

Title: Quantum Fields and Strings Lecture - 230301

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Collection: Quantum Fields and Strings (2022/2023)

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Anomalies

Noether's theorem is a LIE!

classical symmetry $\times$ conservation law in quantum theory

UV - impossibility of finding regulator that preserves all symmetries simultaneously

IR - $\pi^0 \rightarrow \gamma\gamma$

Noether's Theorem

$$S = \int d^d x \mathcal{L}(\phi, \partial\phi)$$

$$\phi \rightarrow \phi + \epsilon(x) X(\phi)$$

$$S \rightarrow S + \int d^d x \epsilon(x) \partial_\mu J^\mu$$

$$\partial_\mu J^\mu = 0$$

$$\boxed{\phi' = \mathcal{D}\phi} \rightarrow \langle \partial_\mu J^\mu \rangle = 0$$

Chiral anomaly in massless QED in 2D

$$\mathcal{L} = i \bar{\Psi} \not{\partial} \Psi$$

$$\begin{array}{ll} \text{vector} & \Psi \rightarrow e^{i\alpha} \Psi \rightarrow J^\mu = \bar{\Psi} \gamma^\mu \Psi \\ \text{axial} & \Psi \rightarrow e^{i\alpha \gamma^5} \Psi \rightarrow J_5^\mu = \bar{\Psi} \gamma^\mu \gamma^5 \Psi \end{array} \leftarrow \begin{array}{l} \text{conflict} \\ \text{in} \\ \text{quantum} \\ \text{theory} \end{array}$$

also QED since $A_\mu \rightarrow A_\mu$ under both
in 2D as in 4D

An

$E = \text{constant} \rightarrow V_{\text{coulomb}} \propto r \rightarrow \text{strong coupling}$

Compactify with periodicity L

$$eL \ll 1$$

Anomaly present if quantize ψ in classical background A_μ

Chiral basis

$$\gamma_5 P_\pm \psi = \pm P_\pm \psi = \psi_\pm$$

$$A_\mu = 0 \rightarrow i(\partial_0 \pm \partial_1) \psi_\pm = 0$$

plane waves moving to left ($-$)
right ($+$) with $v=1$

Gauge fixing

$$A_\mu = \sum a_n(t) \cos\left(\frac{2\pi n}{L} x\right)$$

$$e^{i \oint dx A_1}$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha$$

$$A_1 \rightarrow A_1(t) + \frac{2\pi n}{L}$$

$$\alpha = \sum_{n \neq 0} \frac{L}{2\pi n} a_n(t) \sin\left(\frac{2\pi n}{L} x\right) + \frac{2\pi n}{L} x$$

large
gauge
transformation

A_1 lives on a circle of radius $\frac{2\pi}{L}$

Spectrum

Ψ quantum

A_μ classical

$A_1(t)$ slowly varying

$A_0 = 0 \rightarrow E$ is negligible

$$i\cancel{D}\Psi = 0$$

stationary solutions $\Psi \propto e^{-iE_n t} \Psi_n$

periodicity $\Psi_n \propto e^{2\pi i n x / L}$

$$\rightarrow \boxed{E_n^\pm = \pm \frac{2\pi n}{L} \mp A_1}$$

adiabatic change $\rightarrow$ pair production

right antiparticle	$Q = -1$	$Q_5 = -1$
left particle	$Q = 1$	$Q_5 = -1$
	$\Delta Q = 0$	$\Delta Q_5 = -2$

Anomalous non-conservation equation

$$\partial_\mu J^\mu = ?$$

$$\Delta A_1 = \frac{2\pi}{L} \rightarrow \Delta Q_5 = -2$$

$$\frac{\Delta Q_5}{\Delta t} = -\frac{L}{\pi} \frac{\Delta A_1}{\Delta t}$$

$$\dot{Q}_5 = -\frac{L}{\pi} \dot{A}_1$$

$$Q_5 = \int dx J_5^0$$

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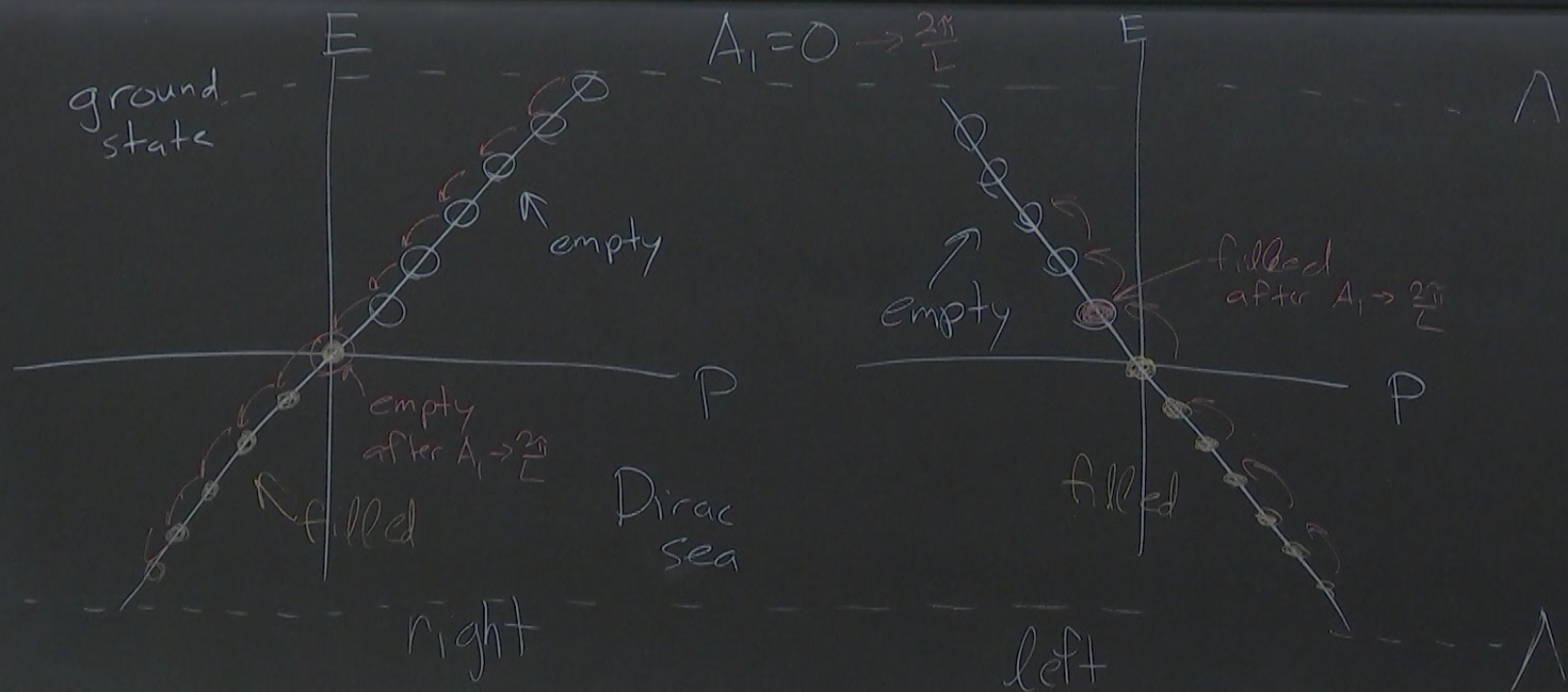
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$$\dot{Q}_5 = -\frac{L}{2\pi} \dot{A}_1$$

$$Q_5 = \int dx J_5^0$$

$$L A_1 = \int dx A_1 = \int \epsilon^{\mu 1} A_\mu dx$$

$$IK - \pi^0 \rightarrow \pi\pi$$



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$$LA_1 = \int dx A_1 = \int \epsilon^{\mu\nu} A_\mu dx$$