Title: Quantum Fields and Strings Lecture - 230306

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Collection: Quantum Fields and Strings (2022/2023)

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CFT is a QFT with additional spacetime symmetries

- Poincaré: QFT has this symmetry

+ additional

- Galilean:

Symmetries: $L = \frac{1}{2}\dot{x}^2 - V(x)$

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Symmetries: $L = \frac{1}{2}\dot{x}^2 - V(x)$

$\exists$ $V(x)$ for which $V(x)$ is scale invariant

- if $V(x)$ is t-independent $E = \frac{1}{2}\dot{x}^2 + V(x)$
- if $V(x)$, then $\exists$ another conserved quantity $Q_1 \Longrightarrow x(E, Q)$

Largest spacetime symmetry that a nontrivial QFT can have

Conformal symmetry
||
symmetry of a CFT

- conformal

↓

- superconformal

susy + conformal

bosons

↕

fermions

Conformal transformations

- scale transformations

$$\vec{x} \to \lambda \vec{x}$$

$$t \to \lambda^{z} t \quad z: \text{critical dynamical exponent}$$

- Relativistic QFT, $z=1$ $x^{\mu} \to \lambda x^{\mu}$
- Nonrelativistic $-\frac{\vec{\nabla}^2}{2}\psi = i\frac{\partial \psi}{\partial t}$ $(z=2)$

Conformal transformations: $x^\mu \to \tilde{x}^\mu(x)$ such that angles are preserved but not necessarily distances.

$$ds^2 \to e^{2\sigma(x)} ds^2$$

Includes:

- Poincaré: $\sigma = 0$
- Scale/Dilatation $ds^2 \to \lambda^2 ds^2$
- The set of such transformations is finite dimensional $(D>2)$

why CFT's?

1. What happens at asymptotically large distances? $\Rightarrow$ scale invariant

1. H of system has a gap

E_1

Δ

$|\Omega\rangle$

$$\langle O^{\dagger}(x)\, O(0)\rangle \sim e^{-x\Delta}$$

local operator

$O(x) \to 0$ $\quad \nexists$ local operators.

$\exists\, V(x)$ for

$-$ if $V(x)$ is t-independent

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- $\exists$ nonlocal operators in the theory.
- Described by a Topological Quantum Field Theory (TQFT)

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2. H of system is gapless $\Rightarrow$ CFT.

$$\langle O(x)\, O(0) \rangle \sim \frac{1}{x^{2\Delta}}$$ Δ scaling dimensions of operators.

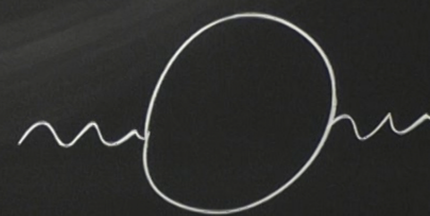
Examples of CFTs:

- QED
- Ising model
- $D=4$ $\qquad \Box\phi + V'(\phi) = 0$

  $V(\phi) = \lambda \phi^n \qquad n=4$

- Yang-Mills $\quad D^\mu F_{\mu\nu} = 0$

classically but



that induce $\lambda(E)$

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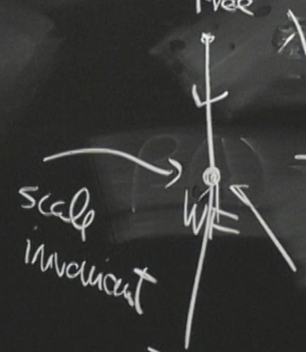
$D = 4 - \epsilon$.

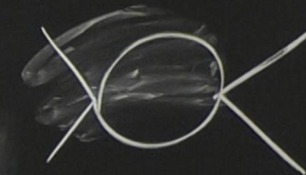
$\lambda\phi^4$ $\quad \lambda$ becomes relevant

$$\mathcal{L} = \mathcal{L}_{free} + \lambda\phi^4$$



Ising CFT in $(D=3)$

$$\beta(\lambda) = -\epsilon\lambda + a\lambda^2 = 0$$



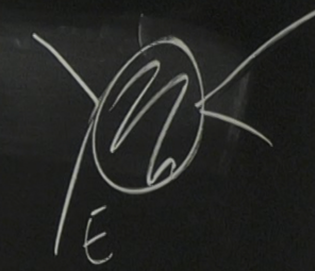
$$\lambda = \epsilon \beta^{1\text{-loop}}_{\phi^4}$$

- Wilson-Fischer
- Banks-Zachs

- CFT's induce an ordering in space of QFTs.

$$S = S_{CFT} + \lambda \mathcal{O}$$

$$\left(E\lambda\right)^{\#}$$



- $\Delta > D$ irrelevant (unimportant in IR)

λ has length dimension

- $\Delta < D$ relevant (important in IR)

λ has mass dimension

$$\left(\frac{\lambda}{E}\right)$$