

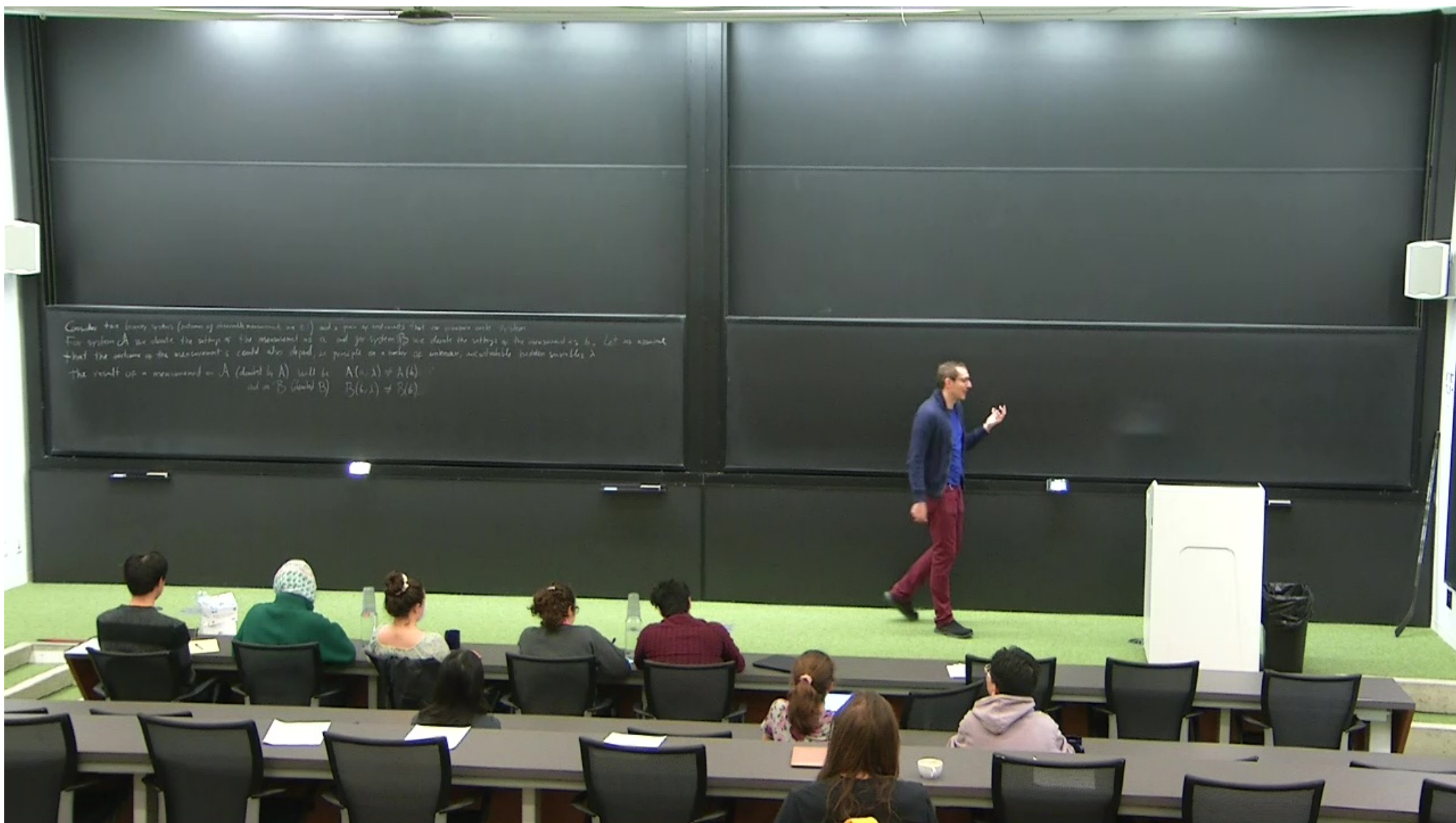
Title: Quantum Information Lecture - 230308

Speakers: Eduardo Martin-Martinez

Collection: Quantum Information (2022/2023)

Date: March 08, 2023 - 9:00 AM

URL: <https://pirsa.org/23030004>



Consider two binary variables (outcomes of observable measurements) a, b and a pair of variables that we presume could be hidden.
For system A we denote the settings of the measurement as a_1 and for system B we denote the settings of the measurement as b_1 . Let us assume
that the outcome of the measurement could also depend, in principle, on a number of unknown, measurable hidden variables λ .
The result of a measurement on A (denoted by A) will be $A(a, \lambda) \neq A(b)$
and on B (denoted by B) $B(b, \lambda) \neq B(a)$.

Consider two binary systems (outcomes of observable measurements are ± 1) and a pair of instruments that can measure each system.
For system A we denote the settings of the measurement as a and for system B we denote the settings of the measurement as b . Let us assume that the outcome of the measurement s could also depend, in principle, on a number of unknown, uncontrollable hidden variables λ .
The result of a measurement on A (denoted by A) will be $A(a, \lambda) \neq A(b)$
and on B (denoted by B) $B(b, \lambda) \neq B(b)$

pair of instruments that can measure each system.

for system B we denote the settings of the measurement as b . Let us assume
dependence on a number of unknown uncontrollable hidden variables λ

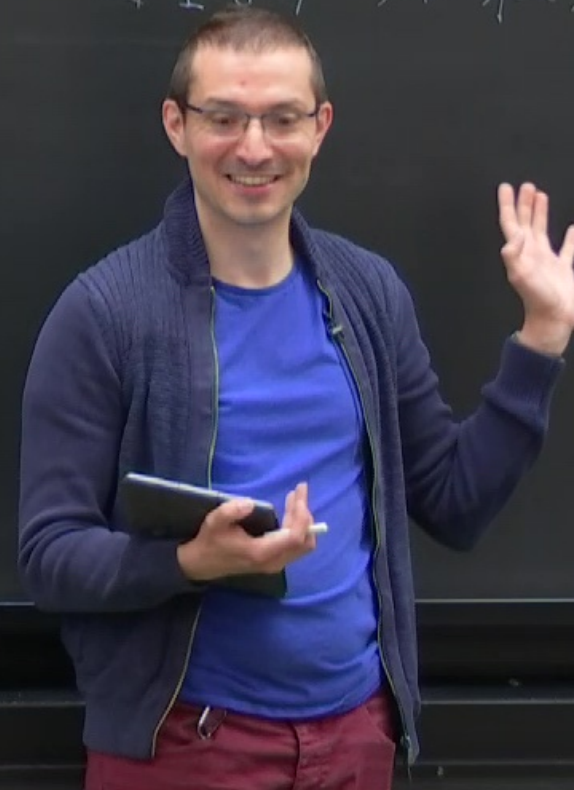
$$\begin{aligned} A(a, \lambda) &\neq A(b) \\ B(b, \lambda) &\neq B(a) \end{aligned} \quad \text{locality} \quad \begin{cases} A(a, b, \lambda) = A(a, \lambda) \\ B(a, b, \lambda) = B(b, \lambda) \end{cases}$$

assumption - A and B could depend on a set of hidden variables

Let us study the correlations between the outcomes of measurements

$$C(a, b) = \int d\lambda \Delta(\lambda) A(a, \lambda) B(b, \lambda) \quad \text{where} \quad \begin{aligned} A(a, \lambda) &= \\ B(b, \lambda) &= \end{aligned}$$

between the outcomes of measurements on A and B
 $(a, \lambda) B(b, \lambda)$ where $A(a, \lambda) = \pm 1$ $B(b, \lambda) = \pm 1$ $\Lambda(\lambda)$ satisfies $\int d\lambda \Lambda(\lambda)$



The result of a measurement on A (denoted by A) will be $A(a, \lambda) \neq A(b)$
 and on B (denoted by B) $B(b, \lambda) \neq B(a)$

Locality $\left\{ \begin{array}{l} A(a, b, \lambda) = A(a, \lambda) \\ B(a, b, \lambda) = B(b, \lambda) \end{array} \right.$

Realism - A and B can depend on a set of hidden variables

Let us study the correlations between outcomes of measurements on A and B

$$C(a, b) = \int d\lambda \Lambda(\lambda) A(a, \lambda) B(b, \lambda) \quad \text{where} \quad \begin{array}{l} A(a, \lambda) = \pm 1 \\ B(b, \lambda) = \pm 1 \end{array} \quad \Lambda(\lambda) \text{ satisfies } \int d\lambda \Lambda(\lambda) = 1$$

$$\begin{aligned} C(a, b) - C(a, b') &= \int d\lambda \Lambda(\lambda) (A(a, \lambda) B(b, \lambda) - A(a, \lambda) B(b', \lambda)) = \\ &= \int d\lambda \Lambda(\lambda) (A(a, \lambda) B(b, \lambda) [1 \pm A(a', \lambda) B(b', \lambda)] - A(a, \lambda) B(b', \lambda) [1 \pm A(a', \lambda) B(b, \lambda)]) \end{aligned}$$



Let us study the correlations between outcomes of measurements on A and B

Realism - A and B can depend on a set of hidden variables

Let us study the correlations between outcomes of measurements on A and B

$$C(a, b) = \int d\lambda \Lambda(\lambda) A(a, \lambda) B(b, \lambda) \quad \text{where} \quad A(a, \lambda) = \pm 1 \quad B(b, \lambda) = \pm 1 \quad \Lambda(\lambda) \text{ satisfies } \int d\lambda \Lambda(\lambda) = 1$$

$$C(a, b) - C(a, b') = \int d\lambda \Lambda(\lambda) (A(a, \lambda) B(b, \lambda) - A(a, \lambda) B(b', \lambda)) =$$

$$= \int d\lambda \Lambda(\lambda) (A(a, \lambda) B(b, \lambda) [1 \pm A(a', \lambda) B(b', \lambda)] - A(a, \lambda) B(b', \lambda) [1 \pm A(a', \lambda) B(b, \lambda)])$$

$$\text{Since } |A(a, \lambda)| \leq 1 \\ |B(b, \lambda)| \leq 1$$

and on λ (determined B) $B(b, \lambda) \neq B(a, \lambda)$

$(\lambda(a, b, \lambda) = B(b, \lambda))$

Realism - A and B could depend on a set of hidden variables

Let us study the correlations between outcomes of measurements on A and B

$$C(a, b) = \int d\lambda \Lambda(\lambda) A(a, \lambda) B(b, \lambda) \quad \text{where} \quad \begin{matrix} A(a, \lambda) = \pm 1 \\ B(b, \lambda) = \pm 1 \end{matrix} \quad \Lambda(\lambda) \text{ satisfies } \int d\lambda \Lambda(\lambda) = 1$$

$$\begin{aligned} C(a, b) - C(a, b') &= \int d\lambda \Lambda(\lambda) (A(a, \lambda) B(b, \lambda) - A(a, \lambda) B(b', \lambda)) = \\ &= \int d\lambda \Lambda(\lambda) \left(A(a, \lambda) B(b, \lambda) [1 \pm A(a', \lambda) B(b', \lambda)] - A(a, \lambda) B(b', \lambda) [1 \pm A(a', \lambda) B(b, \lambda)] \right) \end{aligned}$$

Since $\begin{matrix} A(a, \lambda) = \pm 1 \\ |B(b, \lambda)| = 1 \end{matrix}$

nts that can measure each system.

B we denote the settings of the measurement as b . Let us assume
number of unknown, uncontrollable hidden variables λ

$$\begin{array}{l} A(b) \\ B(a) \end{array} \quad \text{Locality} \left\{ \begin{array}{l} A(a, b, \lambda) = A(a, \lambda) \\ B(a, b, \lambda) = B(b, \lambda) \end{array} \right.$$

Realism - A and B can depend on a set of hidden variables

min QM as the state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle + |1_A 0_B\rangle)$

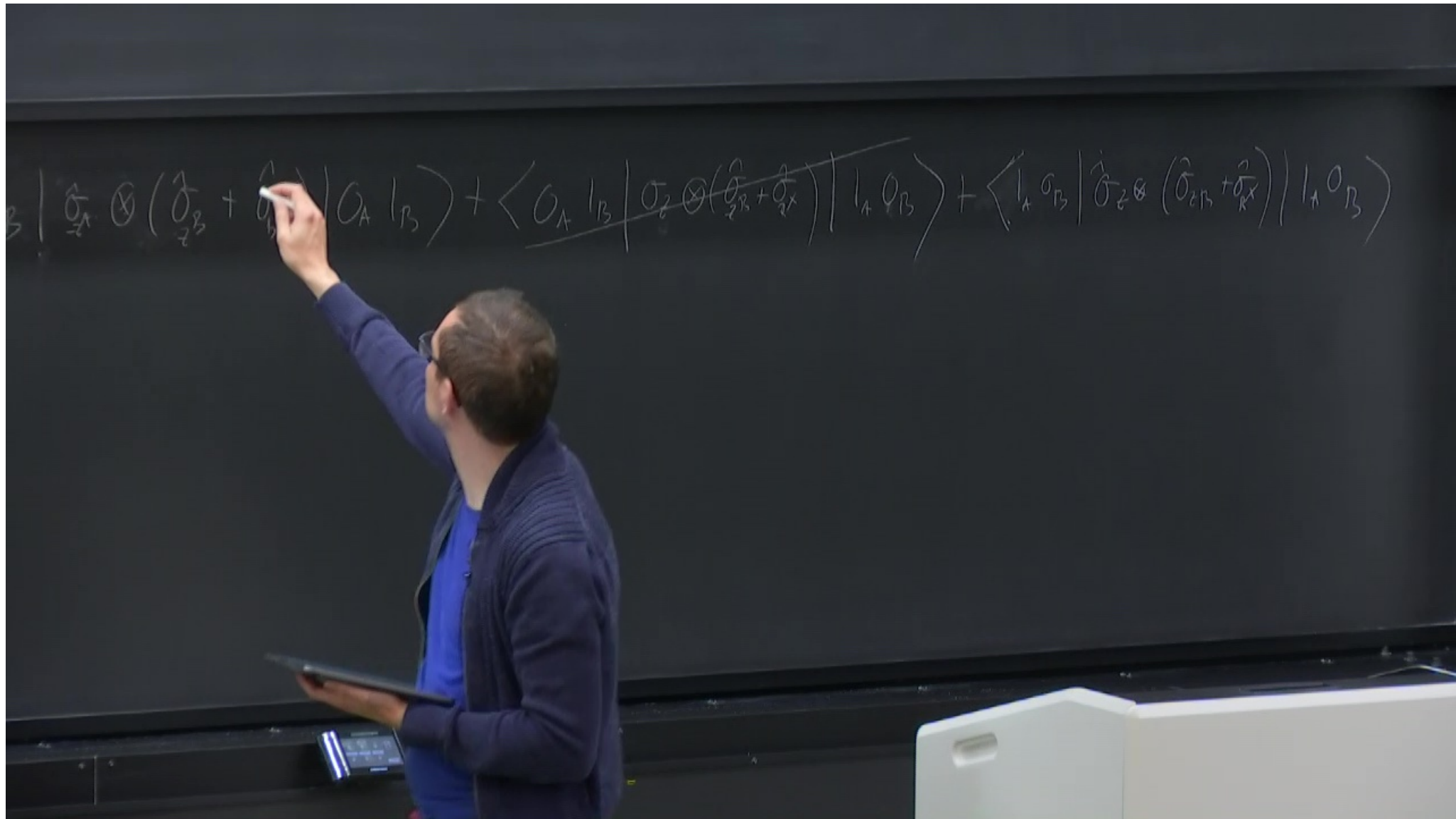
settings $a \equiv$ measure spin in the z axis, $a' \equiv$ measure spin in the x axis

$b \equiv$ ~~measure spin in the z axis~~

min QM as the state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle + |1_A 0_B\rangle)$

settings $a \equiv$ measure spin in the z axis, $a' \equiv$ measure spin in the x axis

$b \equiv$ measures $\frac{1}{\sqrt{2}}(1_A \otimes (\hat{\sigma}_z + \hat{\sigma}_x))$, $b' \equiv$ measures $\frac{1}{\sqrt{2}}(1_A \otimes (\hat{\sigma}_z - \hat{\sigma}_x))$

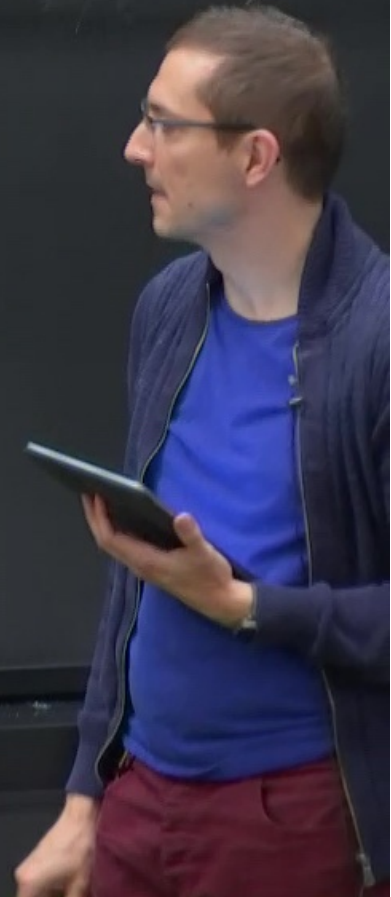


$$\sigma_z |0\rangle = -|0\rangle$$

$$\frac{\sqrt{2}\sigma_z}{2}$$

$$3 \left| \hat{\sigma}_x \otimes (\hat{\sigma}_z + \hat{\sigma}_x) \right| 0_A 1_B \rangle + \left\langle 0_A 1_B \left| \sigma_z \otimes (\hat{\sigma}_z + \hat{\sigma}_x) \right| 1_A 0_B \right\rangle + \left\langle 1_A 0_B \left| \hat{\sigma}_z \otimes (\hat{\sigma}_z + \hat{\sigma}_x) \right| 1_A 0_B \right\rangle$$

$$\begin{aligned} \langle \hat{A}(a) \hat{B}(b) \rangle &= \frac{1}{\sqrt{2}} = \langle \hat{A}(a') \hat{B}(b') \rangle = \langle \hat{A}(a') \hat{B}(b) \rangle = -\langle \hat{A}(a) \hat{B}(b') \rangle \\ \underbrace{|\langle \hat{A}(a) \hat{B}(b) \rangle - \langle \hat{A}(a) \hat{B}(b') \rangle| + |\langle \hat{A}(a') \hat{B}(b') \rangle + \langle \hat{A}(a') \hat{B}(b) \rangle|}_{\frac{2}{\sqrt{2}}} &= \end{aligned}$$



$$\langle \hat{A}(a) \hat{B}(b) \rangle = \frac{1}{\sqrt{2}} = \langle \hat{A}(a') \hat{B}(b') \rangle = \langle \hat{A}(a') \hat{B}(b) \rangle = -\langle \hat{A}(a) \hat{B}(b') \rangle$$

$$\underbrace{|\langle \hat{A}(a) \hat{B}(b) \rangle - \langle \hat{A}(a) \hat{B}(b') \rangle|}_{\frac{2}{\sqrt{2}}} + \underbrace{|\langle \hat{A}(a') \hat{B}(b') \rangle + \langle \hat{A}(a') \hat{B}(b) \rangle|}_{\frac{2}{\sqrt{2}}} = 2\sqrt{2} > 2$$