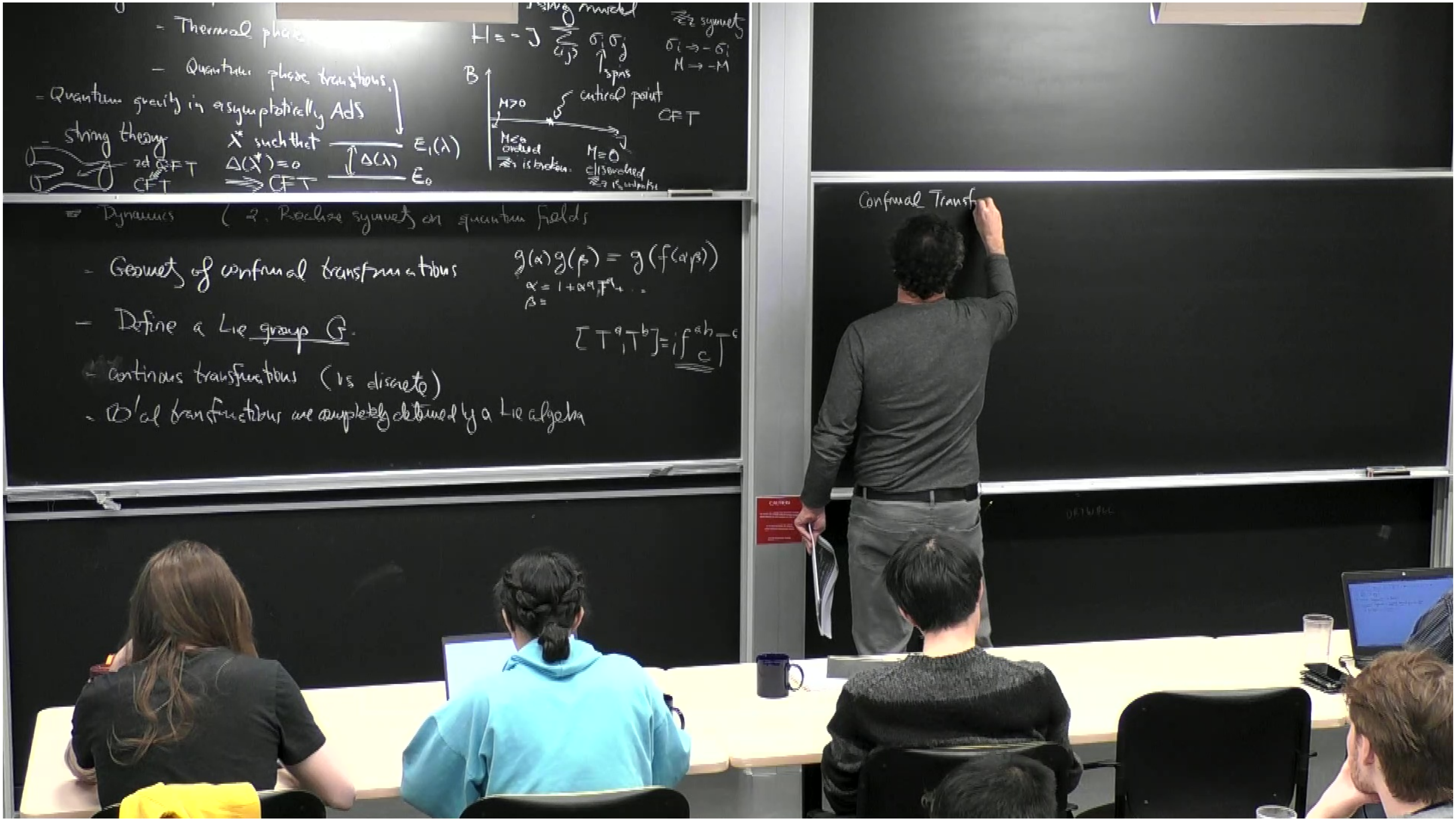
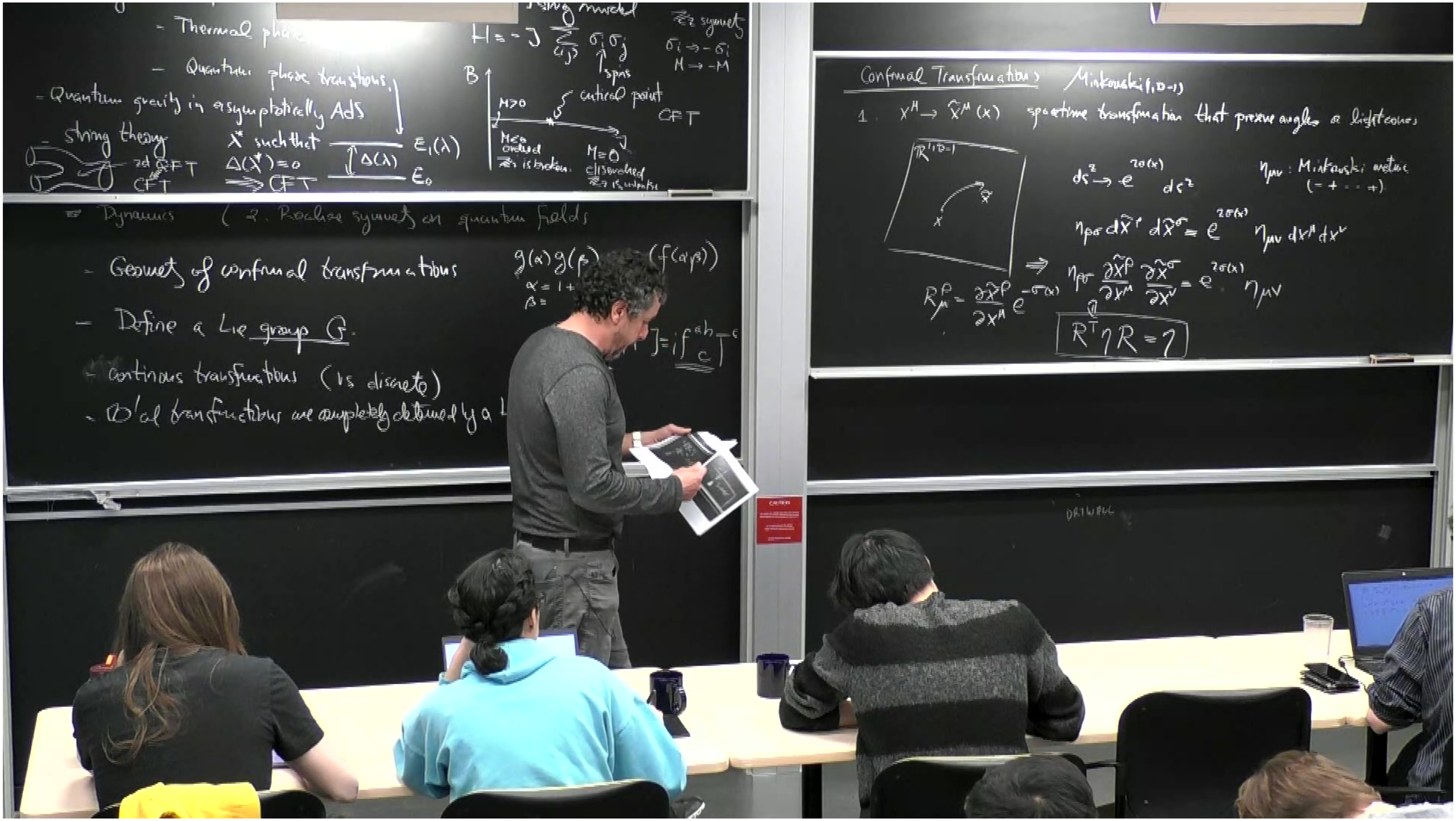
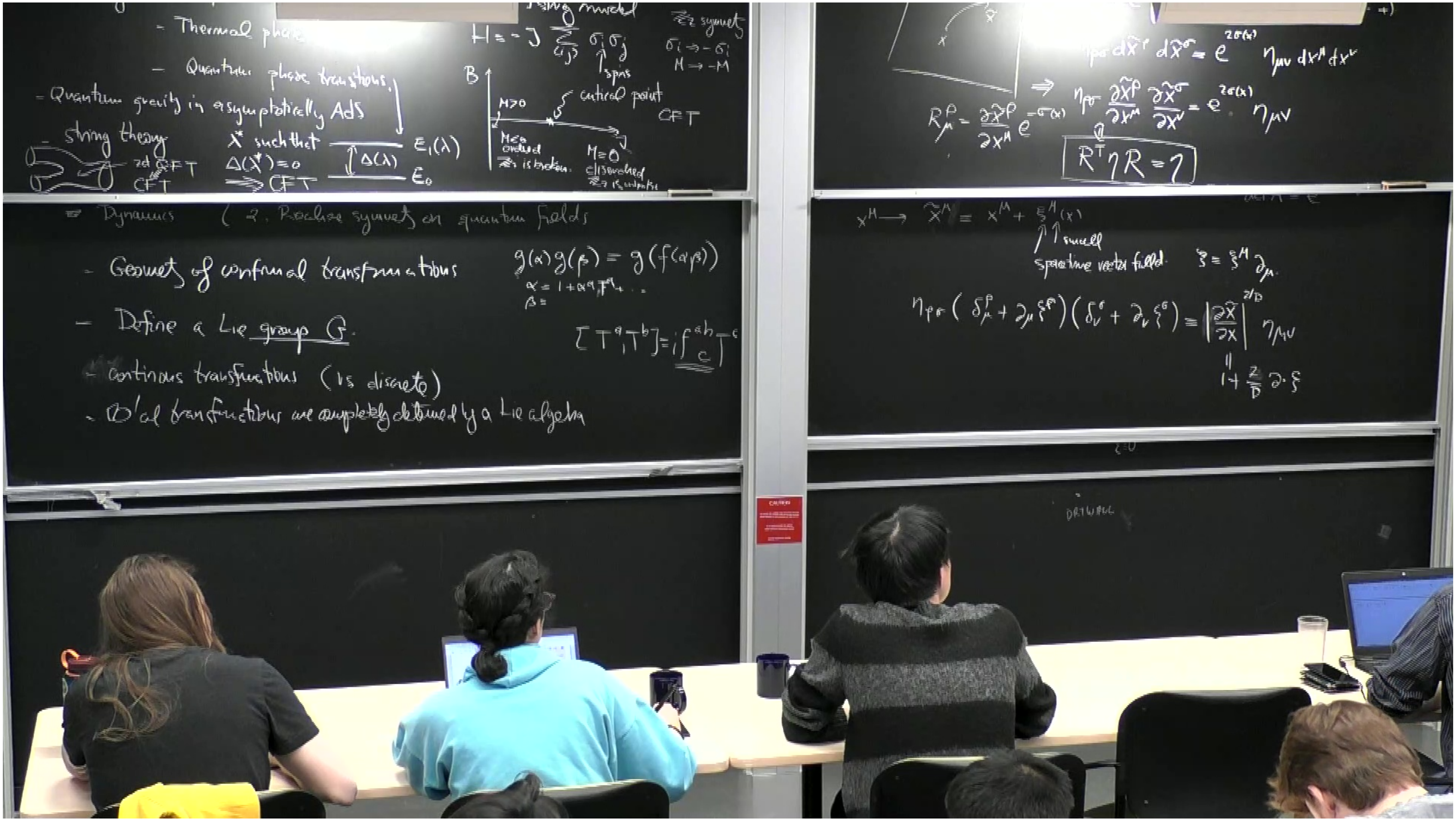

Why CFTs?
- Asymptotic IR limit of QFT { TQFT, CFT
- slogan: a QFT is deformation of a CFT
- Directly relevant for nature
- Thermal phase transitions
- Quantum phase transitions
- Quantum gravity in asymptotically AdS
- string theory
2d QFT
CFT
λ^* such that
$\Delta(\lambda^*) = 0$
$\Rightarrow$ CFT
$E_1(\lambda)$
$\Delta(\lambda)$
E_0
e.g. 3d Ising model
$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$
spins
$\mathbb{Z}_2$ symmetry
$\sigma_i \to -\sigma_i$
$M \to -M$
B
M≠0
critical point
CFT
M≠0 ordered
$\mathbb{Z}_2$ is broken
M=0 disordered
$\mathbb{Z}_2$ is unbroken
CAUTION

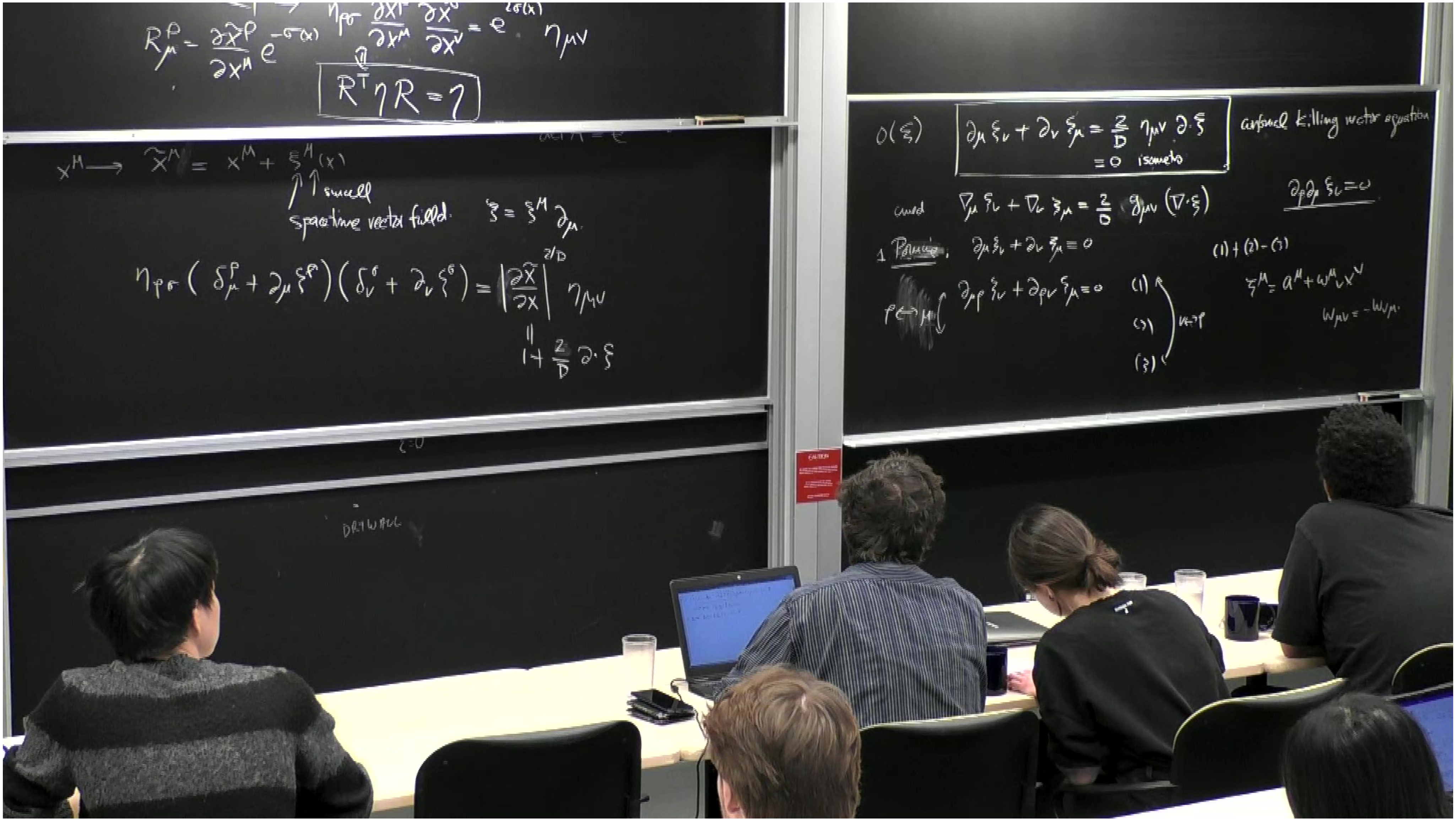

- Thermal phase
- Quantum phase transitions
- Quantum gravity in asymptotically AdS
- string theory
2d CFT
CFT
critical point
CFT
Kinematics
Dynamics
1. Geometry
2. Realize symmetry on quantum fields
- Geometry of conformal transformations
- Define a Lie group G.
- Continuous transformations (vs discrete)

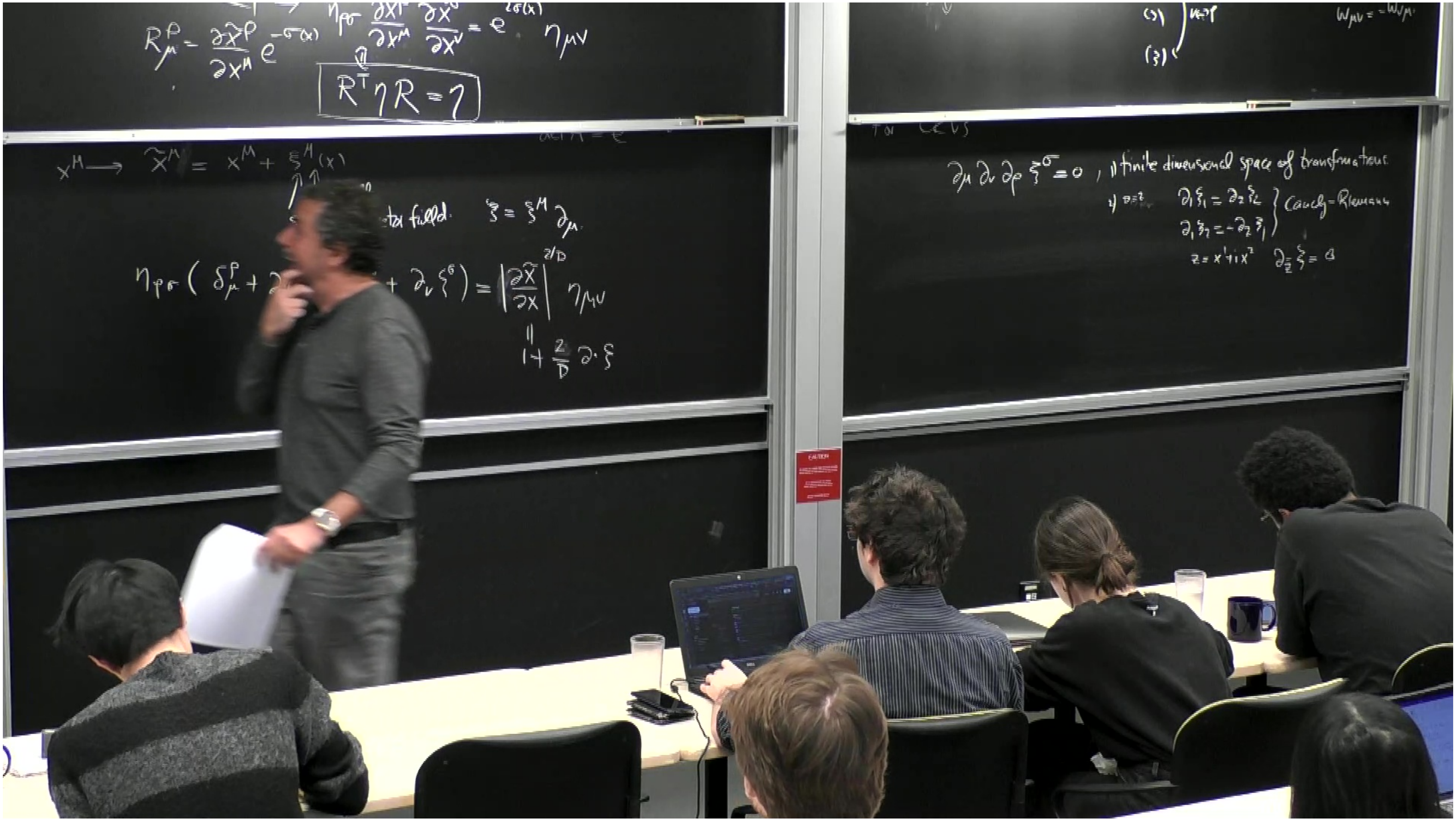

- Thermal pha
- Quantum phase transitions
- Quantum gravity in asymptotically AdS
- string theory
2d CFT
CFT
Ising model
Z2 symmetry
critical point
CFT
Dynamics
(2. Realise symmetry on quantum fields
- Geometry of conformal transformations
- Define a Lie group G.
- continous transformations (vs discrete)
Conformal Transf

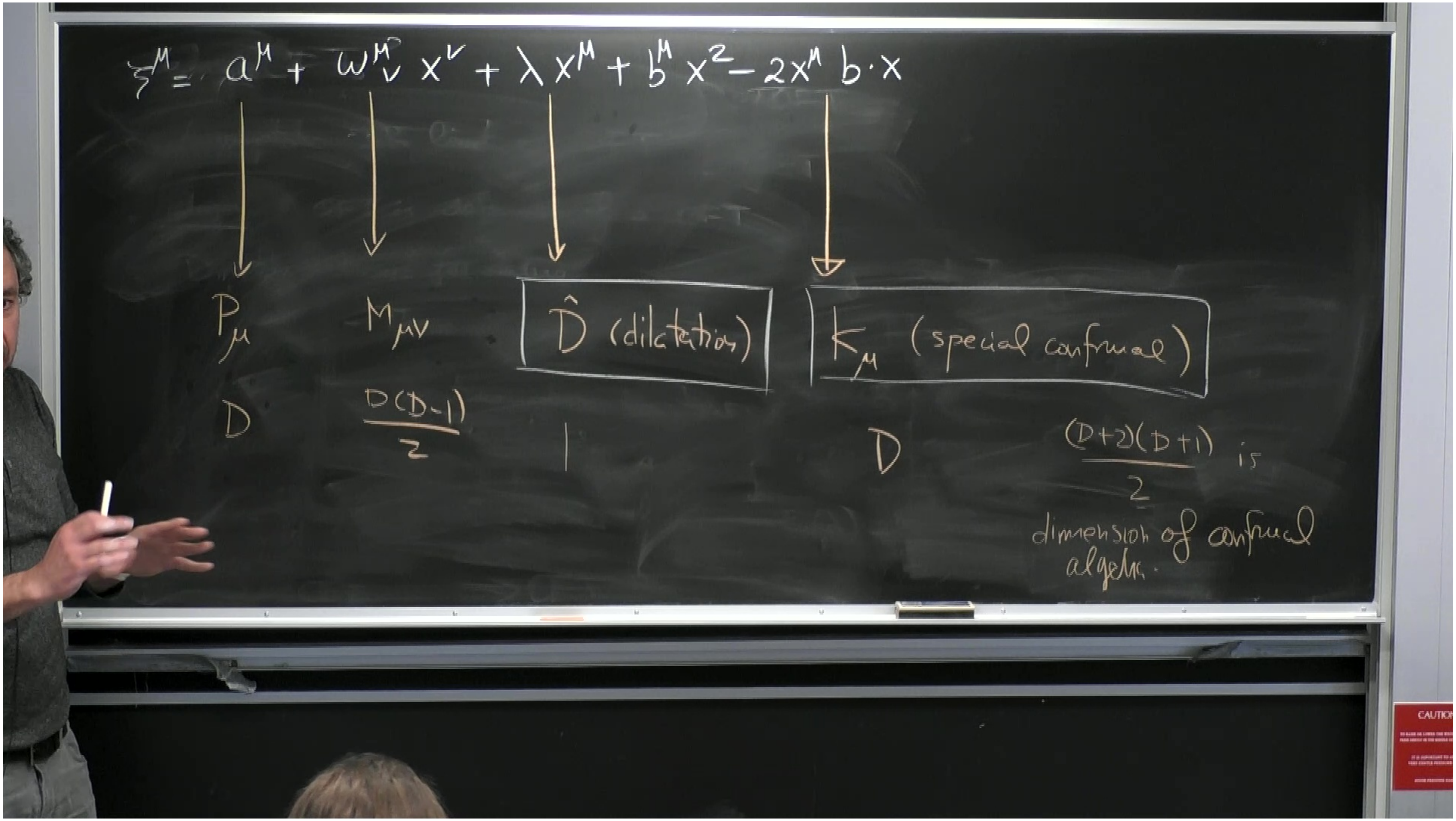


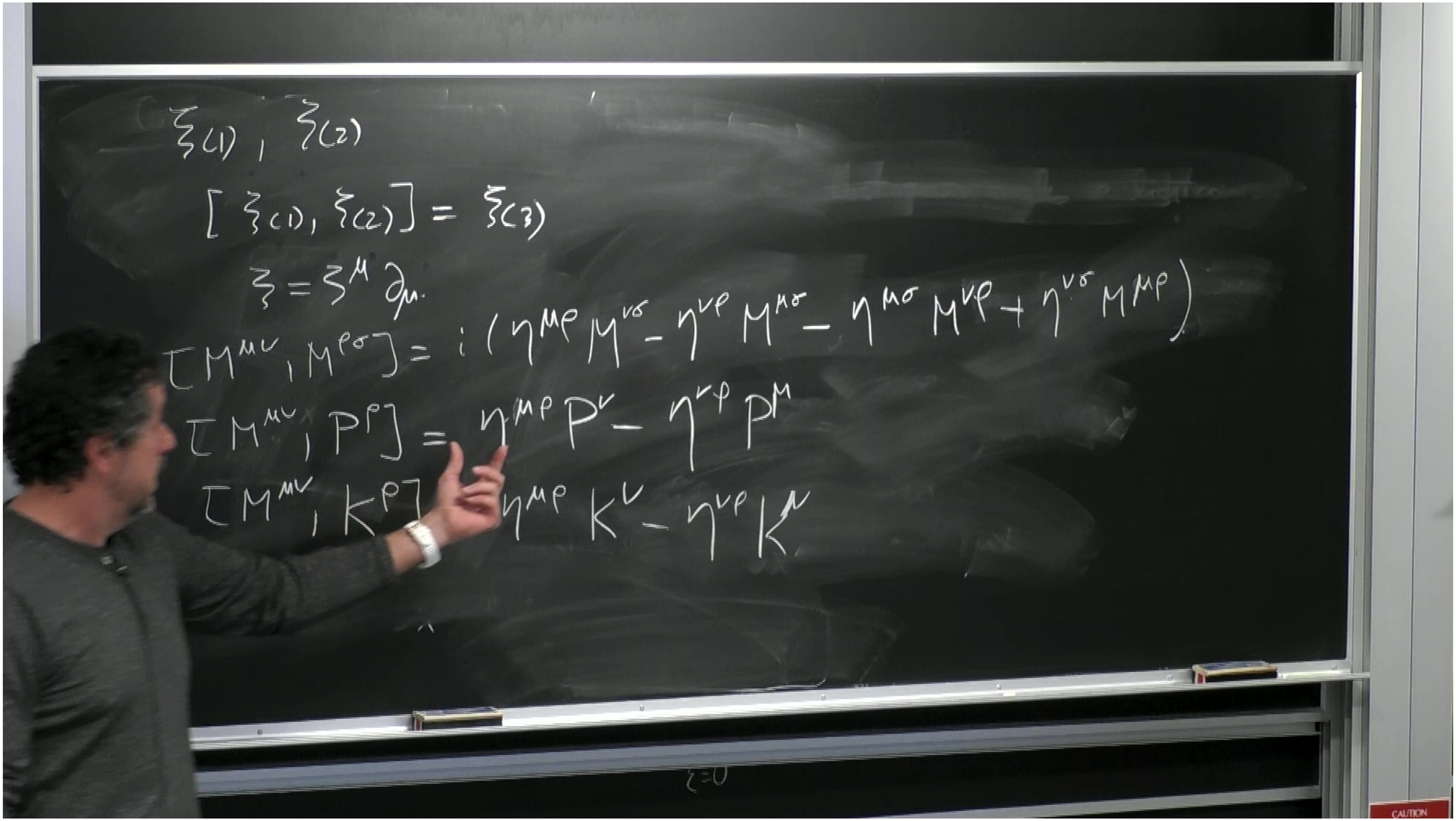
- Thermal pha
- Quantum phase transitions
- Quantum gravity in asymptotically AdS
- string theory
2d CFT
CFT
X* such that
Δ(X*) = 0
⟹ CFT
E₁(λ)
Δ(λ)
E₀
H = -J Σ σᵢ σⱼ
spins
B
M>0
critical point
CFT
M≠0 ordered
is broken
M=0 disordered
Z₂ symmetry
σᵢ → -σᵢ
M → -M
Dynamics
2. Realise symmetry on quantum fields
- Geometry of conformal transformations
- Define a Lie group G
- continuous transformations (vs discrete)
- ∞'al transformations are completely determined by a L
g(α) g(β)
Conformal Transformations
Minkowski(1,D-1)
1. x^μ → x̃^μ (x) spacetime transformation that preserve angles & lightcones
ds² → e^{2σ(x)} ds²
η_μν : Minkowski metric
(- + ... +)
η_ρσ dx̃^ρ dx̃^σ = e^{2σ(x)} η_μν dx^μ dx^ν
⟹ η_ρσ ∂x̃^ρ/∂x^μ ∂x̃^σ/∂x^ν = e^{2σ(x)} η_μν
R^ρ_μ = ∂x̃^ρ/∂x^μ e^{-σ(x)}
R^T η R = η


- Thermal phase
- Quantum phase transitions
- Quantum gravity in asymptotically AdS
- string theory
2d CFT
CFT
X* such that
Δ(X*) = 0
⇒ CFT
E1(λ)
Δ(λ)
E0
Ising model
H = -J Σ σi σj
spins
Z2 symmetry
σi → -σi
M → -M
B
M>0
critical point
CFT
M=0
disordered
Dynamics
Geometry of conformal transformations
g(α)g(β) = g(f(α,β))
- Define a Lie group G.
[T^a, T^b] = i f^{ab}_c T^c
continuous transformations (vs discrete)
R^T η R = η
spacetime vector field
small


R^T η R = η
small
spacetime vector field
conformal Killing vector equation
=0 isometry
DRYWALL
CAUTION


$R^T \eta R = \eta$
finite dimensional space of transformations
Cauchy-Riemann


$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + b^\mu x^2 - 2x^\mu b\cdot x$
P_μ
$M_{\mu\nu}$
$\hat{D}$ (dilatation)
K_μ (special conformal)
D
$\frac{D(D-1)}{2}$
1
D
$\frac{(D+2)(D+1)}{2}$ is dimension of conformal algebra.


$\xi_{(1)}, \xi_{(2)}$
$[\xi_{(1)}, \xi_{(2)}] = \xi_{(3)}$
$\xi = \xi^\mu \partial_\mu$
$[M^{\mu\nu}, M^{\rho\sigma}] = i(\eta^{\mu\rho} M^{\nu\sigma} - \eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\sigma} M^{\nu\rho} + \eta^{\nu\sigma} M^{\mu\rho})$
$[M^{\mu\nu}, P^\rho] = \eta^{\mu\rho} P^\nu - \eta^{\nu\rho} P^\mu$
$[M^{\mu\nu}, K^\rho] = \eta^{\mu\rho} K^\nu - \eta^{\nu\rho} K^\mu$

$$[D, P^\mu] = -iP^\mu \qquad [D, K^\mu] = i K^\mu$$

$$x^\nu \partial_\nu \qquad {}^{\prime\prime}\partial_\mu$$

$$[P^\mu, K^\nu] = -2i\left(\eta^{\mu\nu} D + M^{\mu\nu}\right)$$

$$X^M = (x^\mu, x^D, x^{D+1})$$

Minkowski

$$M = \underbrace{0,1,2,\ldots,D-1}, D, D+1$$

$$[L_{MN}, L_{PQ}] = i\,\hat{\eta}_{MP} L_{NQ} + \ldots$$

$$L_{MN} = \begin{cases} L_{\mu\nu} = M_{\mu\nu} \\ L_{D\,D+1} = \hat{D} \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu\,D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$$

$M = \underbrace{0,1,2,\ldots,D-1}_{\mu}, D, D+1$

$X^M = (X^\mu, X^D, X^{D+1})$

Minkowski

$[L_{MN}, L_{PQ}] = i\hat{\eta}_{MP} L_{NQ} + \ldots$

$$L_{MN} = \begin{cases} L_{\mu\nu} = M_{\mu\nu} \\ L_{D\,D+1} = \hat{D} \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$$

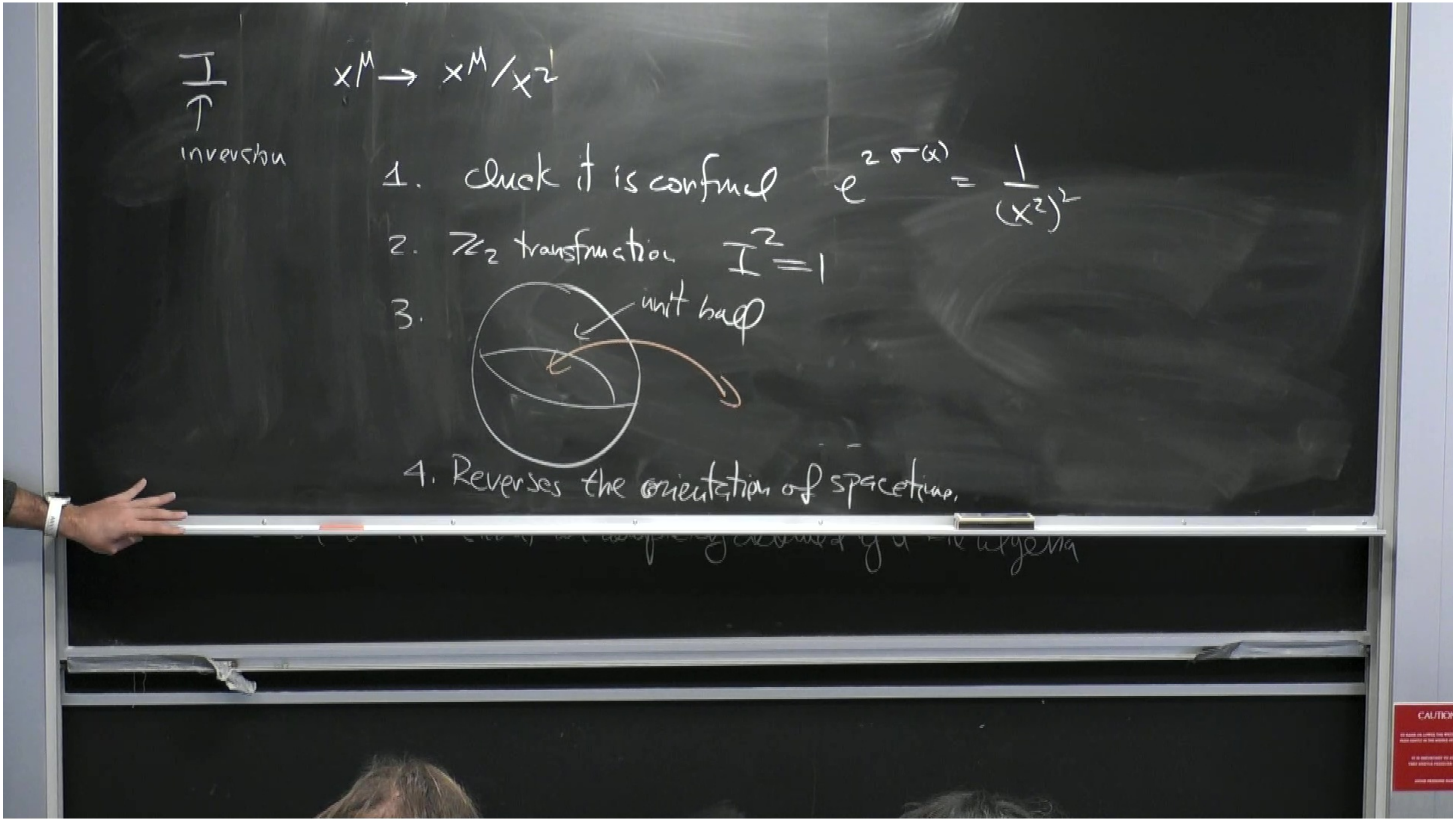
$$\hat{\eta} = \begin{pmatrix} - & & & & & \\ & + & & & & \\ & & + & & & \\ & & & + & & \\ & & & & + & \\ & & & & & - \end{pmatrix}$$

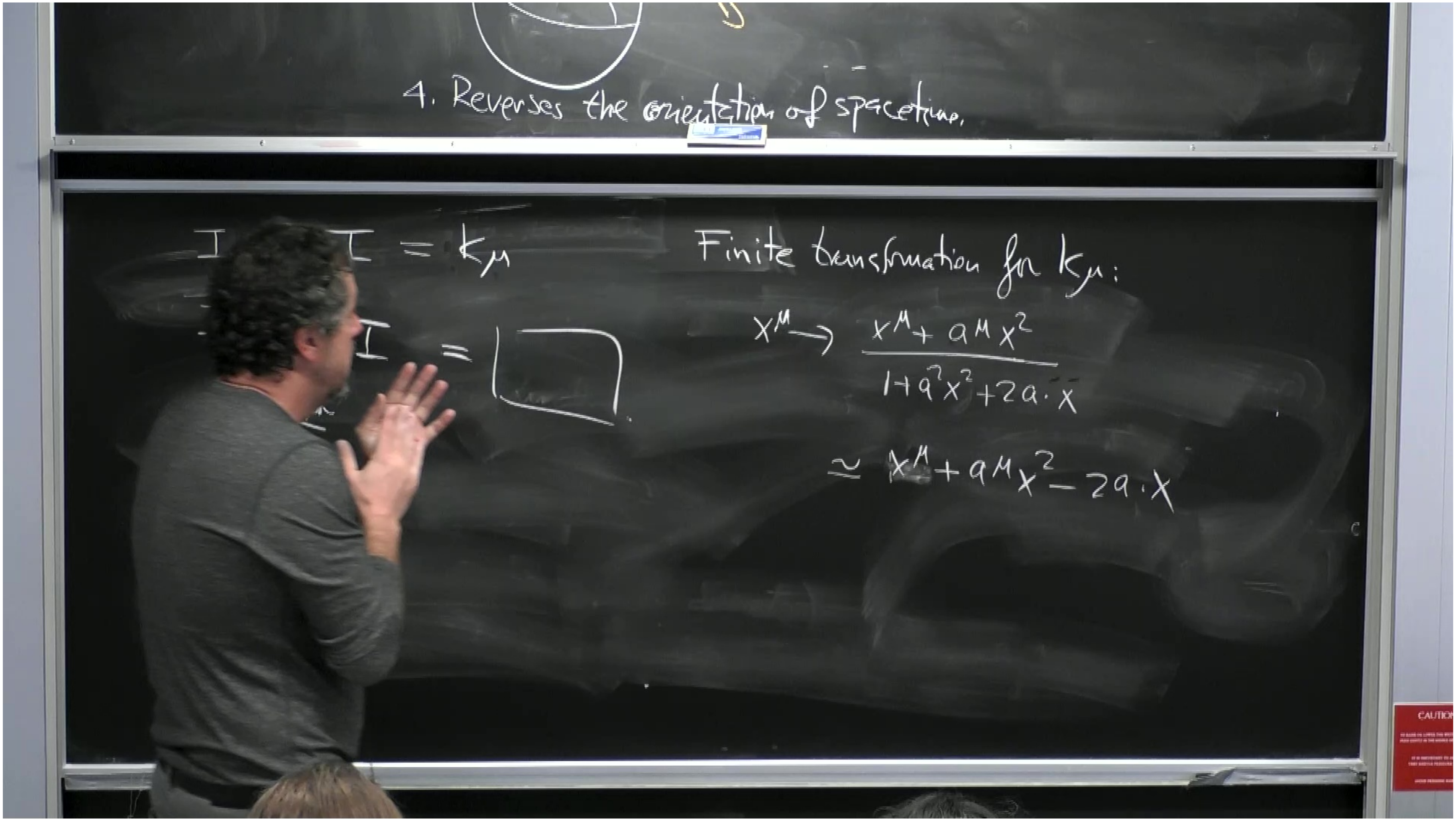
conformal algebra in D dim is

$SO(2,D)$

$\updownarrow$ CFT_D and

AdS_{D+1}


I
inversion
$x^\mu \to x^\mu / x^2$
1. check it is conformal $e^{2\sigma(x)} = \frac{1}{(x^2)^2}$
2. $\mathbb{Z}_2$ transformation $I^2 = 1$
3.
unit ball
4. Reverses the orientation of spacetime.



4. Reverses the orientation of spacetime.
Finite transformation for K_μ:
$x^\mu \to \frac{x^\mu + a^\mu x^2}{1 + a^2 x^2 + 2a\cdot x}$
$\simeq x^\mu + a^\mu x^2 - 2a\cdot x$
CAUTION

- Discrete conformal

Group of conformal transf

$SO(2,D)$

$I \quad x^\mu \to x^\mu / x^2$

inversion

$P = I \, \Lambda$

connected component

1. check it is conformal $e^{2\sigma(x)} = \frac{1}{(x^2)^2}$
2. $\mathbb{Z}_2$ transformation $I^2 = 1$
3. 

unit ball

$I(dx^0 \wedge dx^1 \wedge \dots \wedge dx^{D-1}) = -(dx^0 \wedge \dots \wedge dx^{D-1})$

4. Reverses the orientation of spacetime.