Title: Quantum Information Lecture - 230322

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Collection: Quantum Information (2022/2023)

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# Classical Logic

## NOT

| in | out |
|---|---|
| 0 | 1 |
| 1 | 0 |


## OR

| in | | out | |
|---|---|---|---|
| a | b | a | a OR b |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

## NAND

| in | | out | |
|---|---|---|---|
| a | b | a | a NAND b |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 |

## NOR

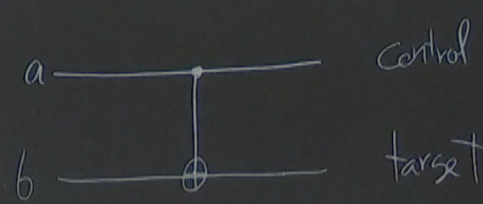
| in | | out | |
|---|---|---|---|
| a | b | a | a NOR b |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

## AND

| in | | out | |
|---|---|---|---|
| a | b | a | $a \cdot b$ |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 |

## XOR (CNOT)

| in | | out | |
|---|---|---|---|
| a | b | a | $a \oplus b$ |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 |

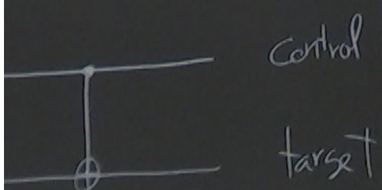


NAND

| in |  | out |  |
|---|---|---|---|
| b | | a | a NAND b |
| 0 | | 0 | 1 |
| 1 | | 0 | 1 |
| 0 | | 1 | 1 |
| 1 | | 1 | 0 |

NOR

| in |  | out |  |
|---|---|---|---|
| a | b | a | a NOR b |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |




TOFFOLI (Doubly controlled NOT)

| in |  |  | out |  |  |
|---|---|---|---|---|---|
| a | b | c | a | b | $c \oplus a \cdot b$ |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |

controlled NOT)

| $c \oplus a \cdot b$ |
|---|
| 0 |
| 1 |
| 0 |
| 1 |
| 0 |
| 1 |
| 1 |
| 0 |

## Universal set of gates

A set of gates is universal if it can be used to build any functions

$$f : \mathbb{Z}_2^n \longrightarrow \mathbb{Z}_2^m \qquad f(x_1, \dots x_n) = (x_1', \dots, x_m')$$

e.g. $\{XOR, AND\}$ is universal

$\{NOT, AND\}$

$\{NAND\}$

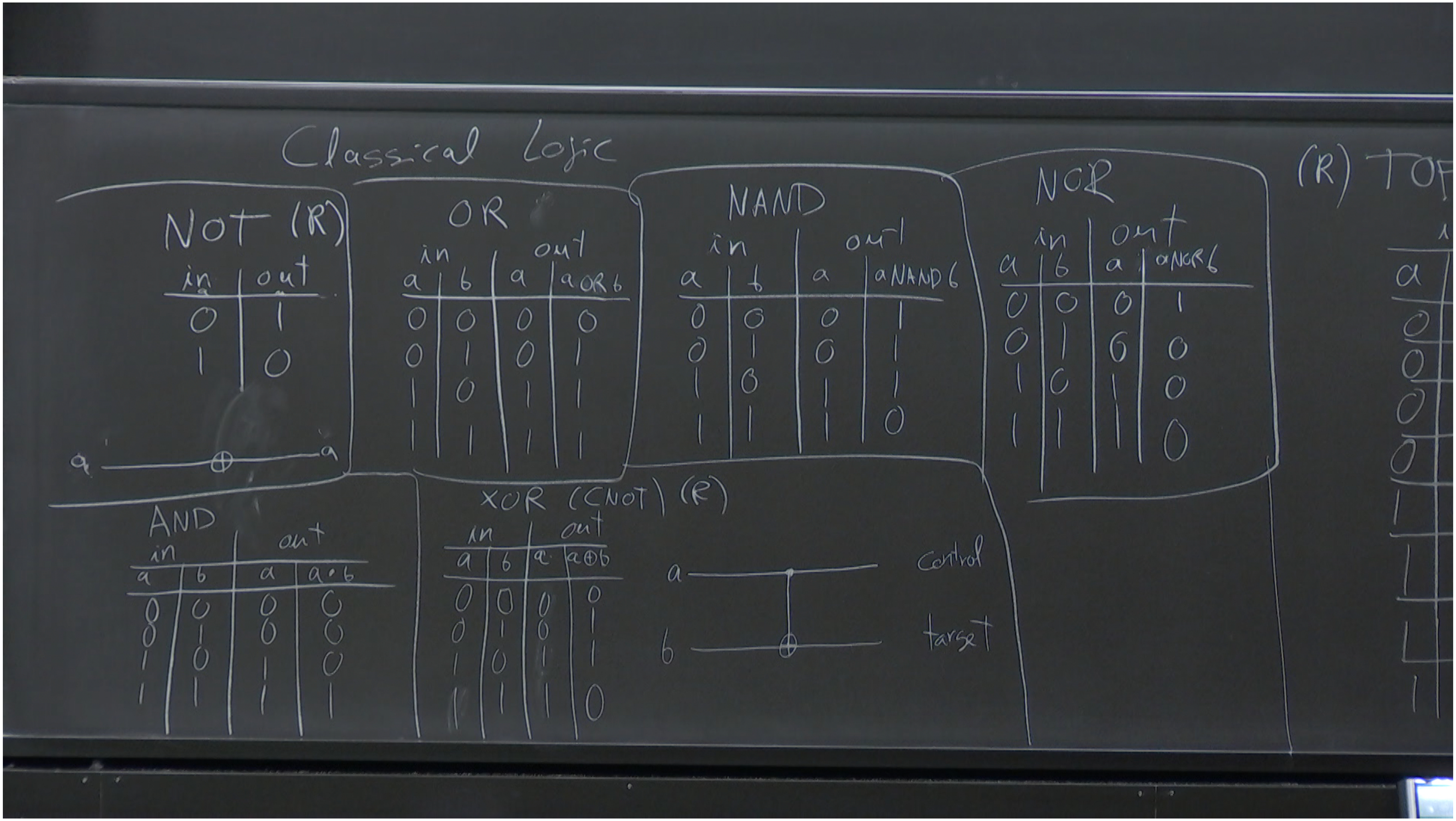
$\{TOFFOLI\}$

$$a \oplus b = a \text{ XOR } b = (a \text{ AND } (\text{NOT } b)) \text{ OR } ((\text{NOT } a) \text{ AND } b)$$

$$a \text{ OR } b = a \cdot b \oplus a \oplus b = \text{NOT}(\text{NOT } a \text{ AND } \text{NOT } b)$$

$$\text{NOT } a = (a \text{ NAND } a)$$

$$a \text{ AND } b = (a \text{ NAND } b) \text{ NAND } (a \text{ NAND } b)$$


Classical Logic
NOT (R)
in
out
OR
in
out
a
b
a
a OR b
NAND
in
out
a
b
a
a NAND b
NOR
in
out
a
b
a
a NOR b
(R) TOF
AND
in
out
a
b
a
a·b
XOR (CNOT) (R)
in
out
a
b
a
a⊕b
control
target

(Not b)) OR ((Not a) AND b)

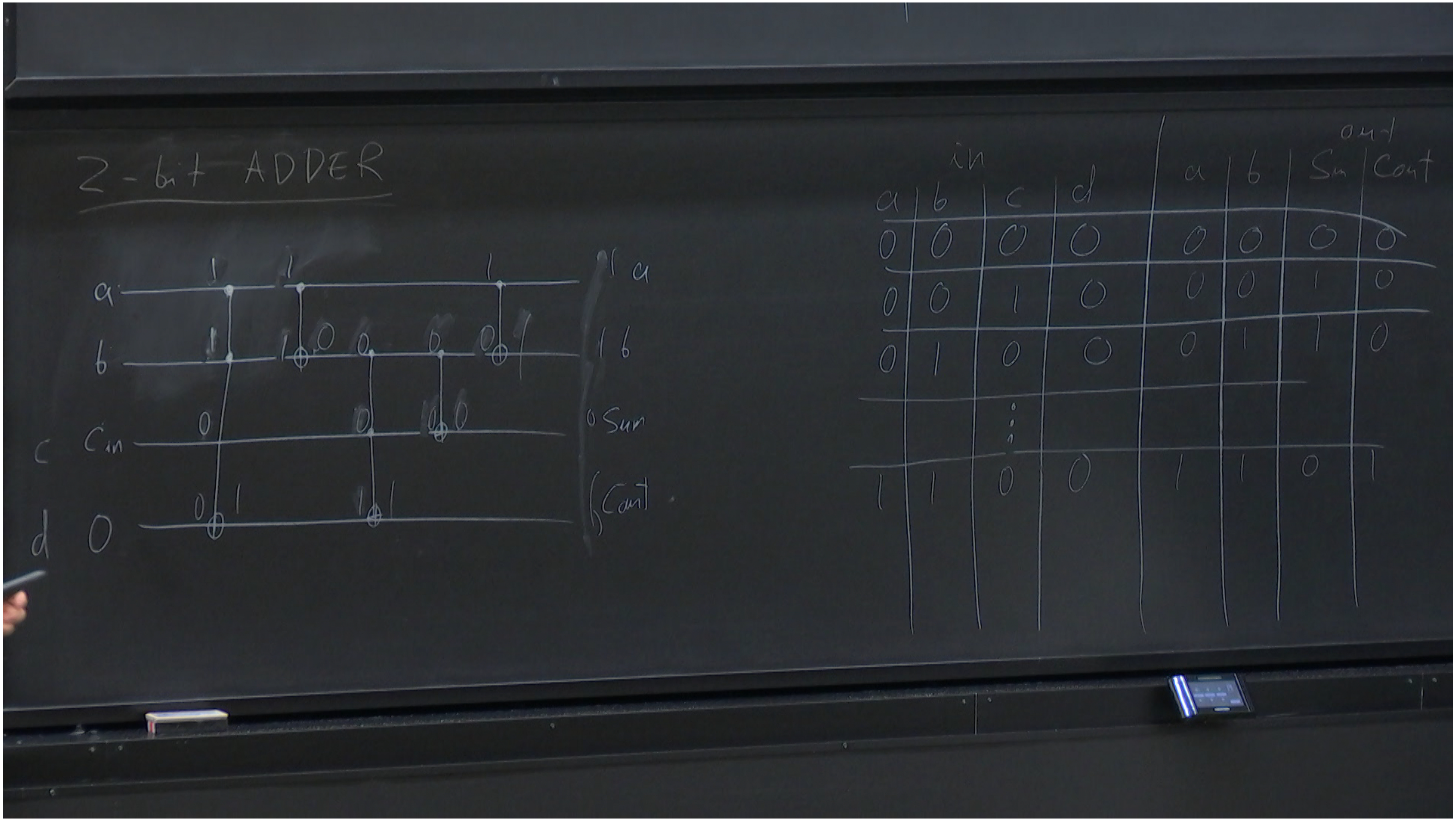
H (Not a AND Not b)

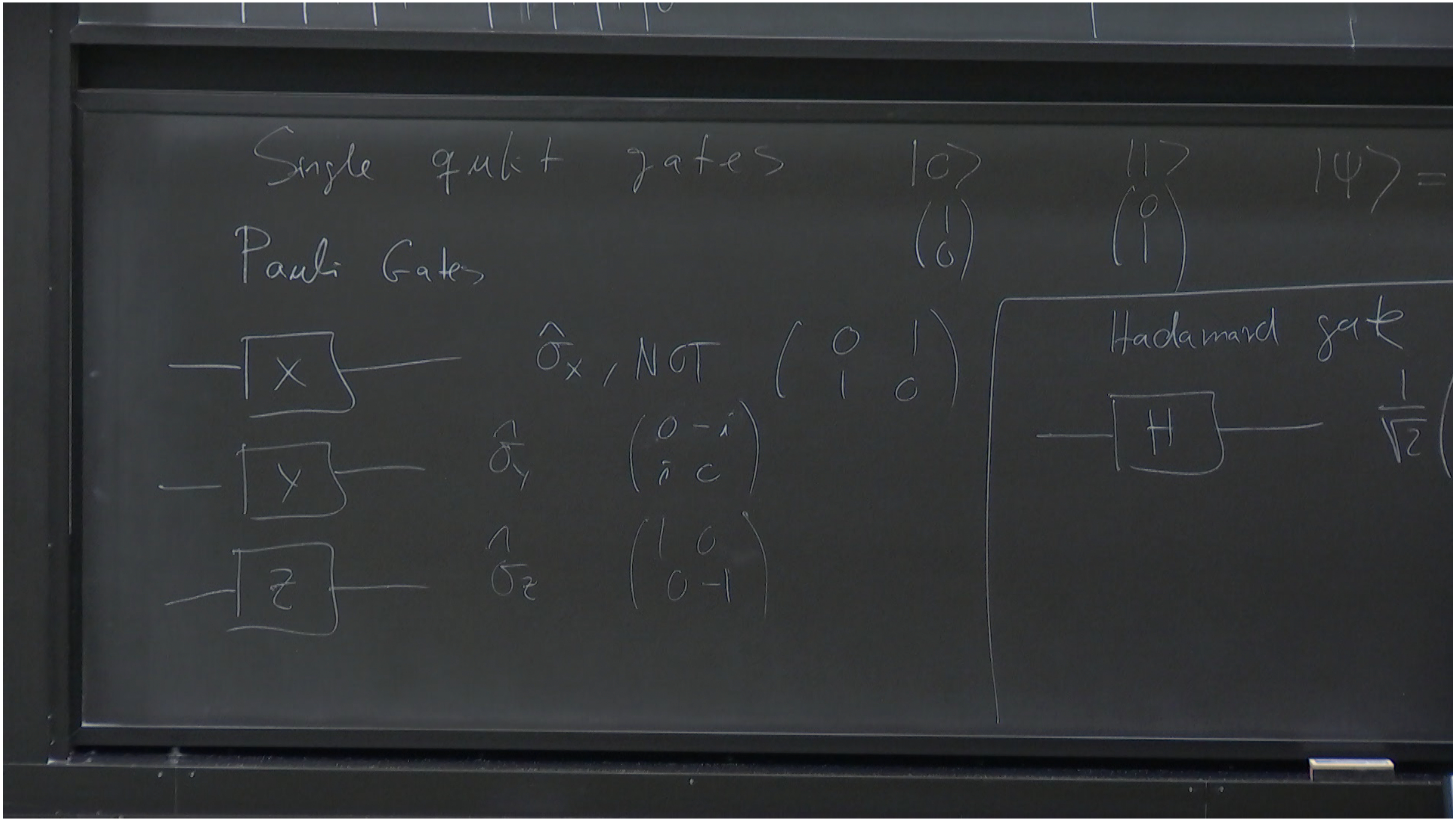
ND (a NAND b))

Not a = a NOR a

a AND b = (a NOR a) NOR (b NOR b)

a NAND b = TOFFOLI (a, b, 1)


2-bit ADDER
a
b
c Cin
d 0
Sum
Cout
in
out
a b c d
a b Sum Cout
0 0 0 0 0 0 0 0
0 0 1 0 0 0 1 0
0 1 0 0 0 1 1 0
1 1 0 0 1 1 0 1


Single qubit gates
$|0\rangle$ $\begin{pmatrix}1\\0\end{pmatrix}$
$|1\rangle$ $\begin{pmatrix}0\\1\end{pmatrix}$
$|\psi\rangle =$
Pauli Gates
X
$\hat{\sigma}_x$, NOT $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
Y
$\hat{\sigma}_y$ $\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$
Z
$\hat{\sigma}_z$ $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$
Hadamard gate
H
$\frac{1}{\sqrt{2}}$

$|0\rangle$ $\begin{pmatrix}1\\0\end{pmatrix}$ $\quad |1\rangle$ $\begin{pmatrix}0\\1\end{pmatrix}$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \to \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$

Hadamard gate

$H$ $\quad \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$

$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \equiv |+\rangle$$

$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \equiv |-\rangle$$

$R_\theta$ $\quad \begin{pmatrix}1 & 0\\ 0 & e^{i\alpha}\end{pmatrix}$

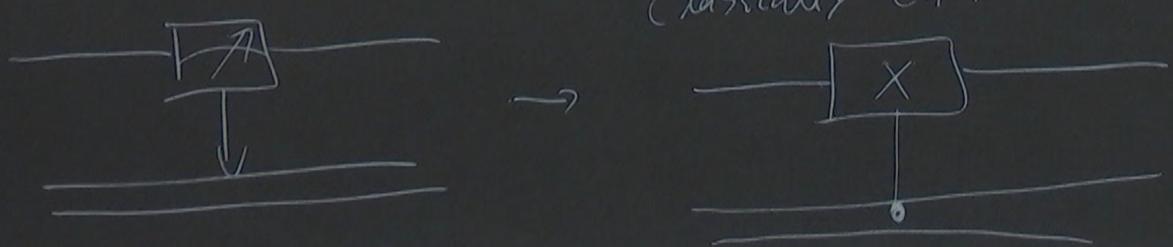
$\longrightarrow$

$\beta|1\rangle \to \begin{pmatrix}\alpha \\ \beta\end{pmatrix}$

$$|0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \equiv |+\rangle$$

$$|1\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \equiv |-\rangle$$

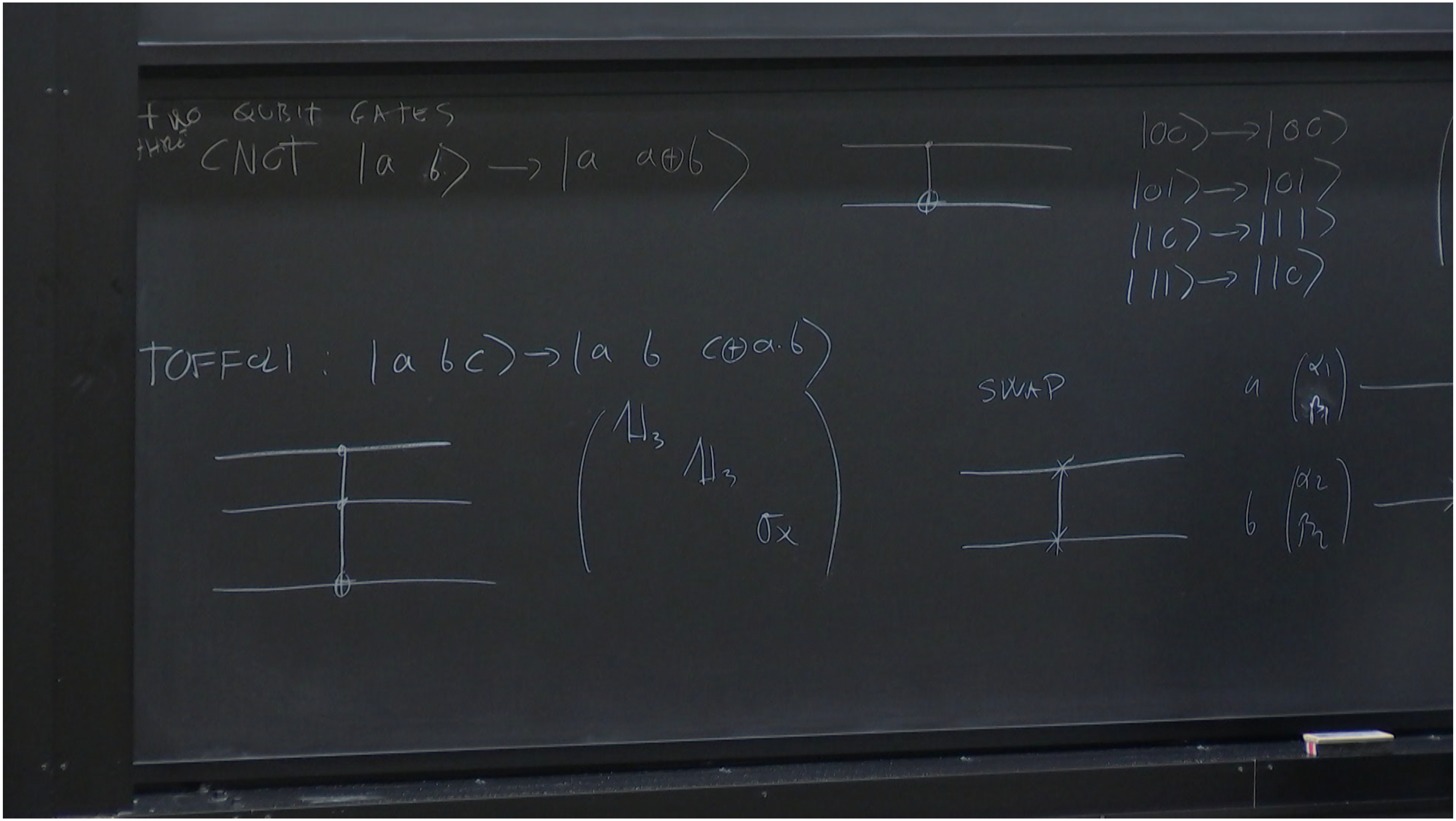
classically controlled NOT

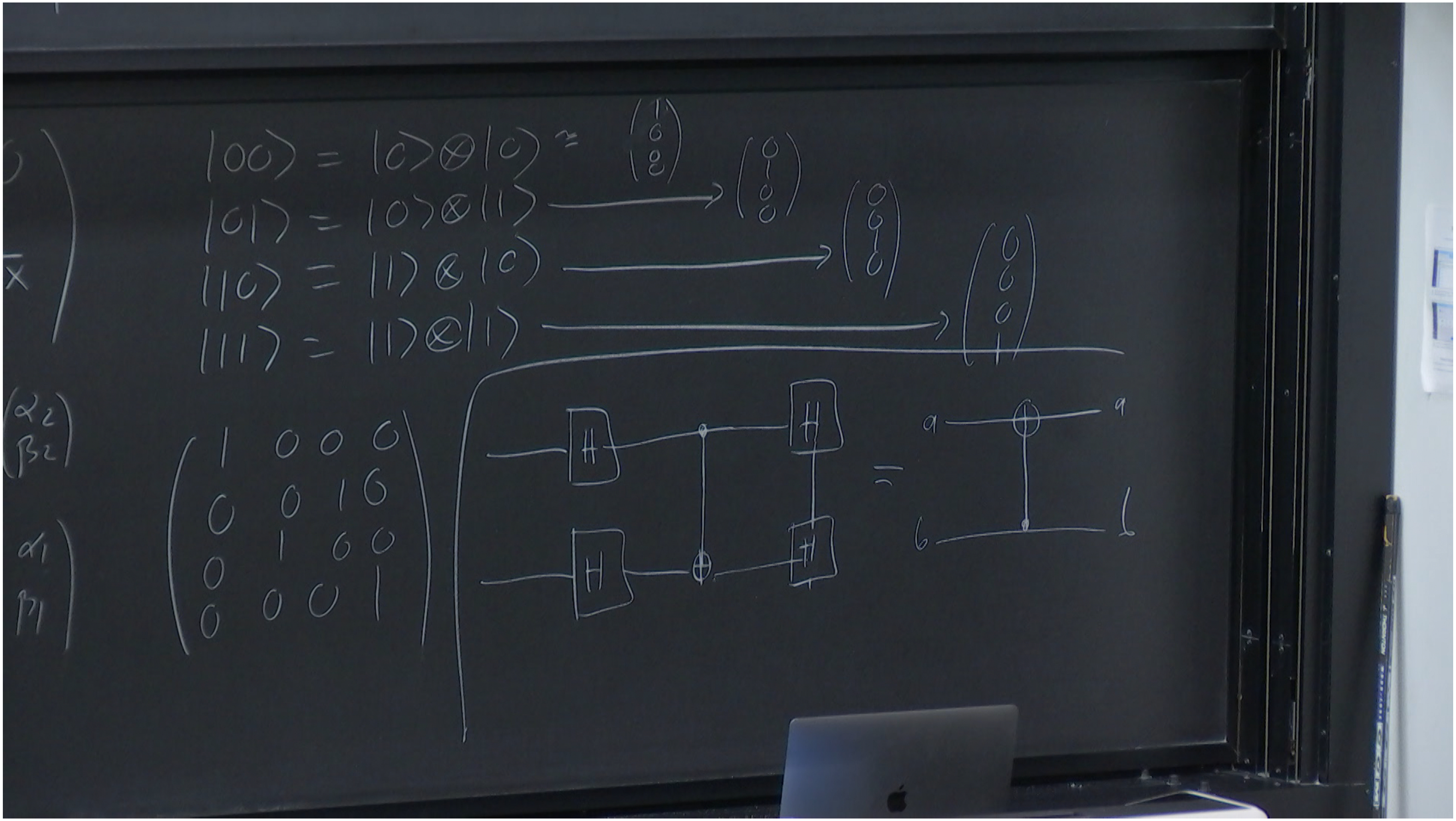


$$|00\rangle \to |00\rangle$$
$$|01\rangle \to |01\rangle$$
$$|10\rangle \to |11\rangle$$
$$|11\rangle \to |10\rangle$$

$$\begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & \sigma_x \end{pmatrix}$$

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$
$$|01\rangle = |0\rangle \otimes |1\rangle \longrightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$
$$|10\rangle = |1\rangle \otimes |0\rangle \longrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$
$$|11\rangle = |1\rangle \otimes |1\rangle \longrightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$


two three QUBIT GATES
CNOT |a b⟩ → |a a⊕b⟩
|00⟩ → |00⟩
|01⟩ → |01⟩
|10⟩ → |11⟩
|11⟩ → |10⟩
TOFFOLI : |a b c⟩ → |a b c⊕a·b⟩
1₃ 1₃ σₓ
SWAP
a (α₁ β₁)
b (α₂ β₂)


|00> = |0>⊗|0> =
|01> = |0>⊗|1>
|10> = |1>⊗|0>
|11> = |1>⊗|1>
H
H
=
a
a
b
b