

Title: Quantum Fields and Strings Lecture - 230306

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Collection: Quantum Fields and Strings (2022/2023)

Date: March 06, 2023 - 10:15 AM

URL: <https://pirsa.org/23030017>

CFT is a QFT with additional spacetime symmetries

- Poincaré: QFT has this symmetry

- Galilean:

Symmetries: $L = \frac{1}{2} \dot{x}^2 - V(x)$

} + additional

CFT is a QFT with additional spacetime symmetries

- Poincaré: QFT has this symmetry

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+ additional

Symmetries: $L = \frac{1}{2} \dot{x}^2 - V(x)$

$\exists V(x)$ for
which $V(x)$

- if $V(x)$ is t -independent $E = \frac{1}{2} \dot{x}^2 + V(x)$

is scale invariant

- if $V(x)$, then \exists another conserved quantity $Q_1 \Rightarrow x(E, Q)$

Largest spacetime symmetry that a nontrivial QFT can have

Conformal symmetry
||
Symmetry of
a QFT

- conformal



- superconformal

susy + conformal

bosons



fermions

Conformal transformations

- scale transformations

$$\vec{x} \rightarrow \lambda \vec{x}$$

$$t \rightarrow \lambda^z t \quad z: \text{critical dynamical exponent}$$

□ Relativistic ϕ Γ , $z=1$ $x^\mu \rightarrow \lambda x^\mu$ \leftarrow

□ Nonrelativistic $-\frac{\vec{\nabla}^2}{2} \psi = i \frac{\partial \psi}{\partial t}$ ($z=2$)

Conformal transformations: $x^M \rightarrow \tilde{x}^M(x)$ such that angles are preserved but not necessarily distances.

Includes:

$$ds^2 \rightarrow e^{2\sigma(x)} ds^2$$

- Poincaré: $\sigma = 0$

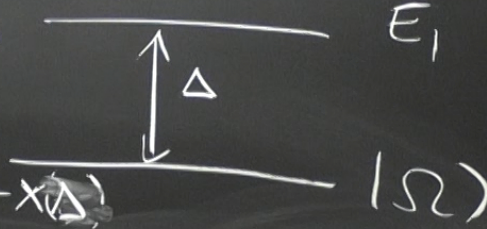
- Scale/Dilatation $ds^2 \rightarrow \lambda^2 ds^2$

- The set of such transformations is finite dimensional (D72)

why CFT's?

1. What happens at asymptotically large distances? \Rightarrow scale invariant

1. H of system has a gap



The diagram shows two horizontal lines representing energy levels. The upper line is labeled E_1 and the lower line is labeled $|\Omega\rangle$. A vertical double-headed arrow between the two lines is labeled Δ , representing the energy gap.

$$\langle O(x) O(0) \rangle \sim e^{-x/\Delta}$$

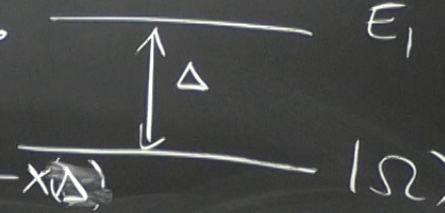
\uparrow
 local operator

$O(x) \rightarrow 0 \quad \neq$ local operators.

$L = \frac{1}{2} \dot{x}^2 - V(x)$
 $\exists V(x)$ for
 - if $V(x)$ is t -independent $F = 1, \dot{x}^2, V(x)$

1. What happens at asymptotically large distances? \Rightarrow scale invariant

1. H of system has a gap



The diagram shows two horizontal lines representing energy levels. The upper line is labeled E_1 and the lower line is labeled $|\Omega\rangle$. A vertical double-headed arrow between them is labeled Δ , representing the energy gap. Below the lower line, the expression $\langle O(x) O(0) \rangle \sim e^{-x/\xi}$ is written, with an arrow pointing from the ξ term to the text 'local operator'.

local operator $O(x) \rightarrow a \neq$ local operators.

- \exists nonlocal operators in the theory

- Described by a Topological Quantum Field Theory (TQFT)

Hamiltonian: $L = \frac{1}{2} \dot{x}^2 - V(x)$

$\exists V(x)$ for which $V(x)$ is scale invariant

- if $V(x)$ is x -independent $E = \frac{1}{2} \dot{x}^2 + V(x)$

- if $V(x)$, then \exists another conserved quantity $Q_1 \Rightarrow X(E, Q)$

local operator $O(x) \rightarrow 0$ \nexists local operators.

- \exists nonlocal operators in the theory.
- Described by a Topological Quantum Field Theory (TQFT)

2. H of system is gapless \Rightarrow CFT.

$$\langle O(x) O(0) \rangle \sim \frac{1}{x^{2\Delta}} \quad \text{scaling dimensions of operators.}$$

Examples of QFTs:

- QED

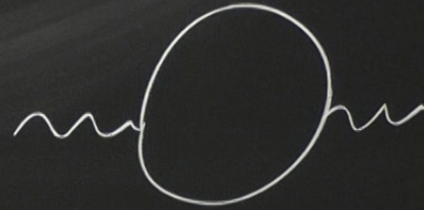
- Ising model

- $D=4$ $\square\phi + V'(\phi) = 0$

$V(\phi) = \lambda\phi^n$ $n=4$

- Yang-Mills $D^\mu F_{\mu\nu} = 0$

} classically
but



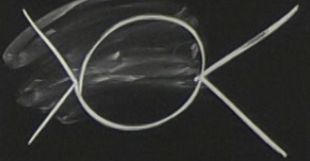
that induce $\lambda(E)$

Examples of CFTs:

- QED
- Ising model
- $D=4$ $\square\phi + V(\phi) = 0$
 $V(\phi) = \lambda\phi^n$ $n=4$
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classical
but

$$\beta(\lambda) = -\epsilon\lambda + a\lambda^2 = 0$$



$$D=4-\epsilon$$

$\lambda\phi^4$ λ becomes relevant

$$\mathcal{L} = \mathcal{L}_{free} + \lambda\phi^4$$

Free

$$\lambda=0$$

turn $m^2 \rightarrow 0$

scale invariance

WF

λ^* is small

$$\lambda = \epsilon\beta_{\phi^4}^{1-loop}$$

- Wilson-Fisher
- Banks-Zaks

Ising CFT in $(D=3)$

- CFTs induce an ordering in space of QFTs.

$$S = S_{\text{CFT}} + \lambda \mathcal{O}$$

• $\Delta > D$ irrelevant (unimportant in IR)

• $\Delta < D$ relevant (important in IR)

$$\left(\frac{E\lambda}{\mu}\right)^\#$$

λ has length dimension

λ has mass dimension

$$\left(\frac{\lambda}{E}\right)$$

