Title: Quantum Information Lecture - 230331

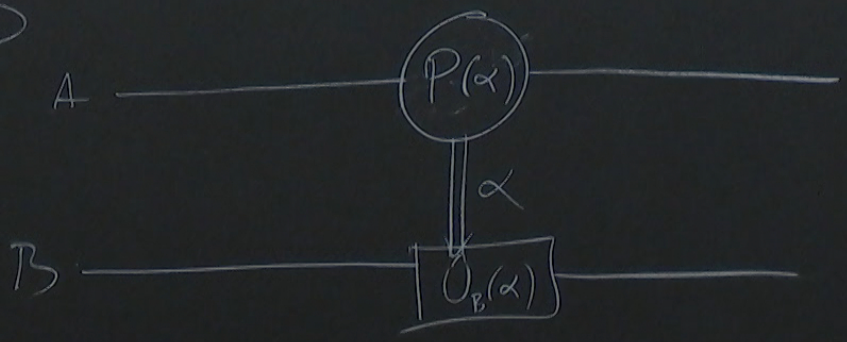
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Collection: Quantum Information (2022/2023)

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can implement any local unitary on qubit B



$(h,k)\,\mathbb{1}\Big]$

$\hat{V}|g\rangle = 0$

$\hat{\sigma}_z^V |0_V\rangle = -|0_V\rangle$

$\hat{\sigma}_z^V |1_V\rangle = |1_V\rangle$

$|1_B\rangle + \sqrt{1+\frac{\delta(h,k)}{n}}\,|0_A\rangle|0_B\rangle\Big)$

Step 1: Projective measurement on A

Alice measures $\hat{\sigma}_x$, obtains a bit of classical info ($\alpha = \pm 1$). State updates

in a single shot: $|\psi_{PM}(\alpha)\rangle = \frac{1}{\sqrt{P_A(\alpha)}} \hat{P}_A(\alpha)|\psi\rangle$, $P_A(\alpha) = \langle\psi|\hat{P}_A|\psi\rangle$

$$\hat{\rho}_{11} = \sum_{\alpha=\pm 1} P_A(\alpha) |\psi_{PM}(\alpha)\rangle\langle\psi_{PM}(\alpha)| = \sum_{\alpha=\pm 1} \hat{P}_A(\alpha)|\psi\rangle\langle\psi|\hat{P}(\alpha)$$

on A

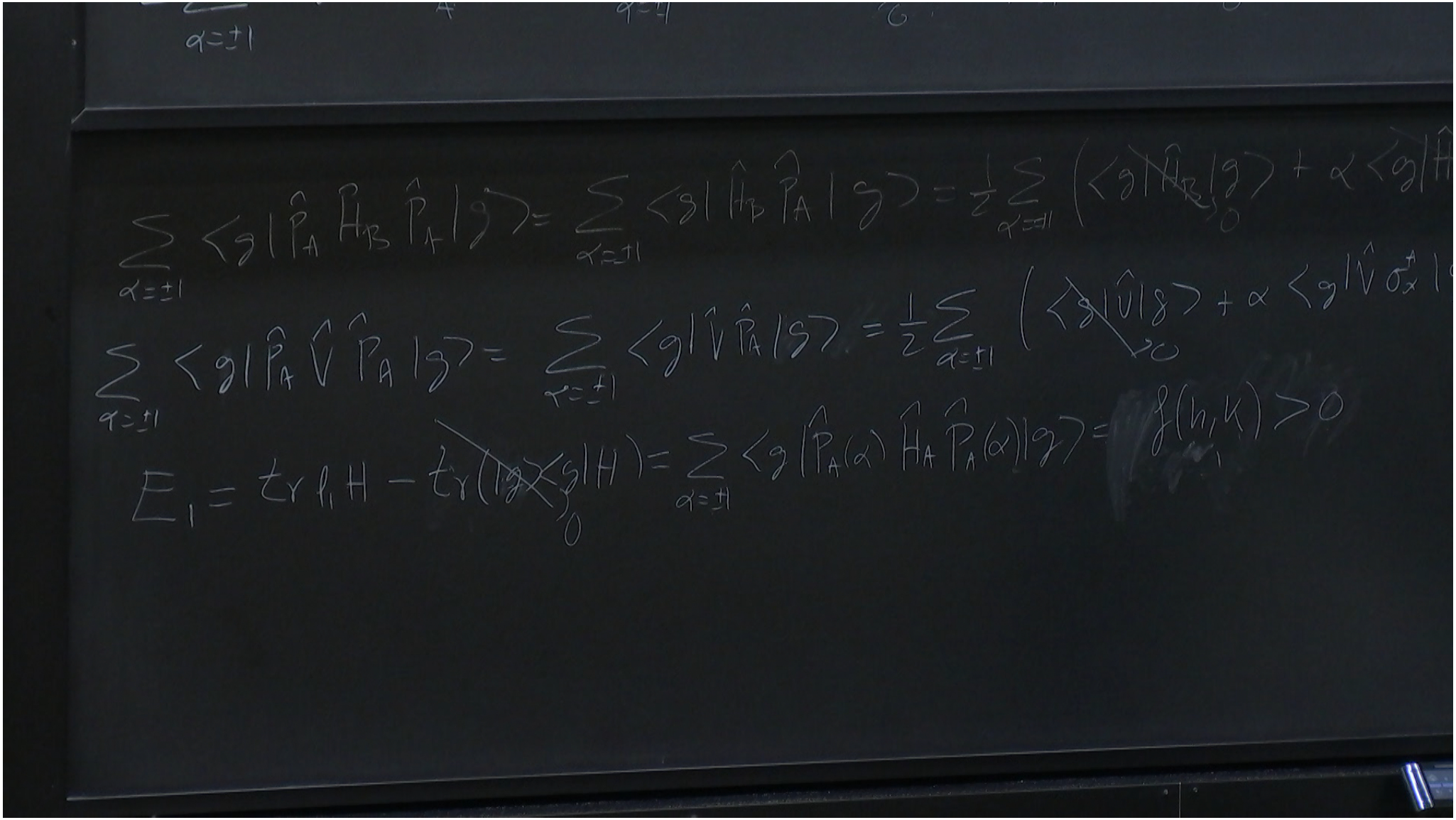
bit of classical info ($\alpha = \pm 1$). State updates: $\hat{P}_A(\alpha) = \frac{1}{2}(1 + \alpha \hat{\sigma}_x^A)$

$\frac{1}{P_A(\alpha)} \hat{P}_A(\alpha)|g\rangle$, $P_A(\alpha) = \langle g|\hat{P}_A|g\rangle$

$= \sum_{\alpha=\pm 1} \hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}(\alpha) \Rightarrow E_1 = \mathrm{Tr}\,\hat{\rho}_1 \hat{H} = \sum_{\alpha=\pm 1} \langle g|\hat{P}_A(\alpha)\hat{H}\hat{P}_A(\alpha)|g\rangle =$

$\langle g|\hat{P}_A \hat{H}_S \hat{P}_A|g\rangle + \sum_{\alpha=\pm 1} \langle g|\hat{P}_A \hat{V} \hat{P}_A|g\rangle$, $[\hat{P}_A(\alpha), \hat{V}] = 0$, $[\hat{P}_A(\alpha), \hat{H}_B] = 0$

0 0


$\alpha=\pm1$
$\sum_{\alpha=\pm1} \langle g|\hat{P}_A \hat{H}_B \hat{P}_A|g\rangle = \sum_{\alpha=\pm1} \langle g|\hat{H}_B \hat{P}_A|g\rangle = \frac{1}{2}\sum_{\alpha=\pm1}$
$\sum_{\alpha=\pm1} \langle g|\hat{P}_A \hat{V} \hat{P}_A|g\rangle = \sum_{\alpha=\pm1} \langle g|\hat{V} \hat{P}_A|g\rangle = \frac{1}{2}\sum_{\alpha=\pm1}$
$E_1 = \mathrm{tr}\,\rho_1 H - \mathrm{tr}(|g\rangle\langle g|H) = \sum_{\alpha=\pm1} \langle g|\hat{P}_A(\alpha)\hat{H}_A\hat{P}_A(\alpha)|g\rangle = f(h,k) > 0$

$$\sum_{\alpha=1}^{} \left(\langle g|\hat{H}_B|g\rangle + \alpha \langle g|\hat{H}_B \sigma_x^A|g\rangle \right) \qquad \langle g|H_B \hat{\sigma}_x|g\rangle \alpha \langle g|\sigma_B^z \sigma_x^A|g\rangle = 0$$

$$\left(\langle g|\hat{U}|g\rangle + \alpha \langle g|\hat{V}\sigma_x^A|g\rangle = 0 \right. \Leftarrow \langle g|\hat{U}\hat{\sigma}_x^A|g\rangle \alpha \langle g|\sigma_x^B|g\rangle = 0$$

$$(\alpha)|g\rangle = f(h,k) > 0$$

θ is given by $\cos(2\theta) = \dfrac{h^2 + 2k^2}{\sqrt{(h^2+2k^2)^2 + h^2k^2}}$ $\sin(2\theta) = \dfrac{hk}{\sqrt{(h^2+2k^2)^2 + h^2k^2}}$

in a single shot $\hat{U}_B(\alpha)|\psi_{PM}\rangle = \dfrac{1}{\sqrt{P_A(\alpha)}} \hat{U}_B \hat{P}_A(\alpha)|g\rangle \Rightarrow$ in an ensemble of experiments

Step 2: Classical commication and local

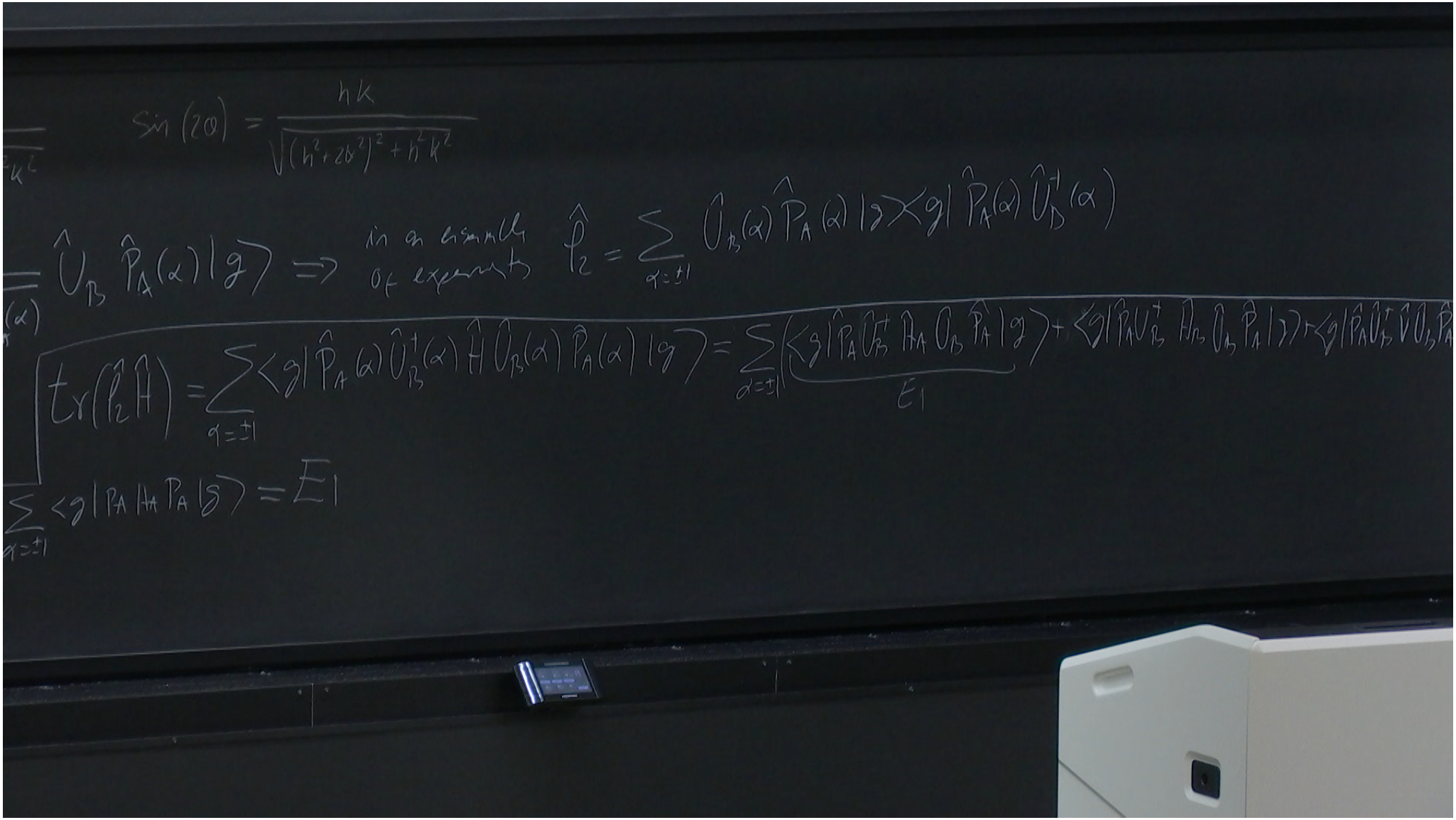
Bob Applies $\hat{U}_B(\alpha) = \cos\theta\, \mathbb{1} - i\alpha \sin\theta\, \sigma_y^B$

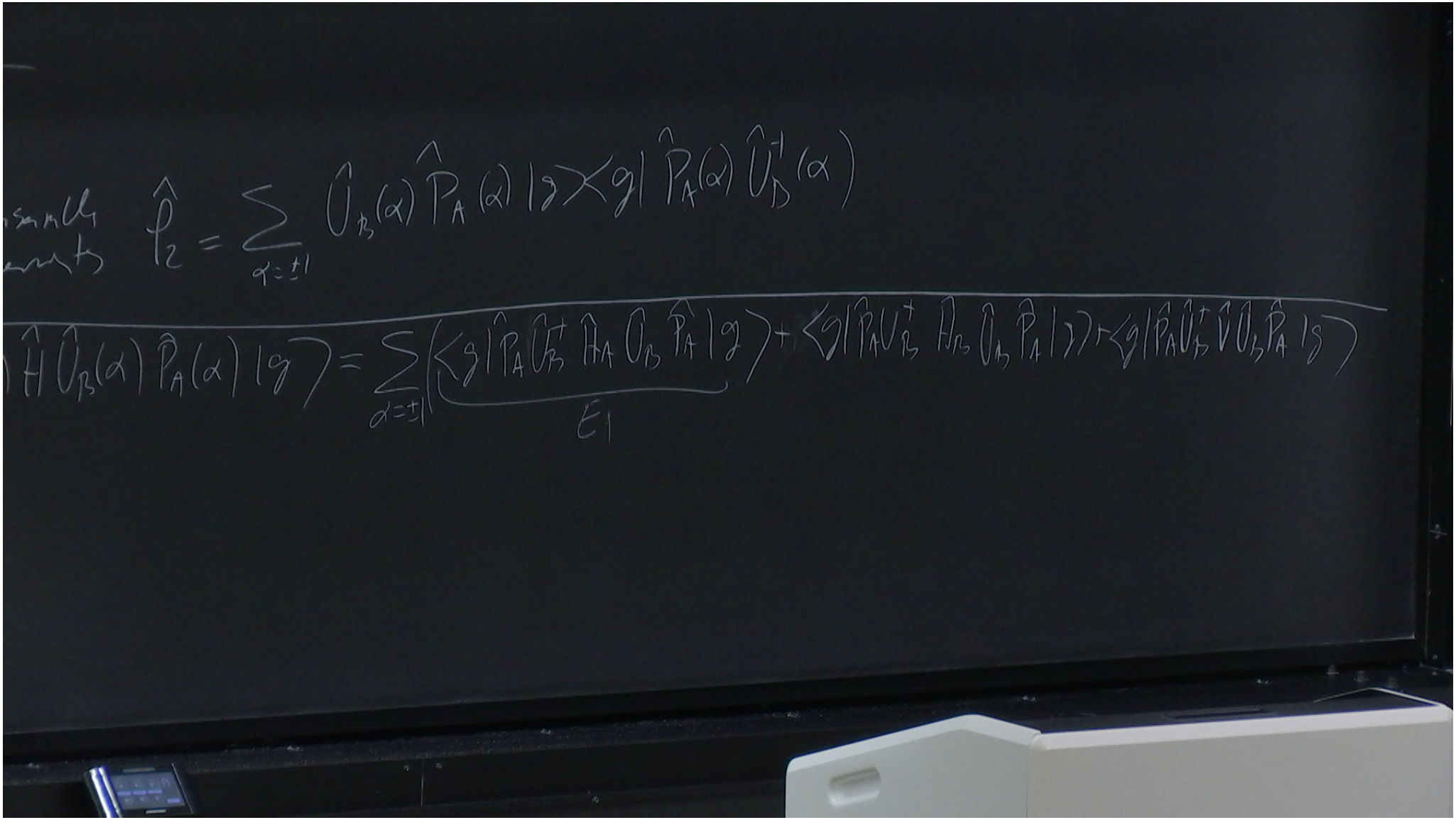
In a single shot $\hat{U}_B(\alpha)|\psi_{PM}\rangle = \frac{1}{\sqrt{P_\alpha(\alpha)}} \hat{U}_B \hat{P}_A(\alpha)|g\rangle \Rightarrow$ in an ensemble of experiments $\hat{\rho}_2 = \sum_\alpha$

$E_2 = tr(\hat{\rho}_2 \hat{H}) - tr(\hat{\rho}_1 \hat{H}) = tr(\hat{\rho}_2 \hat{H}) - E_1$

$tr(\hat{\rho}_2 \hat{H}) = \sum_{\alpha=\pm1} \langle g|\hat{P}_A(\alpha)\hat{U}_B^\dagger(\alpha)\hat{H}\hat{U}_B(\alpha)\hat{P}_A(\alpha)|g\rangle$

$[\hat{U}_B(\alpha), \hat{H}_A] = 0 \Rightarrow \sum_{\alpha=\pm} \langle g|\hat{P}_A \hat{U}_B^\dagger H_A \hat{U}_B \hat{P}_A|g\rangle = \sum_{\alpha=\pm1} \langle g|P_A H_A P_A|g\rangle = E_1$

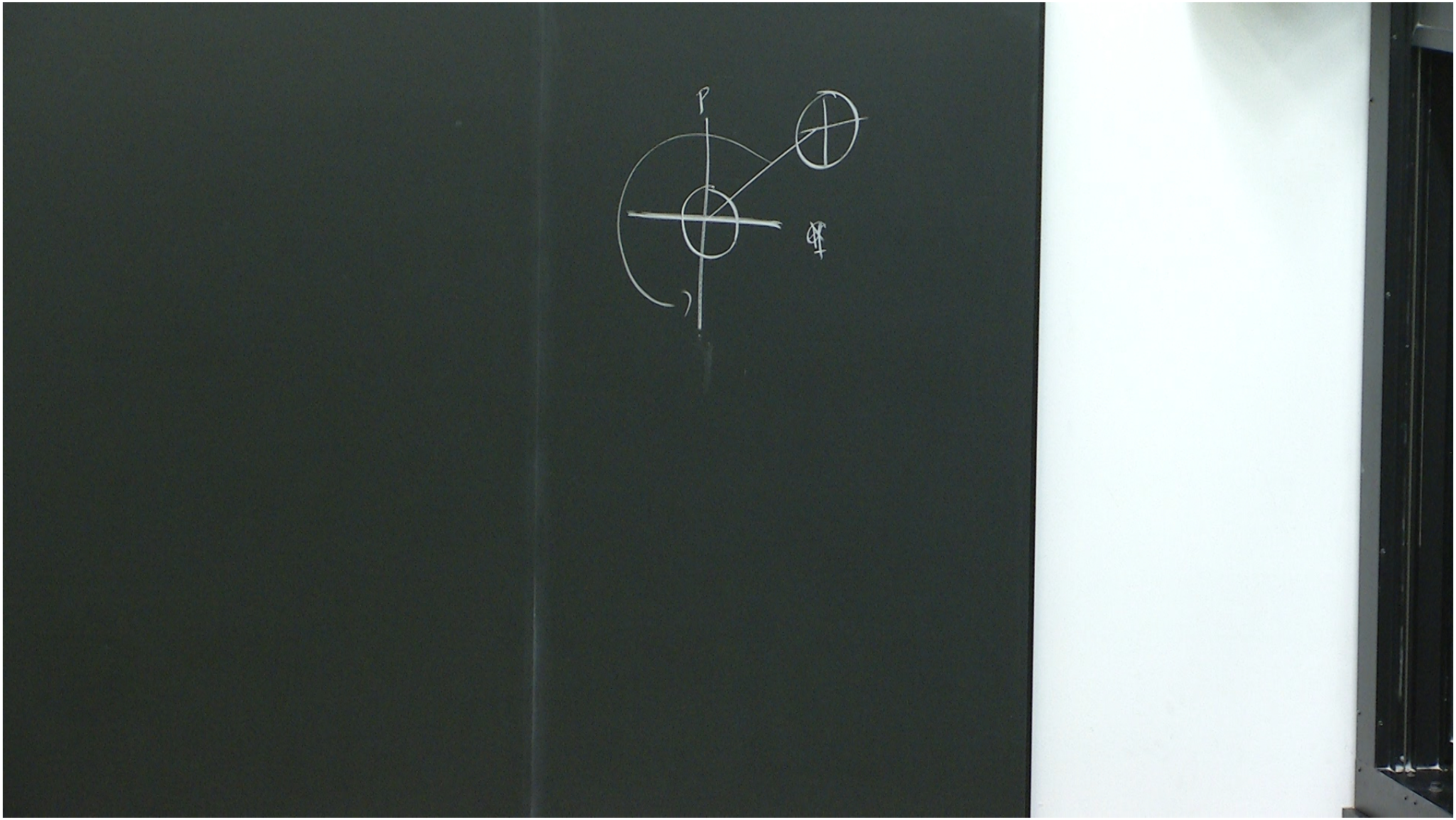

$\sin(2\alpha) = \frac{hk}{\sqrt{(h^2+2\alpha^2)^2 + h^2k^2}}$
$\hat{U}_B \hat{P}_A(\alpha)|g\rangle \Rightarrow$ in an ensemble of experiments $\hat{P}_2 = \sum_{\alpha=\pm1} \hat{U}_B(\alpha)\hat{P}_A(\alpha)|g\rangle\langle g|\hat{P}_A(\alpha)\hat{U}_B^\dagger(\alpha)$
$\mathrm{tr}(\hat{P}_2\hat{H}) = \sum_{\alpha=\pm1}\langle g|\hat{P}_A(\alpha)\hat{U}_B^\dagger(\alpha)\hat{H}\hat{U}_B(\alpha)\hat{P}_A(\alpha)|g\rangle = \sum_{\alpha=\pm1}(\underbrace{\langle g|\hat{P}_A\hat{U}_B^\dagger\hat{P}_A\hat{U}_B\hat{P}_A|g\rangle}_{E_1} + \langle g|\hat{P}_A\hat{U}_B^\dagger\hat{H}_B\hat{U}_B\hat{P}_A|g\rangle + \langle g|\hat{P}_A\hat{U}_B^\dagger\hat{V}\hat{U}_B\hat{P}_A$
$\sum_{\alpha=\pm1}\langle g|P_A H_A P_A|g\rangle = E_1$

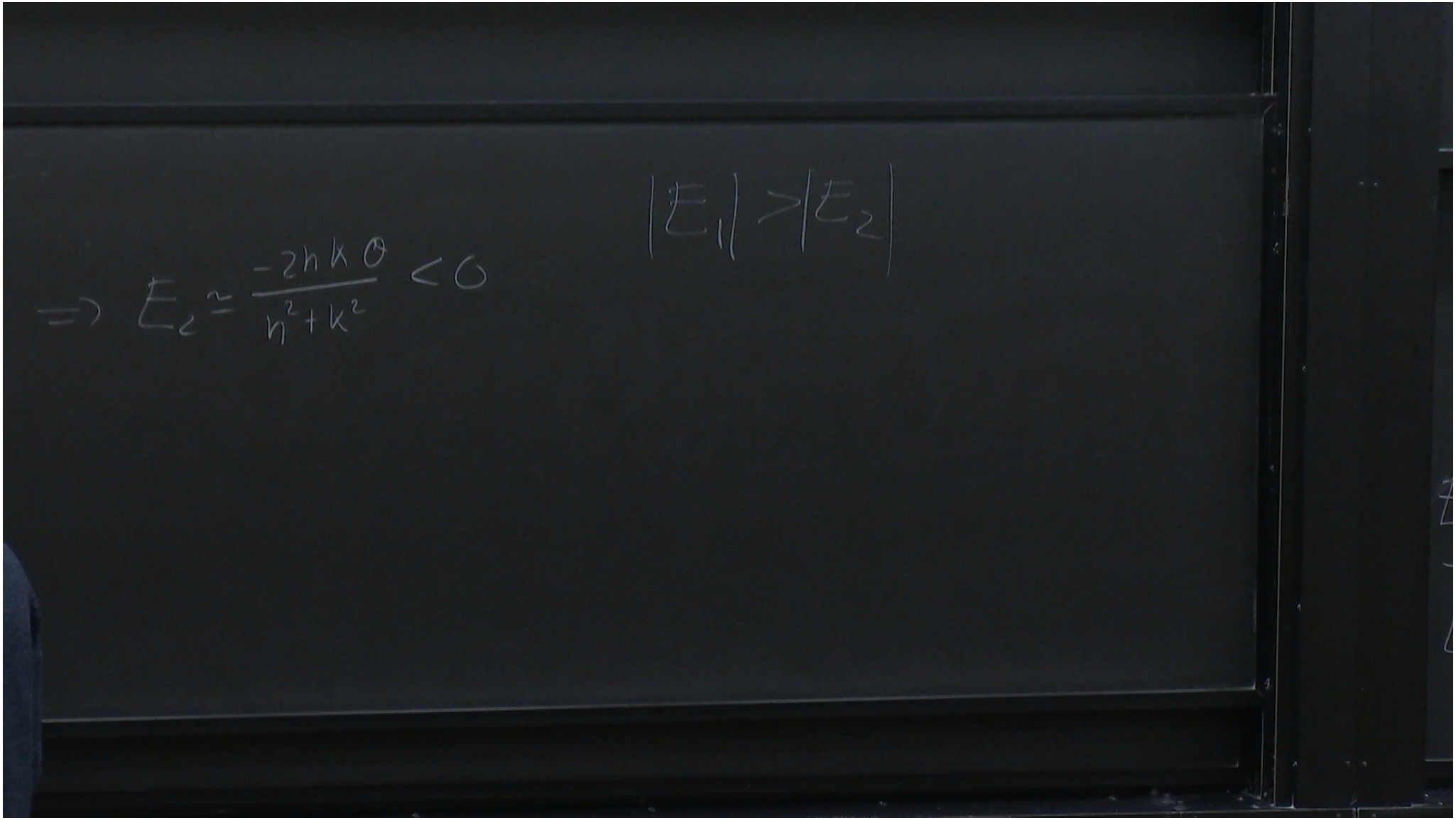

$\hat{\rho}_2 = \sum_{\alpha=\pm 1} \hat{U}_B(\alpha)\hat{P}_A(\alpha)|\psi\rangle\langle\psi|\hat{P}_A(\alpha)\hat{U}_B^{-1}(\alpha)$
$\hat{H}\hat{U}_B(\alpha)\hat{P}_A(\alpha)|\psi\rangle = \sum_{\alpha=\pm 1}(\underbrace{\langle\psi|\hat{P}_A\hat{U}_B^{\dagger}\hat{H}_A\hat{U}_B\hat{P}_A|\psi\rangle}_{E_1} + \langle\psi|\hat{P}_A\hat{U}_B^{\dagger}\hat{H}_B\hat{U}_B\hat{P}_A|\psi\rangle + \langle\psi|\hat{P}_A\hat{U}_B^{\dagger}\hat{V}\hat{U}_B\hat{P}_A|\psi\rangle$

$$E_2 = \mathrm{Tr}(\hat{P}_2 \hat{H}) - \mathrm{Tr}(\hat{P}_1 \hat{H}) = \mathrm{Tr}\left(\hat{P}_2 (\hat{H}_B + \hat{V})\right) = \sum_{\alpha} \langle g | \hat{P}_A \hat{U}_B^{\dagger} (\hat{H}_B + \hat{V}) \hat{U}_B | g \rangle$$

$$[\hat{P}_A, \hat{H}_B] = 0 = [\hat{P}_A, \hat{V}] = [\hat{P}_A, \hat{U}_B] = 0$$

$$E_2 = \frac{-1}{h^2+k^2}\left[hk\sin(2\theta) - (h^2+2k^2)(1-\cos(2\theta))\right] \qquad 0 < \theta \ll 1 \Rightarrow$$


P


$\Rightarrow E_2 = \frac{-2nk\theta}{n^2+k^2} < 0$
$|E_1| > |E_2|$

$\langle g | P_A U_B^\dagger (H_B + V) U_{\cdots} | g \rangle$

$= 0$

$\cos(2\theta)\Big]\Big\}$ $\quad 0 < \theta \ll 1 \Rightarrow E_2 \simeq \frac{-2\hbar k \theta}{\eta^2 + k^2} < 0$ $\quad |2_{11}\rangle$

+ classical info exchange

$\text{pendent}) \Rightarrow \hat{W}_B |\psi_{PM}\rangle = \frac{1}{\sqrt{P_A(a)}} \hat{W}_A \hat{P}_A(a) |g\rangle$

$$\hat{\rho}_2 = W_B \hat{\rho}_1 W_B^\dagger = W_B \left(\sum_{\alpha=1} \hat{P}(\alpha) |\psi\rangle\langle\psi| \hat{P}(\alpha) \right) W_B^\dagger \Rightarrow$$

$$E_{W_B} = Tr(\hat{\rho}_2 \hat{H}) - Tr(\hat{\rho}_1 H) = \sum_\alpha \langle\psi| P_A(\alpha) W_B^\dagger (\hat{H}_B + \hat{U}) W_B |\psi\rangle,$$

$$E_{W_B} = \langle\psi| W_B^\dagger (\hat{H}_B + \hat{U}) W_B |\psi\rangle. \quad \text{We know } [\hat{H}_A, W_B] = 0, \quad \langle\psi|$$

$$E_{W_B} = \langle\psi| W_B^\dagger \hat{H} W_B |\psi\rangle \geq 0$$

$\langle s | \hat{P}(\alpha) | W_B^\dagger \Rightarrow$

$P_A(\alpha) \hat{W}_B^\dagger (\hat{H}_B + \hat{V}) \hat{W}_B |s\rangle$, $\sum_\alpha \hat{P}(\alpha) = \underline{1}$

We know $[\hat{H}_A, \hat{W}_B] = 0$, $\langle s | W_B^\dagger H_A W_B | s \rangle = \langle s | H_A | s \rangle = c$

0

$\hat{P}(\alpha)|\psi\rangle\langle\psi|\hat{P}(\alpha)\,W_B^\dagger \Rightarrow$

$\sum_\alpha \langle\psi| P_A(\alpha)\, W_B^\dagger (\hat{H}_B + \hat{V})\, W_B |\psi\rangle$, $\quad \sum_\alpha \hat{P}(\alpha) = \mathbb{1}$

7. We know $[\hat{H}_A, W_B] = 0$, $\quad \langle\psi| W_B^\dagger H_A W_B |\psi\rangle = \langle\psi| H_A |\psi\rangle = 0$

≥ 0

Natural Energy flow from A to B:

$\langle \hat{H}_B(t) \rangle = \sum_{\alpha=\pm} \langle\psi| \hat{P}_A\, e^{iHt} \hat{H}_B e^{-iHt} P_A |\psi\rangle = \frac{1}{2} g(h/\mu)\left(1 - \cos(4\mu)\right)$

$\langle \hat{V}(t) \rangle = \sum_{\alpha=\pm} \langle\psi| \hat{P}_A\, e^{iHt} \hat{V} e^{-iHt} P_A |\psi\rangle = 0$