Title: Quantum Information Lecture - 230306

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Finding the Gibbs equilibrium state. Extremalize $S(\hat{\rho}) = -\mathrm{tr}(\hat{\rho}\,\mathrm{Log}\,\hat{\rho})$ at $\langle \hat{H} \rangle = E$. Hypothesis: S is a thermodynamic entropy

$$\frac{\partial}{\partial \hat{\rho}}\left(S - \beta\left(\mathrm{tr}(\hat{\rho}\hat{H}) - E\right) - \mu\left(\mathrm{tr}(\hat{\rho}) - 1\right)\right) = 0 \qquad \hat{\rho} = \sum_i p_i \hat{P}_i, \quad S = \sum_i n_i p_i \mathrm{Log}\, p_i, \quad E = \sum_i n_i E_i p_i, \quad \mathrm{tr}(\hat{\rho}) = \sum_i n_i p_i = 1$$

trace constraint

we know $[\hat{H}, \hat{P}_i] = 0$, $E_i = \mathrm{Tr}(\hat{P}_i \hat{H})$

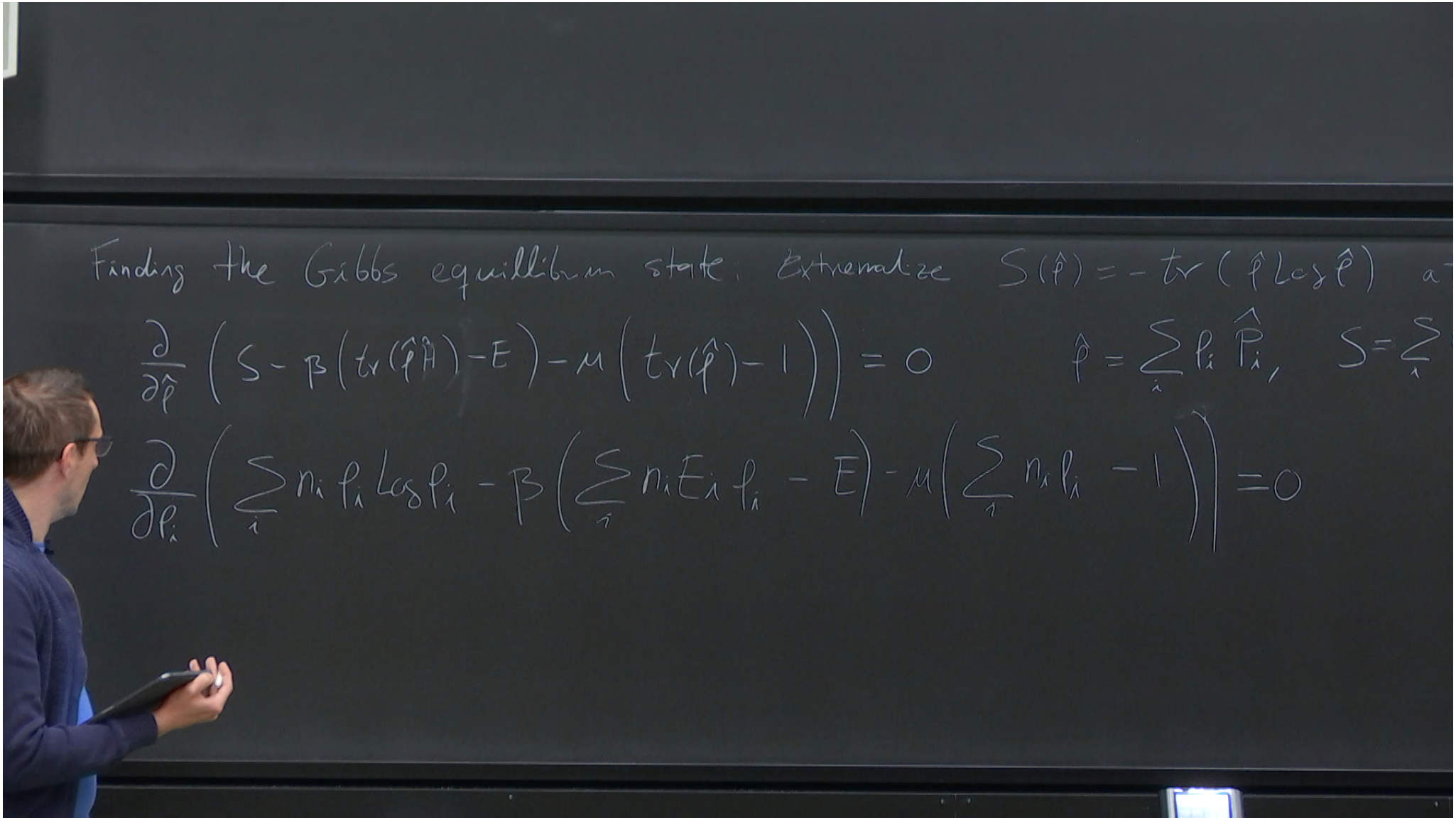
Finding the Gibbs equilibrium state: Extremalize $S(\hat{\rho}) =$

$$\frac{\partial}{\partial \hat{\rho}} \left(S - \beta \left(\mathrm{tr}(\hat{\rho}\hat{H}) - E \right) - \mu \left(\mathrm{tr}(\hat{\rho}) - 1 \right) \right) = 0 \qquad \hat{\rho} =$$

$S(\hat{\rho}) = -\mathrm{tr}(\hat{\rho}\,\mathrm{Log}\,\hat{\rho})$ at $\langle \hat{H} \rangle = E$. Hypothesis: S is a thermodynamic entropy

$\hat{\rho} = \sum_i p_i \hat{P}_i$, $S = \sum_i n_i p_i \mathrm{Log}\, p_i$, $E = \sum n_i E_i p_i$, $\mathrm{tr}(\hat{\rho}) = \sum_i n_i p_i = 1$ trace constraint

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$\hat{\rho} = \sum_i p_i \hat{P}_i$, $S = \sum_i$
$\frac{\partial}{\partial p_i}\left(\sum_i n_i p_i \log p_i - \beta\left(\sum_i n_i E_i p_i - E\right) - \mu\left(\sum_i n_i p_i - 1\right)\right) = 0$

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$$\frac{\partial}{\partial p_i}\left(\sum_i n_i p_i \log p_i - \beta\left(\sum_i n_i E_i p_i - E\right) - \mu\left(\sum_i n_i p_i - 1\right)\right) = 0$$

$$-\mathrm{Log}\,p_i - 1 - \beta E_i - \mu = 0 \xrightarrow[\text{and sum}]{\text{multiply by } \hat{P}_i} -\mathrm{Log}\,\hat{\rho} - \mathbb{1} - \beta\hat{H} - \mu\mathbb{1} = 0$$

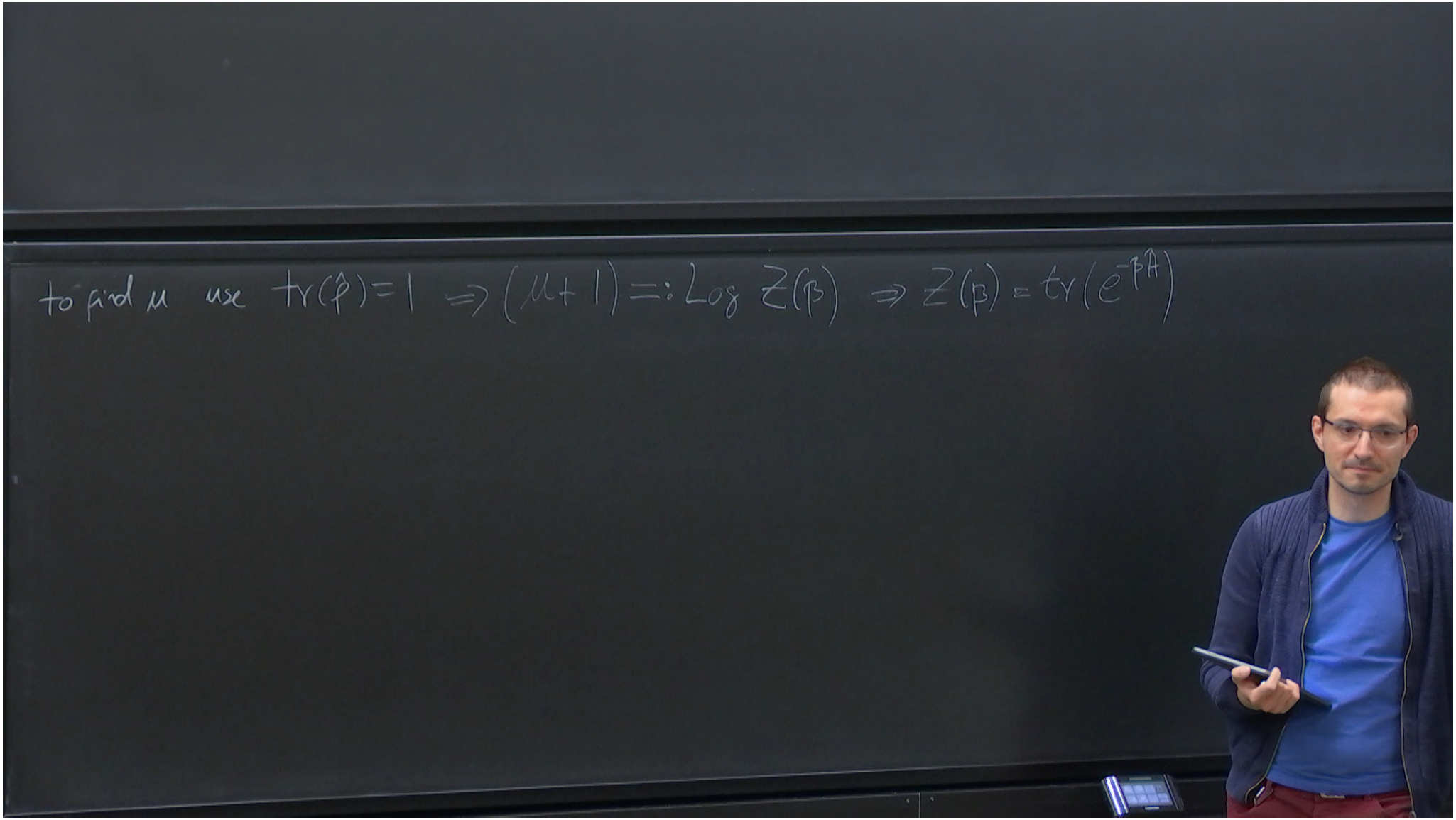
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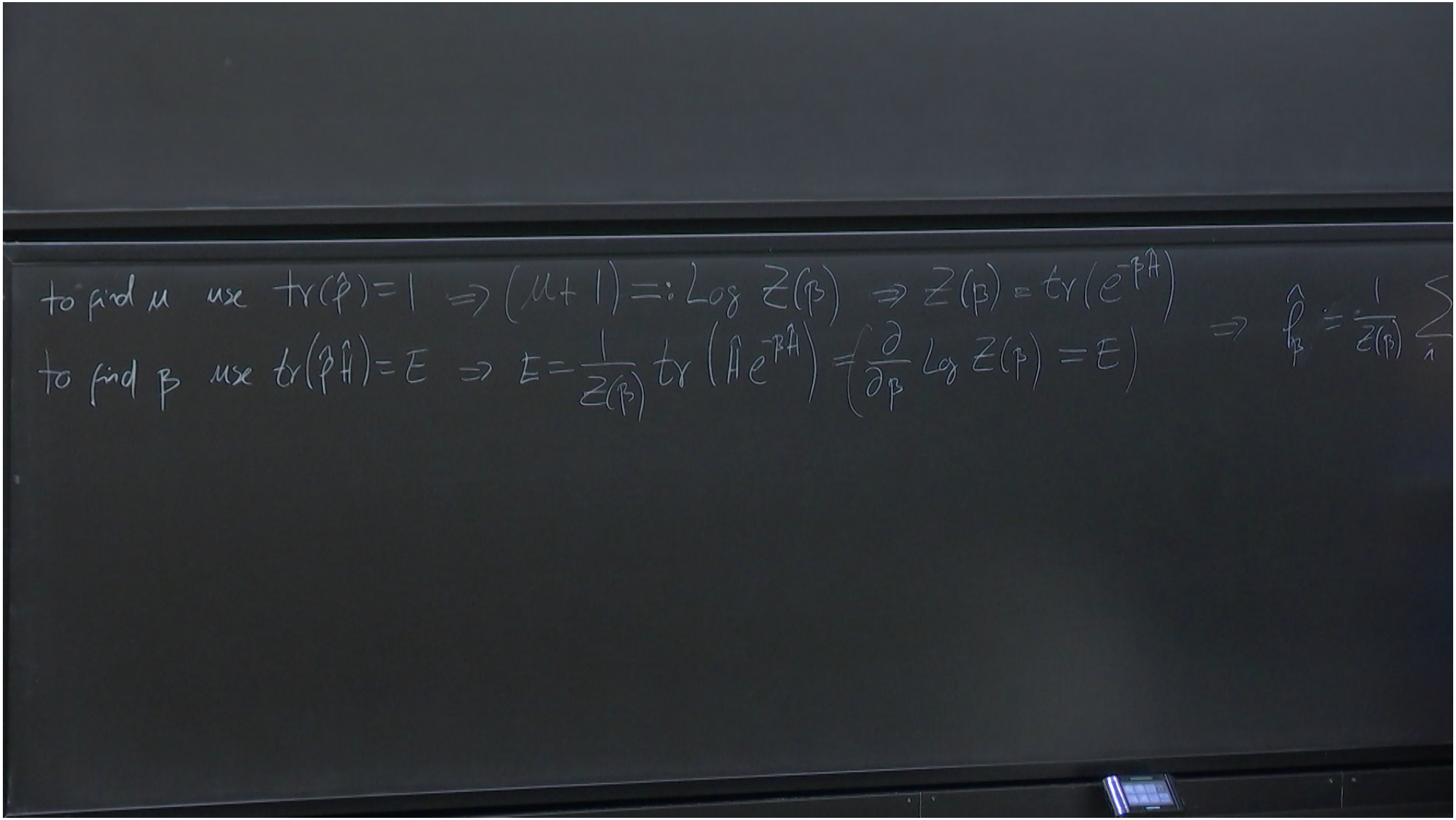
$\hat{\rho} = \sum_i p_i \hat{P}_i$, $S = \sum_i n_i p_i \mathrm{Log}\, p_i$, $E = \sum n_i E_i p_i$, $\mathrm{tr}(\hat{\rho}) = \sum_i n_i p_i = 1$ trace constraint

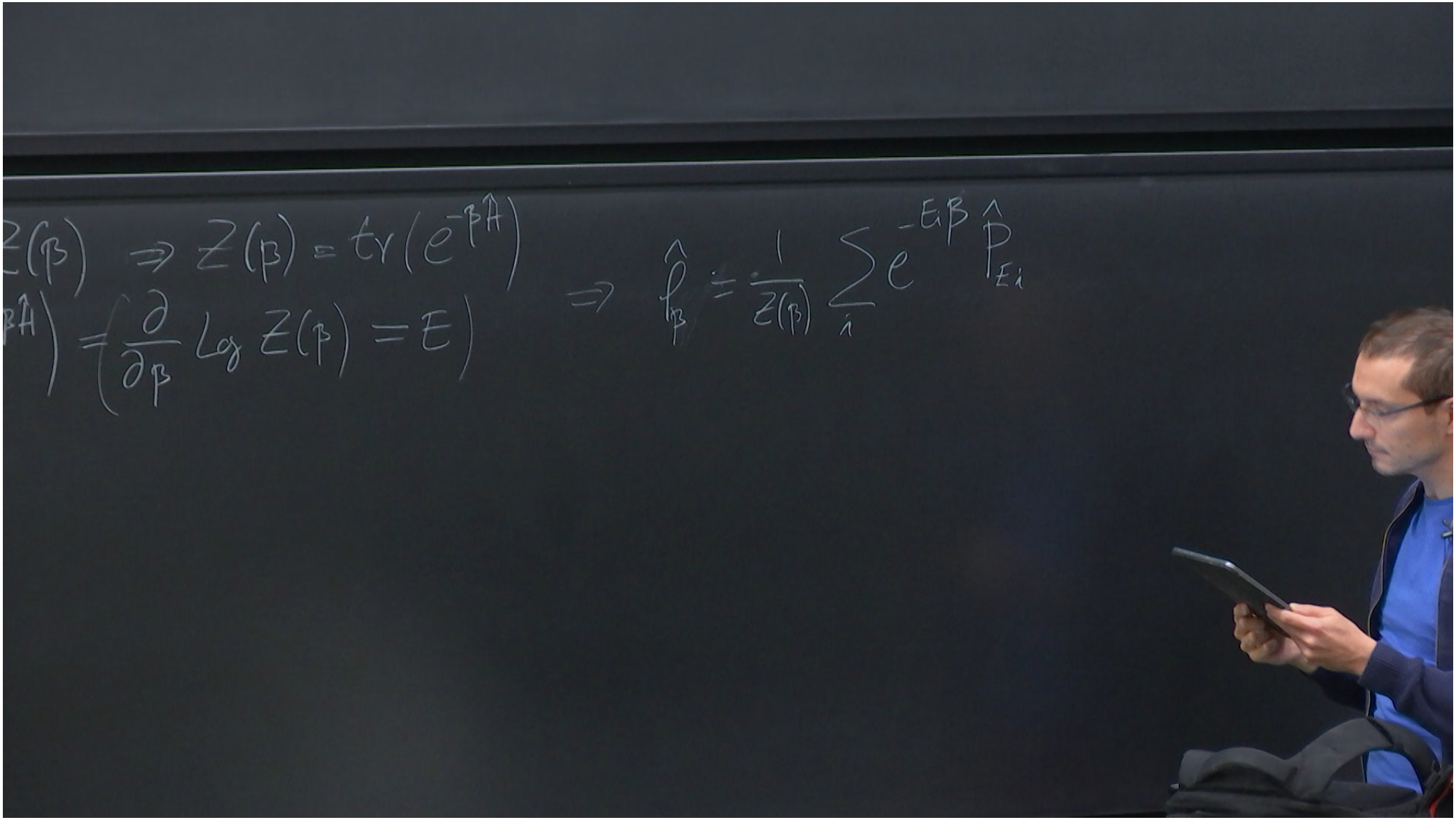
we know $[\hat{H}, \hat{\rho}] = 0$, $E_i = \mathrm{Tr}(\hat{P}_i \hat{H})$

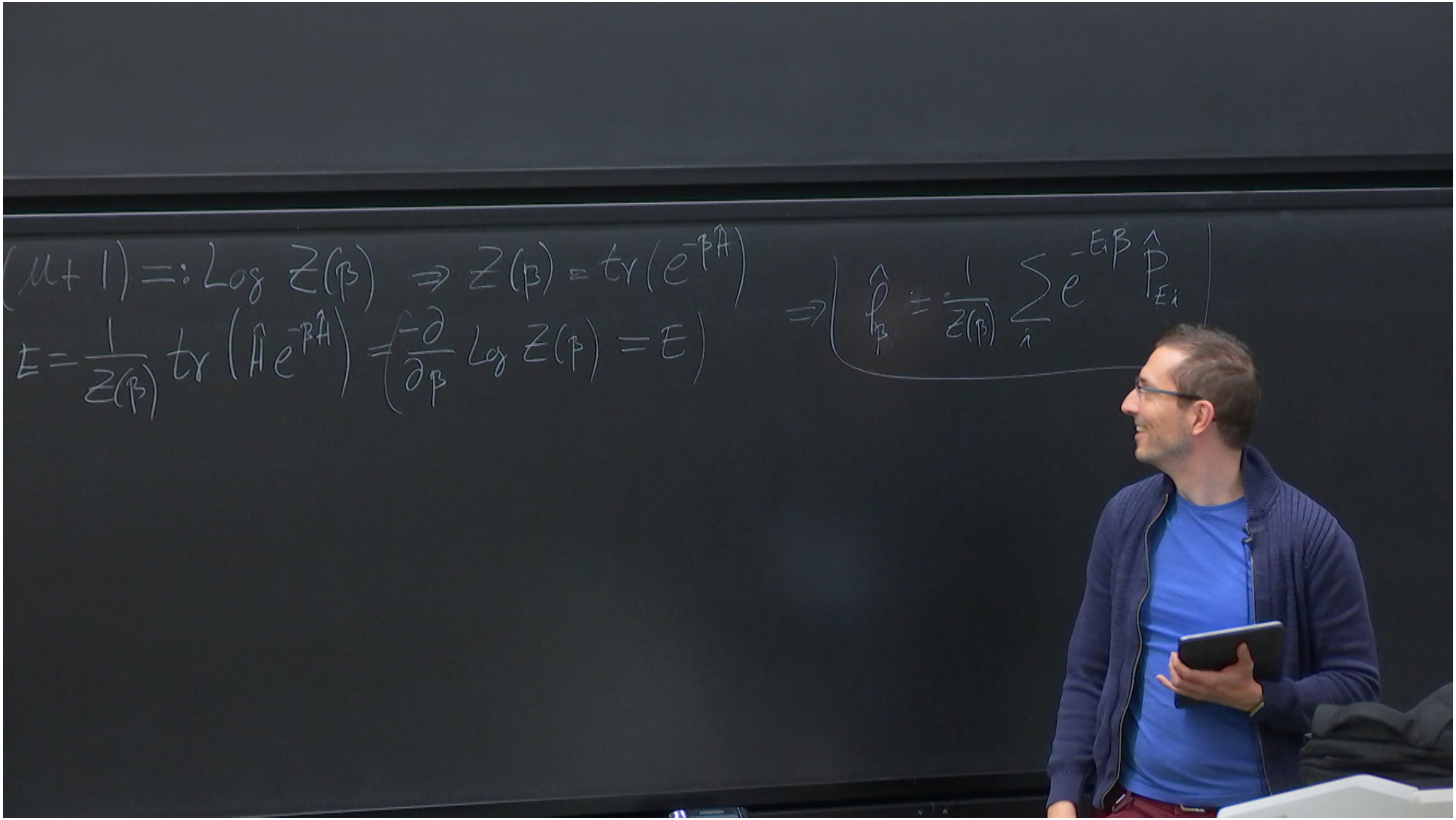
$n_i p_i - 1))) = 0$

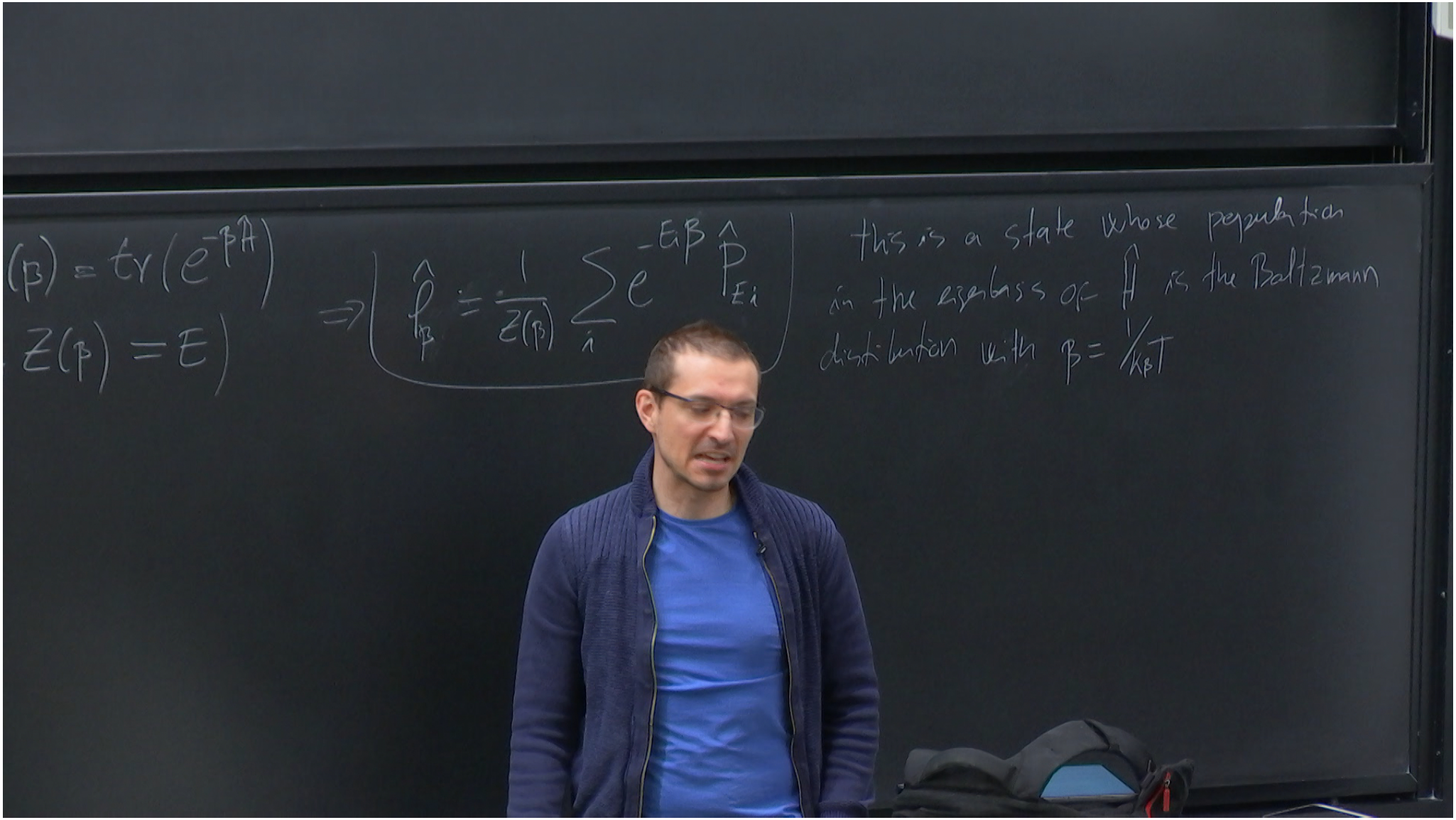
$-\mathrm{Log}\,\hat{\rho} - \mathbb{1} - \beta\hat{H} - \mu\mathbb{1} = 0 \Rightarrow \hat{\rho} = \exp\left[-\beta\hat{H} - (\mu+1)\mathbb{1}\right] = e^{-(\mu+1)\mathbb{1}}\, e^{-\beta\hat{H}}$


to find μ use tr(ρ̂) = 1 ⇒ (μ + 1) =: Log Z(β) ⇒ Z(β) = tr(e^{−βĤ})


to find μ use $\mathrm{tr}(\hat{\rho}) = 1 \Rightarrow (\mu + 1) =: \mathrm{Log}\, Z(\beta) \Rightarrow Z(\beta) = \mathrm{tr}(e^{-\beta \hat{H}})$
to find β use $\mathrm{tr}(\hat{\rho}\hat{H}) = E \Rightarrow E = \frac{1}{Z(\beta)} \mathrm{tr}(\hat{H} e^{-\beta \hat{H}}) = \left(\frac{\partial}{\partial \beta} \mathrm{Log}\, Z(\beta) = E\right)$
$\Rightarrow \hat{\rho}_\beta = \frac{1}{Z(\beta)} \sum$


$Z(\beta) \Rightarrow Z(\beta) = tr(e^{-\beta \hat{A}})$
$\beta \hat{A}) = \left(\frac{\partial}{\partial \beta} Log\, Z(\beta) = E\right)$
$\Rightarrow \hat{\rho}_\beta = \frac{1}{Z(\beta)} \sum_i e^{-E_i \beta} \hat{P}_{Ei}$


$(\mu+1) =: \log Z(\beta) \Rightarrow Z(\beta) = \mathrm{tr}(e^{-\beta \hat{H}})$
$E = \frac{1}{Z(\beta)} \mathrm{tr}(\hat{H} e^{-\beta \hat{H}}) = \left(\frac{-\partial}{\partial \beta} \log Z(\beta) = E\right)$
$\Rightarrow \hat{\rho}_\beta = \frac{1}{Z(\beta)} \sum_i e^{-E_i \beta} \hat{P}_{E_i}$


$Z(\beta) = \mathrm{tr}(e^{-\beta \hat{H}})$
$Z(\beta) = E$
$\Rightarrow \hat{\rho}_\beta = \frac{1}{Z(\beta)} \sum_i e^{-E_i \beta} \hat{P}_{E_i}$
this is a state whose population in the eigenbasis of $\hat{H}$ is the Boltzmann distribution with $\beta = 1/k_B T$

to find μ use $\mathrm{tr}(\hat{\rho}) = 1 \Rightarrow (\mu + 1) =: \log Z(\beta) \Rightarrow Z(\beta) = \mathrm{tr}(e^{-\beta \hat{H}})$

to find β use $\mathrm{tr}(\hat{\rho}\hat{H}) = E \Rightarrow E = \frac{1}{Z(\beta)} \mathrm{tr}(\hat{H} e^{-\beta \hat{H}}) = \left(\frac{-\partial}{\partial \beta} \log Z(\beta) = E\right) \Rightarrow \hat{\rho}_\beta =$

$\beta \to 0 \Rightarrow \hat{\rho}_{\beta \to 0} = \frac{1}{Z(\beta \to 0)} \sum_1 \hat{P}_{E_1} = \frac{1}{d} \underline{\underline{1}}$

Finding the Gibbs equilibrium state: Extremalize $S(\hat\rho) = -\mathrm{tr}(\hat\rho \log \hat\rho)$ at $\langle \hat H \rangle = E$. Hypothesis: S is a thermodynamic entropy

$$\frac{\partial}{\partial \hat\rho}\Big(S - \beta\big(\mathrm{tr}(\hat\rho \hat H) - E\big) - \mu\big(\mathrm{tr}(\hat\rho) - 1\big)\Big) = 0 \qquad \hat\rho = \sum_i p_i \hat P_i, \quad S = \sum_i n_i p_i \log p_i, \quad E = \sum n_i E_i p_i, \quad \mathrm{tr}(\hat\rho) = \sum_i n_i p_i = 1 \text{ (trace constraint)}$$

we know $[\hat H, \hat\rho] = 0$, $E_i = \mathrm{Tr}(\hat P_i \hat H)$

$$\frac{\partial}{\partial p_i}\Big(\sum_i n_i p_i \log p_i - \beta\Big(\sum_i n_i E_i p_i - E\Big) - \mu\Big(\sum_i n_i p_i - 1\Big)\Big) = 0$$

$$-\log p_i - 1 - \beta E_i - \mu = 0 \xrightarrow[\text{and sum}]{\text{multiply by } \hat P_i} -\log\hat\rho - \mathbb{1} - \beta\hat H - \mu\mathbb{1} = 0 \Rightarrow \hat\rho = \exp\left[-\beta\hat H - (\mu+1)\mathbb{1}\right] = e^{-(\mu+1)\mathbb{1}}\, e^{-\beta\hat H}$$

Multipartite systems. Consider a bipartite system AB, the Hilbert space associated with A (or B) is $\mathcal{H}_A$ (or $\mathcal{H}_B$). The joint Hilbert space is then $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$, if $\{|a_i\rangle\}$ and $\{|b_i\rangle\}$ are orthonormal bases of $\mathcal{H}_A$ and $\mathcal{H}_B$ then

$$\{|a_i\rangle \otimes |b_j\rangle\} \equiv \{|a_i b_i\rangle\} \text{ is an orthonormal basis of } \mathcal{H}_{AB}$$

∂p_i

$$-\log p_i - 1 - \beta E_i - \mu = 0 \xrightarrow[\text{and sum}]{\text{multiply by } \hat{P}_i} -\log \hat{\rho} - \mathbb{1} - \beta \hat{H} - \mu \mathbb{1} = 0 \Rightarrow \hat{\rho} =$$

Multipartite systems. Consider a bipartite system AB, the Hilbert space associated

Hilbert space is then $\mathcal{L}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$, if $\{|a_i\rangle\}$ and $\{|b_i\rangle\}$ are orthonor

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We define the partial state of $\hat{\rho}_A$ over the state $\hat{\rho}_{AB}$

∂p_i

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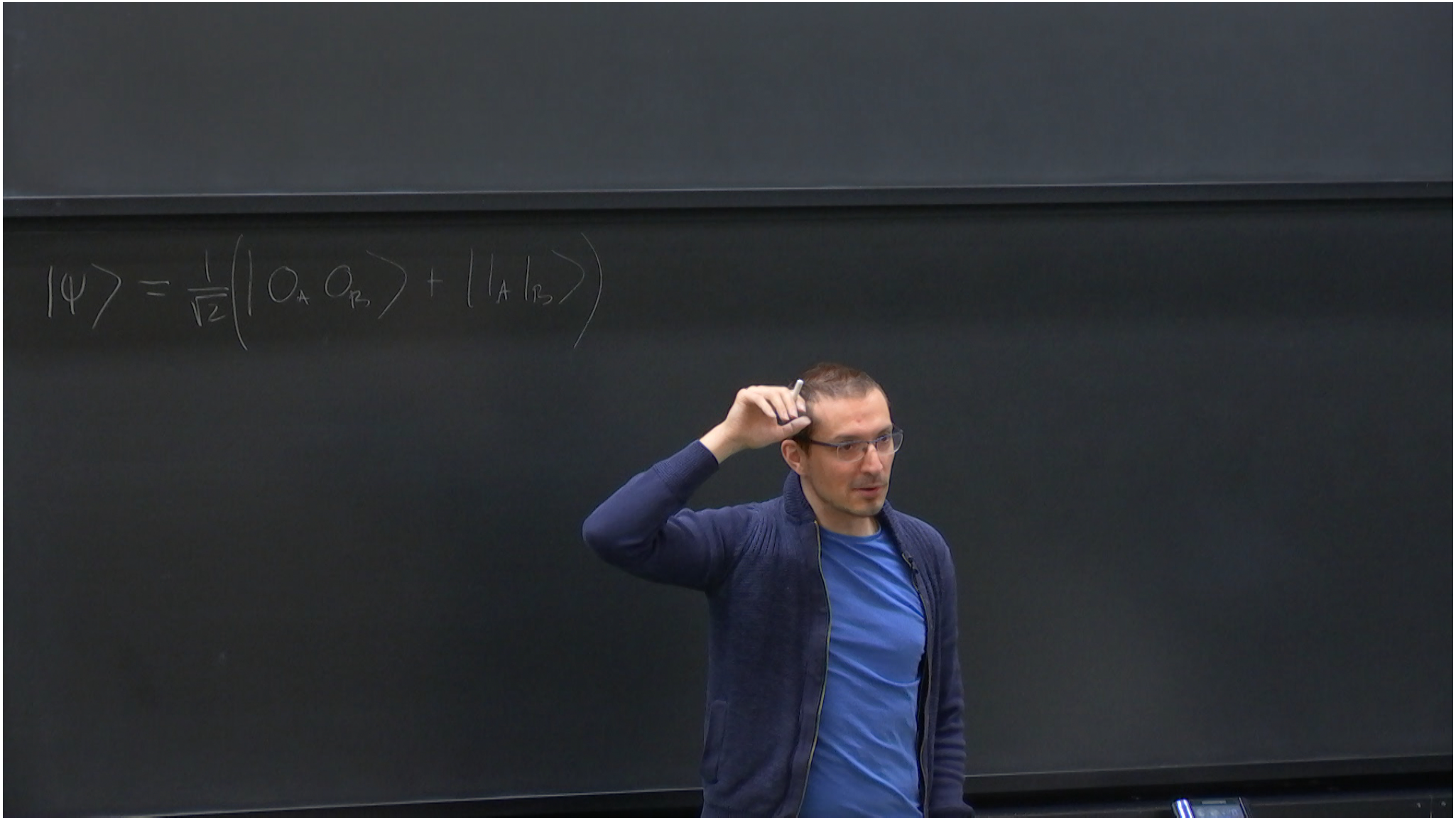
$$\hat{\rho}_A = \mathrm{tr}_B(\hat{\rho}_{AB}) = \sum_i \langle b_i | \hat{\rho}_{AB} | b_i \rangle \equiv \sum_i (\mathbb{1}_A \otimes \langle b_i |) \hat{\rho}_{AB} (\mathbb{1}_A \otimes |b_i\rangle)$$

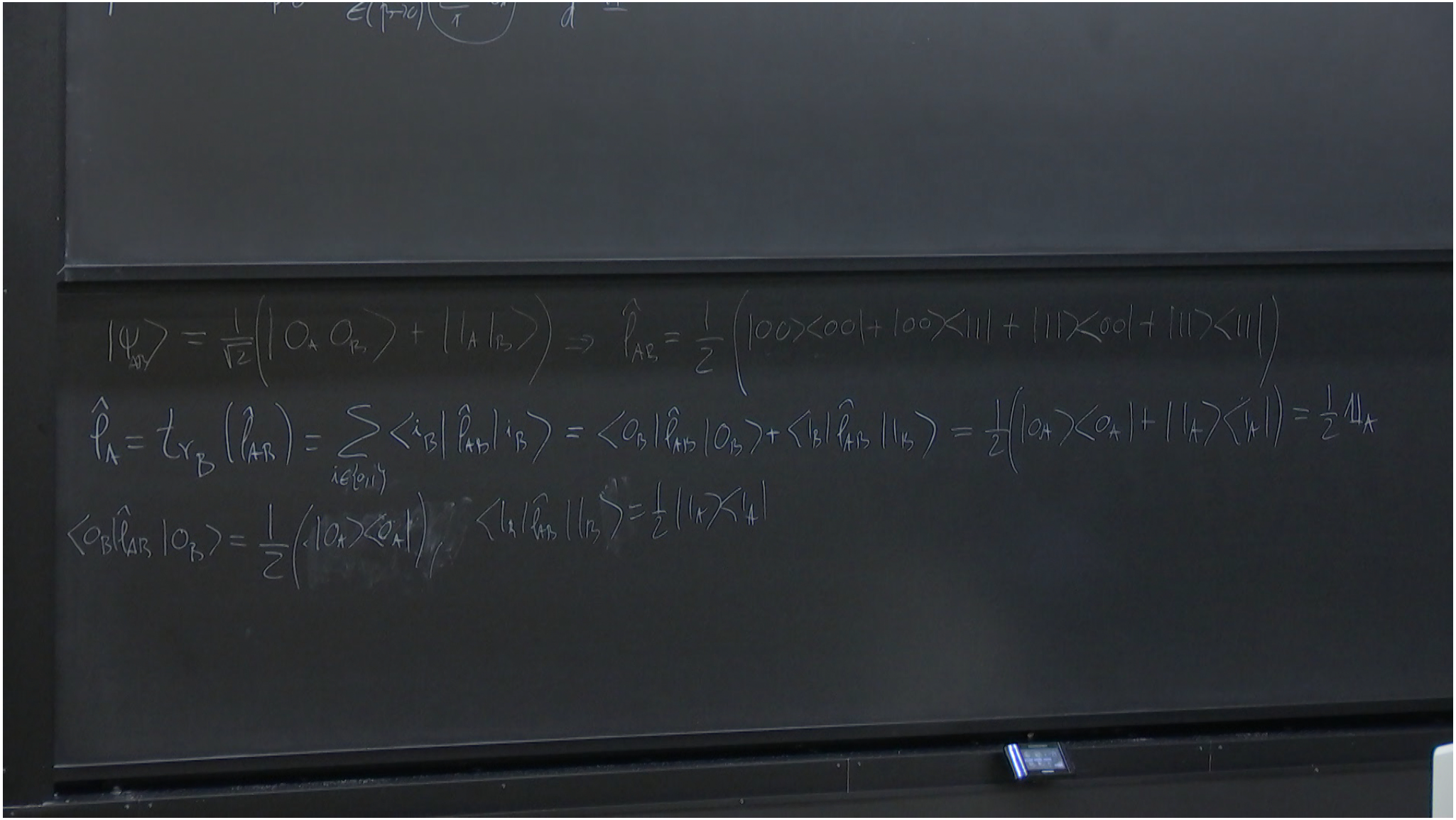
$$\Rightarrow \quad -\log\hat{\rho} - \mathbb{1} - \beta\hat{H} - \mu\mathbb{1} = 0 \Rightarrow \hat{\rho} = \exp\left[-\beta\hat{H} - (\mu+1)\mathbb{1}\right] = e^{-(\mu+1)\mathbb{1}} e^{-\beta\hat{H}}$$

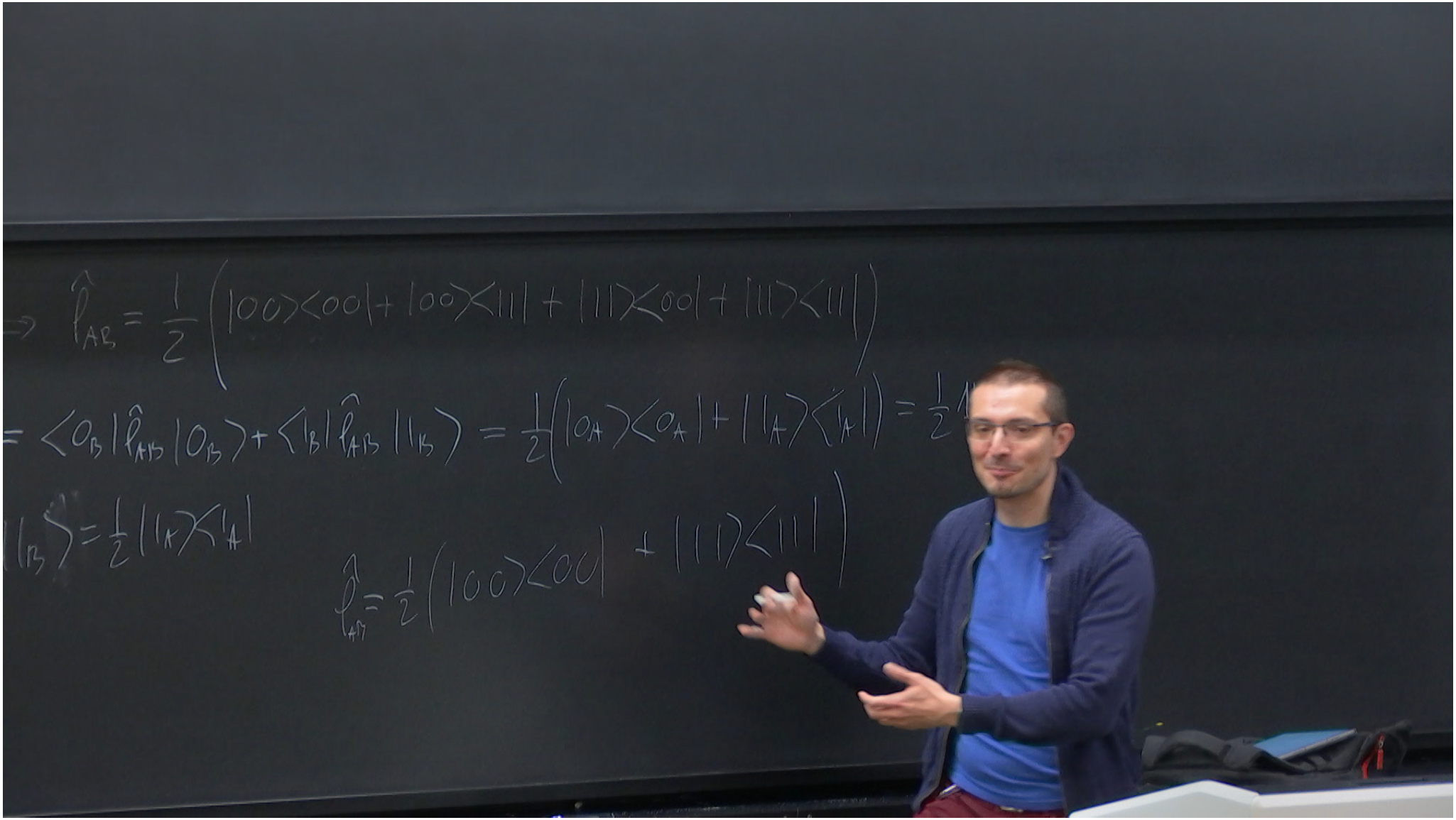
AB, the Hilbert space associated with A (or B) is $\mathcal{H}_A$ (or $\mathcal{H}_B$). the joint $\{|a_i\rangle\}$ and $\{|b_i\rangle\}$ are orthonormal bases of $\mathcal{H}_A$ and $\mathcal{H}_B$ then an orthonormal basis of $\mathcal{H}_{AB}$

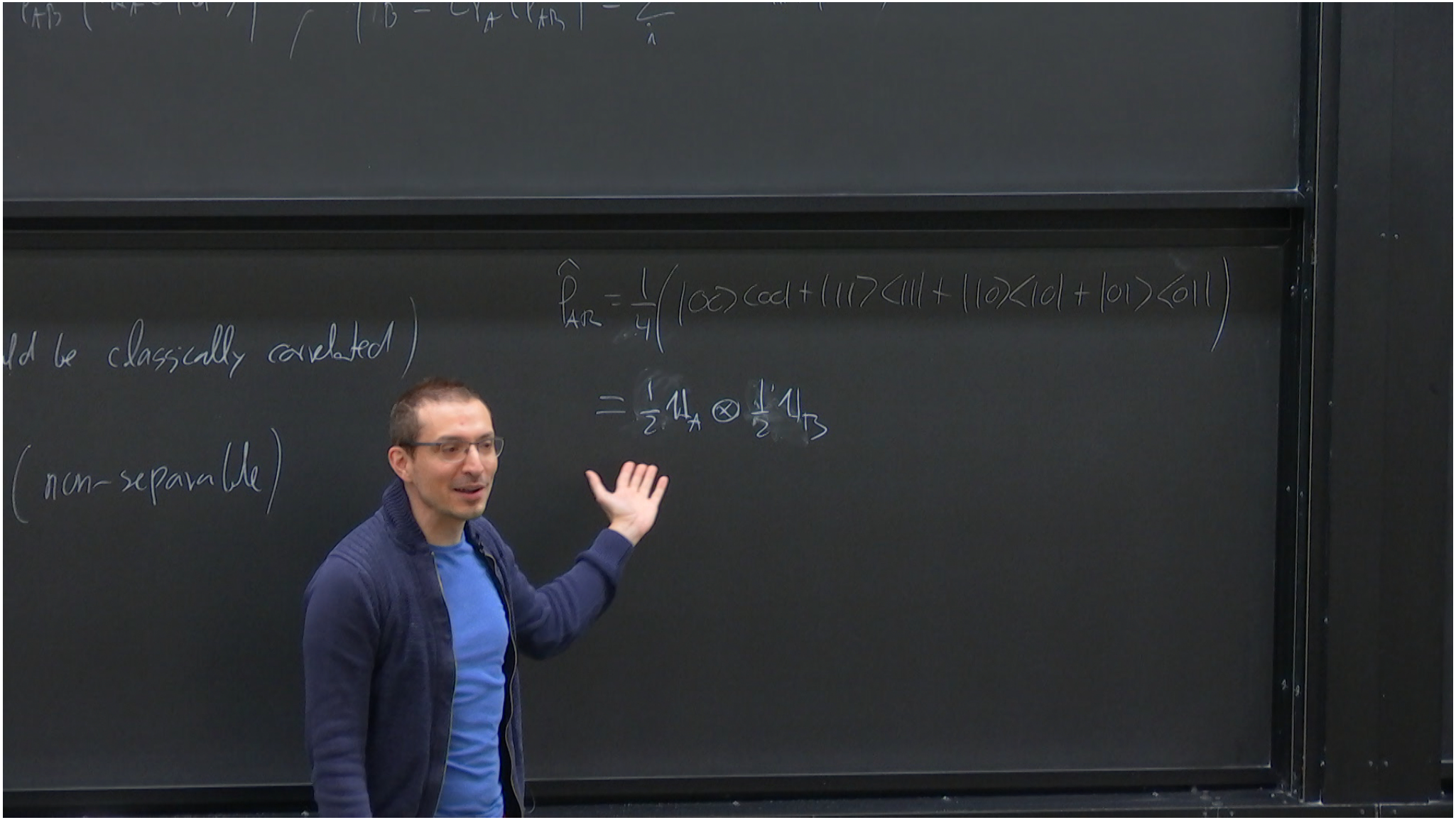
state $\hat{\rho}_{AB}$

$$(\mathbb{1}_A \otimes \langle b_i|)\,\hat{\rho}_{AB}\,(\mathbb{1}_A \otimes |b_i\rangle), \qquad \hat{\rho}_B = tr_A(\hat{\rho}_{AB}) = \sum_i \langle a_i|\hat{\rho}_{AB}|a_i\rangle$$


$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0_A 0_B\rangle + |1_A 1_B\rangle\right)$


$|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}\left(|0_A 0_B\rangle + |1_A 1_B\rangle\right) \Rightarrow \hat{\rho}_{AB} = \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$
$\hat{\rho}_A = tr_B(\hat{\rho}_{AB}) = \sum_{i\in\{0,1\}} \langle i_B|\hat{\rho}_{AB}|i_B\rangle = \langle 0_B|\hat{\rho}_{AB}|0_B\rangle + \langle 1_B|\hat{\rho}_{AB}|1_B\rangle = \frac{1}{2}\left(|0_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A|\right) = \frac{1}{2}\mathbb{1}_A$
$\langle 0_B|\hat{\rho}_{AB}|0_B\rangle = \frac{1}{2}\left(|0_A\rangle\langle 0_A|\right)$
$\langle 1_B|\hat{\rho}_{AB}|1_B\rangle = \frac{1}{2}|1_A\rangle\langle 1_A|$


$\Rightarrow \hat{\rho}_{AB} = \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$
$= \langle 0_B|\hat{\rho}_{AB}|0_B\rangle + \langle 1_B|\hat{\rho}_{AB}|1_B\rangle = \frac{1}{2}\left(|0_A\rangle\langle 0_A| + |1_A\rangle\langle 1_A|\right) = \frac{1}{2}$
$|1_B\rangle = \frac{1}{2}|1_A\rangle\langle 1_A|$
$\hat{\rho}_{AB} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$


$\hat{\rho}_{AB} = \frac{1}{4}\left(|00\rangle\langle 00| + |11\rangle\langle 11| + |10\rangle\langle 10| + |01\rangle\langle 01|\right)$
$= \frac{1}{2}\mathbb{1}_A \otimes \frac{1}{2}\mathbb{1}_B$
ld be classically correlated)
(non-separable)