Title: Quantum Information Lecture - 230303

Speakers: Eduardo Martin-Martinez

Collection: Quantum Information (2022/2023)

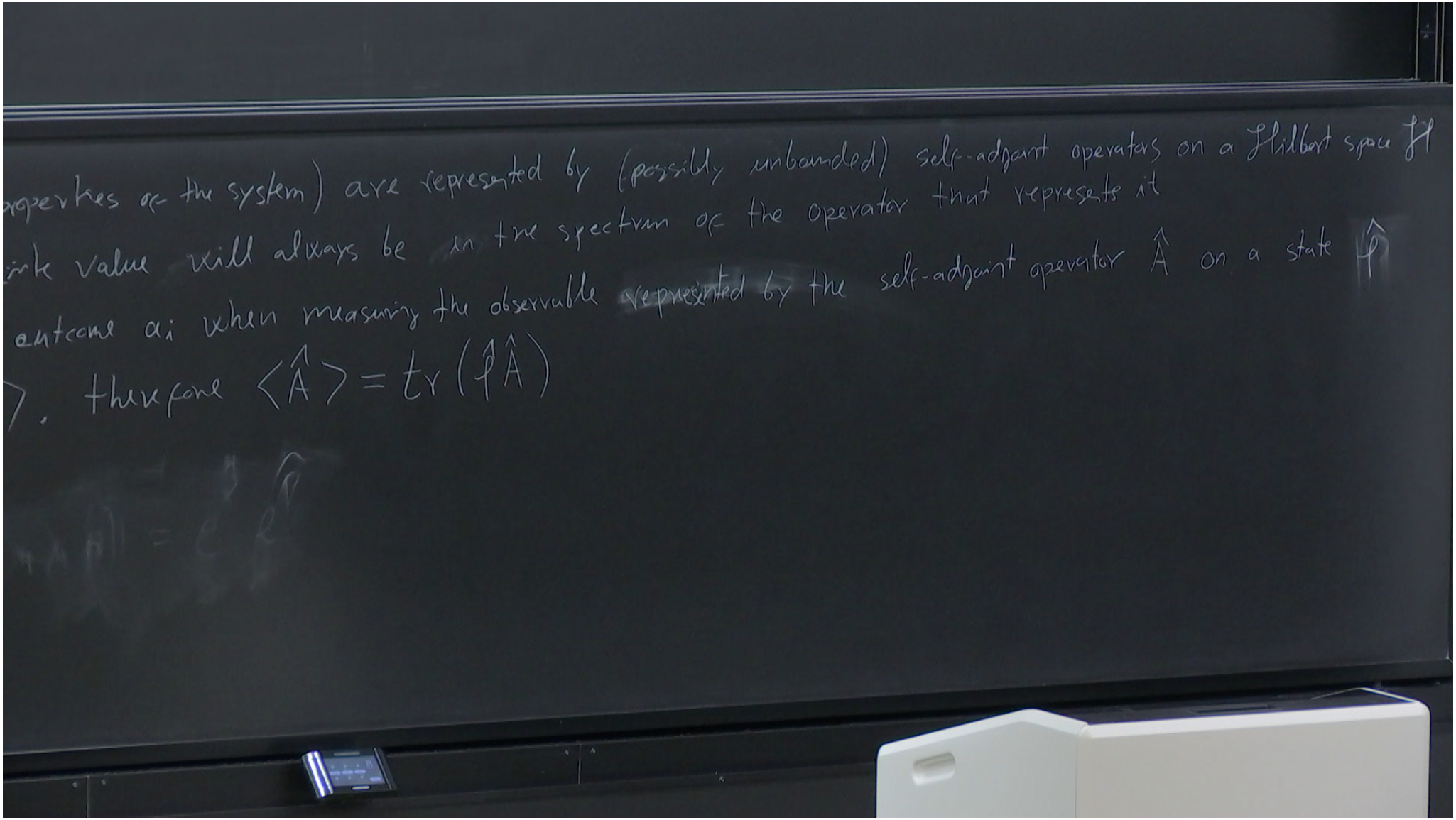
Date: March 03, 2023 - 9:00 AM

URL: https://pirsa.org/23030002

Postulates of QM

Postulate 2: Physical observables (in principle measurable properties of the system) ar
the result of a measurement of an observable that produces a definite value will always
Postulate 3 (Born's rule): the probability of measuring an outcome a_i when measu
is given by $\mathrm{tr}(|a_i\rangle\langle a_i|\hat{\rho})$ where $\hat{A}|a_i\rangle = a_i|a_i\rangle$. therefore $\langle\hat{A}\rangle =$

$$H = \frac{p^2}{2m}$$


roperties of the system) are represented by (possibly unbounded) self-adjoint operators on a Hilbert space $\mathcal{H}$
ible value will always be in the spectrum of the operator that represents it
outcome a_i when measuring the observable represented by the self-adjoint operator $\hat{A}$ on a state $\hat{\rho}$
7. therefore $\langle \hat{A} \rangle = tr(\hat{\rho}\hat{A})$

Postulates of QM

Postulate 2: Physical observables (in principle measurable properties of the system) the result of a measurement of an observable that produces a definite value will alw

Postulate 3 (Born's rule): the probability of measuring an outcome a_i when m is given by $P_{(A=a_i)} = \mathrm{tr}(|a_i\rangle\langle a_i|\hat{\rho})$ where $\hat{A}|a_i\rangle = a_i|a_i\rangle$. therefore $\langle\hat{A}$

Postulate 1: to each physical state there corresponds a unique st

1. $\langle\psi|\hat{\rho}|\psi\rangle \geq 0 \quad |\psi\rangle \in \mathcal{H}$ (all eigenvalues are positi

2. $\mathrm{Tr}(\hat{\rho}) \left(= \sum_i \langle e_i|\hat{\rho}|e_i\rangle\right) = 1$ (where $\{|e_i\rangle\}$ is any orth

3. It is self adjoint $\hat{\rho} = \hat{\rho}^\dagger$

ands a unique state operator (density operator) $\hat{\rho}$ which has to satisfy
eigenvalues are positive semidefinite)

$\{|e_i\rangle\}$ is any orthonormal basis of $\mathcal{H}$)

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Postulate 1: to each physical state there corresponds a unique state

1. $\langle\psi|\hat{\rho}|\psi\rangle \geq 0 \quad |\psi\rangle \in \mathcal{H}$ (all eigenvalues are positive

2. $\mathrm{tr}(\hat{\rho}) \left(= \sum_i \langle e_i|\hat{\rho}|e_i\rangle\right) = 1$ (where $\{|e_i\rangle\}$ is any orthon

3. It is self adjoint $\hat{\rho} = \hat{\rho}^\dagger$

$$\langle\hat{O}\rangle = \mathrm{tr}(\hat{\rho}\hat{O}) = \sum_j \langle e_j|\hat{\rho}\hat{O}|e_j\rangle = \sum_i \sum_j p_i \langle e_j|\phi_i\rangle\langle\phi_i|\hat{O}|e_j\rangle =$$

nique state operator (density operator) $\hat{\rho}$ which has to satisfy
are positive semidefinite)

is any orthonormal basis of $\mathcal{H}$)

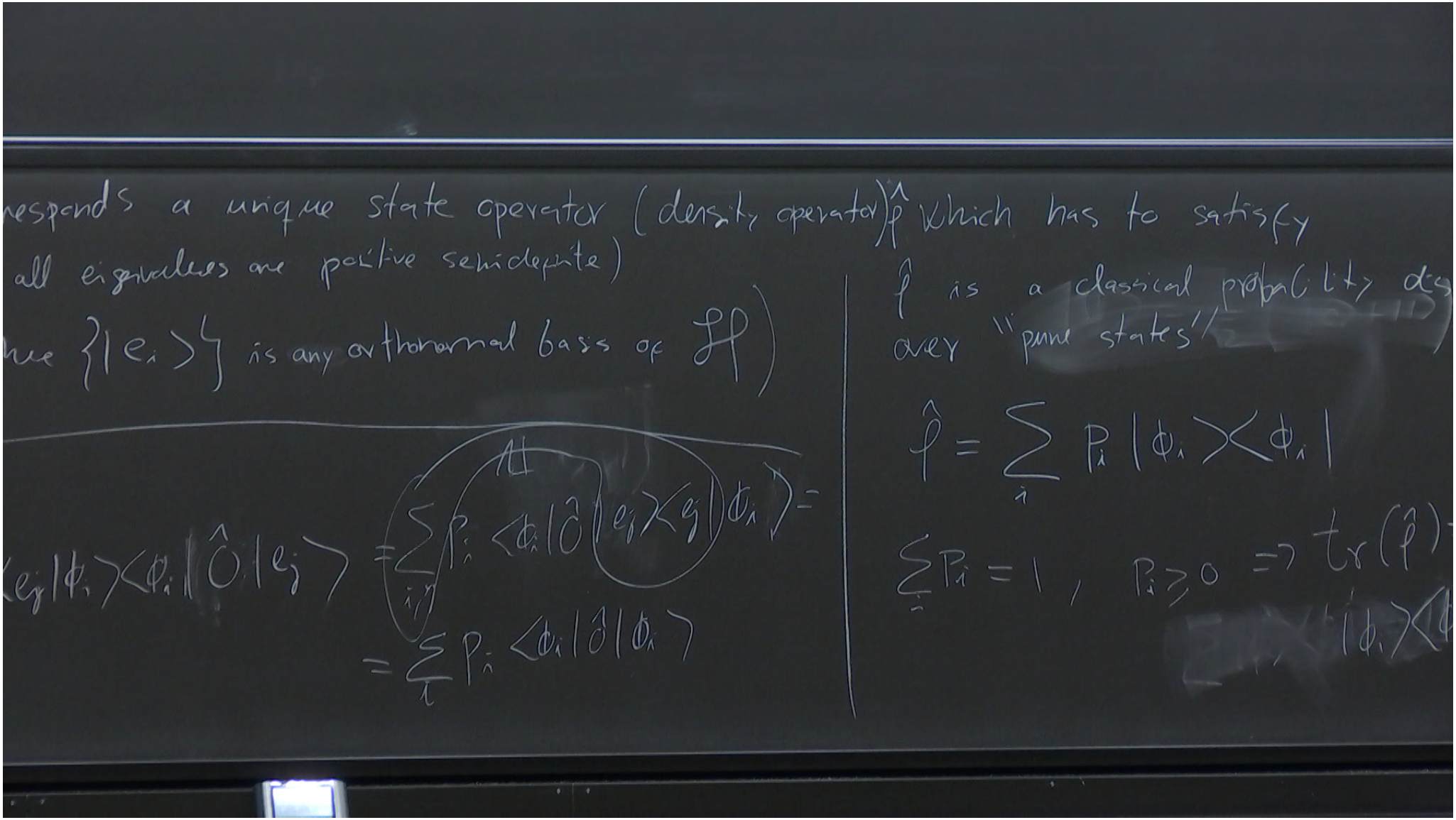
$|e_j\rangle =$

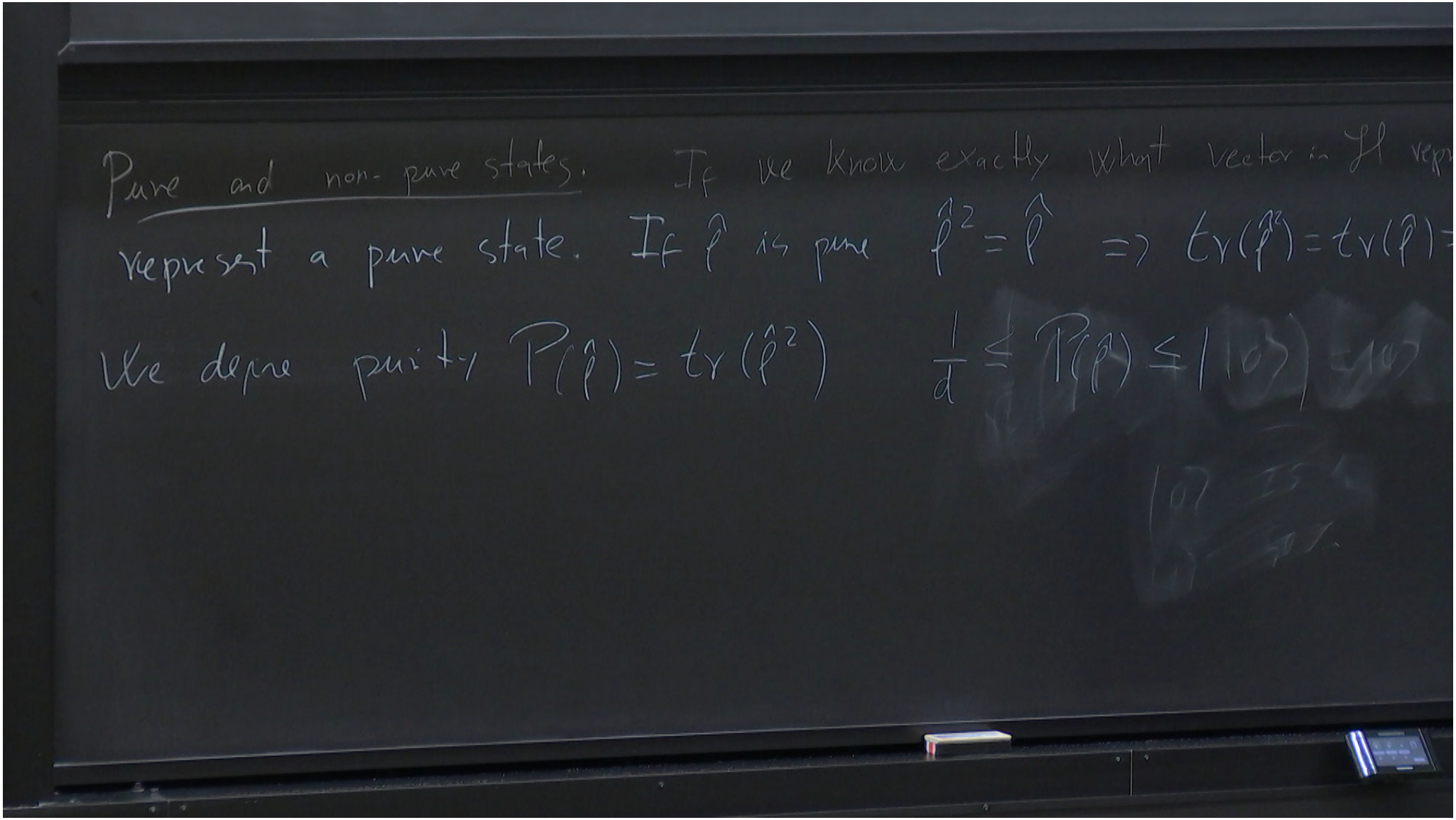
$\hat{\rho}$ is a classical probability distribution over "pure states"

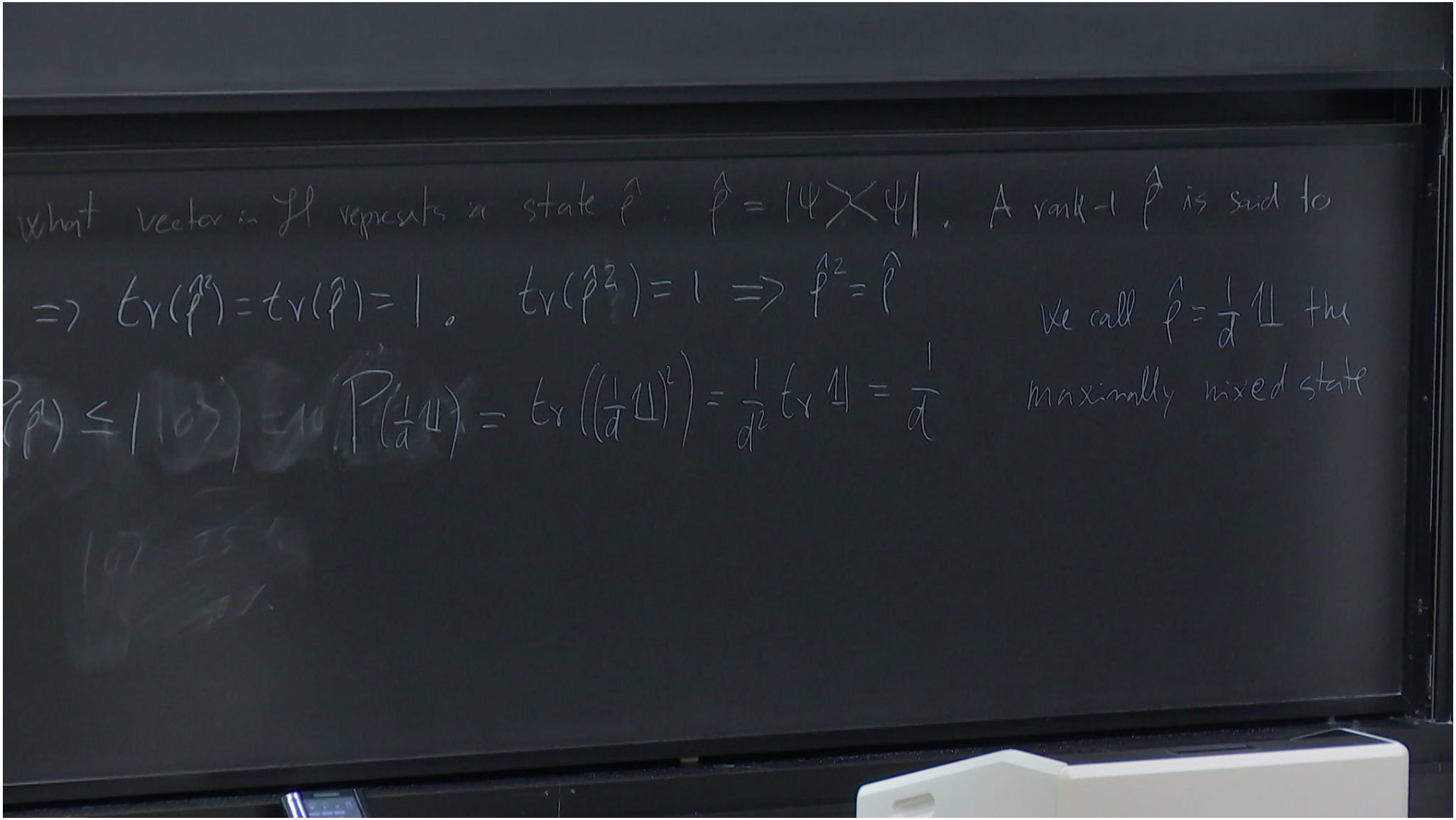
$$\hat{\rho} = \sum_i p_i |\phi_i\rangle\langle\phi_i|$$

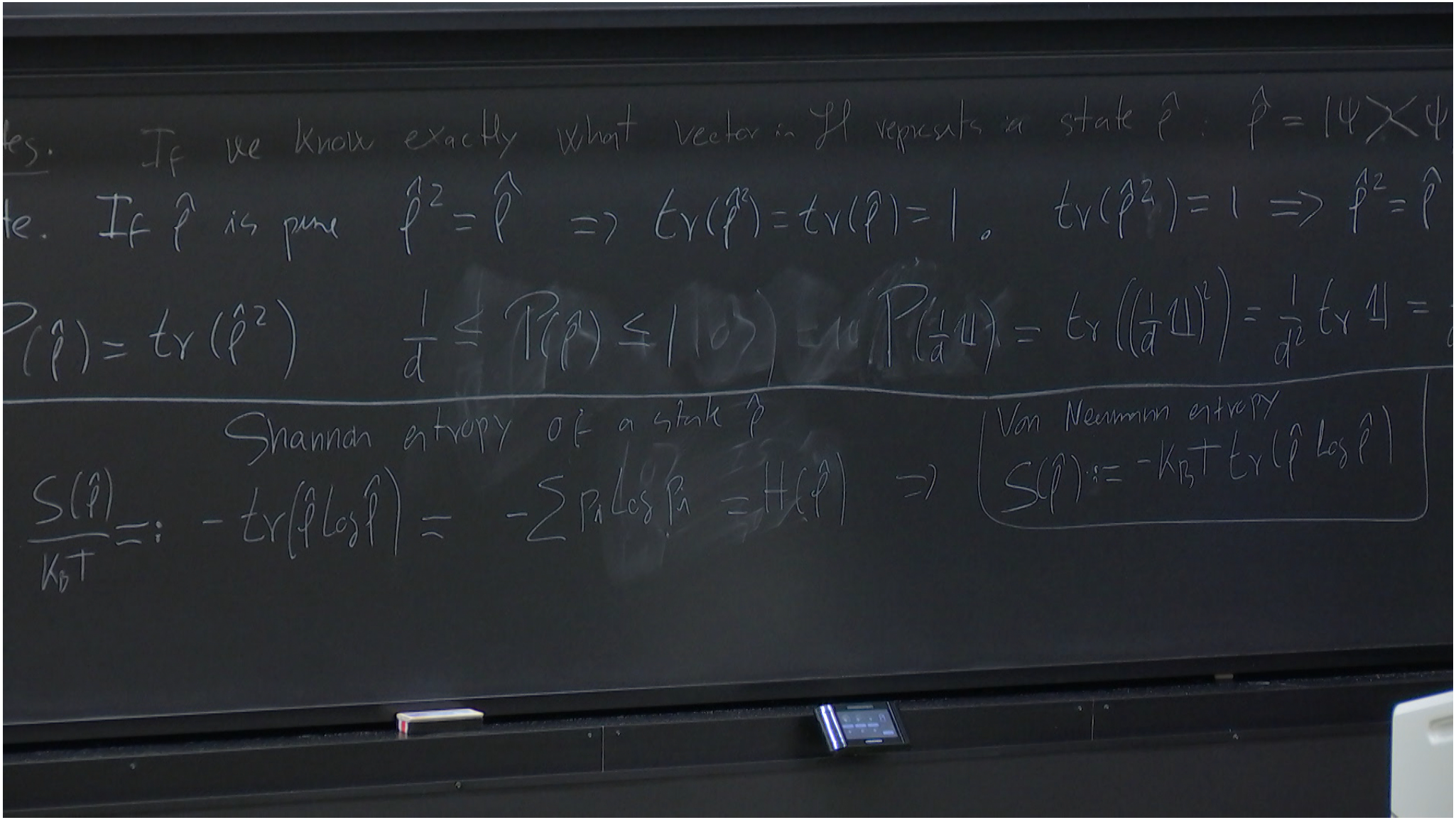
$$\sum_i p_i = 1, \quad p_i \geq 0 \Rightarrow \mathrm{tr}(\hat{\rho}) = 1$$

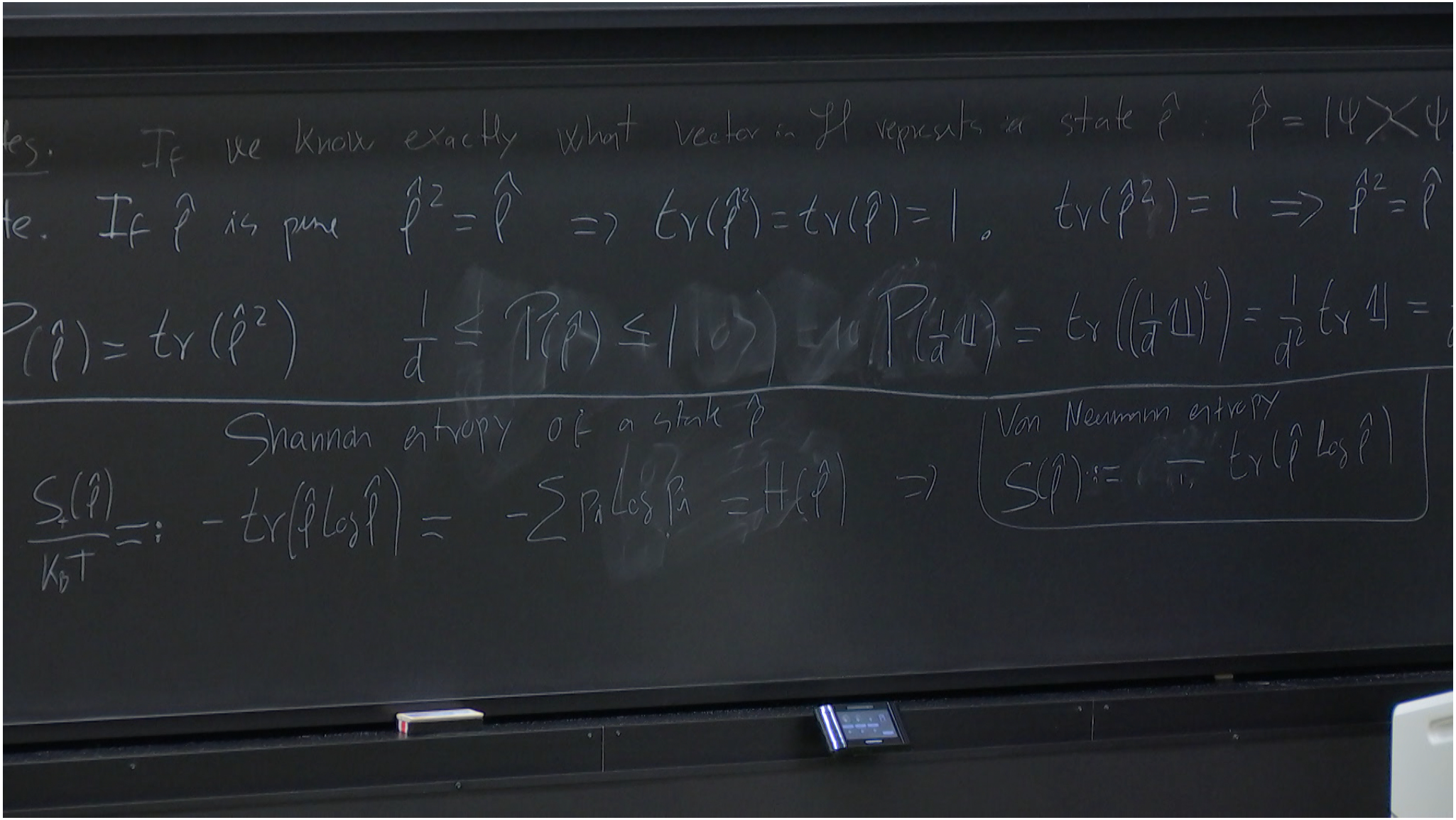
$|\phi_i\rangle\langle\phi_i|$


nesponds a unique state operator (density operator) $\hat{\rho}$ which has to satisfy
all eigenvalues are positive semidefinite)
where $\{|e_i\rangle\}$ is any orthonormal basis of $\mathcal{H}$)
$\hat{\rho}$ is a classical probability dis
over "pure states"
$\hat{\rho} = \sum_i p_i |\phi_i\rangle\langle\phi_i|$
$\sum_i p_i = 1$, $p_i \geq 0$ => $tr(\hat{\rho})$
$\langle e_j|\phi_i\rangle\langle\phi_i|\hat{O}|e_j\rangle = \sum_{ij} p_i \langle\phi_i|\hat{O}|e_j\rangle\langle e_j|\phi_i\rangle =$
$= \sum_i p_i \langle\phi_i|\hat{O}|\phi_i\rangle$


Pure and non-pure states. If we know exactly what vector in $\mathcal{H}$ repr
represent a pure state. If $\hat{\rho}$ is pure $\hat{\rho}^2 = \hat{\rho}$ $\Rightarrow$ $tr(\hat{\rho}^2) = tr(\hat{\rho}) =$
We define purity $P(\hat{\rho}) = tr(\hat{\rho}^2)$ $\frac{1}{d} \leq P(\hat{\rho}) \leq 1$


What vector in $\mathcal{H}$ represents a state $\hat{\rho}$: $\hat{\rho} = |\psi\rangle\langle\psi|$. A rank-1 $\hat{\rho}$ is said to
$\Rightarrow tr(\hat{\rho}^2) = tr(\hat{\rho}) = 1$. $tr(\hat{\rho}^2) = 1 \Rightarrow \hat{\rho}^2 = \hat{\rho}$
We call $\hat{\rho} = \frac{1}{d}\mathbb{1}$ the
maximally mixed state
$P(\hat{\rho}) \leq |\langle\psi|$
$P(\frac{1}{d}\mathbb{1}) = tr\left(\left(\frac{1}{d}\mathbb{1}\right)^2\right) = \frac{1}{d^2} tr\,\mathbb{1} = \frac{1}{d}$


es. If we know exactly what vector in $\mathcal{H}$ represents a state $\hat{\rho}$: $\hat{\rho} = |\psi\rangle\langle\psi|$
te. If $\hat{\rho}$ is pure $\hat{\rho}^2 = \hat{\rho} \Rightarrow tr(\hat{\rho}^2) = tr(\hat{\rho}) = 1$. $tr(\hat{\rho}^2) = 1 \Rightarrow \hat{\rho}^2 = \hat{\rho}$
$P(\hat{\rho}) = tr(\hat{\rho}^2)$ $\frac{1}{d} \leq P(\hat{\rho}) \leq 1$ $|\psi\rangle$ $P(\frac{1}{d}\mathbb{1}) = tr\left(\left(\frac{1}{d}\mathbb{1}\right)^2\right) = \frac{1}{d^2} tr\,\mathbb{1} =$
Shannon entropy of a state $\hat{\rho}$
$\frac{S(\hat{\rho})}{K_B T} =: -tr(\hat{\rho}\,Log\,\hat{\rho}) = -\sum p_i Log\, p_i = H(\hat{\rho})$ $\Rightarrow$
Von Neumann entropy
$S(\hat{\rho}) := -K_B T\, tr(\hat{\rho}\,Log\,\hat{\rho})$


es. If we know exactly what vector in $\mathcal{H}$ represents a state $\hat{\rho}$: $\hat{\rho} = |\psi\rangle\langle\psi|$
te. If $\hat{\rho}$ is pure $\hat{\rho}^2 = \hat{\rho} \Rightarrow \mathrm{tr}(\hat{\rho}^2) = \mathrm{tr}(\hat{\rho}) = 1$. $\mathrm{tr}(\hat{\rho}^2) = 1 \Rightarrow \hat{\rho}^2 = \hat{\rho}$
$P(\hat{\rho}) = \mathrm{tr}(\hat{\rho}^2)$ $\frac{1}{d} \leq P(\hat{\rho}) \leq 1$ $|\psi\rangle$ $P(\frac{1}{d}\mathbb{1}) = \mathrm{tr}\left(\left(\frac{1}{d}\mathbb{1}\right)^2\right) = \frac{1}{d^2}\mathrm{tr}\,\mathbb{1} =$
Shannon entropy of a state $\hat{\rho}$
$\frac{S_s(\hat{\rho})}{k_B T} =: -\mathrm{tr}(\hat{\rho}\log\hat{\rho}) = -\sum p_i \log p_i = H(\hat{\rho})$ $\Rightarrow$
Von Neumann entropy
$S(\hat{\rho}) := \frac{1}{\ln 2}\mathrm{tr}(\hat{\rho}\log\hat{\rho})$

$\hat{\rho} = |\psi\rangle\langle\psi|$. A rank-1 $\hat{\rho}$ is said to

$\mathrm{tr}(\hat{\rho}) = 1$. $\mathrm{tr}(\hat{\rho}^2) = 1 \Rightarrow \hat{\rho}^2 = \hat{\rho}$

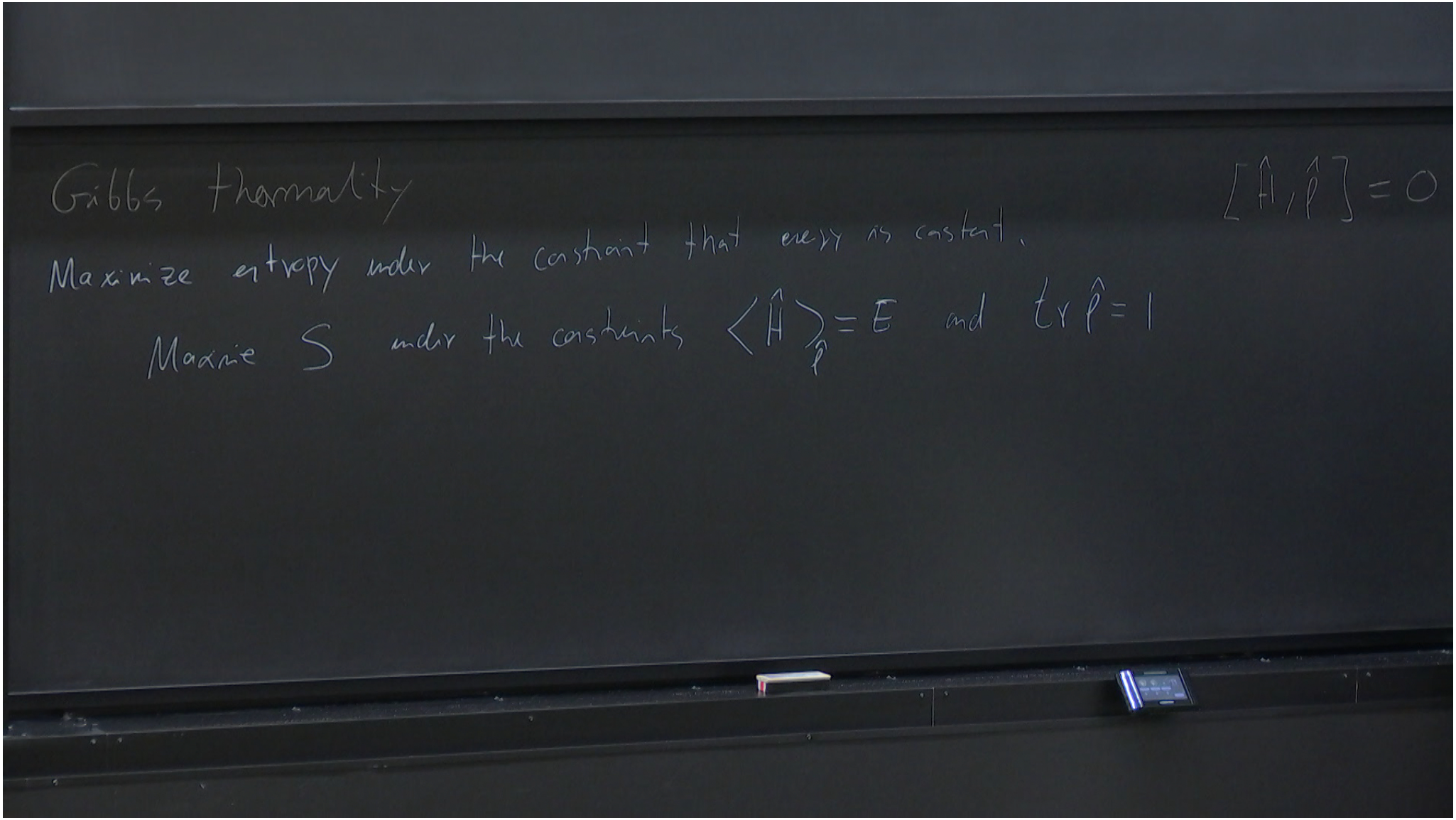
We call $\hat{\rho} = \frac{1}{d}\mathbb{1}$ the maximally mixed state

$P\left(\frac{1}{d}\mathbb{1}\right) = \mathrm{tr}\left(\left(\frac{1}{d}\mathbb{1}\right)^2\right) = \frac{1}{d^2}\mathrm{tr}\,\mathbb{1} = \frac{1}{d}$

$\Rightarrow$ **Von Neumann entropy**

$S(\hat{\rho}) := -\mathrm{tr}(\hat{\rho}\log\hat{\rho})$

$0 \leq S(\hat{\rho}) \leq \log(d)$


Gibbs thermality
$[\hat{H}, \hat{\rho}] = 0$
Maximize entropy under the constraint that energy is constant.
Maxmie S under the constraints $\langle \hat{H} \rangle_{\hat{\rho}} = E$ and $tr\,\hat{\rho} = 1$

Gibbs thermality

$[\hat{H}, \hat{\rho}] = 0$

Maximize entropy under the constraint that energy is constant.

Maxmie S under the constraints $\langle \hat{H} \rangle_{\hat{\rho}} = E$ and $\mathrm{tr}\,\hat{\rho} = 1$

$$\frac{\partial}{\partial \hat{\rho}} \left[S - \beta \left(\mathrm{tr}(\hat{\rho}\hat{H}) - E \right) - \mu \left(\mathrm{tr}\,\hat{\rho} - 1 \right) \right] = 0$$

β and μ are Lagrange multipliers