Title: Quantum Information Lecture - 230324

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Collection: Quantum Information (2022/2023)

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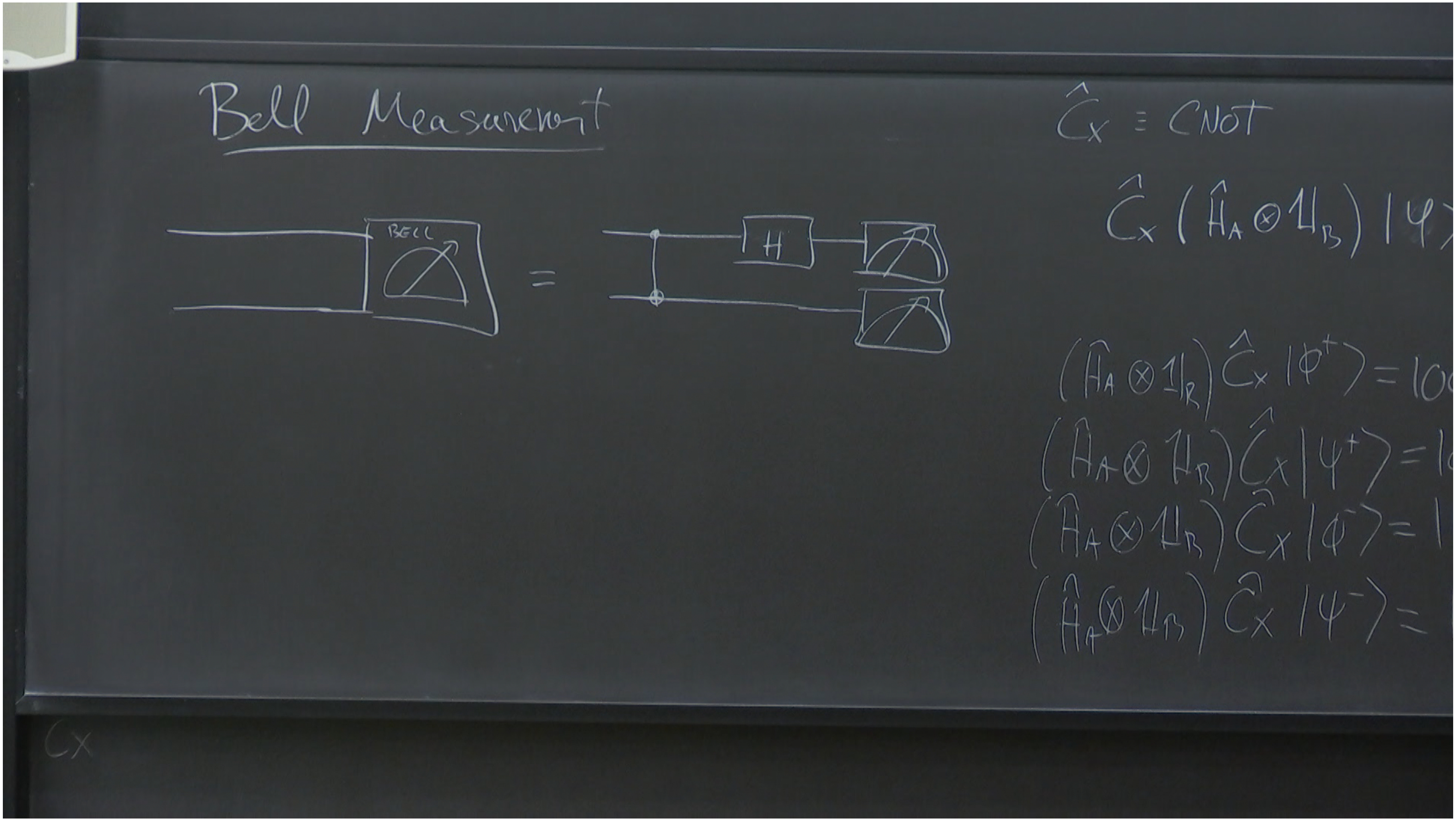

Bell Measurement
BELL
H
=
$\hat{C}_x \equiv$ CNOT
$\hat{C}_x (\hat{H}_A \otimes 1\!\!1_B) |\psi\rangle$
$(\hat{H}_A \otimes 1\!\!1_B) \hat{C}_x |\phi^+\rangle = |0$
$(\hat{H}_A \otimes 1\!\!1_B) \hat{C}_x |\psi^+\rangle = |$
$(\hat{H}_A \otimes 1\!\!1_B) \hat{C}_x |\phi^-\rangle = |$
$(\hat{H}_A \otimes 1\!\!1_B) \hat{C}_x |\psi^-\rangle =$
Cx

$$\hat{C}_X \equiv CNOT$$

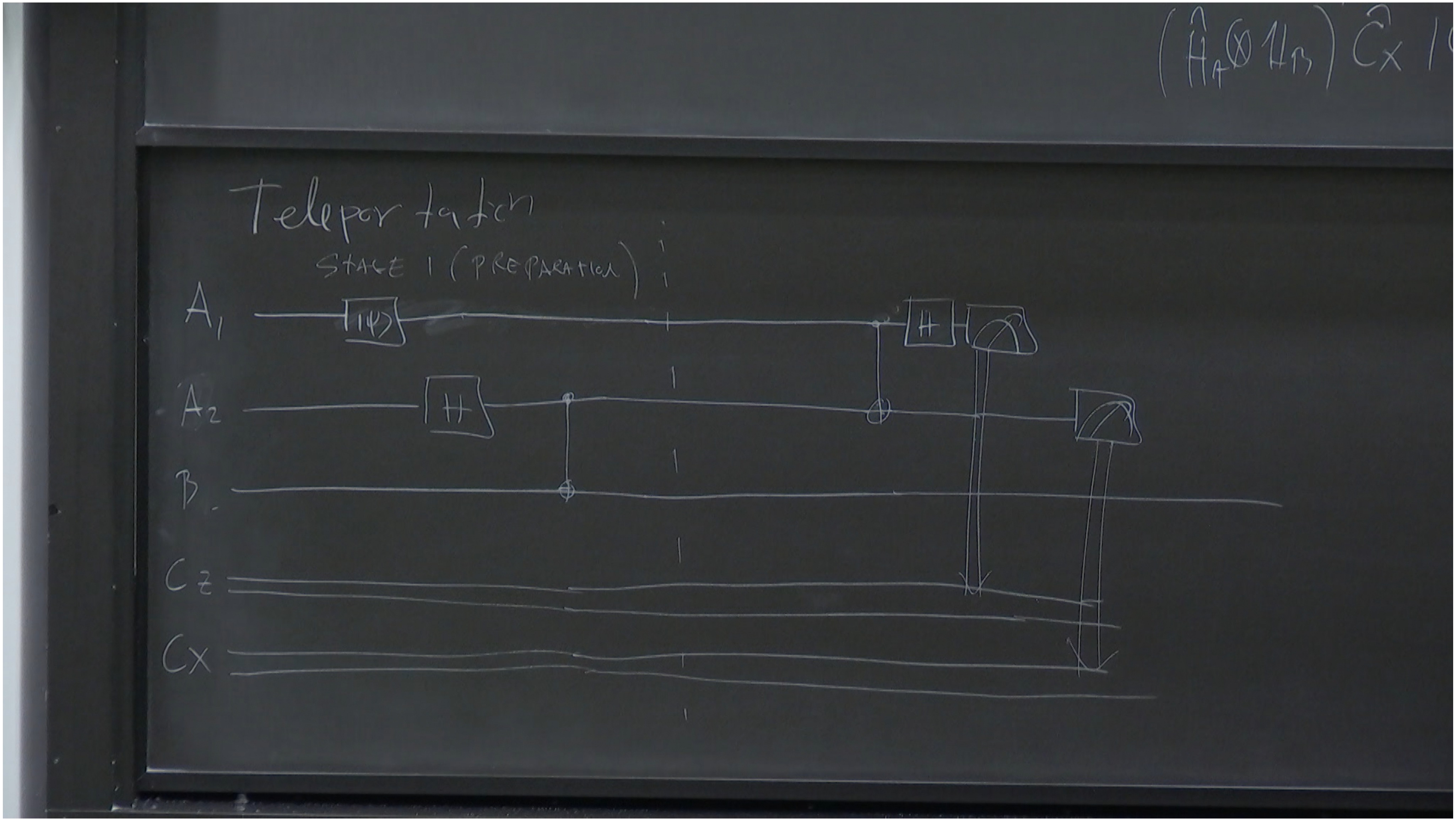
$$\hat{C}_X \left(\hat{H}_A \otimes \mathbb{1}_B\right) |\psi\rangle = \begin{cases} \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = |\phi^+\rangle & \text{if } |\psi\rangle = |00\rangle \\ |\psi^+\rangle & \text{if } |\psi\rangle = |01\rangle \\ |\phi^-\rangle & \text{if } |\psi\rangle = |10\rangle \\ |\psi^-\rangle & \text{if } |\psi\rangle = |11\rangle \end{cases}$$

$$\left(\hat{H}_A \otimes \mathbb{1}_B\right) \hat{C}_X |\phi^+\rangle = |00\rangle$$

$$\left(\hat{H}_A \otimes \mathbb{1}_B\right) \hat{C}_X |\psi^+\rangle = |01\rangle$$

$$\left(\hat{H}_A \otimes \mathbb{1}_B\right) \hat{C}_X |\phi^-\rangle = |10\rangle$$

$$\left(\hat{H}_A \otimes \mathbb{1}_B\right) \hat{C}_X |\psi^-\rangle = |11\rangle$$


Teleportation
STAGE 1 (PREPARATION)
A1
|ψ⟩
H
A2
H
B
Cz
Cx
H

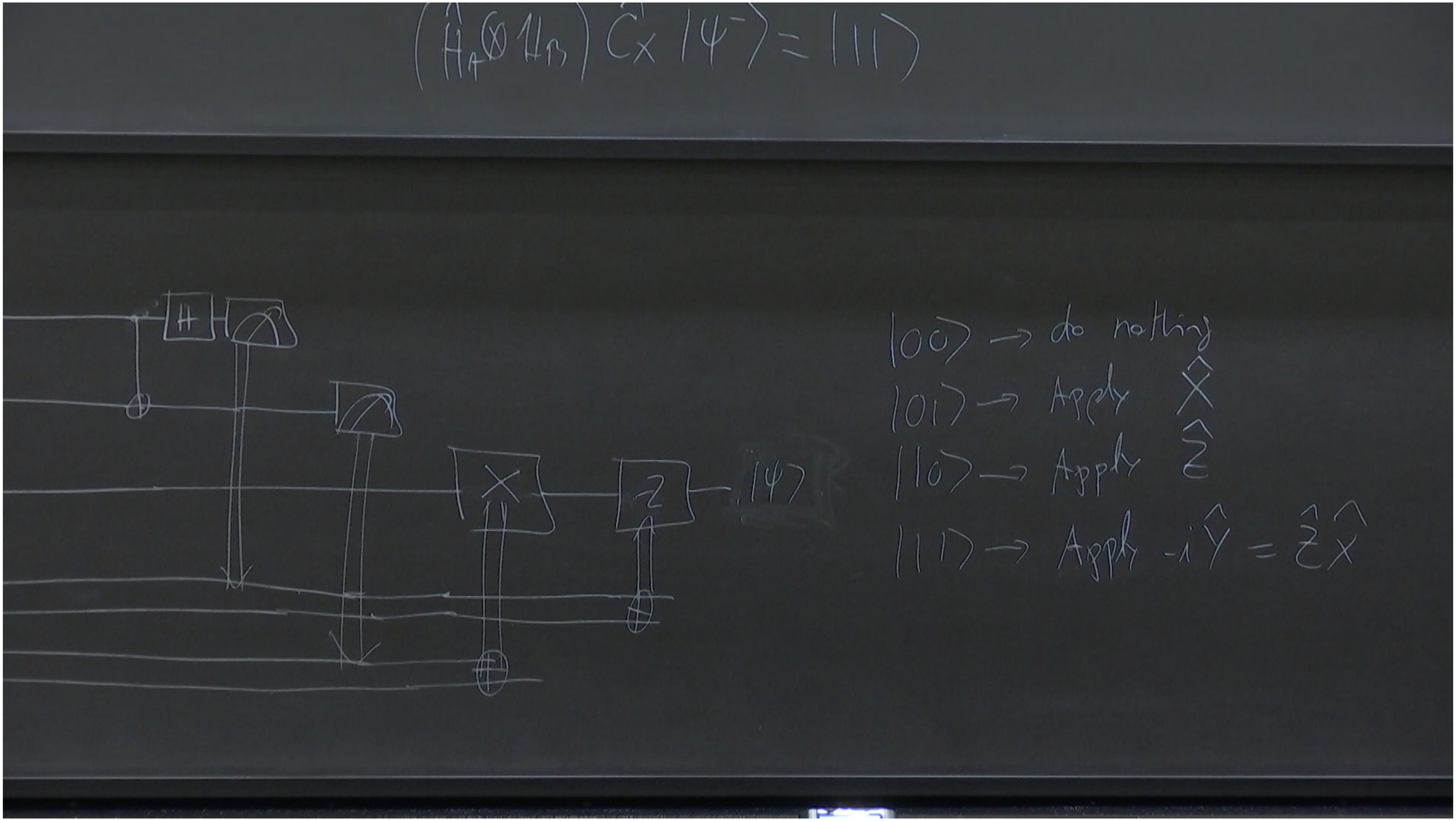
$(1_B) \, C_X |\psi^-\rangle = |11\rangle$

$|00\rangle \to$ do nothing

$|01\rangle \to$ Apply $\hat{X}$

$|10\rangle \to$ Apply $\hat{Z}$

$|11\rangle \to$ Apply $-i\hat{Y} = \hat{Z}\hat{X}$


$(\hat{H}_A \otimes 1\!\!1_B)\,\hat{C}_X |\psi^-\rangle = |11\rangle$
H
X
Z
$|\psi\rangle$
$|00\rangle \to$ do nothing
$|01\rangle \to$ Apply $\hat{X}$
$|10\rangle \to$ Apply $\hat{Z}$
$|11\rangle \to$ Apply $-i\hat{Y} = \hat{Z}\hat{X}$

thing

$\hat{X}$

$\hat{Z}$

$-i\hat{Y} = \hat{Z}\hat{X}$

A set S is universel if there exists a prodct of gates in S that yields any $\hat{U} \in SU(2^n)$

Example CNOT + local unitaries

$|\psi\rangle$

$|00\rangle \to$ do nothing

$|01\rangle \to$ Apply $\hat{X}$

$|10\rangle \to$ Apply $\hat{Z}$

$|11\rangle \to$ Apply $-i\hat{Y} = \hat{Z}\hat{X}$

A set S is universal if there exists a product of gates in S that yields any $\hat{U} \in SU(2^n)$

Example CNOT + local unitaries

A set S of gates is approx. universal if $\exists$ a product of gates in S such that one can be arbitrarily close to any $\hat{U} \in SU(2^n)$

Example: $\{\hat{H}, \hat{R}_{\pi/2}, \text{Toff}\}$

thing

$\hat{X}$

$\hat{Z}$

$-i\hat{Y} = \hat{Z}\hat{X}$

A set S is universal if there exists a product of gates in S that yields any $\hat{U} \in SU(2^n)$

Example CNOT + local unitaries

A set S of gates is approx. universal if $\exists$ a product of gates in S such that one can be arbitrarily close to any $\hat{U} \in SU(2^n)$

Examples: $\{\hat{H}, \hat{R}_{\pi/2}, \text{Toff}\}$, $\{\hat{H}, \text{CNOT}, \hat{R}_{\pi/4}\}$

Grover's Algorithm (Unstructured search)

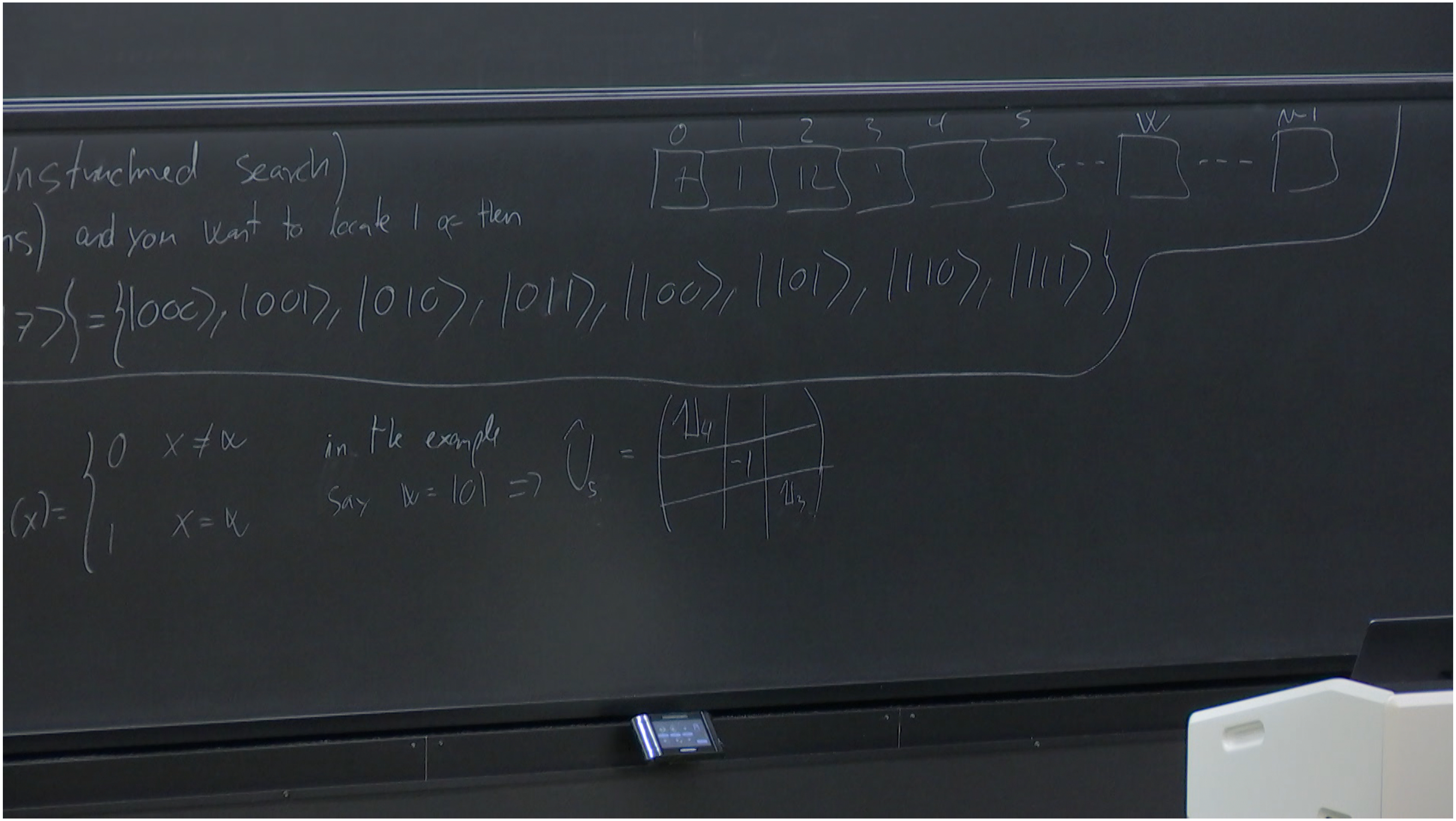
Large list of items (N items) and you want to locate 1 of them

example 8 elements $\{|0\rangle, |1\rangle, \dots |7\rangle\} = \{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |$

$$|s\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle = H^{\otimes n} |0 \dots 0\rangle$$

Oracle: $\hat{U}_w := (-1)^{f(x)} |x\rangle, \quad f(x) = \begin{cases} 0 & x \neq w \\ 1 & x = w \end{cases}$

in the example say $w = 101$


Unstructured search)
ns) and you want to locate 1 of them
0 1 2 3 4 5 w N-1
|?⟩} = {|000⟩, |001⟩, |010⟩, |011⟩, |100⟩, |101⟩, |110⟩, |111⟩}
(x) = 0 x ≠ w
1 x = w
in the example
say w = 101 ⇒ U_s =
1_4
-1
1_3

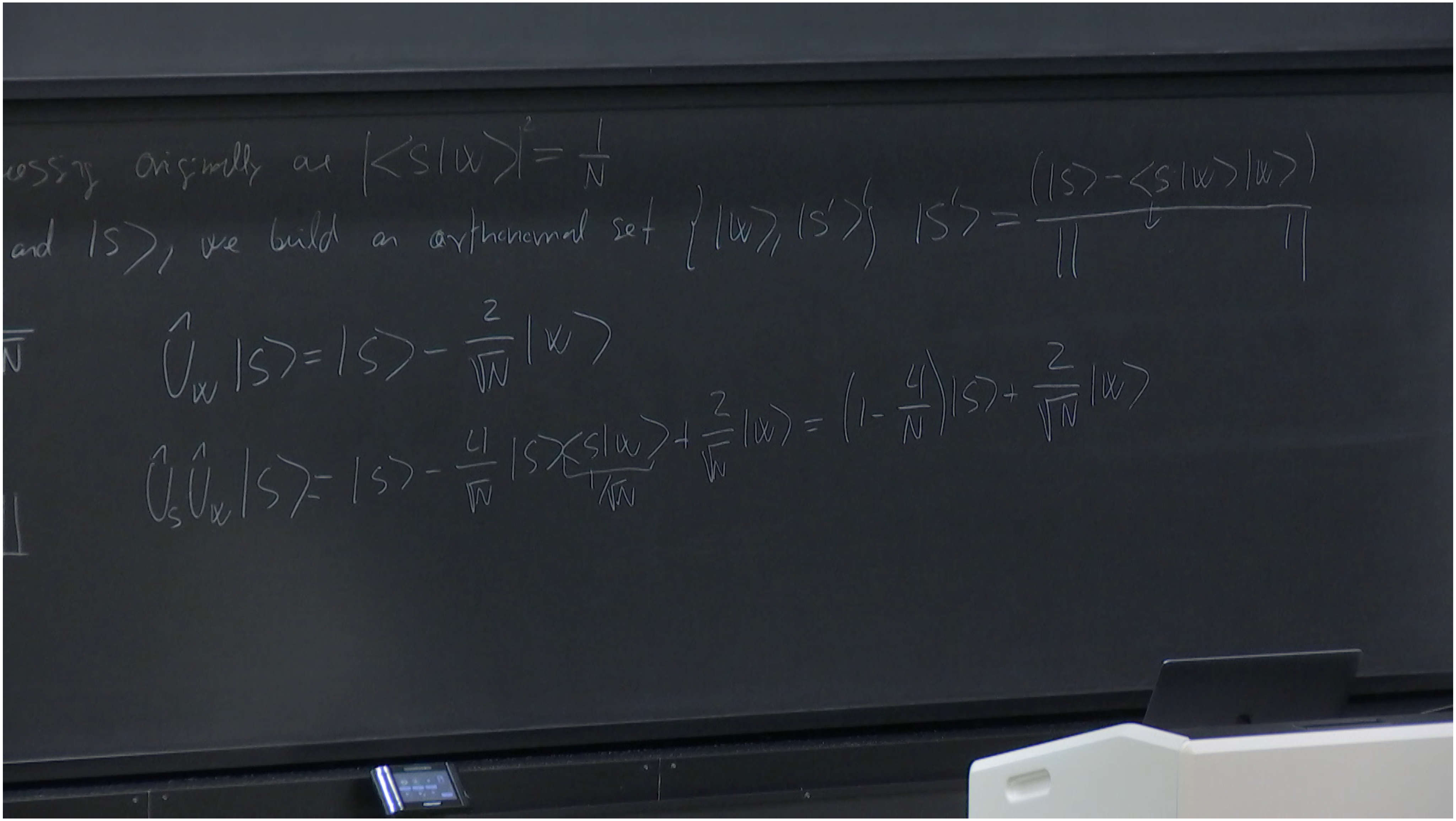
AMPLITUDE AMPLIFICATION: Chances of correct guessing originally are $|\langle$

Consider the two-dimensional space spanned by $|w\rangle$ and $|s\rangle$, we build an

$$|s\rangle = \cos\theta\,|s'\rangle + \sin\theta\,|w\rangle, \quad \sin\theta = \frac{1}{\sqrt{N}} \qquad \hat{U}_w|s\rangle = |s\rangle$$

1- Apply the oracle to $|s\rangle$: $\hat{U}_w|s\rangle$

2- Apply a reflection about $|s\rangle$: $\hat{U}_s = 2|s\rangle\langle s| - \mathbb{1}$



essing originally as $|\langle s|w\rangle|^2 = \frac{1}{N}$
and $|s\rangle$, we build an orthonormal set $\{|w\rangle, |s'\rangle\}$ $|s'\rangle = \frac{(|s\rangle - \langle s|w\rangle|w\rangle)}{\| \quad \|}$
$\hat{U}_w|s\rangle = |s\rangle - \frac{2}{\sqrt{N}}|w\rangle$
$\hat{U}_s\hat{U}_w|s\rangle = |s\rangle - \frac{4}{\sqrt{N}}|s\rangle\underbrace{\langle s|w\rangle}_{1/\sqrt{N}} + \frac{2}{\sqrt{N}}|w\rangle = \left(1 - \frac{4}{N}\right)|s\rangle + \frac{2}{\sqrt{N}}|w\rangle$

AMPLITUDE AMPLIFICATION: Chances of correct guessing originally

Consider the two-dimensional space spanned by $|w\rangle$ and $|s\rangle$, we

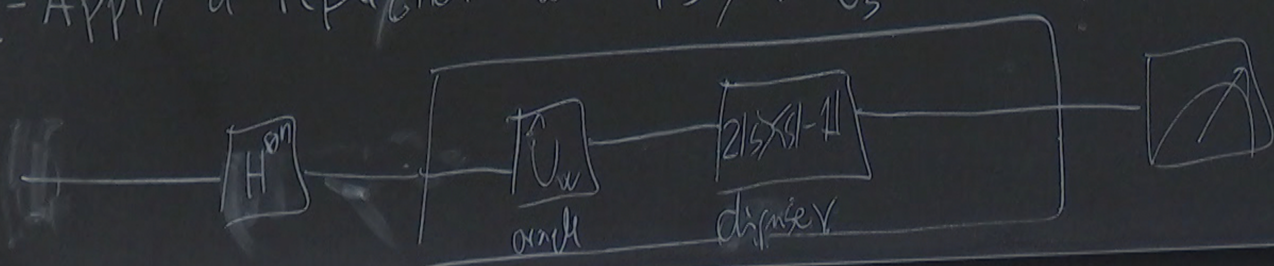
$$|s\rangle = \cos\theta\,|s'\rangle + \sin\theta\,|w\rangle, \quad \sin\theta = \frac{1}{\sqrt{N}}$$

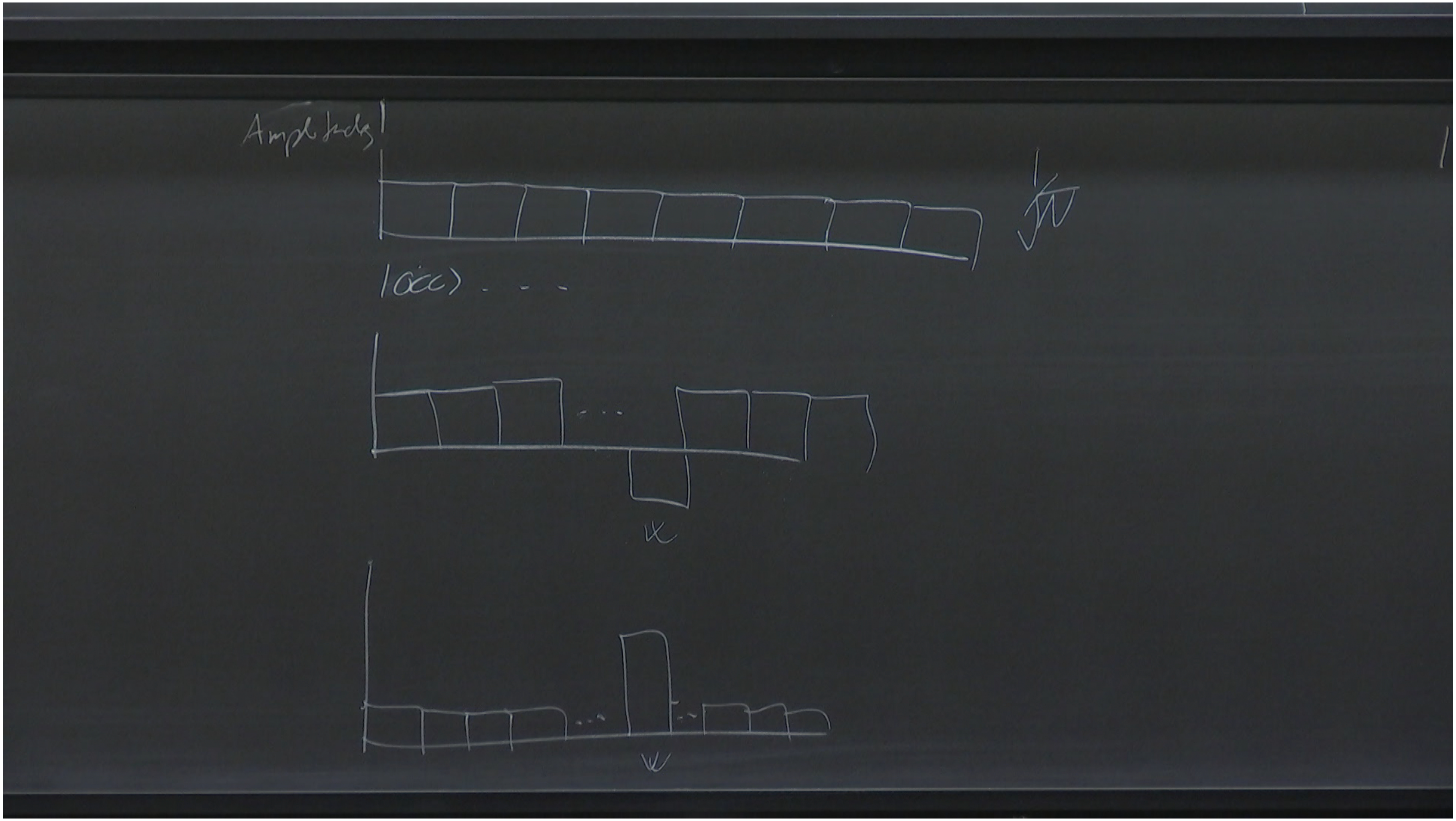
$\hat{U}_w |s$

1- Apply the oracle to $|s\rangle$: $\hat{U}_w |s\rangle$

$\hat{U}_s \hat{U}_w |s$

2- Apply a reflection about $|s\rangle$: $\hat{U}_s = 2|s\rangle\langle s| - \mathbb{1}$



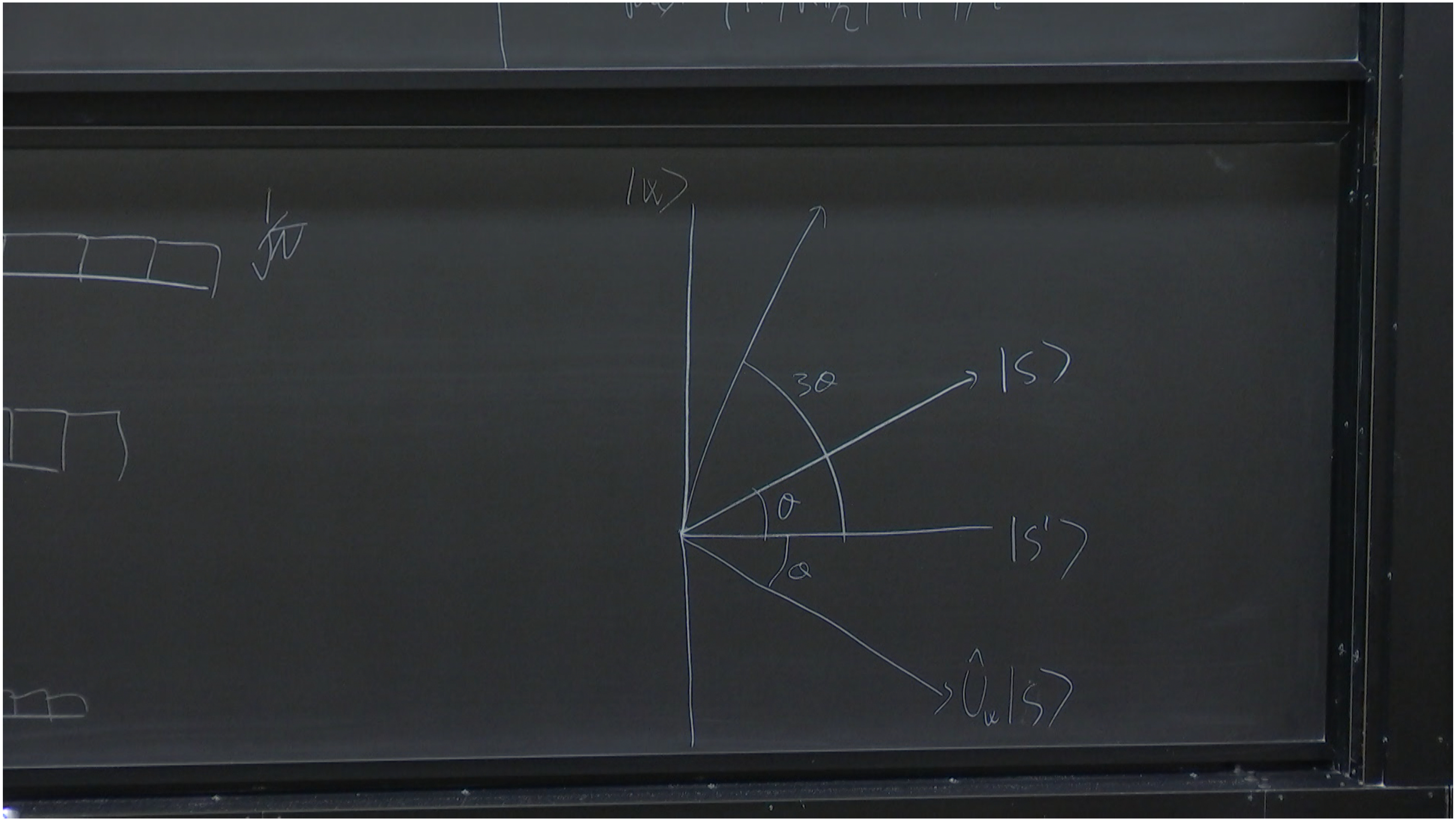

Amplitude

lly are $|\langle s|w\rangle|^2 = \frac{1}{N}$

e build an orthonormal set $\{|w\rangle, |s'\rangle\}$ $\quad |s'\rangle = \frac{(|s\rangle - \langle s|w\rangle|w\rangle)}{\|\quad\|}$

$$|s\rangle = |s\rangle - \frac{2}{\sqrt{N}}|w\rangle$$

$$|s\rangle = |s\rangle - \frac{4}{\sqrt{N}}|s\rangle\underbrace{\langle s|w\rangle}_{1/\sqrt{N}} + \frac{2}{\sqrt{N}}|w\rangle = \left(1 - \frac{4}{N}\right)|s\rangle + \frac{2}{\sqrt{N}}|w\rangle$$


$|w\rangle$
$|s\rangle$
$|s'\rangle$
$\hat{U}_w|s\rangle$
3θ
θ
θ

```
In[1]:= (*Basis elements*)
K0 = (1
      0);
K1 = (0
      1);
K00 = KroneckerProduct[K0, K0];
K01 = KroneckerProduct[K0, K1];
K10 = KroneckerProduct[K1, K0];
K11 = KroneckerProduct[K1, K1];
{MatrixForm[K00], MatrixForm[K01], MatrixForm[K10], MatrixForm[K11]}
```

```
In[7]:= (*Pauli Gates*)
```

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$; $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$;

```
(*Hadamard and Phase Rotation*)
```

$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$; R[θ_] := $\begin{pmatrix} 1 & 0 \\ 0 & \mathrm{Exp}[i\,\theta] \end{pmatrix}$;

```
(*CNOT AND SWAP*)
```

$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$; $SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$;

```
(*Toffoli*)
```

$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Assuming a matrix | Use as *suppressed output* or a *list of pairs* instead

matrix plot | display as | determinant | inverse | more...

```
(*Playing with Rotations*)

In[11]:= MatrixForm[FullSimplify[H.K0, Assumptions → θ ∈ Reals]]

(*Swaping the control! Who controls who?*)

In[12]:= MatrixForm[KroneckerProduct[H, H].CNOT.KroneckerProduct[H, H]]

In[13]:= MatrixForm[KroneckerProduct[H, H].CNOT.KroneckerProduct[H, H].K11]

In[ ]:=
(* Grover Example with three qubits *)
```

$$\text{OPTn} = \text{N}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{8}}\right]} - \frac{1}{2}\right]$$

```
Out[ ]= 1.67341

In[ ]:= (* Initialization *)
MatrixForm[K000 = KroneckerProduct[K0, K0, K0]]

Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$$

In[274]:=

```
MatrixForm[KroneckerProduct[H, H].CNOT.KroneckerProduct[H, H].K11]
```

Out[274]//MatrixForm=

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Assuming a matrix | Use as a list of lists instead

transpose | display as | flatten | total | more...

In[•]:=

```
(* Grover Example with three qubits *)
```

$$\text{OPTn} = \text{N}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{8}}\right]} - \frac{1}{2}\right]$$

Out[•]= 1.67341

In[•]:=

```
(* Initialization *)
MatrixForm[K000 = KroneckerProduct[K0, K0, K0]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[275]:=

```
(* Grover Example with three qubits *)
```

$$\text{OPTn} = \text{N}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{8}}\right]} - \frac{1}{2}\right]$$

Out[275]=

1.67341

show all digits | integer part | nth digit... | digits | more...

In[•]:=

```
(* Initialization *)
MatrixForm[K000 = KroneckerProduct[K0, K0, K0]]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[•]:=

```
MatrixForm[S = KroneckerProduct[H, H, H].K000]
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[275]:=

```
(* Grover Example with three qubits *)
```

$$\text{OPTn} = \text{N}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{8}}\right]} - \frac{1}{2}\right]$$

Out[275]=

1.67341

show all digits | integer part | nth digit... | digits | more...

In[ ]:=

```
(* Initialization *)
MatrixForm[K000 = KroneckerProduct[K0, K0, K0]]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[ ]:=

```
MatrixForm[S = KroneckerProduct[H, H, H].K000]
```

Out[ ]//MatrixForm=

```
1.67341
```

In[277]:=

```
(* Initialization *)
MatrixForm[K000 = KroneckerProduct[K0, K0, K0]]
```

Out[277]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Assuming a matrix | Use as a list of lists instead

transpose | display as | flatten | total | more...

In[ ]:=

```
MatrixForm[S = KroneckerProduct[H, H, H].K000]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix}$$

In[278]:=

```
(* Initialization *)
MatrixForm[K000 = KroneckerProduct[K0, K0, K0]]
```

Out[278]//MatrixForm=

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[ ]:=

```
MatrixForm[S = KroneckerProduct[H, H, H].K000]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix}$$

In[ ]:=

```
(* Oracle *)
MatrixForm[Winner = KroneckerProduct[K1, K1, K0]]
```

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

In[279]:=

```
MatrixForm[S = KroneckerProduct[H, H, H].K000]
```

Out[279]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix}$$

Assuming a matrix | Use as a list of lists instead

transpose | display as | flatten | total | more...

In[ ]:=

```
(* Oracle *)
MatrixForm[Winner = KroneckerProduct[K1, K1, K0]]
MatrixForm[Uw = IdentityMatrix[8] - 2 Winner.Winner†]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

In[280]:=

```
(* Oracle *)
MatrixForm[Winner = KroneckerProduct[K1, K1, K0]]
MatrixForm[Uw = IdentityMatrix[8] - 2 Winner.Winner†]
```

Out[280]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Out[281]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Assuming a matrix | Use as a list of lists instead

matrix plot | display as | determinant | inverse | more...

In[ ]:=

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Out[281]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:=

```
MatrixForm[Uw.S]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix}$$

In[ ]:=

```
(*Difusser*)
```

```
MatrixForm[Uw.S]
```

Out[284]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{pmatrix}$$

In[∘]:=

```
(*Difusser*)
MatrixForm[Us = 2 S.S† - IdentityMatrix[8]]
```

In[∘]:=

```
(* Grover Cell *)
Steps[n_] := MatrixPower[Us.Uw, n].S
```

In[∘]:=

```
i = 2;
MatrixForm[Steps[i]†]
BarChart[Abs[Steps[i]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & \frac{11}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} \end{pmatrix}$$

1.0

```
MatrixForm[Us = 2 S.S† - IdentityMatrix[8]]
```

Out[285]//MatrixForm=

$$\begin{pmatrix} -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix}$$

Assuming a matrix | Use as a *list of lists* instead

matrix plot | display as | determinant | inverse | more...

In[∘]:=
```
(* Grover Cell *)
Steps[n_] := MatrixPower[Us.Uw, n].S
```

In[∘]:=
```
i = 2;
MatrixForm[Steps[i]†]
BarChart[Abs[Steps[i]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & \frac{11}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} \end{pmatrix}$$

```
(* Grover Cell *)
Steps[n_] := MatrixPower[Us.Uw, n].S
```

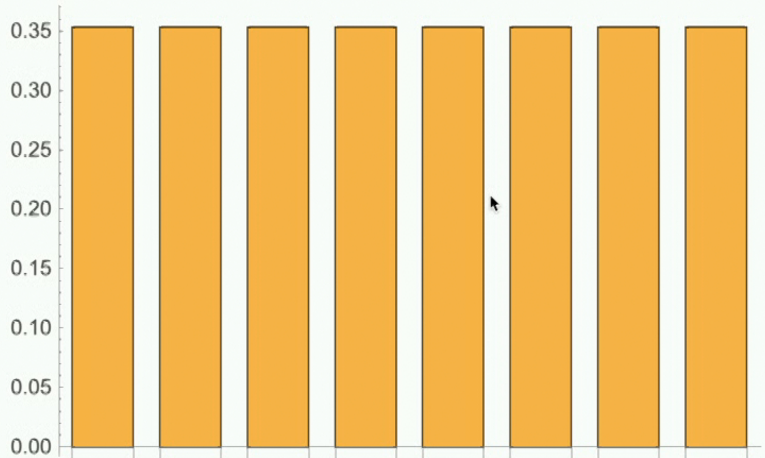
In[287]:=

```
i = 0;
MatrixForm[Steps[i]†]
BarChart[Abs[Steps[i]]]
```

Out[288]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} \end{pmatrix}$$

Out[289]=



theme... frame... labels... axes more...

In[ ]:= (* Example with four qubits *)

$$\text{OPTn} = \text{N}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{2^{10}}}\right]} - \frac{1}{2}\right]$$

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix}$$

In[286]:=

```
(* Grover Cell *)
Steps[n_] := MatrixPower[Us.Uw, n].S
```

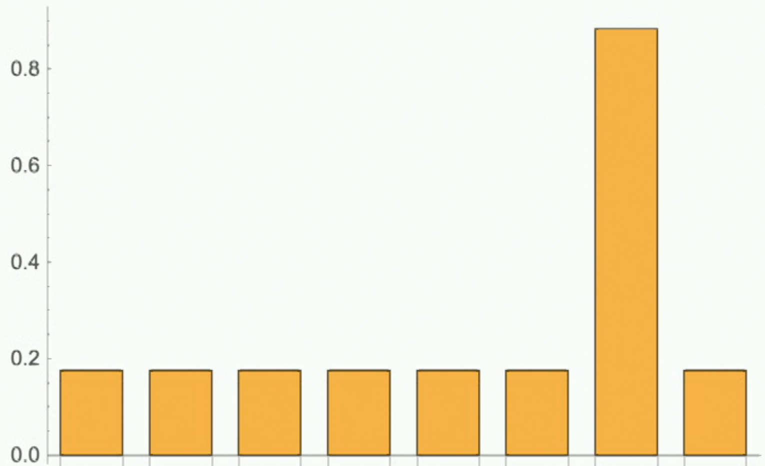
In[290]:=

```
i = 1;
MatrixForm[Steps[i]†]
BarChart[Abs[Steps[i]]]
```

Out[291]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} & \frac{5}{4\sqrt{2}} & \frac{1}{4\sqrt{2}} \end{pmatrix}$$

Out[292]=



theme... frame... labels... axes more...

In[ ]:= (* Example with four qubits *)

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix}$$

In[286]:=

```
(* Grover Cell *)
Steps[n_] := MatrixPower[Us.Uw, n].S
```

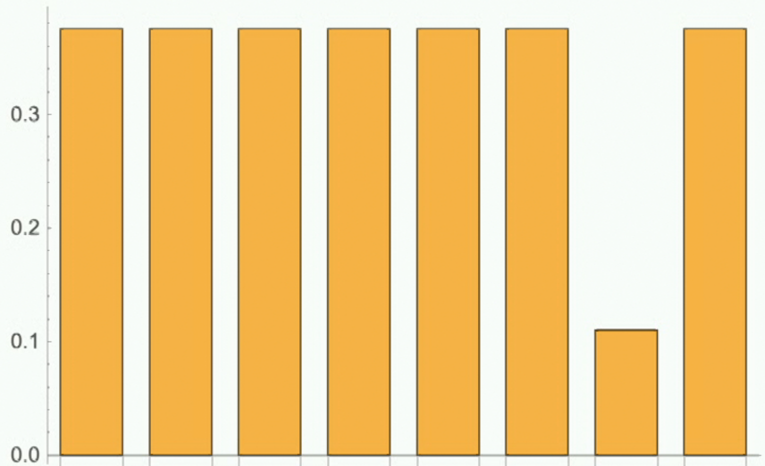
In[299]:=

```
i = 4;
MatrixForm[Steps[i]†]
BarChart[Abs[Steps[i]]]
```

Out[300]//MatrixForm=

$$\begin{pmatrix} -\frac{17}{32\sqrt{2}} & -\frac{17}{32\sqrt{2}} & -\frac{17}{32\sqrt{2}} & -\frac{17}{32\sqrt{2}} & -\frac{17}{32\sqrt{2}} & -\frac{17}{32\sqrt{2}} & -\frac{5}{32\sqrt{2}} & -\frac{17}{32\sqrt{2}} \end{pmatrix}$$

Out[301]=



theme... frame... labels... axes more...

In[∘]:= (* Example with four qubits *)

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix}$$

In[286]:=

```
(* Grover Cell *)
Steps[n_] := MatrixPower[Us.Uw, n].S
```

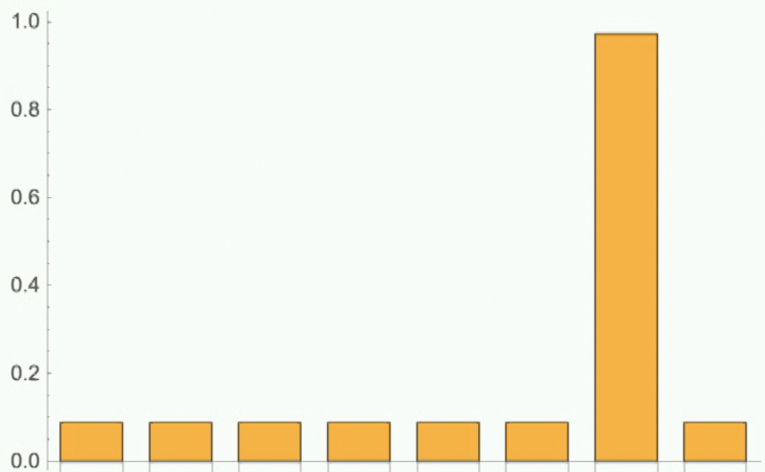
In[302]:=

```
i = 2;
MatrixForm[Steps[i]†]
BarChart[Abs[Steps[i]]]
```

Out[303]//MatrixForm=

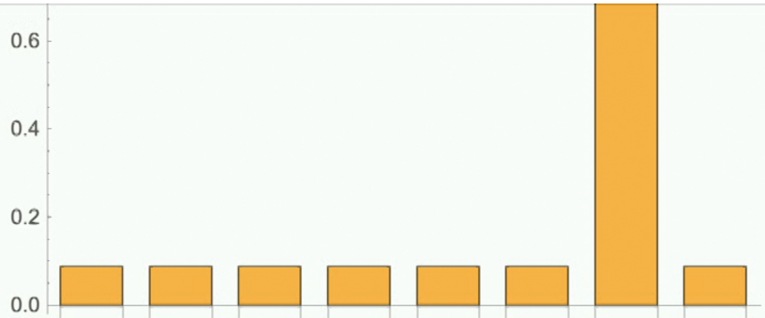
$$\begin{pmatrix} -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} & \frac{11}{8\sqrt{2}} & -\frac{1}{8\sqrt{2}} \end{pmatrix}$$

Out[304]=



theme... frame... labels... axes more...

```
In[ ]:= (* Example with four qubits *)
```



In[305]:=

```
(* Example with four qubits *)
```

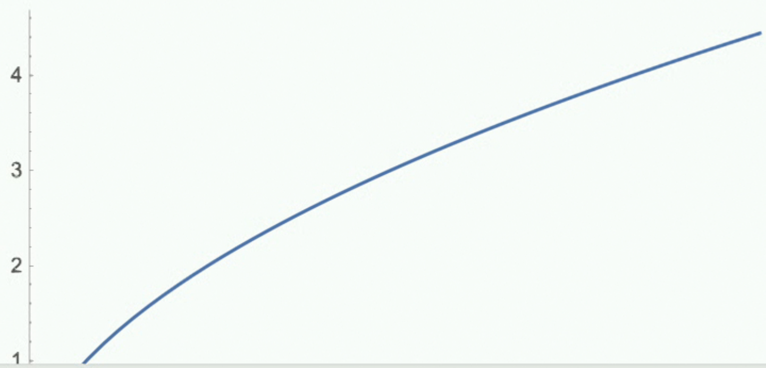
$$\text{OPTn} = \text{N}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{2^{10}}}\right]} - \frac{1}{2}\right]$$

Out[305]=

24.6286

$$\text{Plot}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]} - \frac{1}{2}, \{x, 1, 40\}\right]$$

Out[ ]=



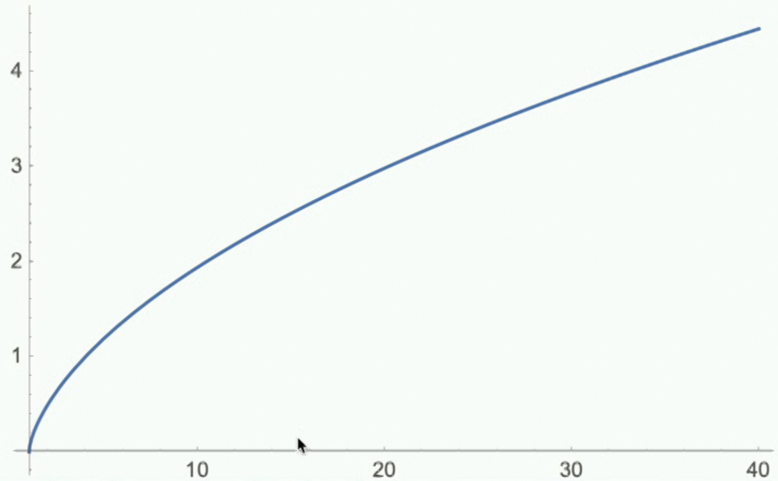
Out[307]=

2.60827

In[308]:=

$$\text{Plot}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]}-\frac{1}{2},\ \{x,\ 1,\ 40\}\right]$$

Out[308]=

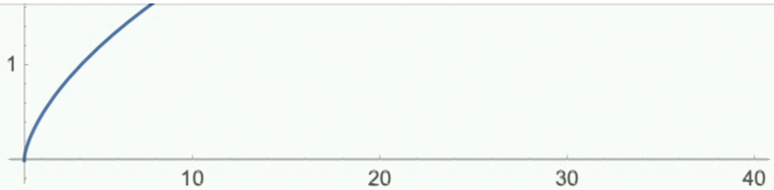


theme... labels... axes image size more...

In[ ]:=

$$\text{N}\left[\text{Solve}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]}-\frac{1}{2}==1,\ x\right]\right]$$

$$\text{N}\left[\text{Solve}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]}-\frac{1}{2}==3,\ x\right]\right]$$



In[311]:=

$$N\left[\text{Solve}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]}-\frac{1}{2}==1,\ x\right]\right]$$

$$N\left[\text{Solve}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]}-\frac{1}{2}==3,\ x\right]\right]$$

··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[311]=

$\{\{x \to 4.\}\}$

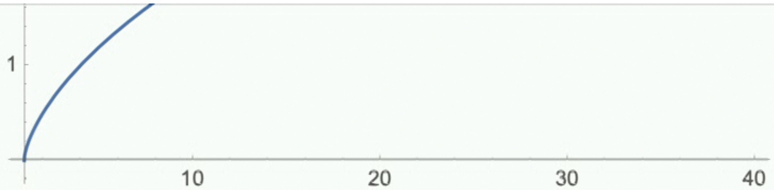
··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[312]=

$\{\{x \to 20.1957\}\}$

Assuming a list of rules | Use as a two-dimensional array instead

apply rules to variable | apply rules to expr...



In[315]:=

$$N\left[\text{Solve}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]}-\frac{1}{2}==1,\ x\right]\right]$$

$$N\left[\text{Solve}\left[\frac{\pi}{4\,\text{ArcSin}\left[\frac{1}{\sqrt{x}}\right]}-\frac{1}{2}==3,\ x\right]\right]$$

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[315]=

{{x → 4.}}

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[316]=

{{x → 20.1957}}

Assuming a list of rules | Use as a two-dimensional array instead

apply rules to variable | apply rules to expr...