Title: Quantum Information Lecture - 230310

Speakers: Eduardo Martin-Martinez

Collection: Quantum Information (2022/2023)

Date: March 10, 2023 - 9:00 AM

URL: https://pirsa.org/23030005

$$\text{Local realism} \Rightarrow \left| C(a,b) - C(a,b') \right| + \left| C(a',b') + C(a',b) \right| \leq 2$$

$$A = A(a,\lambda)$$

$$B = B(b,\lambda)$$

$$\hat{A}(a) = \hat{\sigma}_{z_A} \otimes \mathbb{1}_B \qquad \hat{B}(b) = \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B} \right) \right)$$

$$\hat{A}(a') = \hat{\sigma}_{x_A} \otimes \mathbb{1}_B \qquad \hat{B}(b') = \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B} \right) \right)$$

$$\left\langle \hat{A}(a)\hat{B}(b) \right\rangle = \langle \psi^- | \; \hat{\sigma}_{z_A} \otimes \frac{-1}{\sqrt{2}}\left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B} \right) | \psi^- \rangle = \frac{-1}{2\sqrt{2}} \Big(\langle 0_A 1_B | \; \hat{\sigma}_{z_A} \otimes \left(\hat{\sigma}_{z_B} + \cancel{\hat{\sigma}_{x_B}} \right) | 0_A 1_B \rangle$$

$$\left\langle \hat{A}(a)\hat{B}(b') \right\rangle = \langle \psi^- | \; \hat{\sigma}_{z_A} \otimes \frac{1}{\sqrt{2}}\left(\sigma_{z_B} - \sigma_{x_B} \right) | \psi^- \rangle = -\frac{1}{\sqrt{2}}$$

$$\left\langle \hat{A}(a')\hat{B}(b) \right\rangle = \langle \psi^- | \hat{\sigma}_{x_A} \otimes \frac{1}{\sqrt{2}}\left(\sigma_{z_B} - \sigma_{x_B} \right) | \psi^- \rangle = \frac{1}{2\sqrt{2}} \Big(\langle 0_A 1_B | \hat{\sigma}_{x_A} \otimes \left(\cancel{\hat{\sigma}_{z_B}} + \hat{\sigma}_{x_B} \right) | 1_A 0_B \rangle + \langle 1_A 0_B | \hat{\sigma}_{x_A}$$

$$\ldots, b') + C(a', b)| \leq 2 \qquad |\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - |1_A 0_B\rangle\right)$$

$$= \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right)\right)$$

$$\langle 0_A 1_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \cancel{\hat{\sigma}_{x_B}}) | 0_A 1_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \cancel{\hat{\sigma}_{x_B}}) | 1_A 0_B\rangle \Big) = \frac{-1}{2\sqrt{2}}\left(-1 + (-1)\right) = \frac{1}{\sqrt{2}}$$

$$\langle \hat{A}(a') \hat{B}(b') \rangle =$$

$$=$$

$$\overline{2}$$

$$\otimes (\cancel{\hat{\sigma}_{z_B}} + \hat{\sigma}_{x_B}) | 1_A 0_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{x_A} \otimes (\cancel{\hat{\sigma}_{z_B}} - \hat{\sigma}_{x_B}) | 0_A 1_B\rangle = \frac{1}{2\sqrt{2}}(-1-1) = \frac{-1}{\sqrt{2}}$$

$$\Rightarrow \quad |C(a,b) - C(a,b')| + |C(a',b') + C(a',b)| \leq 2 \qquad |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - \right.$$

$$\hat{A}(a) = \hat{\sigma}_{z_A} \otimes \mathbb{1}_B \qquad \hat{B}(b) = \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B})\right)$$

$$\hat{A}(a') = \hat{\sigma}_{x_A} \otimes \mathbb{1}_B \qquad \hat{B}(b') = \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes (\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B})\right)$$

$$\langle \psi^- | \; \hat{\sigma}_{z_A} \otimes \frac{-1}{\sqrt{2}}(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}) |\psi^-\rangle = \frac{-1}{2\sqrt{2}}\left(\langle 0_A 1_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \cancel{\hat{\sigma}_{x_B}}) | 0_A 1_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{z_A} \otimes (\hat{\sigma}_{z_B} + \cancel{\hat{\sigma}_{x_B}}) | 1_A 0_B \rangle \right.$$

$$\langle \psi^- | \; \hat{\sigma}_{z_A} \otimes \frac{1}{\sqrt{2}}(\sigma_{z_B} - \sigma_{x_B}) |\psi^-\rangle = \frac{-1}{\sqrt{2}}$$

$$-| \hat{\sigma}_{x_A} \otimes \frac{1}{\sqrt{2}}(\sigma_{z_B} + \sigma_{x_B}) |\psi^-\rangle = \frac{1}{2\sqrt{2}}\left(\langle 0_A 1_B | \hat{\sigma}_{x_A} \otimes (\cancel{\hat{\sigma}_{z_B}} + \hat{\sigma}_{x_B}) | 1_A 0_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{x_A} \otimes (\cancel{\hat{\sigma}_{z_B}} + \hat{\sigma}_{x_B}) | 0_A 1_B \rangle \right) = \frac{1}{2\sqrt{2}}(-1-1) = \frac{-1}{\sqrt{2}}$$

$$C(a',b') + C(a',b) | \le 2 \qquad |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - |1_A 0_B\rangle\right)$$

$$(b) = \frac{-1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} + \hat{\sigma}_{x_B}\right)\right)$$

$$B(b') = \frac{1}{\sqrt{2}}\left(\mathbb{1}_A \otimes \left(\hat{\sigma}_{z_B} - \hat{\sigma}_{x_B}\right)\right)$$

$$\frac{-1}{2\sqrt{2}}\left(\langle 0_A 1_B | \hat{\sigma}_{z_A} \otimes \left(\hat{\sigma}_{z_B} + \cancel{\hat{\sigma}_{x_B}}\right) | 0_A 1_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{z_A} \otimes \left(\hat{\sigma}_{z_B} + \cancel{\hat{\sigma}_{x_B}}\right) | 1_A 0_B\rangle\right) = \frac{-1}{2\sqrt{2}}\left(-1 + (-1)\right) = \frac{1}{\sqrt{2}}$$

$$= \frac{-1}{\sqrt{2}} \qquad \langle \hat{A}(a') \hat{B}(b') \rangle = \frac{+1}{\sqrt{2}}$$

$$| \hat{\sigma}_{x_A} \otimes \left(\cancel{\hat{\sigma}_{z_B}} + \hat{\sigma}_{x_B}\right) | 1_A 0_B\rangle + \langle 1_A 0_B | \hat{\sigma}_{x_A} \otimes \left(\cancel{\hat{\sigma}_{z_B}} + \hat{\sigma}_{x_B}\right) | 0_A 1_B\rangle = \frac{1}{2\sqrt{2}}(+1+1) = \frac{+1}{\sqrt{2}}$$

$$\ldots,b)| \le 2 \qquad |\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|0_A 1_B\rangle - |1_A 0_B\rangle\right)$$

$$\ldots\hat{\sigma}_{z_B}+\hat{\sigma}_{x_B}))$$

$$\ldots\hat{\sigma}_{z_B}-\hat{\sigma}_{x_B}))$$

$$\boxed{|\underbrace{\langle \hat{A}(a)\hat{B}(b)\rangle}_{1/\sqrt{2}} - \underbrace{\langle \hat{A}(a)\hat{B}(b')\rangle}_{-1/\sqrt{2}}| + |\underbrace{\langle \hat{A}(a')\hat{B}(b')\rangle}_{1/\sqrt{2}} + \underbrace{\langle \hat{A}(a')\hat{B}(b')\rangle}_{1/\sqrt{2}}| = 2\sqrt{2} > 2}$$

$$\ldots\hat{\sigma}_{z_B}+\cancel{\hat{\sigma}_{x_B}})|0_A 1_B\rangle + \langle 1_A 0_B|\hat{\sigma}_{z_A}\otimes(\hat{\sigma}_{z_B}+\cancel{\hat{\sigma}_{x_B}})|1_A 0_B\rangle\Big) = \frac{-1}{2\sqrt{2}}\left(-1+(-1)\right) = \frac{1}{\sqrt{2}}$$

$$\langle \hat{A}(a')\hat{B}(b')\rangle = \frac{+1}{\sqrt{2}}$$

$$\ldots 0_B\rangle + \langle 1_A 0_B|\hat{\sigma}_{x_A}\otimes(\cancel{\hat{\sigma}_{z_B}}+\hat{\sigma}_{x_B})|0_A 1_B\rangle = \frac{1}{2\sqrt{2}}(+1+1) = \frac{+1}{\sqrt{2}}$$

Peres Criterion:

partial transpose: $\hat{\rho}_{AB} = \sum_{i,j,k,l} \rho_{ij,kl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$

Peres Criterion:

partial transpose: $\hat{\rho}_{AB} = \sum_{i,j,k,l} \rho_{ijkl} \, |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B$ $\quad \hat{\rho}^{T_B}$, $\hat{\rho}^{T_A}$

$$|0_A 1_B\rangle\langle 1_A 0_B| \xrightarrow[\text{transp B}]{\text{Partial}} |0_A 0_B\rangle\langle 1_A 1_B|$$

iff $\hat{\rho}_{AB}$ is separable (NOT ENTANGLED) $\hat{\rho}_{AB} = \sum_i c_i \, \hat{\rho}_A^i \otimes \hat{\rho}_B^i \Longrightarrow \hat{\rho}_{AB}^{T_B} = \sum_i c_i \, \hat{\rho}_A^i \otimes \hat{\rho}_B^{i\,T}$

$$\hat{\rho}^{T_B} = \sum_{ijkl} \rho_{ijkl} \, |i\rangle_A |l\rangle_B \langle k|_A \langle j|_B$$

$$\hat{\rho}^{T_A} = \sum_{ijkl} \rho_{ijkl} \, |k\rangle_A |j\rangle_B \langle i|_A \langle l|_B$$

$\otimes \hat{\rho}_B^{i\,\dagger}$ $\hat{\rho}_{AB}$ is a positive semidefinite operator

$\rho_{AB}^{T_{A/B}}$ has at least one negative eigenvalue then $\hat{\rho}_{AB}$ is entangled

$$|0_A 1_B\rangle\langle 1_A 0_B| \xrightarrow[\text{transp } B]{\text{Partial}} |0_A 0_B\rangle\langle 1_A 1_B|$$

iff $\hat{\rho}_{AB}$ is separable (NOT ENTANGLED) $\hat{\rho}_{AB} = \sum_i c_i \, \hat{\rho}_A^i \otimes \hat{\rho}_B^i \Longrightarrow \hat{\rho}_{AB}^{T_B} = \sum_i c_i \, \hat{\rho}_A^i \otimes \hat{\rho}_B^{i\,T}$

if $\mathcal{H}_{AB}$ is dim 2×2 or 3×2 then $\hat{\rho}^{T_{A/B}}$ not being positive semidefinite $\Longleftrightarrow \hat{\rho}^{AB}$

Distillable entanglement: the number of maximally entangled states of lower di

of $\hat{\rho}_{AB}$ and LOCC

$\hat{\rho}_{AB}^{t_{A|B}}$ has negative eigenvalues $\Longleftrightarrow$ $\hat{\rho}_{AB}$ has distillable entanglement

Entanglement measures $f: \hat{\rho}_{AB} \longrightarrow \mathbb{R}$

- Must be maximal for maximally entangled states
- Must be zero only for separable states
- Must not increase under LOCC