Title: Quantum Information Lecture - 230313

Speakers: Eduardo Martin-Martinez

Collection: Quantum Information (2022/2023)

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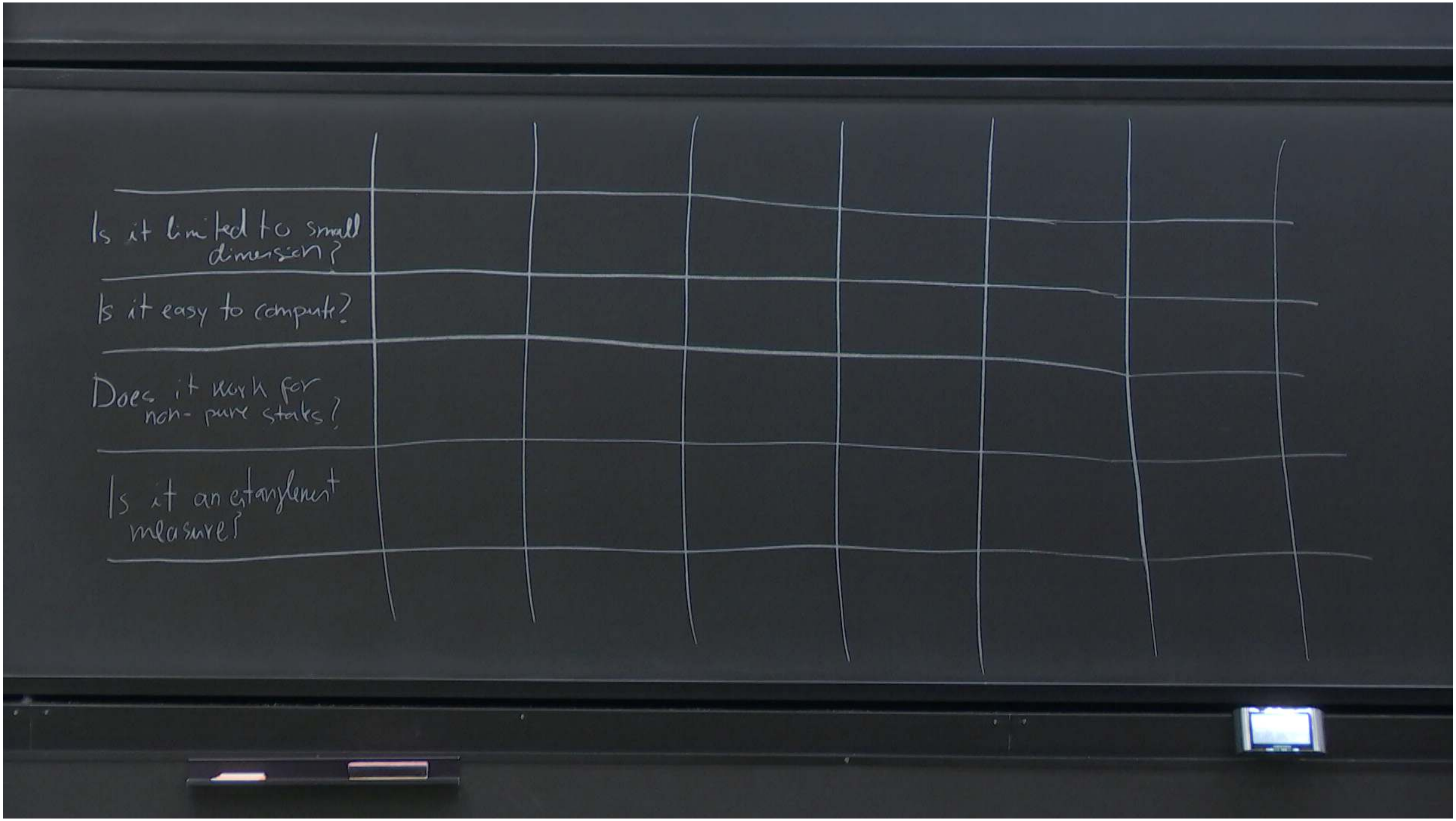
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Is it easy to compute?
Does it work for non-pure states?
Is it an entanglement measure?

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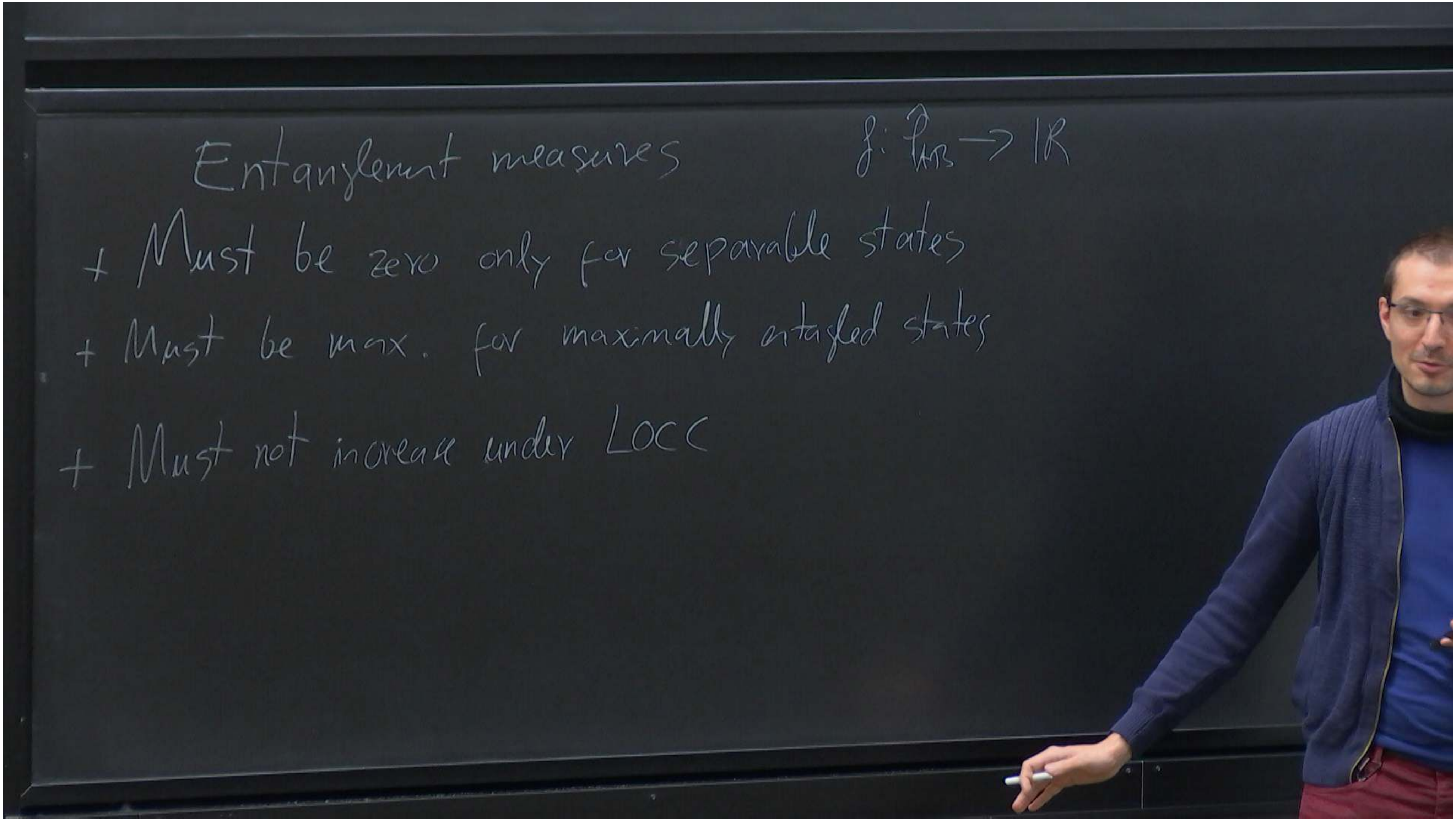
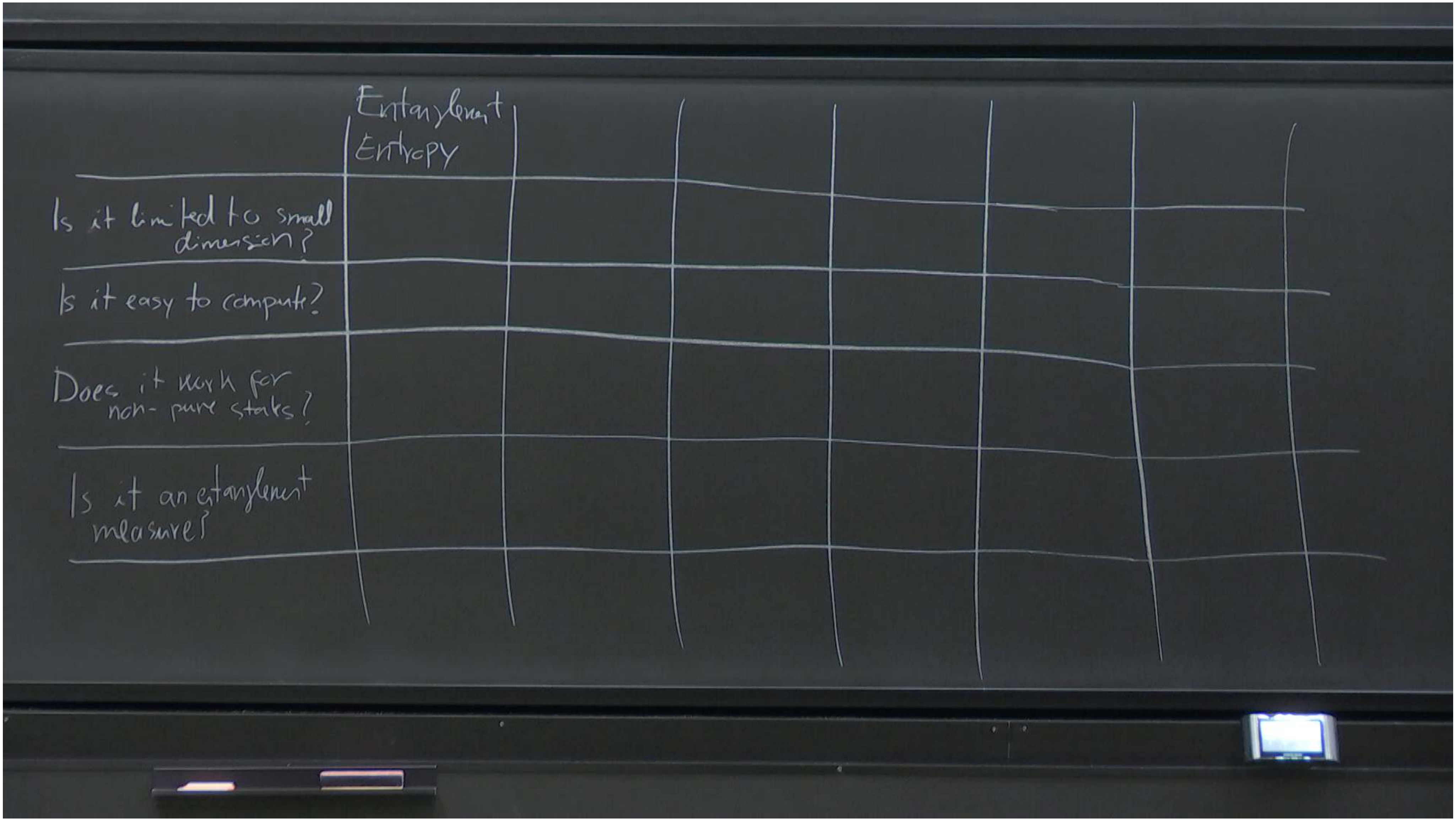
Recall: given $\hat{\rho}_{AB}$ determining if its entangled is NP-Hard

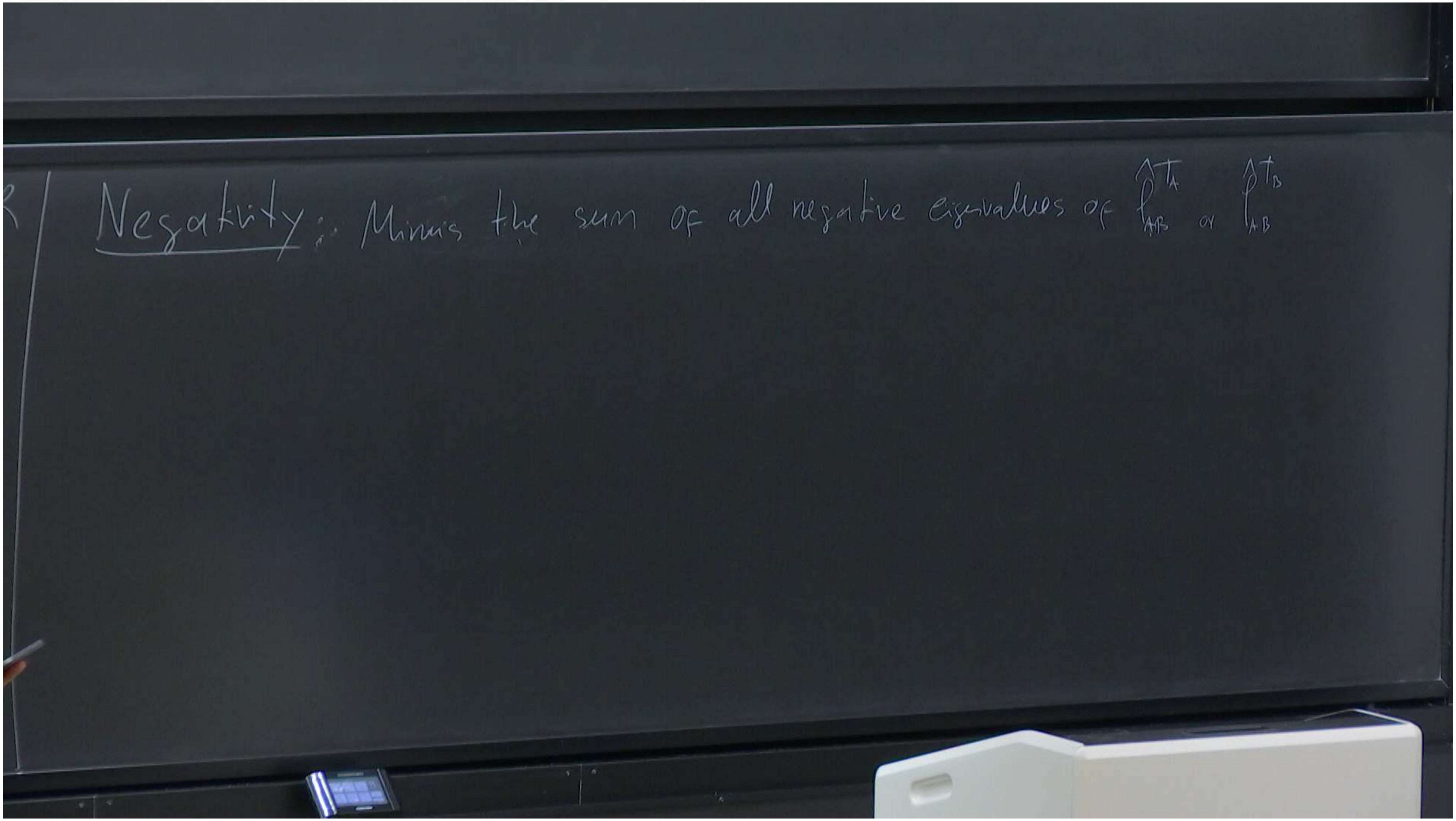
Peres criterion. $\hat{\rho}_{AB}$ entangled $\Leftarrow$ $\hat{\rho}_{AB}^{T_A}$ (or $\hat{\rho}_{AB}^{T_B}$) have negative eigenvalues

Distillable entanglement: Amount of maximally entangled states of dimension d (typically qubits) extractable from N copies of $\hat{\rho}_{AB}$ by LOCC


Entanglement measures
$f: \hat{\rho}_{AB} \to \mathbb{R}$
+ Must be zero only for separable states
+ Must be max. for maximally entangled states
+ Must not increase under LOCC


Entanglement Entropy
Is it limited to small dimension?
Is it easy to compute?
Does it work for non-pure states?
Is it an entanglement measure?

	Entanglement Entropy					
Is it limited to small dimension?	No					
Is it easy to compute?	Yes					
Does it work for non-pure states?	No					
Is it an entanglement measure?	No (Yes for pure states)					
Does it have a nice physical interpretation?	Yes					


Negativity: Minus the sum of all negative eigenvalues of $\hat{\rho}_{AB}^{T_A}$ or $\hat{\rho}_{AB}^{T_B}$

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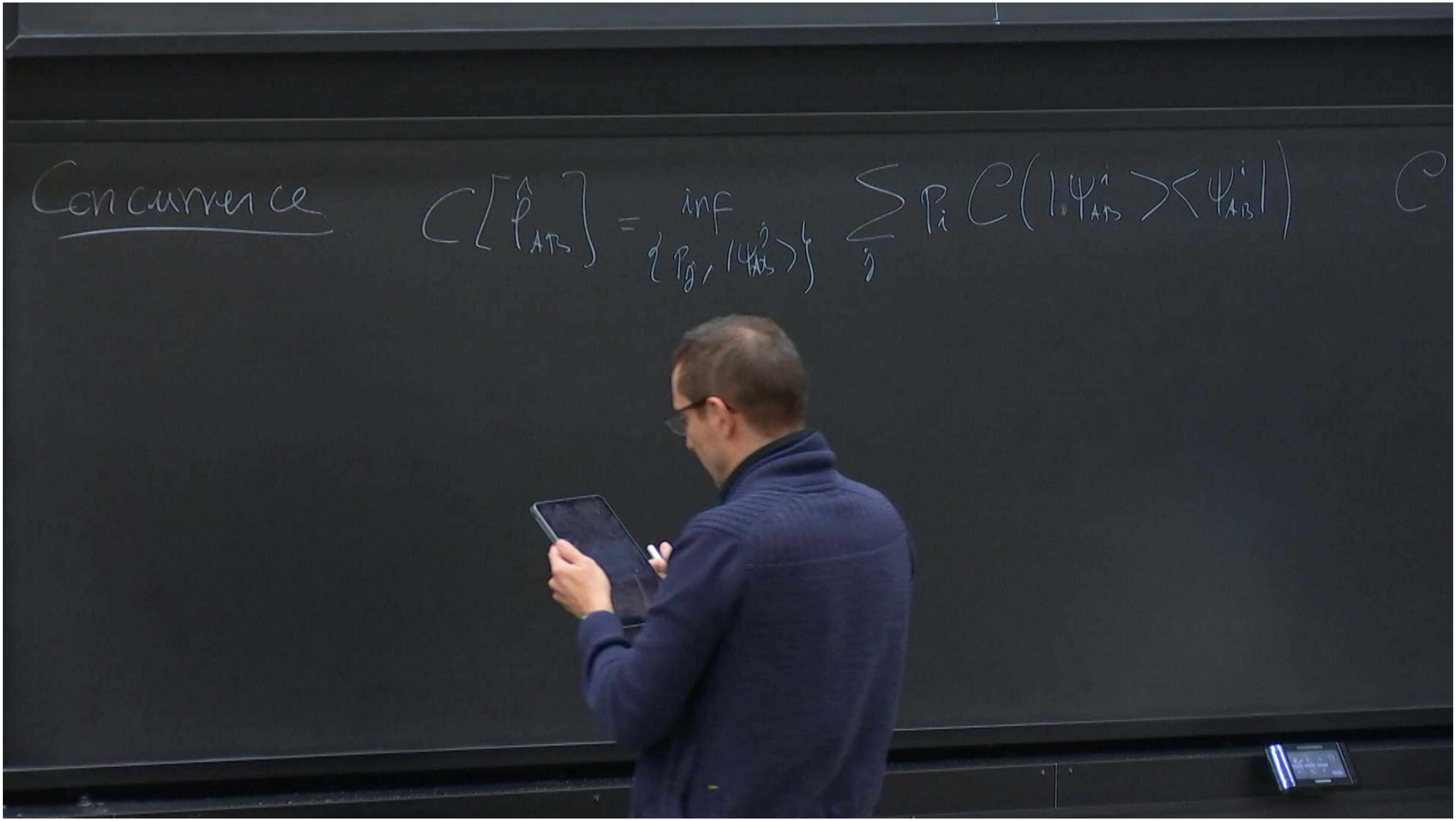
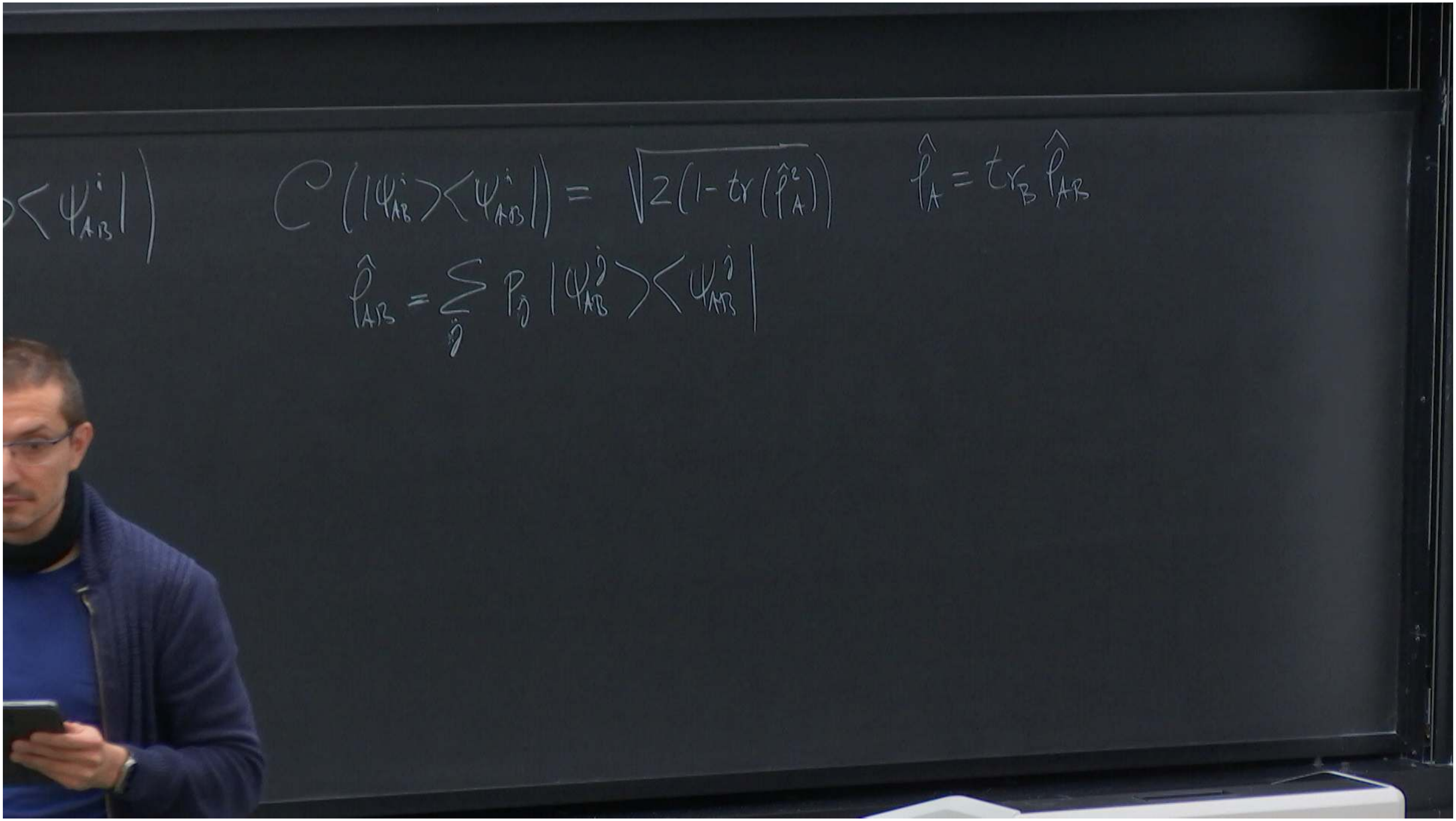
$\mathcal{N}$

Logarithmic Negativity $\mathcal{N}_L := \mathrm{Log}_2(2\mathcal{N}+1)$


Concurrence
$C[\hat{\rho}_{AB}] = \inf_{\{p_j, |\psi^j_{AB}\rangle\}} \sum_j p_i \, C(|\psi^i_{AB}\rangle\langle\psi^i_{AB}|)$



$\rangle\langle\psi^i_{AB}|)$
$C(|\psi^i_{AB}\rangle\langle\psi^i_{AB}|) = \sqrt{2(1-tr(\hat{\rho}_A^2))}$
$\hat{\rho}_A = tr_B\,\hat{\rho}_{AB}$
$\hat{\rho}_{AB} = \sum_j p_j |\psi^j_{AB}\rangle\langle\psi^j_{AB}|$

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$$\hat{\rho}_{AB} = \sum_j p_j\,|\psi^j_{AB}\rangle\langle\psi^j_{AB}|$$

Concurrence $\quad C[\hat{\rho}_{AB}] = \inf_{\{p_j, |\psi^j_{AB}\rangle\}} \sum_j p_j C(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|) \quad C($

for a 2×2 system: $C(\hat{\rho}_{AB}) := \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ where $\lambda_1 \geq \lambda$

$$\hat{R} := \sqrt{\sqrt{\hat{\rho}_{AB}}\,\hat{\tilde{\rho}}\,\sqrt{\hat{\rho}_{AB}}}$$

$\tilde{\rho} := (\hat{\sigma}_{y_A} \otimes \hat{\sigma}_{y_B})\,\hat{\rho}^{*}_{AB}\,(\hat{\sigma}_{y_A} \otimes \hat{\sigma}_{y_A})$ where $\hat{\rho}^{*}_{AB}$ is the complex conjugate of the representati

$\mathcal{C}(|\psi^j_{AB}\rangle\langle\psi^j_{AB}|)$ $\quad\quad$ $\mathcal{C}(|\psi^i_{AB}\rangle\langle\psi^i_{AB}|) = \sqrt{2(1-tr(\hat{\rho}_A^2))}$ $\quad$ $\hat{\rho}_A = tr_B\, \hat{\rho}_{AB}$

$$\hat{\rho}_{AB} = \sum_j p_j\, |\psi^j_{AB}\rangle\langle\psi^j_{AB}|$$

$-\lambda_2 - \lambda_3 - \lambda_4\}$ where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ are the eigenvalues of the operator $\hat{R}$

the complex conjugate of the representation of $\hat{\rho}_{AB}$ in the computational basis (eigenvectors of $\hat{\sigma}_z$)

Recall: given $\hat{\rho}_{AB}$ determining if its entangled is NP-Hard

Peres criterion. $\hat{\rho}_{AB}$ entangled $\Leftarrow$ $\hat{\rho}_{AB}^{T_A}$ (or $\hat{\rho}_{AB}^{T_B}$) have negative eigenvalues

Distillable entanglement: Amount of maximally entangled states of dimension d (typically qubits) extractable from N copies of $\hat{\rho}_{AB}$ by LOCC

$$C \geq 2\mathcal{N} \geq \sqrt{(1-C)^2 + C^2} - (1-C)$$

$2 \times 2 \Leftrightarrow C = 2\mathcal{N}$

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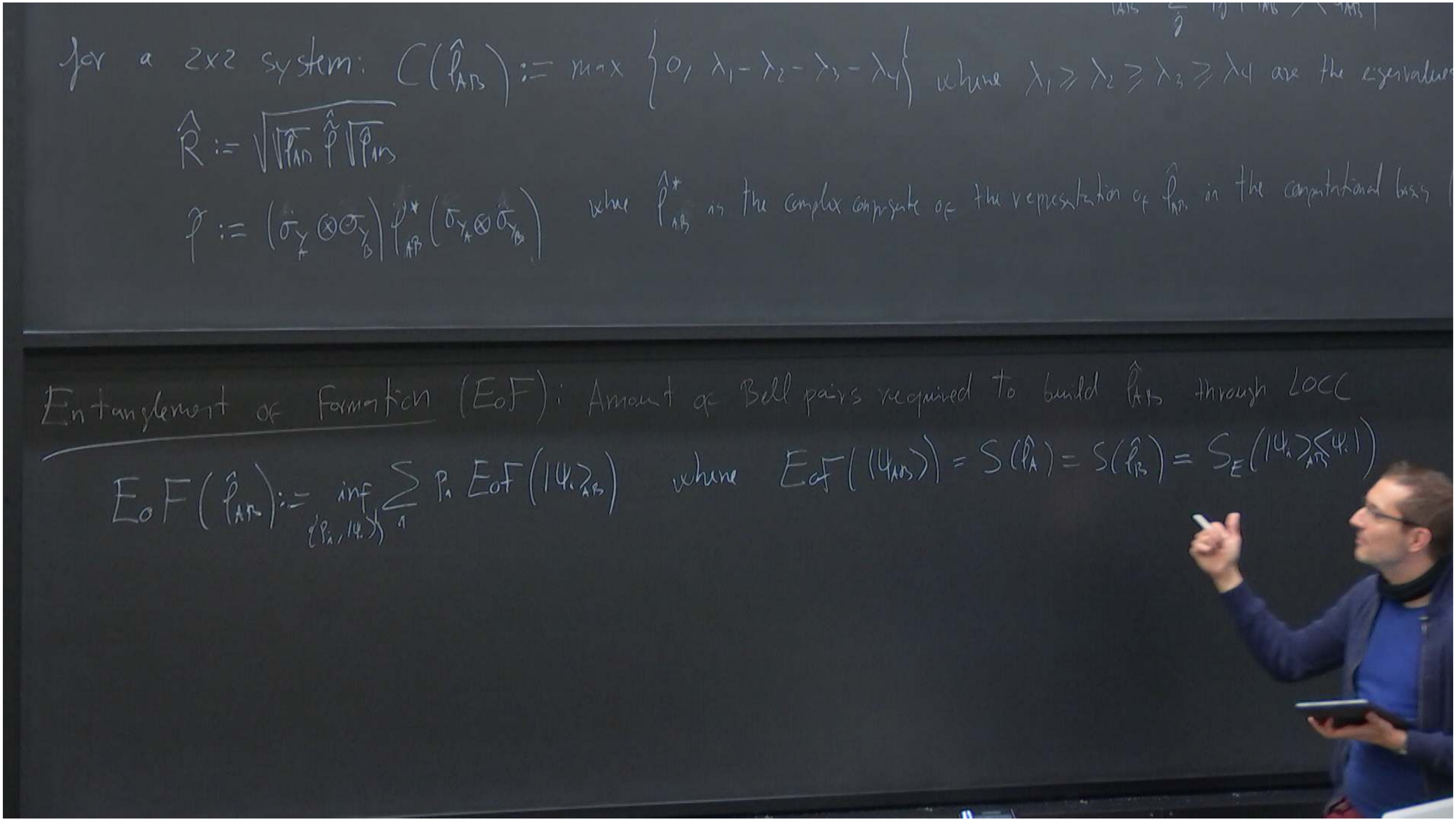
$$\begin{pmatrix} \cdot & & & \cdot \\ & \cdot & \cdot & \\ & \cdot & \cdot & \\ \cdot & & & \cdot \end{pmatrix} \quad 2\times 2 \Leftrightarrow C = 2\mathcal{N}$$



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$\hat{R} := \sqrt{\sqrt{\hat{\rho}_{AB}}\,\hat{\tilde{\rho}}\,\sqrt{\hat{\rho}_{AB}}}$
$\tilde{\rho} := (\sigma_{y_A} \otimes \sigma_{y_B})\, \rho^*_{AB}\, (\sigma_{y_A} \otimes \sigma_{y_B})$ where $\hat{\rho}^*_{AB}$ is the complex conjugate of the representation of $\hat{\rho}_{AB}$ in the computational basis
Entanglement of formation (EoF): Amount of Bell pairs required to build $\hat{\rho}_{AB}$ through LOCC
$EoF(\hat{\rho}_{AB}) := \inf_{\{p_i, |\psi_i\rangle\}} \sum_i p_i\, EoF(|\psi_i\rangle_{AB})$ where $EoF(|\psi_{AB}\rangle) = S(\hat{\rho}_A) = S(\hat{\rho}_B) = S_E(|\psi_i\rangle\langle\psi_i|_{AB})$

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$$EoF(\hat{\rho}_{AB}) = h\left(\frac{1 + \sqrt{1 - C(\hat{\rho}_{AB})}}{2}\right)$$ where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$

d to build $\hat{\rho}_{AB}$ through LOCC

$$= S(\hat{\rho}_A) = S(\hat{\rho}_B) = S_E(|\psi_i\rangle_{AB}\langle\psi_i|)$$

$$- (1-x)\, \mathrm{Log}_2\, (1-x)$$

- EoF is faithful

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$$- (1-x) \log_2 (1-x)$$

- EoF is faithful
- For pure states it reduces to S_E

	Entanglement Entropy	Negativity	Concurrence	EoF			
Is it limited to small dimension?	No	No	No but Yes when easy				
Is it easy to compute?	Yes	Yes!	No! (in general) Yes for 2x2				
Does it work for non-pure states?	No	Yes!	Yes				
Is it an entanglement measure?	No (Yes for pure states)	No* *Yes for 2x2, 3x2 It measures distillable entanglement	Yes				
Does it have a nice physical interpretation?	Yes	Maybe...	Meh Yes through EoF				

$$\hat{R} := \sqrt{\sqrt{\hat{\rho}_{AB}}\,\hat{\tilde{\rho}}\,\sqrt{\hat{\rho}_{AB}}}$$

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– EoF is

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– For pure

$I(x,y) := H(x) + H(y) - H(x,y)$

$H(y|x) := -\sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)}$ $\quad p(A|B) = \frac{P(A \cap B)}{P(B)}$

$I(x,y) := H(y) - H(y|x)$