Title: Quantum Information Lecture - 230310

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Collection: Quantum Information (2022/2023)

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Local realism $\Rightarrow$ $|C(a,b) - C(a,b')| + |C(a',b') + C(a',b)| \leq 2$

$A = A(a, \lambda)$

$B = B(b, \lambda)$

$\hat{A}(a) = \hat{\sigma}_{z_A} \otimes \mathbb{1}_B$ $\quad \hat{B}(b) = \frac{-1}{\sqrt{2}}(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}))$

$\hat{A}(a') = \hat{\sigma}_{x_A} \otimes \mathbb{1}_B$ $\quad \hat{B}(b') = \frac{1}{\sqrt{2}}(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}))$

$\langle \hat{A}(a)\hat{B}(b) \rangle = \langle \psi^- | \hat{\sigma}_{z_A} \otimes \frac{-1}{\sqrt{2}}(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | \psi^- \rangle = \frac{-1}{2\sqrt{2}} \left(\langle O_A | B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | O_A | B \right)$

$\langle \hat{A}(a)\hat{B}(b') \rangle = \langle \psi^- | \hat{\sigma}_{z_A} \otimes \frac{1}{\sqrt{2}}(\sigma_{z_B} - \sigma_{x_B}) | \psi^- \rangle = -\frac{1}{\sqrt{2}}$

$\langle \hat{A}(a')\hat{B}(b) \rangle = \langle \psi^- | \hat{\sigma}_{x_A} \otimes \frac{1}{\sqrt{2}}(\sigma_{z_B} - \sigma_{x_B}) | \psi^- \rangle = \frac{1}{2\sqrt{2}} \left(\langle O_A | B | \hat{\sigma}_{x_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | A O_B \rangle + \langle A O_B | \hat{\sigma}_{x_A} \right.$

$$|C(a',b') + C(a',b)| \leq 2 \qquad |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - |1_A 0_B\rangle\right)$$

$$= \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right)\right)$$

$$\left(\langle 0_A 1_B | \hat{\sigma}_{z_A} \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}\right) | 0_A 1_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{z_A} \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}\right) | 1_A 0_B\rangle\right) = \frac{-1}{2\sqrt{2}}\left(-1 + (-1)\right) = \frac{1}{\sqrt{2}}$$

$$\langle \hat{A}(a')\hat{B}(b')\rangle =$$

$$\frac{1}{\sqrt{2}} \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right) | 1_A 0_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{x_A} \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right) | 0_A 1_B\rangle = \frac{1}{2\sqrt{2}}(-1-1) = \frac{-1}{\sqrt{2}}$$

$\Rightarrow \quad |C(a,b) - C(a,b')| + |C(a',b') + C(a',b)| \leq 2 \qquad |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0_A 1_B\rangle -$

$\hat{A}(a) = \hat{\sigma}_{z_A} \otimes \mathbb{1}_B \qquad \hat{B}(b) = \frac{-1}{\sqrt{2}}(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}))$

$\hat{A}(a') = \hat{\sigma}_{x_A} \otimes \mathbb{1}_B \qquad \hat{B}(b') = \frac{1}{\sqrt{2}}(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}))$

$\langle\Psi^-| \; \hat{\sigma}_{z_A} \otimes \frac{-1}{\sqrt{2}}(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) |\Psi^-\rangle = \frac{-1}{2\sqrt{2}}\left(\langle 0_A 1_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | 0_A 1_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | 1_A 0_B\rangle\right)$

$\langle\Psi^-| \; \hat{\sigma}_{z_A} \otimes \frac{1}{\sqrt{2}}(\sigma_{z_B} - \sigma_{x_B}) |\Psi^-\rangle = \; ? \; \frac{-1}{\sqrt{2}}$

$-|\hat{\sigma}_{x_A} \otimes \frac{1}{\sqrt{2}}(\sigma_{z_B} + \sigma_{x_B})|\Psi^-\rangle = \frac{1}{2\sqrt{2}}\left(\langle 0_A 1_B | \hat{\sigma}_{x_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | 1_A 0_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{x_A} \otimes (\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}) | 0_A 1_B\rangle\right) = \frac{1}{2\sqrt{2}}(1+1) = \frac{+1}{\sqrt{2}}$

$C(a',b') + C(a',b) \leq 2$ $\quad |\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - |1_A 0_B\rangle\right)$

$(b) = \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B})\right)$

$(b') = \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B})\right)$

$\frac{-1}{2\sqrt{2}}\left(\langle 0_A 1_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | 0_A 1_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | 1_A 0_B\rangle\right) = \frac{-1}{2\sqrt{2}}(-1 + (-1)) = \frac{1}{\sqrt{2}}$

$: \frac{-1}{\sqrt{2}}$

$\langle \hat{A}(a')\hat{B}(b')\rangle = \frac{+1}{\sqrt{2}}$

$| \hat{\sigma}_{x_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | 1_A 0_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{x_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) | 0_A 1_B\rangle = \frac{1}{2\sqrt{2}}(+1+1) = \frac{+1}{\sqrt{2}}$

$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - |1_A 0_B\rangle\right)$

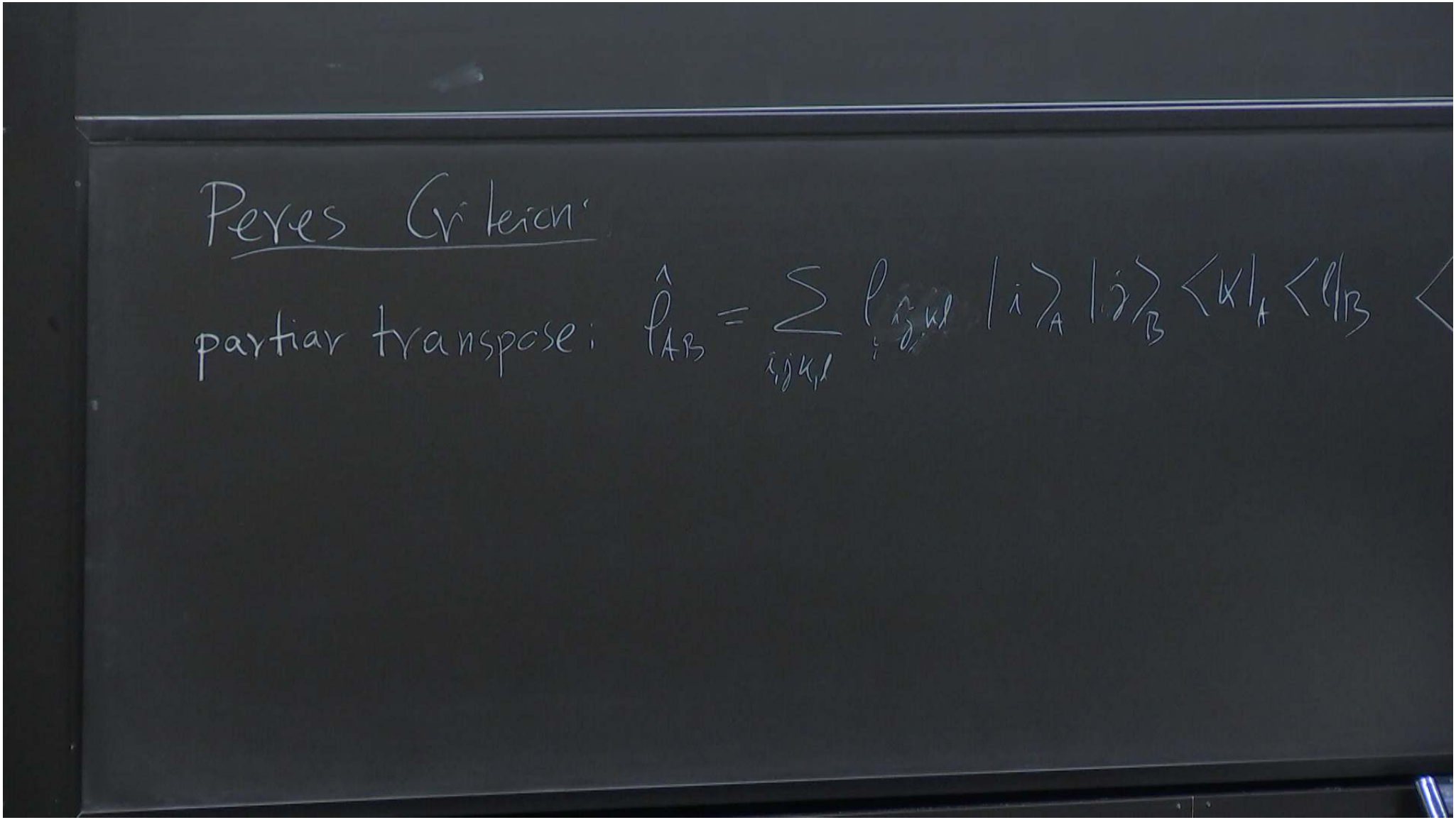
$|\langle \hat{A}(a)\hat{B}(b)\rangle - \langle \hat{A}(a)\hat{B}(b')\rangle| + |\langle \hat{A}(a')\hat{B}(b')\rangle + \langle \hat{A}(a')\hat{B}(b')\rangle| = 2\sqrt{2} > 2$

$\langle \hat{A}(a)\hat{B}(b)\rangle = \frac{1}{\sqrt{2}}$, $\langle \hat{A}(a)\hat{B}(b')\rangle = -\frac{1}{\sqrt{2}}$, $\langle \hat{A}(a')\hat{B}(b')\rangle = \frac{1}{\sqrt{2}}$, $\langle \hat{A}(a')\hat{B}(b')\rangle = \frac{1}{\sqrt{2}}$

$\left(\langle 0_A 1_B| \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B})|0_A 1_B\rangle + \langle 1_A 0_B| \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B})|1_A 0_B\rangle\right) = \frac{-1}{2\sqrt{2}}(-1 + (-1)) = \frac{1}{\sqrt{2}}$

$\langle \hat{A}(a')\hat{B}(b')\rangle = \frac{+1}{\sqrt{2}}$

$\langle 1_A 0_B| \hat{\sigma}_{x_A} \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B})|0_A 1_B\rangle = \frac{1}{2\sqrt{2}}(+1+1) = \frac{+1}{\sqrt{2}}$

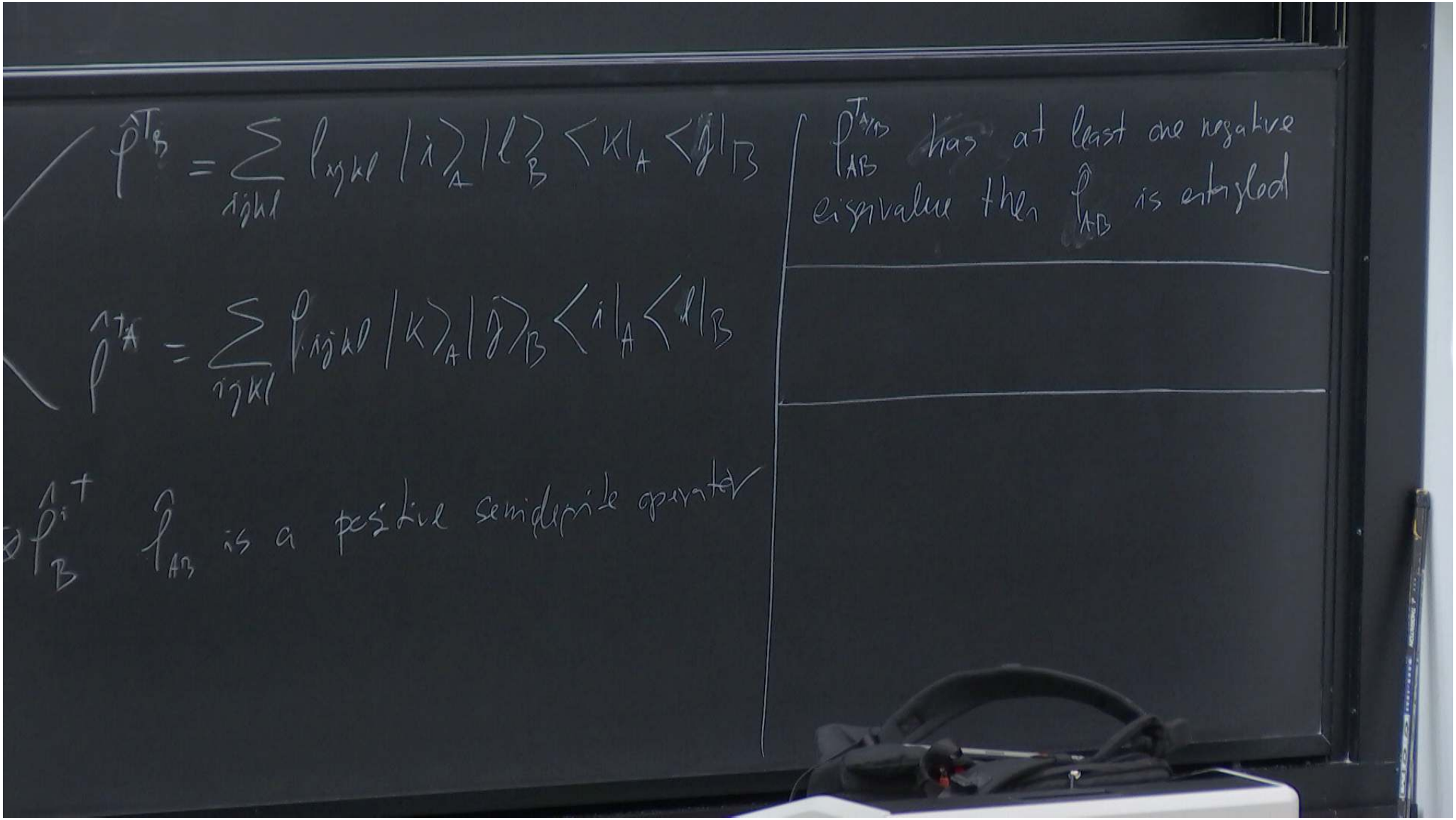

Peres Criterion:
partiar transpose:

Peres Criterion

partial transpose: $\hat{\rho}_{AB} = \sum_{ijkl} \rho_{ijkl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$ $\Rightarrow$ $\hat{\rho}^{T_B}$, $\hat{\rho}^{T_A}$

$|O_A l_B\rangle\langle l_A O_B| \xrightarrow{\text{Partial transp } B} |O_A O_B\rangle\langle l_A l_B|$

iff $\hat{\rho}_{AB}$ is separable (Not ENTANGLED) $\hat{\rho}_{AB} = \sum_i c_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i \Rightarrow \hat{\rho}_{AB}^{T_B} = \sum_i c_i \hat{\rho}_A^i \otimes \hat{\rho}_B^{i\,T}$


$\hat{\rho}^{T_B} = \sum_{ijkl} \rho_{ijkl} |i\rangle_A |l\rangle_B \langle k|_A \langle j|_B$
$\hat{\rho}^{T_A} = \sum_{ijkl} \rho_{ijkl} |k\rangle_A |j\rangle_B \langle i|_A \langle l|_B$
$\hat{\rho}_B^{\dagger} \quad \hat{\rho}_{AB}$ is a positive semidefinite operator
$\hat{\rho}_{AB}^{T_A/T_B}$ has at least one negative eigenvalue then $\hat{\rho}_{AB}$ is entangled

$|O_A l_b\rangle\langle l_A O_B| \xrightarrow{\text{Partial transp } B} |O_A O_B\rangle\langle l_A l_B|$

iff $\hat{\rho}_{AB}$ is separable (Not ENTANGLED) $\hat{\rho}_{AB} = \sum_i c_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i \Rightarrow \hat{\rho}_{AB}^{T_B} = \sum_i c_i \hat{\rho}_A^i \otimes \hat{\rho}_B^{i\,T}$

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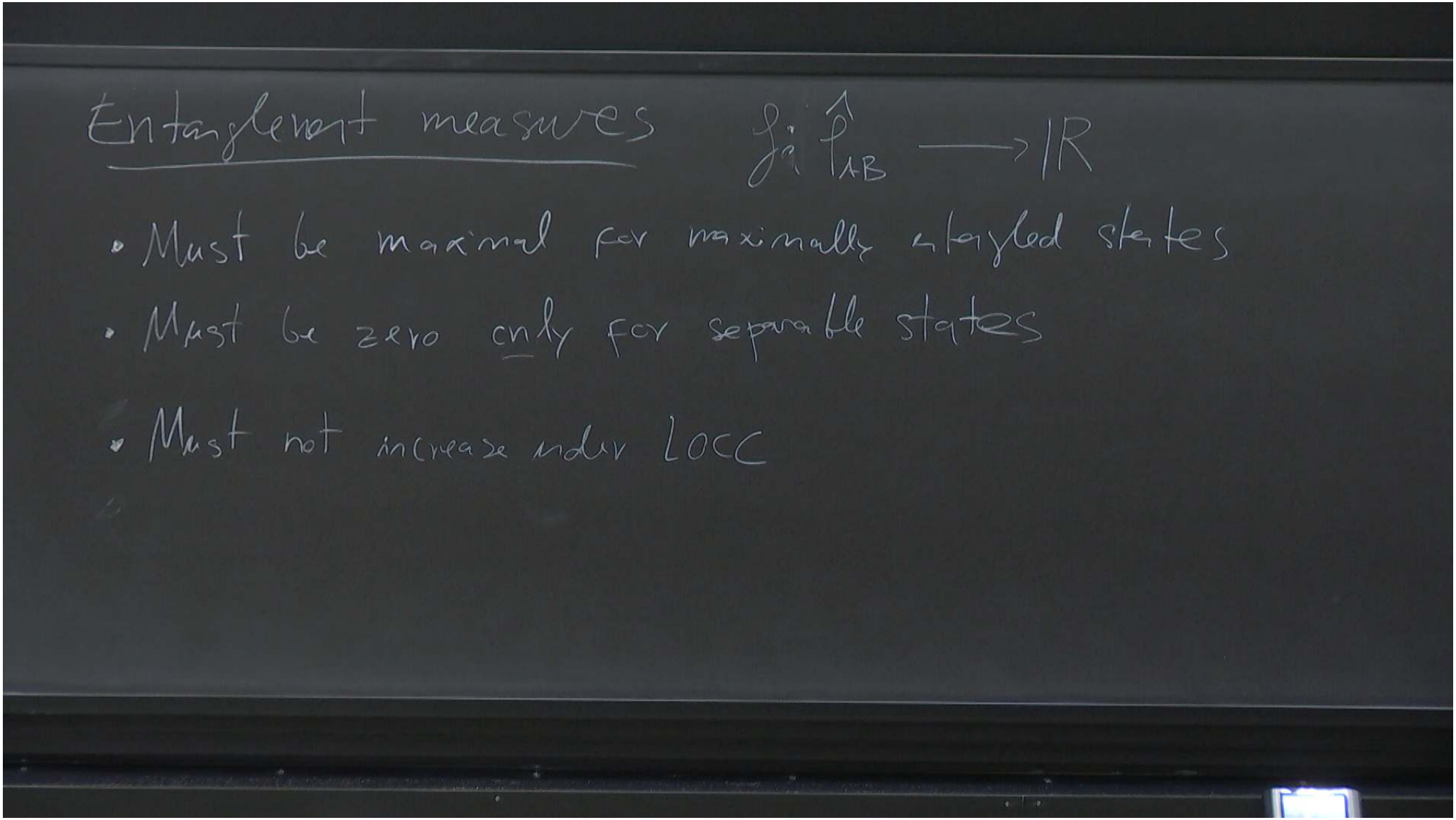
if $\mathcal{H}_{AB}$ is dim 2×2 or 3×2 then $\hat{\rho}^{T_{A/B}}$ not being positive semidefinite $\Leftrightarrow$ $\hat{\rho}^{AB}$

Distillable entanglement: the number of maximally entangled states of lower d

of $\hat{\rho}_{AB}$ and LOCC

$\hat{\rho}_{AB}^{T_{AB}}$ has negative eigenvalues $\Longleftrightarrow$ $\hat{\rho}_{AB}$ has distillable entanglement


Entanglement measures
$f: \hat{\rho}_{AB} \longrightarrow \mathbb{R}$
• Must be maximal for maximally entangled states
• Must be zero only for separable states
• Must not increase under LOCC