Title: Quantum Information Lecture - 230315

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Collection: Quantum Information (2022/2023)

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Mutual information

$$I(X,Y) := H(X) + H(Y) - H(X,Y)$$

$$\parallel$$

$$I(X,Y) := H(Y) - H(Y|X)$$

$$H(Y|X) = -\sum_{\substack{x\in X\\ y\in Y}} p(x,y)\, \mathrm{Log}\, \underbrace{\frac{P(x,y)}{P(x)}}_{P(y|x)}$$

Quantum Mutual info.

$$I(\hat{\rho}_{AB}) := S(\hat{\rho}_A) + S(\hat{\rho}_B) - S(\hat{\rho}_{AB})$$

$$J_A(\hat{\rho}_{AB}) := S(\hat{\rho}_B) - \underbrace{S(\hat{\rho}_B \mid \hat{\rho}_A)}_{\min_{(\Pi_j^A)} S(\hat{\rho}_B \mid \{\Pi_j^A\})}$$

$$\begin{array}{ccc} J_B & \neq & J_A \\ \neq & & \neq \\ & I(\rho_{AB}) & \end{array}$$

Quantum Mutual info.

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$$\begin{array}{ccc} J_B & \neq & J_A \\ \neq & & \neq \\ I(\rho_{AB}) & & \end{array}$$

$$D_A(\hat{\rho}_{AB}) := I(\hat{\rho}_{AB}) - J_A(\hat{\rho}_{AB})$$ ← Quantum Discord

$\|\alpha\rangle$	$\pm\|0\rangle$	$\neq\pm\|0\rangle$	$\pm\|0\rangle$	$\pm\|1\rangle$	$\neq\pm\{\|0\rangle,\|1\rangle\}$	$\neq\pm\{\|0\rangle,\|1\rangle\}$	$\pm\|1\rangle$
$\|\beta\rangle$	$\pm\|0\rangle$	$\pm\|0\rangle$	$\neq\pm\|0\rangle$	$\neq\pm\{\|0\rangle,\|1\rangle\}$	$\pm\|1\rangle$	$\neq\pm\{\|0\rangle,\|1\rangle\}$	$\pm\|1\rangle$
Entanglement structure							
$D(A,B)$	0	0	0	>0	0	>0	0
$D(B,A)$	0	0	0	0	>0	>0	0

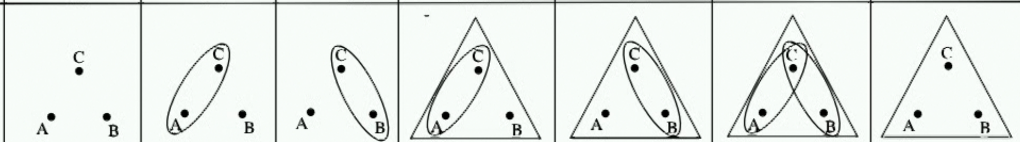
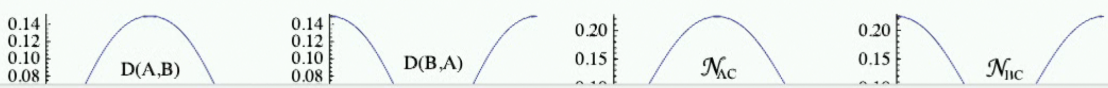
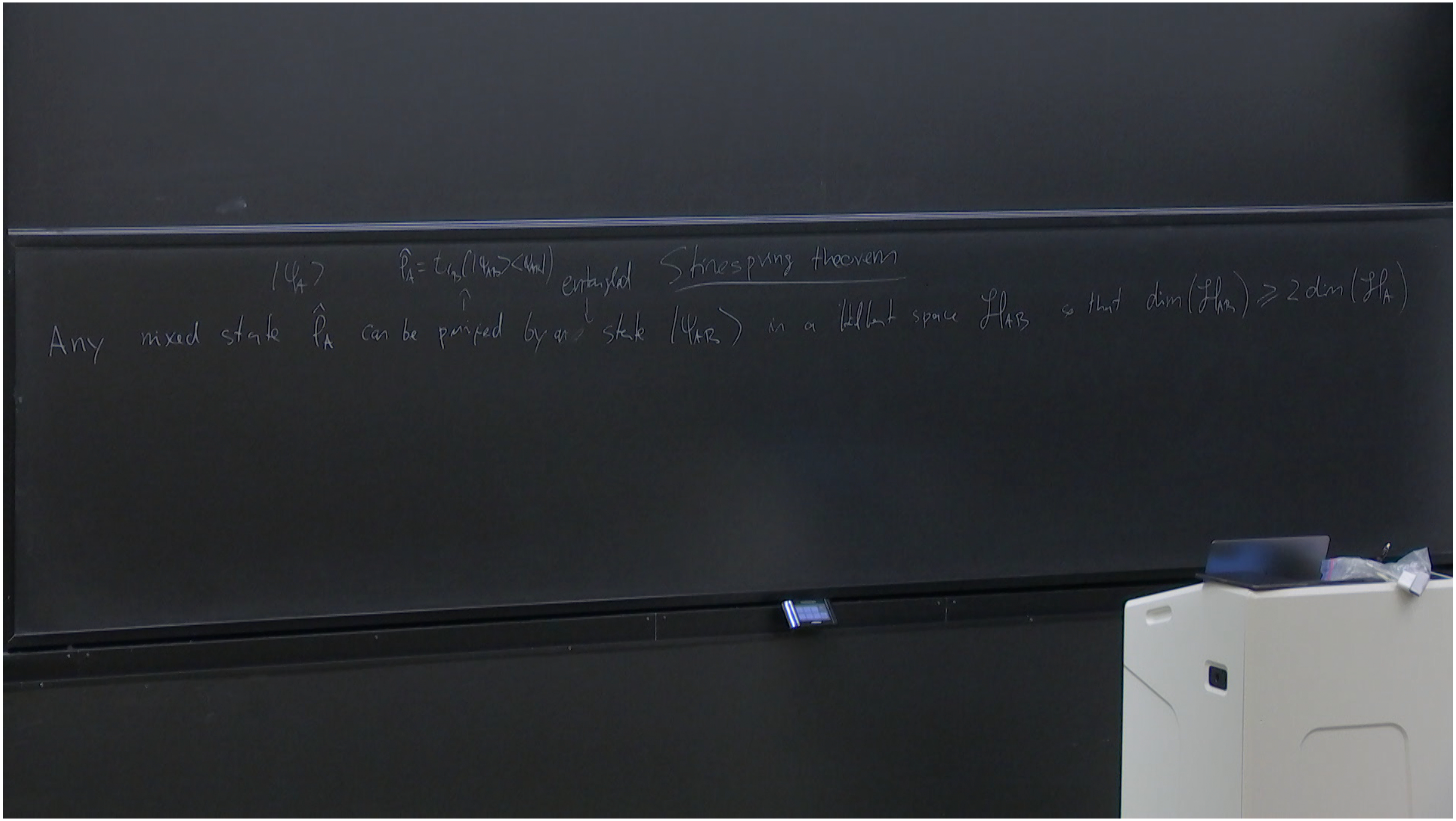


FIG. 1. The relationship between the entanglement structure of $|\psi\rangle_{ABC}$ and the discord in ρ_{AB}. For given conditions on $|\alpha\rangle$ and $|\beta\rangle$ we display the resulting entanglement structure and the results for the discords $D(A,B)$ and $D(B,A)$. In the structure diagrams an ellipse represents the presence of bipartite entanglement while a triangle represents the presence of tripartite entanglement.





$|\psi_A\rangle$
$\hat{\rho}_A = tr_B(|\psi_{AB}\rangle\langle\psi_{AB}|)$
entangled
Stinespring theorem
Any mixed state $\hat{\rho}_A$ can be purified by an state $|\psi_{AB}\rangle$ in a Hilbert space $\mathcal{H}_{AB}$ so that $dim(\mathcal{H}_{AB}) \geq 2 \, dim(\mathcal{H}_A)$

$= I(\rho_{AB}) - S_A(\rho_{AB})$

Discord

$$\hat{\rho} = \frac{1}{2}\left(|B\rangle\langle B| + |W\rangle\langle W|\right)$$

$D_A(\rho_{AB}) := I(\rho_{AB}) - S_A(\rho_{AB})$ ← Quantum Discord

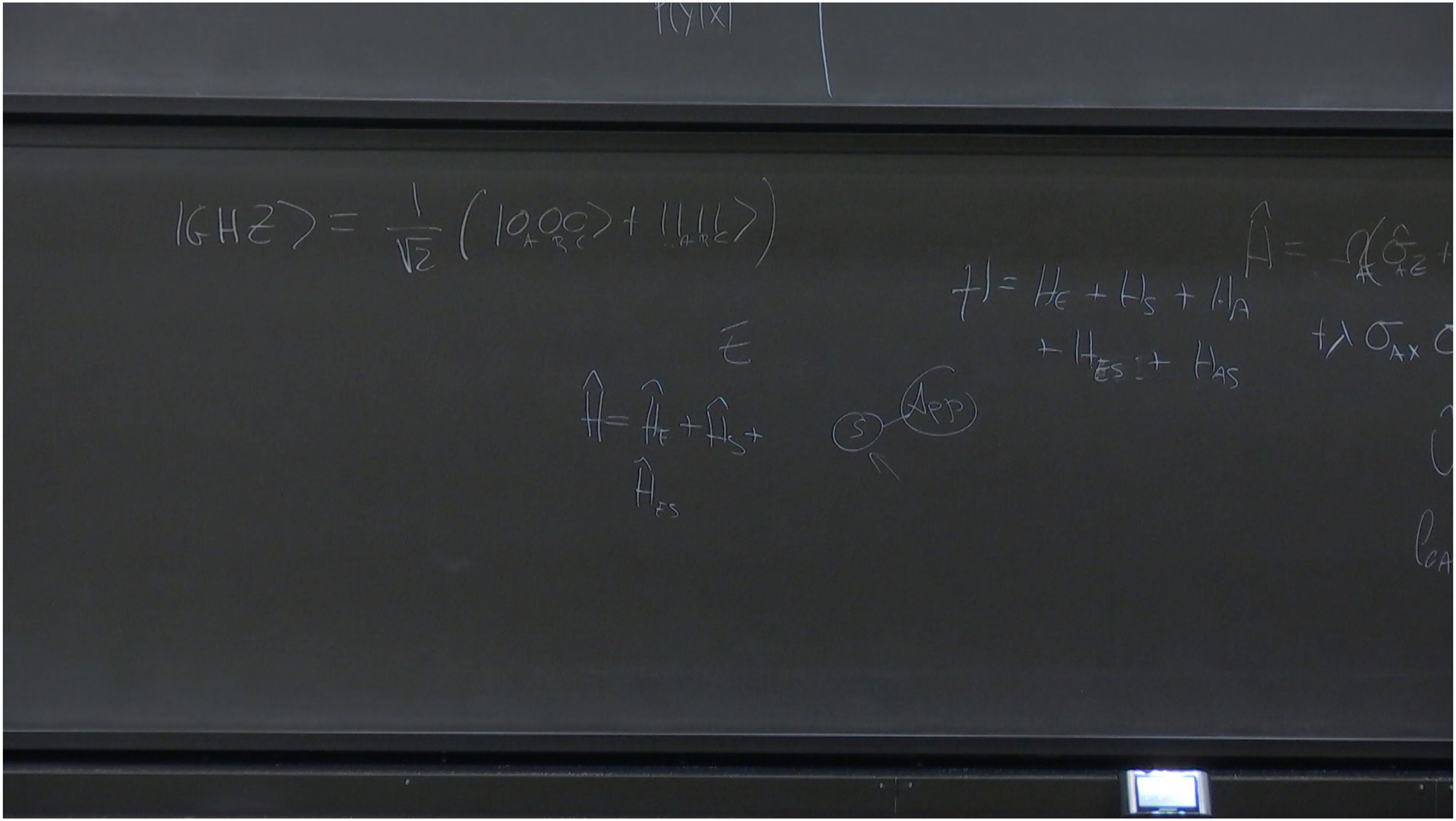
$$\hat{A} = \Omega_A(\hat{\sigma}_{Az} + 1) + \Omega_B(\hat{\sigma}_{Bz} + 1) + \lambda\, \sigma_{Ax}\, \sigma_{Bx}$$

$$\hat{\rho} = \frac{1}{2}\left(|B\rangle\langle B| + |W\rangle\langle W|\right) =$$

(5)

$$\hat{U} \rho_0 \hat{U}^\dagger$$

$$\rho_{CA} \otimes \rho_{bAD}$$


$P(y|x)$
$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|0_A 0_B 0_C\rangle + |1_A 1_B 1_C\rangle\right)$
E
$\hat{H} = \hat{H}_E + \hat{H}_S + \hat{H}_{ES}$
S
App
$H = H_E + H_S + H_A + H_{ES} + H_{AS}$
$+\lambda\,\sigma_{Ax}$

…ted by a set $\{\hat{M}_n\}$ of operators on the Hilbert space of the system

satisfies $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$. the index n refers to the possible outcome of the

Postulate 4 : Measurements of a quantum system are represented b

one per each of the possible outcomes of the measurements, that satisfie

measurements.

If the state of the system is $|\psi\rangle$, immediately after the measurement:

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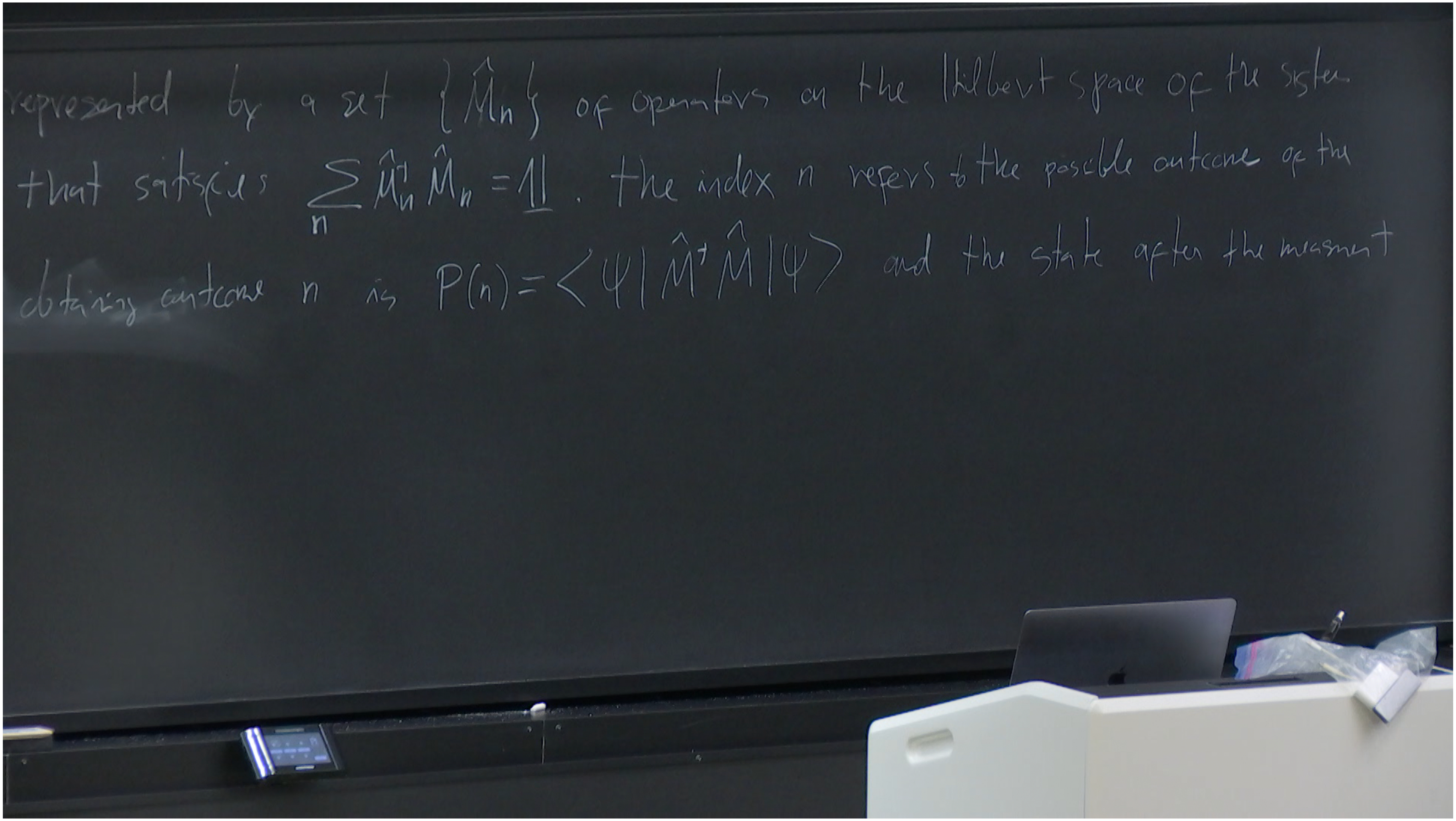
If the state of the system is $|\psi\rangle$, the probability of obtaining outcom


represented by a set $\{\hat{M}_n\}$ of operators on the Hilbert space of the system
that satisfies $\sum_n \hat{M}_n^\dagger \hat{M}_n = \mathbb{1}$. the index n refers to the possible outcome of the
obtaining outcome n is $P(n) = \langle\psi|\hat{M}^\dagger \hat{M}|\psi\rangle$ and the state after the measurement

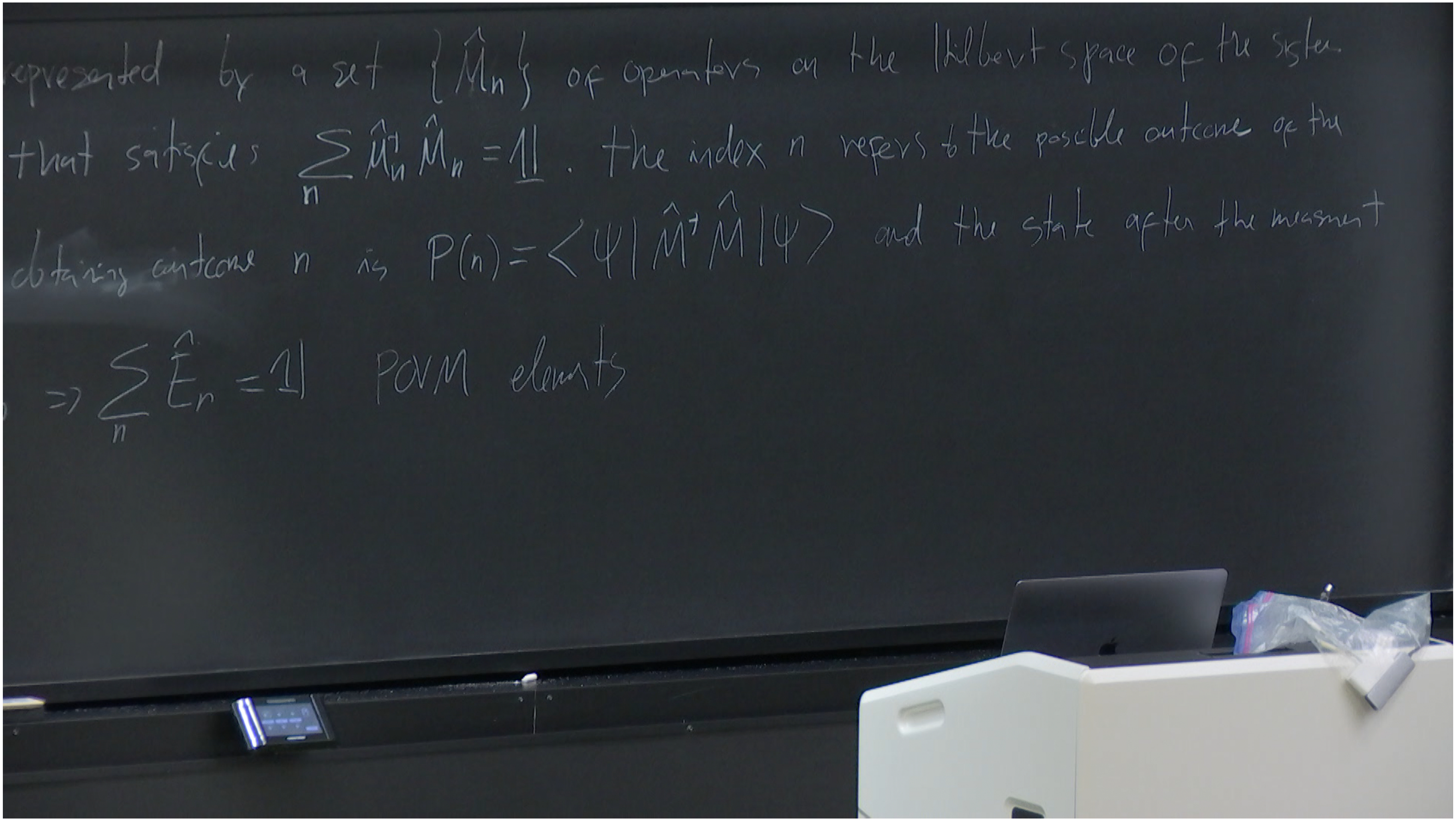
Postulate 4 : Measurements of a quantum system are represented

one per each of the possible outcomes of the measurements, that satis

measurements.

If the state of the system is $|\psi\rangle$, the probability of obtaining an

is $$|\psi'\rangle = \frac{\hat{M}_n|\psi\rangle}{\sqrt{\langle\psi|\hat{M}_n^\dagger \hat{M}_n|\psi\rangle}}$$, we call $\hat{E}_n := \hat{M}_n^\dagger \hat{M}_n \Rightarrow \sum_n \hat{E}$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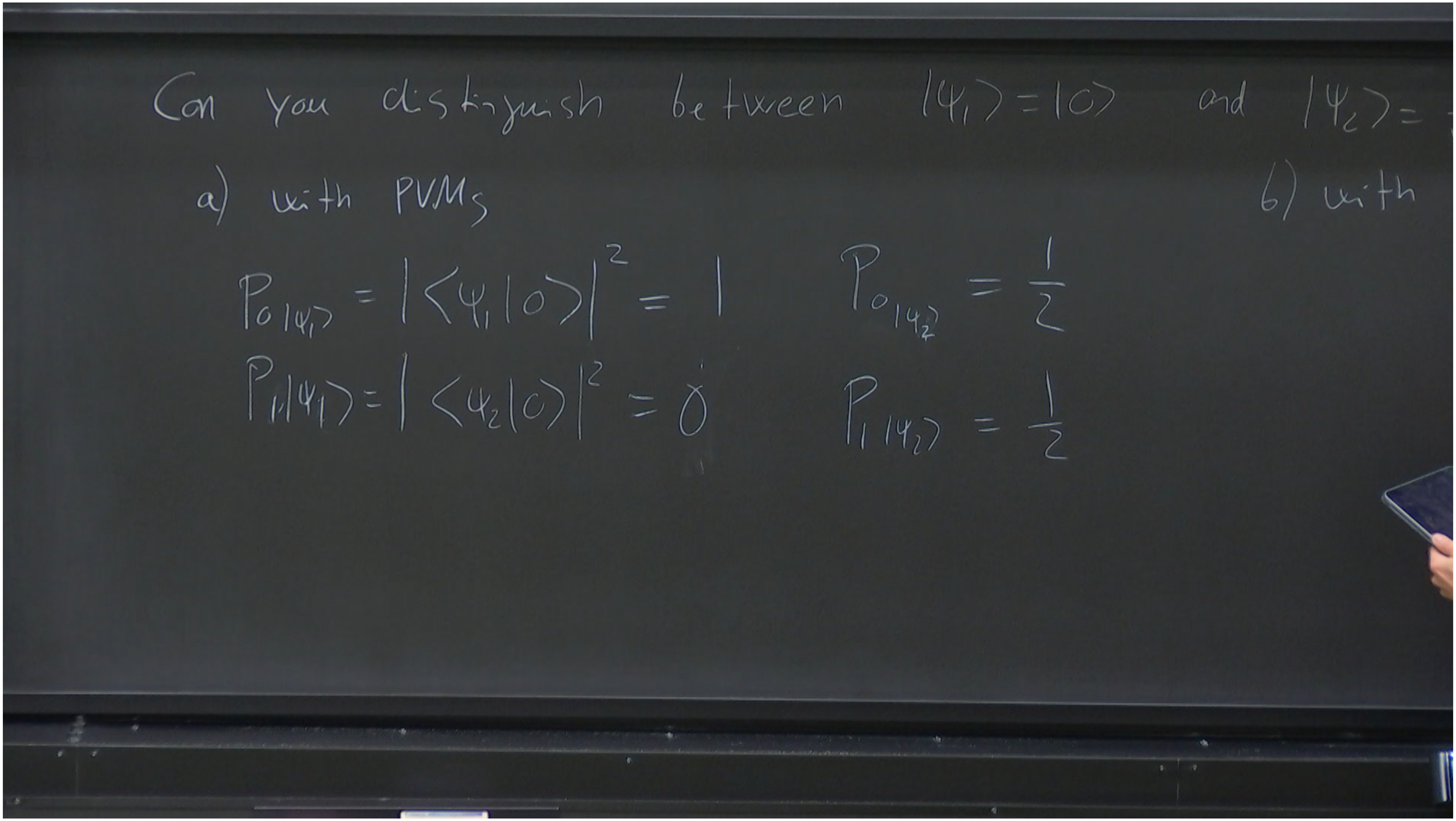
$\Rightarrow \sum_n \hat{E}_n = \mathbb{1}$ POVM elements


Can you distinguish between $|\psi_1\rangle = |0\rangle$ and $|\psi_2\rangle =$
a) with PVMs
b) with
$P_{0|\psi_1\rangle} = |\langle\psi_1|0\rangle|^2 = 1$
$P_{1|\psi_1\rangle} = |\langle\psi_2|0\rangle|^2 = 0$
$P_{0|\psi_2\rangle} = \frac{1}{2}$
$P_{1|\psi_2\rangle} = \frac{1}{2}$

and $|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$ in a sigle shot measment?)

b) with a POVM

$$\hat{E}_1 = \frac{\sqrt{2}}{1+\sqrt{2}} |1\rangle\langle 1| \,, \quad \hat{E}_2 = \frac{\sqrt{2}}{1+\sqrt{2}} \left(|0\rangle - |1\rangle\right)\left(\langle 0| - \langle 1|\right)/2$$

$$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$$

$\rho_{CA} \otimes \rho_{BD}$

Can you distinguish between $|\psi_1\rangle = |0\rangle$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$ in a single shot measurement?

a) with PVMs

$$P_{0|\psi_1\rangle} = |\langle\psi_1|0\rangle|^2 = 1 \qquad P_{0|\psi_2\rangle} = \frac{1}{2}$$

$$P_{1|\psi_1\rangle} = |\langle\psi_2|0\rangle|^2 = 0 \qquad P_{1|\psi_2\rangle} = \frac{1}{2}$$

b) with a POVM

$$\hat{E}_1 = \frac{\sqrt{2}}{1+\sqrt{2}}|1\rangle\langle 1| , \quad \hat{E}_2 = \frac{\sqrt{2}}{1+\sqrt{2}}\left(|0\rangle - |1\rangle\right)\left(\langle 0| - \langle 1|\right)/2$$

$$\hat{E}_3 = \mathbb{1} - \hat{E}_1 - \hat{E}_2$$

$$P_{|\psi_1\rangle}(1) = \langle \psi_1 | \hat{E}_1 | \psi_1 \rangle = 0$$

$$P_{|\psi_1\rangle}(2) = \langle \psi_1 | \hat{E}_2 | \psi_1 \rangle = \frac{1}{2} \frac{\sqrt{2}}{1+\sqrt{2}} = 1 - \frac{1}{\sqrt{2}}$$

$$P_{|\psi_1\rangle}(3) = \langle \psi_1 | \hat{E}_3 | \psi_1 \rangle = \frac{1}{\sqrt{2}}$$

$$P_{|\psi_2\rangle}(1) = \langle \psi_2 | \hat{E}_1 | \psi_2 \rangle = 1 - \frac{1}{\sqrt{2}}$$

$$P_{|\psi_2\rangle}(2) = \langle \psi_2 | \hat{E}_2 | \psi_2 \rangle = 0$$

$$P_{|\psi_2\rangle}(3) = \langle \psi_2 | \hat{E}_3 | \psi_3 \rangle = \frac{1}{\sqrt{2}}$$

$\hat{A}_{FS}$

$U \rho_U U$

$\rho_{CA} \otimes \rho_{BAD}$

Can you distinguish between $|\psi_1\rangle = |0\rangle$ and $|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$ in a single shot measurement?

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b) with a POVM

$$\hat{E}_1 = \frac{\sqrt{2}}{1+\sqrt{2}}|1\rangle\langle 1|, \quad \hat{E}_2 = \frac{\sqrt{2}}{1+\sqrt{2}}\left(|0\rangle - |1\rangle\right)\left(\langle 0| - \langle 1|\right)/2$$

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