

Title: Quantum Information Lecture - 230315

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## Mutual information

$$I(X, Y) := H(X) + H(Y) - H(X, Y)$$

||

$$I(X, Y) := H(Y) - H(Y|X)$$

$$H(Y|X) = - \sum_{\substack{x \in X \\ y \in Y}} P(x, y) \log \frac{P(x, y)}{P(x)}$$

↑  
 $P(y|x)$

Quantum Mutual info.

$$I(\hat{\rho}_{AB}) := S(\hat{\rho}_A) + S(\hat{\rho}_B) - S(\hat{\rho}_{AB})$$

$$J_A(\hat{\rho}_{AB}) := S(\hat{\rho}_B) - \underbrace{S(\hat{\rho}_B | \hat{\rho}_A)}$$

$$\min_{(\pi_j^A)} S(\hat{\rho}_B | \pi_j^A)$$

$$J_B \neq J_A$$

$$I(\hat{\rho}_{AB})$$

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$$D_A(\hat{\rho}_{AB}) := I(\hat{\rho}_{AB}) - J_A(\hat{\rho}_{AB}) \leftarrow \text{Quantum Discord}$$

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Purified discord and multipartite entanglement

- Abstract
- I Introduction
- II Entanglement structure and discord
- III The case of qubits
- IV Why Gaussian states require quantum correlation to have classical correlation
- V Remote activation of entanglement
- VI Conclusions
- VII acknowledgements
- References

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$ \alpha\rangle$	$\pm  0\rangle$	$\neq \pm  0\rangle$	$\pm  0\rangle$	$\pm  1\rangle$	$\neq \pm\{ 0\rangle,  1\rangle\}$	$\neq \pm\{ 0\rangle,  1\rangle\}$	$\pm  1\rangle$
$ \beta\rangle$	$\pm  0\rangle$	$\pm  0\rangle$	$\neq \pm  0\rangle$	$\neq \pm\{ 0\rangle,  1\rangle\}$	$\pm  1\rangle$	$\neq \pm\{ 0\rangle,  1\rangle\}$	$\pm  1\rangle$
Entanglement structure							
$D(A, B)$	0	0	0	$> 0$	0	$> 0$	0
$D(B, A)$	0	0	0	0	$> 0$	$> 0$	0

FIG. 1. The relationship between the entanglement structure of  $|\psi\rangle_{ABC}$  and the discord in  $\rho_{A|B}$ . For given conditions on  $|\alpha\rangle$  and  $|\beta\rangle$  we display the resulting entanglement structure and the results for the discords  $D(A, B)$  and  $D(B, A)$ . In the structure diagrams an ellipse represents the presence of bipartite entanglement while a triangle represents the presence of tripartite entanglement.

0.14 0.12 0.10 0.08 D(A,B)

0.14 0.12 0.10 0.08 D(B,A)

0.20 0.15  $\mathcal{N}_{AC}$

0.20 0.15  $\mathcal{N}_{BC}$

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$|\psi_A\rangle$       $\hat{\rho}_A = \text{tr}_B(|\psi_{AB}\rangle\langle\psi_{AB}|)$  entangled     Schmidt theorem  
 Any mixed state  $\hat{\rho}_A$  can be purified by an  $n$ -state  $|\psi_{AB}\rangle$  in a Hilbert space  $\mathcal{H}_{AB}$  so that  $\dim(\mathcal{H}_{AB}) \geq 2 \dim(\mathcal{H}_A)$

$\rho = \frac{1}{2}(|AB\rangle\langle AB| + |A\bar{B}\rangle\langle A\bar{B}|)$  Discord

$$\hat{\rho} = \frac{1}{2}(|B\rangle\langle B| + |W\rangle\langle W|)$$

$$|A\rangle\langle A|_{AB} := |A\rangle\langle A|_A \otimes |B\rangle\langle B|_B \quad \text{Quantum Discord}$$

$$\hat{A} = \frac{1}{2}(\hat{\sigma}_z + 1) + \mu_B(\hat{\sigma}_z + 1) + \lambda \hat{\sigma}_{Ax} \hat{\sigma}_{Bx}$$

$$\hat{U} \rho \hat{U}^\dagger$$

$$\rho_A \otimes \rho_B$$

(5)

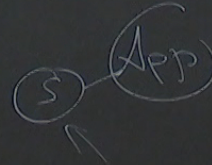


$f(y|x)$

$$|GHZ\rangle = \frac{1}{\sqrt{2}} (|000\rangle_{A,B,C} + |111\rangle_{A,B,C})$$

$\bar{E}$

$$\hat{H} = \hat{H}_E + \hat{H}_S + \hat{H}_{ES}$$



$$\begin{aligned} \hat{H} &= H_E + H_S + H_A \\ &+ H_{ES} + H_{AS} \end{aligned}$$
  
$$\hat{A} = \sigma_x + \sigma_y + \sigma_z + \lambda \sigma_{Ax}$$

ted by a set  $\{\hat{M}_n\}$  of operators on the Hilbert space of the system satisfies  $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$ . The index  $n$  refers to the possible outcome of the

Postulate 4: Measurements of a quantum system are represented by one per. each of the possible outcomes of the measurements, that satisfy measurements.

If the state of the system is  $|\psi\rangle$ , immediately after the measurement:

Postulate 41: Measurements of a quantum system are represented by one per each of the possible outcomes of the measurements, that satisfy measurements.

If the state of the system is  $|\psi\rangle$ , the probability of obtaining outcome

represented by a set  $\{\hat{M}_n\}$  of operators on the Hilbert space of the system that satisfies  $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$ . The index  $n$  refers to the possible outcome of the obtaining outcome  $n$  is  $P(n) = \langle \psi | \hat{M}_n^\dagger \hat{M}_n | \psi \rangle$  and the state after the measurement

Postulate 11: Measurements of a quantum system are represented one per. each of the possible outcomes of the measurements, that satisfy measurements.

If the state of the system is  $|\psi\rangle$ , the probability of obtaining an

is  $|\psi'\rangle = \frac{\hat{M}_n |\psi\rangle}{\sqrt{\langle \psi | \hat{M}_n^\dagger \hat{M}_n | \psi \rangle}}$ , we call  $E_n := \hat{M}_n^\dagger \hat{M}_n \Rightarrow \sum_n E_n$

represented by a set  $\{\hat{M}_n\}$  of operators on the Hilbert space of the system that satisfies  $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$ . The index  $n$  refers to the possible outcome of the obtaining outcome  $n$  is  $P(n) = \langle \psi | \hat{M}_n^\dagger \hat{M}_n | \psi \rangle$  and the state after the measurement

$$\Rightarrow \sum_n \hat{E}_n = \mathbb{1} \quad \text{POVM elements}$$

Can you distinguish between  $|\psi_1\rangle = |0\rangle$  and  $|\psi_2\rangle =$

a) with PVMs

b) with

$$P_{0|\psi_1} = |\langle \psi_1 | 0 \rangle|^2 = 1$$

$$P_{0|\psi_2} = \frac{1}{2}$$

$$P_{1|\psi_1} = |\langle \psi_2 | 0 \rangle|^2 = 0$$

$$P_{1|\psi_2} = \frac{1}{2}$$



and  $|\psi_c\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$  in a single shot measurement?

b) with a POVM

$$\hat{E}_1 = \frac{\sqrt{2}}{1+\sqrt{2}} |1\rangle\langle 1|, \quad \hat{E}_2 = \frac{\sqrt{2}}{1+\sqrt{2}} (|0\rangle - |1\rangle)(\langle 0| - \langle 1|)/2$$

$$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$$

$\rho_{0A} \otimes \rho_{0B}$ 

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$$P_{|\psi_1\rangle}^{(1)} = \langle \psi_1 | \hat{E}_1 | \psi_1 \rangle = 0$$

$$P_{|\psi_1\rangle}^{(2)} = \langle \psi_1 | \hat{E}_2 | \psi_1 \rangle = \frac{1}{2} \frac{\sqrt{2}}{1+\sqrt{2}} = 1 - \frac{1}{\sqrt{2}}$$

$$P_{|\psi_1\rangle}^{(3)} = \langle \psi_1 | \hat{E}_3 | \psi_1 \rangle = \frac{1}{\sqrt{2}}$$

$$P_{|\psi_2\rangle}^{(1)} = \langle \psi_2 | \hat{E}_1 | \psi_2 \rangle = 1 - \frac{1}{\sqrt{2}}$$

$$P_{|\psi_2\rangle}^{(2)} = \langle \psi_2 | \hat{E}_2 | \psi_2 \rangle = 0$$

$$P_{|\psi_2\rangle}^{(3)} = \langle \psi_2 | \hat{E}_3 | \psi_2 \rangle = \frac{1}{\sqrt{2}}$$

Can you distinguish between  $|\psi_1\rangle = |0\rangle$  and  $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$  in a single shot measurement?

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b) with a POVM

$$\hat{E}_1 = \frac{\sqrt{2}}{1+\sqrt{2}} |1\rangle\langle 1|, \quad \hat{E}_2 = \frac{\sqrt{2}}{1+\sqrt{2}} (|0\rangle - |1\rangle)(\langle 0| - \langle 1|) / 2$$

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