Title: Gravitational Physics Q&A

Speakers: Ruth Gregory

Collection: Gravitational Physics (2022/2023)

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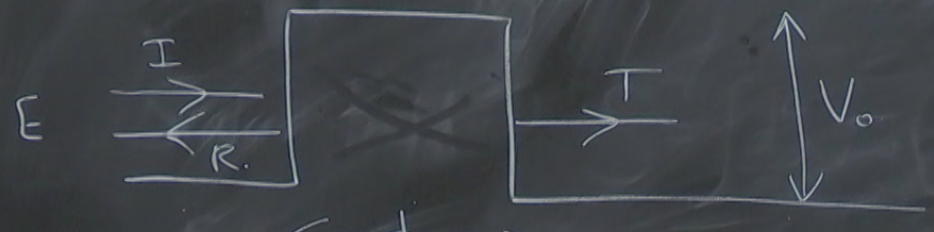
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LECTURE 14: Gravitational Instantons

Coleman "Fate of the false vacuum"
PRD 15 2929 (77)

+ deLuccia PRD 21 3305 (80)

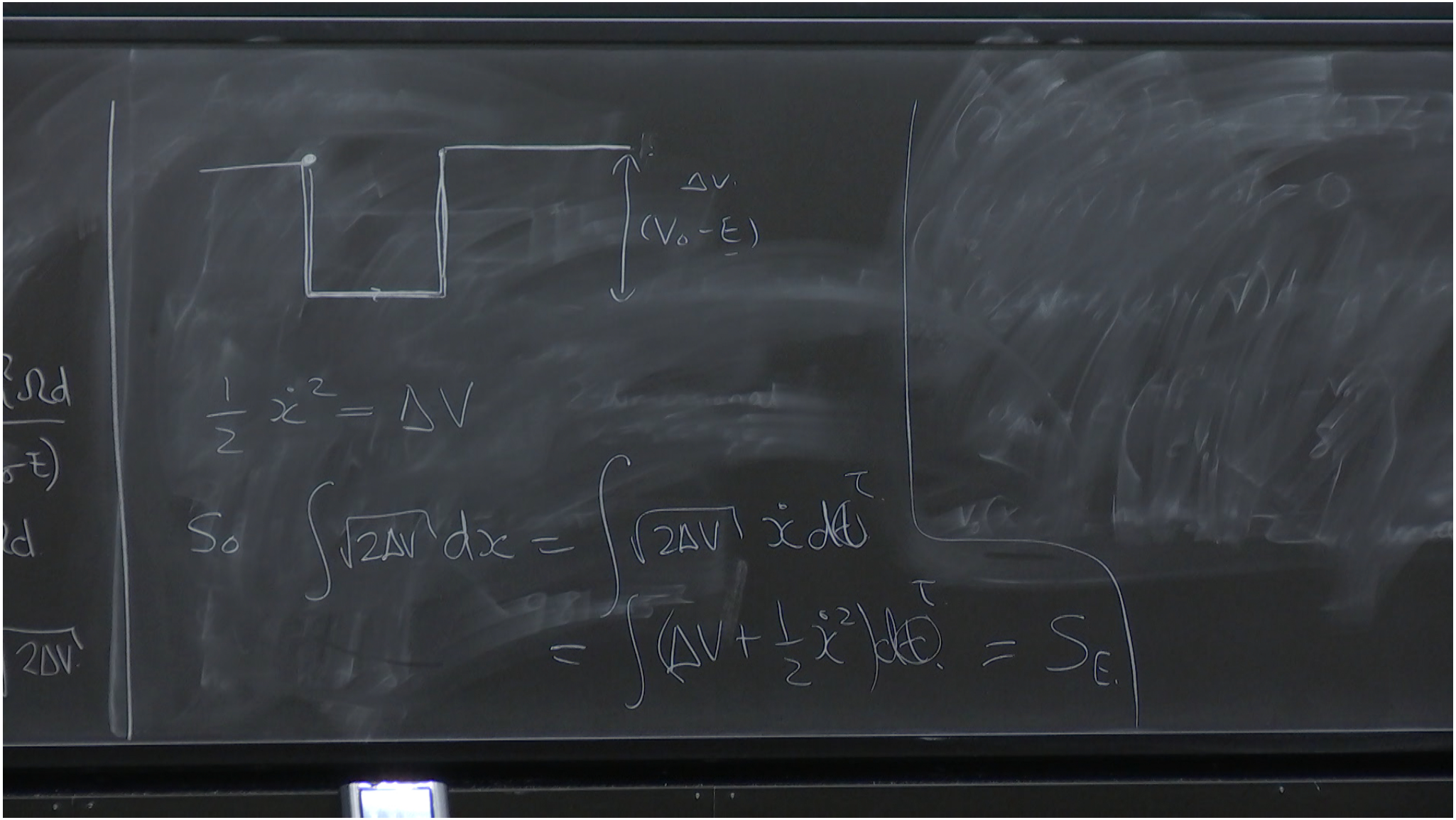
Recall Schrödinger tunneling:



$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}}$$

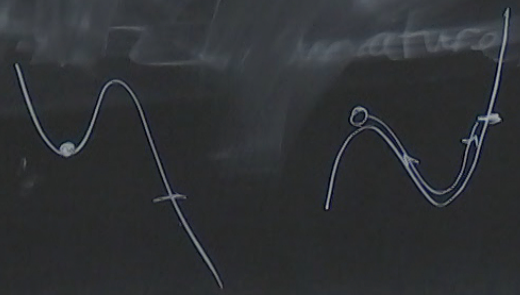
where $\Omega^2 = \frac{2m}{\hbar^2}(V_0 - E) \sim 2\Delta V$ $\quad \psi \sim e^{-\Omega d}$

$$\Omega d = \int_0^d dx \sqrt{2\Delta V}$$

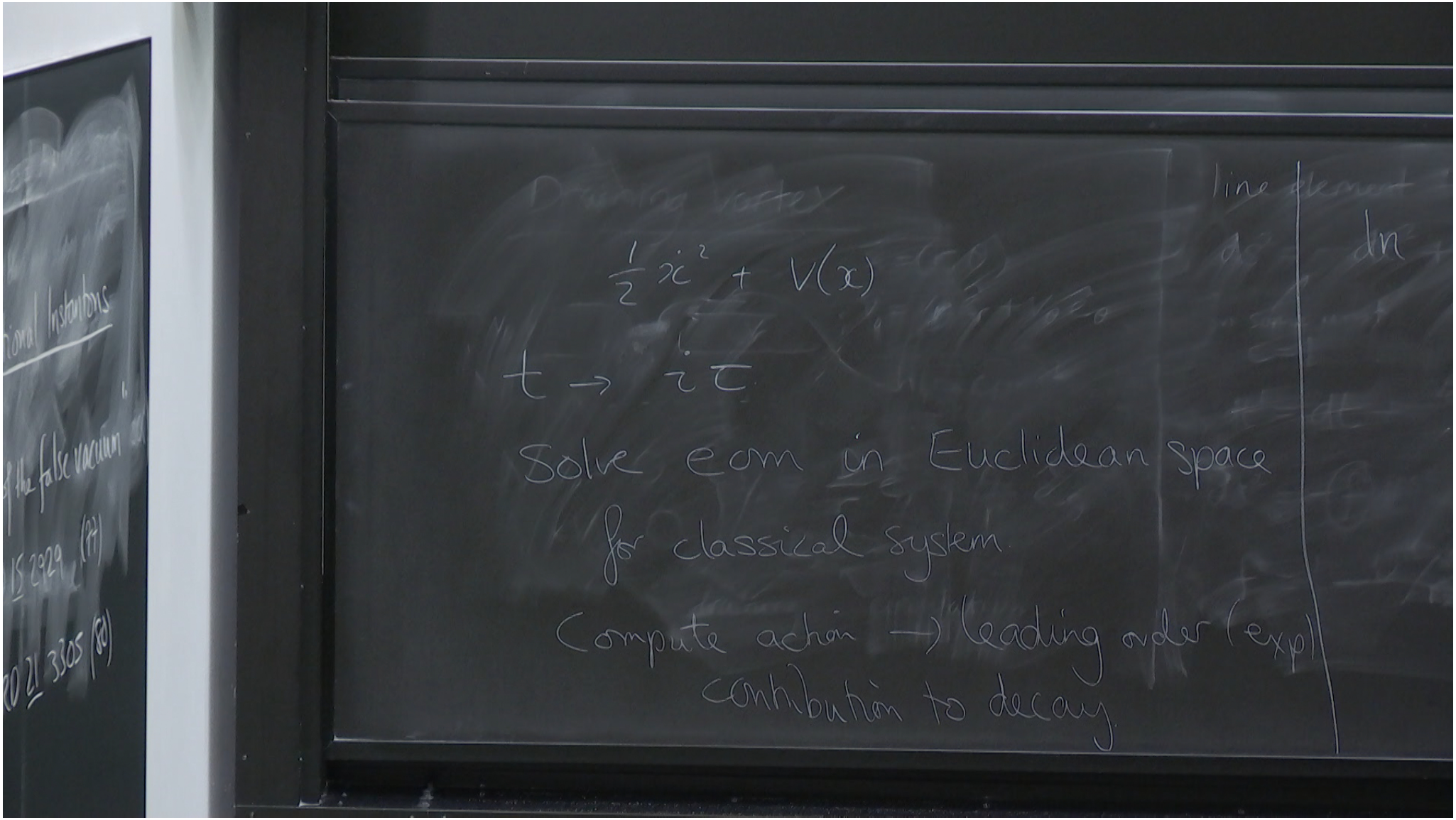

ΔV
(V₀ - E)
½ ẋ² = ΔV
So ∫√(2ΔV) dx = ∫√(2ΔV) ẋ dt
= ∫(ΔV + ½ ẋ²) dt = S_E

In general $P \sim e^{-S_E}$

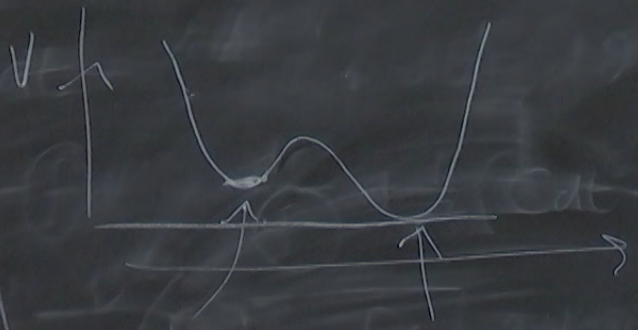
where S_E is the action of a Euclidean trajectory from initial (metastable) pt, to exit.



"Bounce"


$\frac{1}{2}\dot{x}^2 + V(x)$
$t \to i\tau$
Solve eom in Euclidean space
for classical system
Compute action → leading order (exp)
contribution to decay
line element
ds
dn

In Field Theory - Higgs determines our vacuum & its energy.



$\lambda(\phi^2-\eta^2)^2 - \epsilon\frac{(\phi-\eta)}{2\eta}$

$\phi=\eta$ – T.V. $V=0$.

$\phi=-\eta$ – F.V. $V=\epsilon$.

$\lambda(h)(h^2-\eta^2)^2$

ermines

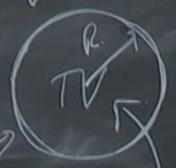
$\epsilon(\phi-\eta)$ / 2η

T.V. $V=0$.

F.V $V=\epsilon$

$R^2)^2$

F.V



WALL
σ per unit area

Volume
NOT
at ξ

TOO SMALL → Recollapse (surface area ; vol ratio)

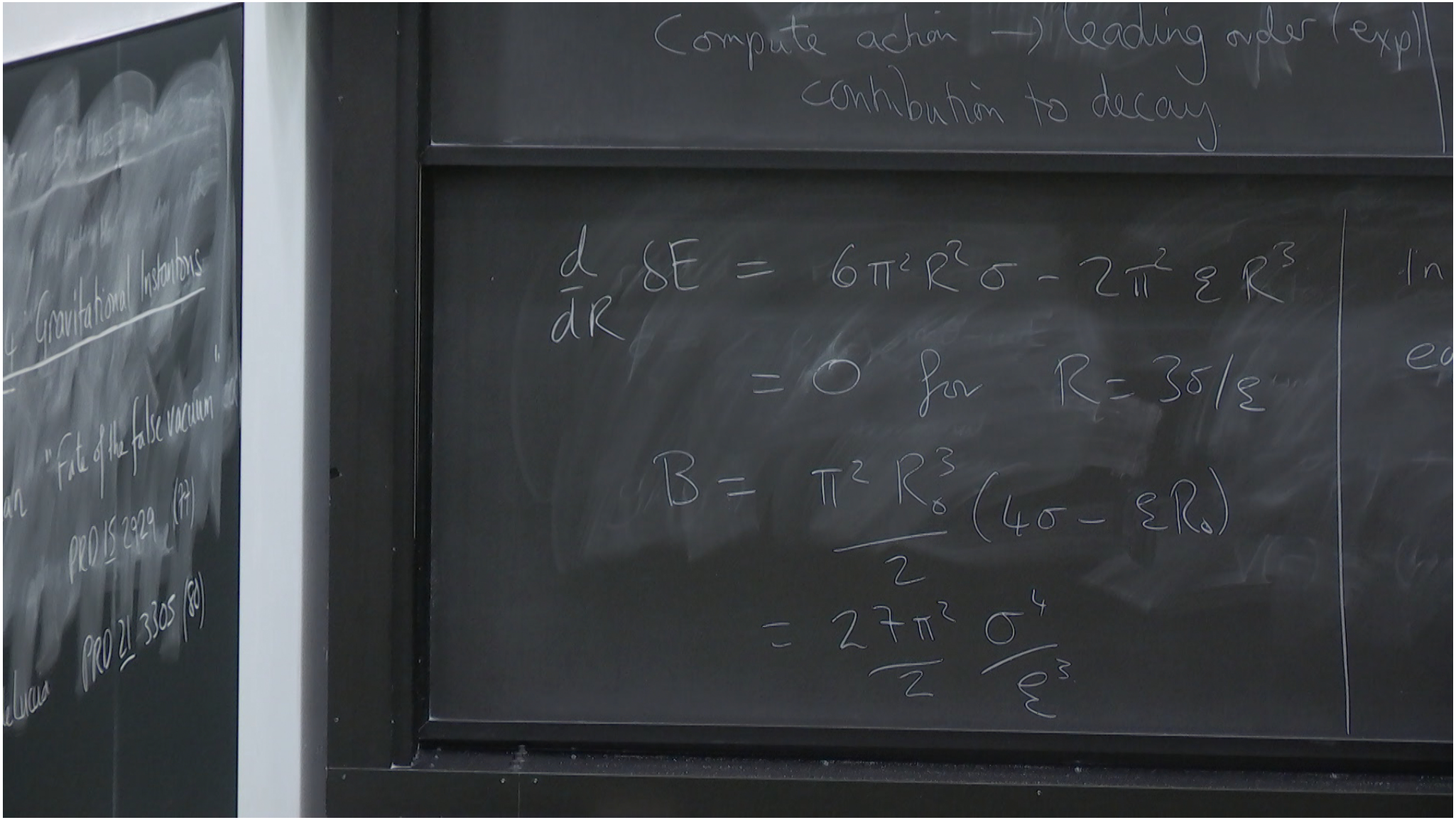
TOO BIG → Hard to form

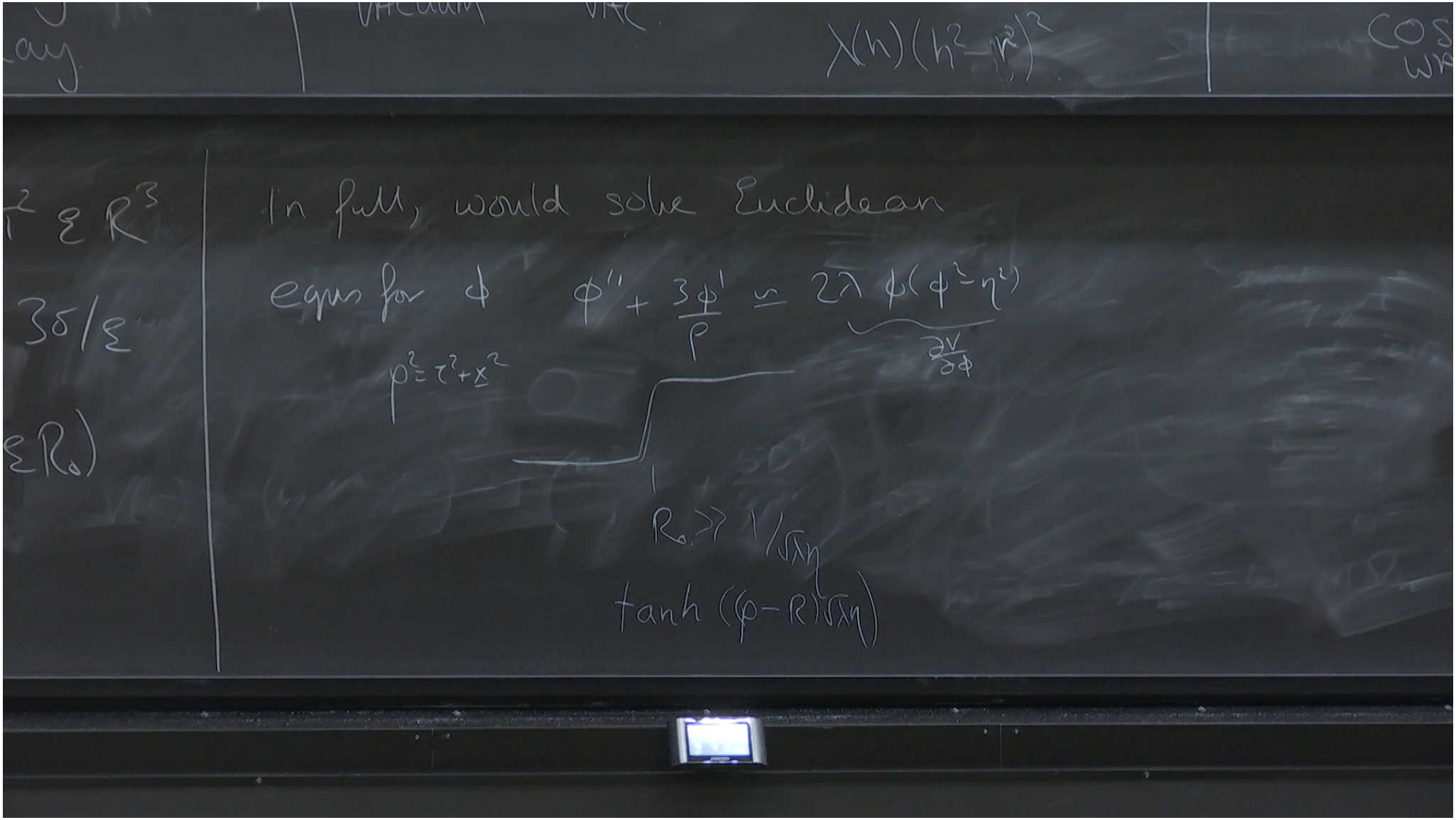
In Euclidean space: τ; $\underline{x}$.

$$2\pi^2 R^3 \sigma \cdot 1 - \frac{\Pi^2}{2} \xi R^4$$

COST WALL

ENERGY GAIN


Compute action → leading order (exp)
contribution to decay
$\frac{d}{dR}\delta E = 6\pi^2 R^2 \sigma - 2\pi^2 \varepsilon R^3$
$= 0$ for $R = 3\sigma/\varepsilon$
$B = \frac{\pi^2 R_0^3}{2}(4\sigma - \varepsilon R_0)$
$= \frac{27\pi^2}{2}\frac{\sigma^4}{\varepsilon^3}$
Gravitational Instantons
"Fate of the false vacuum
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$\lambda(h)(h^2-v^2)^2$
$\in R^3$
$3\sigma/\varepsilon$
$\varepsilon R_0)$
In full, would solve Euclidean
eqns for ϕ
$\phi'' + \frac{3\phi'}{\rho} = 2\lambda\phi(\phi^2-\eta^2)$
$\frac{\partial V}{\partial \phi}$
$\rho^2 = \tau^2 + \underline{x}^2$
$R_0 \gg 1/\sqrt{\lambda}\eta$
$\tanh((\rho-R)\sqrt{\lambda}\eta)$

$\phi = -\eta - F.V \quad V = \epsilon$

$\lambda(h)(h^2 - v^2)^2$

$2\pi^2 R^3 \sigma \cdot \Gamma - \frac{\pi}{2} \epsilon R^4$

COST WALL

ENERGY GAIN

clidean

$2\lambda \underbrace{\phi(\phi^2 - \eta^2)}_{\frac{\partial V}{\partial \phi}}$

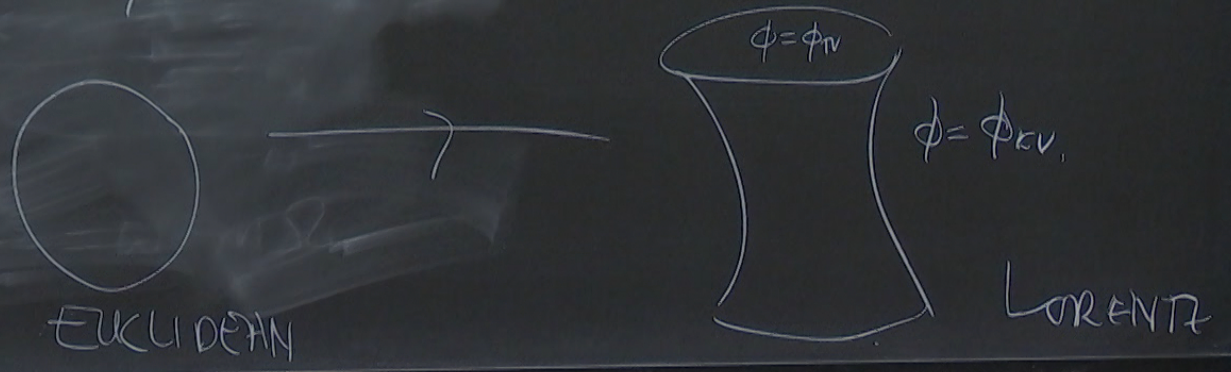
$\sqrt{\lambda}\eta$

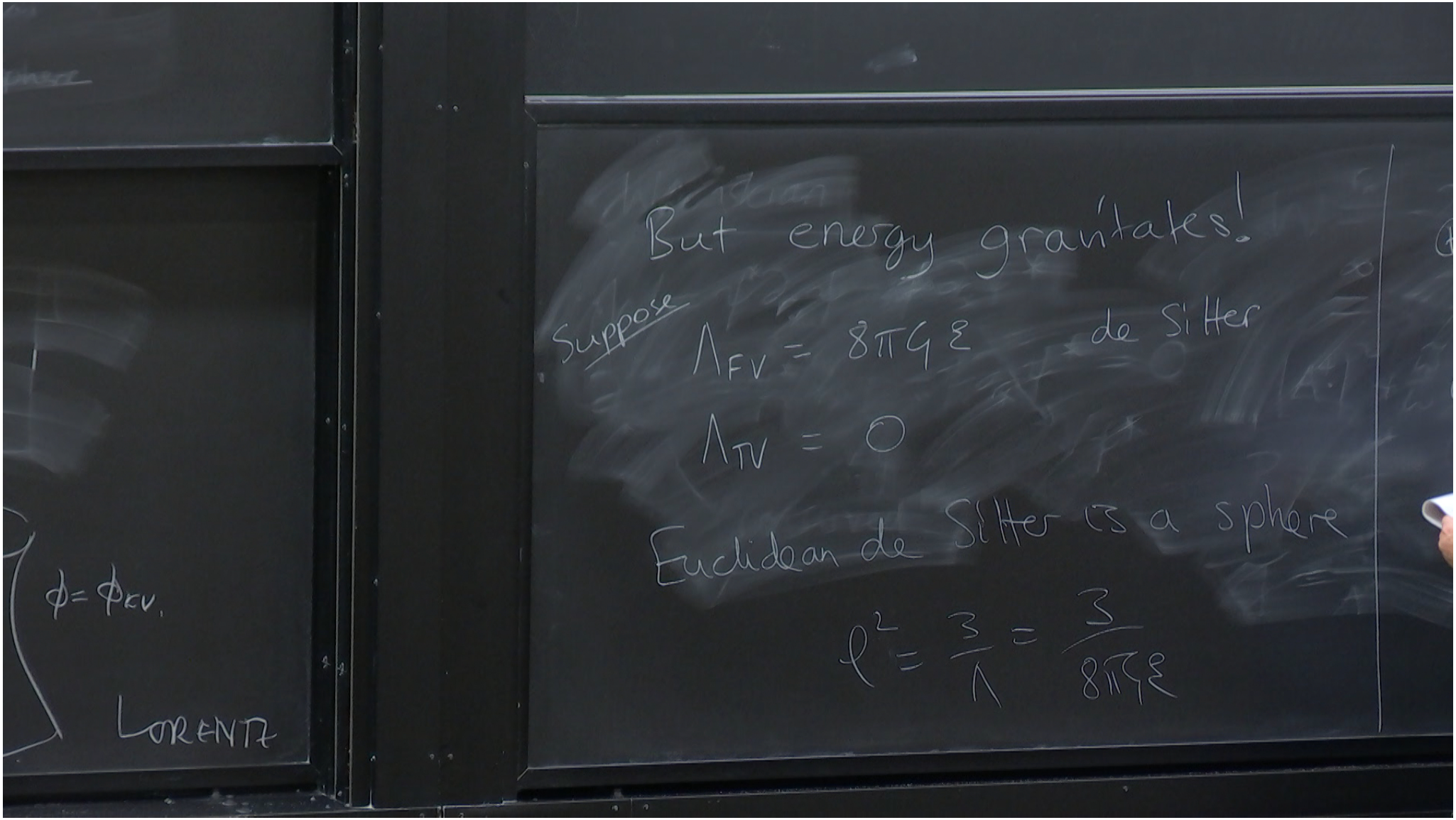
$R)\sqrt{\lambda}\eta)$

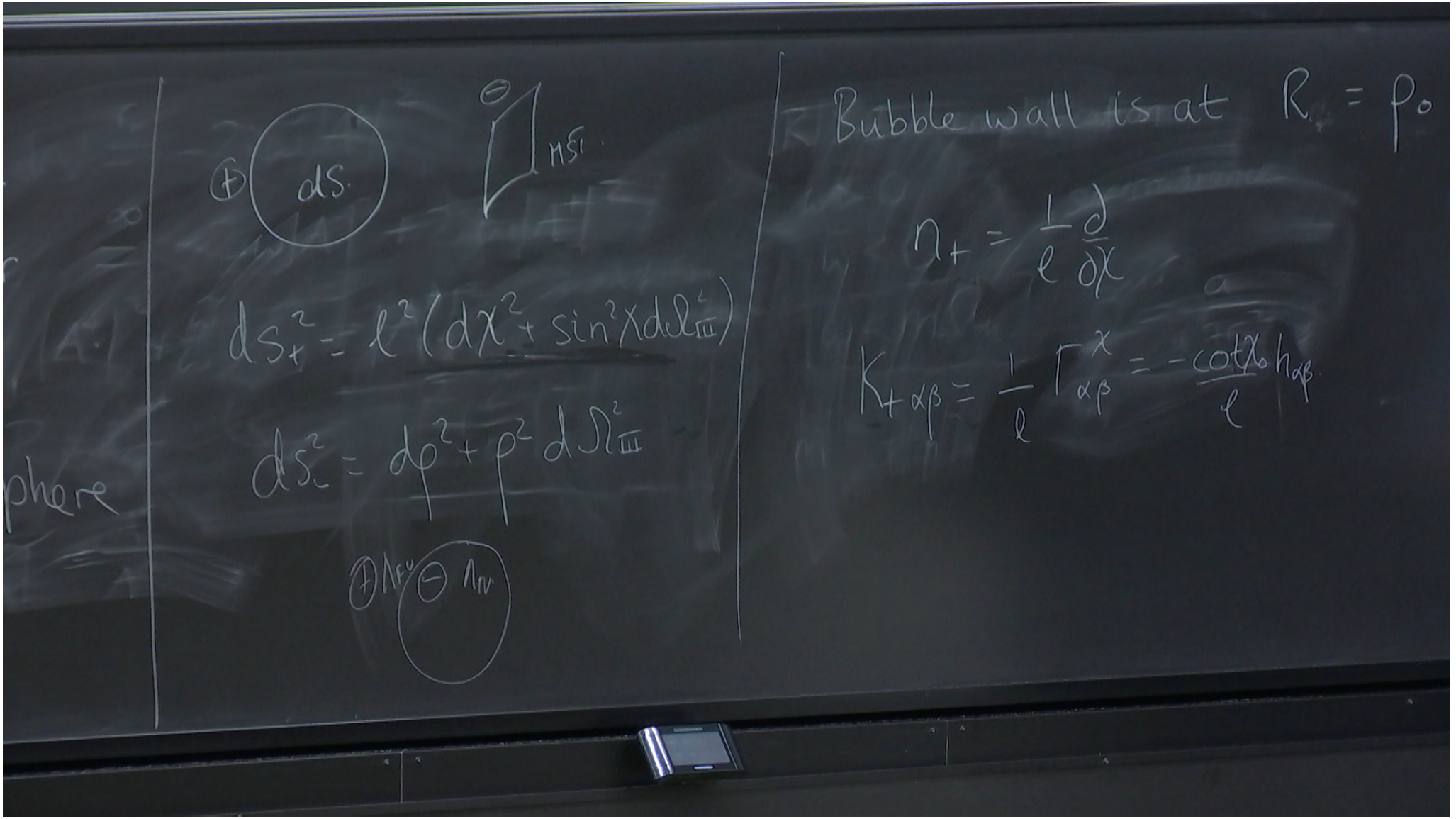
Note, bubble is at:

$r^2 + \tau^2 = R_0^2$

$\Rightarrow r^2 = t^2 = R_0^2$




But energy gravitates!
Suppose
$\Lambda_{FV} = 8\pi G \varepsilon$ de Sitter
$\Lambda_{TV} = 0$
Euclidean de Sitter is a sphere
$\ell^2 = \frac{3}{\Lambda} = \frac{3}{8\pi G \varepsilon}$
$\phi = \phi_{FV}$.
LORENTZ


phere
dS.
MSI
$ds_+^2 = \ell^2(d\chi^2 + \sin^2\chi\, d\Omega_{III}^2)$
$ds_-^2 = d\rho^2 + \rho^2 d\Omega_{III}^2$
Λ_{FV}
Λ_{TV}
Bubble wall is at $R_* = \rho_0$
$n_+ = \frac{1}{\ell}\frac{\partial}{\partial\chi}$
$K_{+\alpha\beta} = \frac{1}{\ell}\Gamma^{\chi}_{\alpha\beta} = -\frac{\cot\chi_0}{\ell} h_{\alpha\beta}$

Bubble wall is at $R = \rho_0 = \ell \sin\chi_0$.

$$n_+ = \frac{1}{\ell}\frac{\partial}{\partial\chi} \qquad n_- = \frac{\partial}{\partial\rho}$$

$$K_{+\alpha\beta} = \frac{1}{\ell}\Gamma^{\chi}_{\alpha\beta} = -\frac{\cot\chi_0}{\ell} h_{\alpha\beta} \qquad K_{-\alpha\beta} = \Gamma^{\rho}_{\alpha\beta} = -\frac{1}{\rho_0} h_{\alpha\beta}$$

Israel eqns.

$$\Delta K_{\alpha\beta} - \Delta K h_{\alpha\beta} = -\frac{2 h_{\alpha\beta}}{\ell \sin\chi_0}(1-\cos\chi_0) = -8\pi G_5 \sigma h_{\alpha\beta}$$

(ISRAEL)

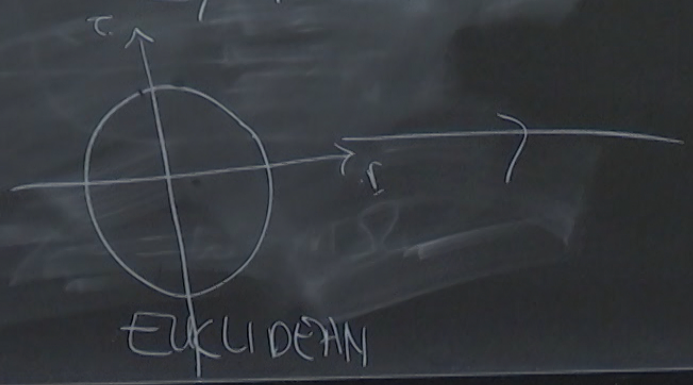
$V = \epsilon$ | $2\pi^2 R^3 \sigma$ | $-\frac{1}{2}\epsilon R^4$

COST WALL | ENERGY GAIN

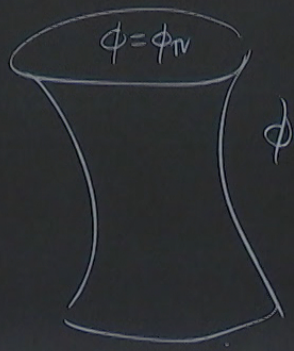
Note, bubble is at

$$r^2 + \tau^2 = R_0^2$$

$$\Rightarrow r^2 - t^2 = R_0^2$$



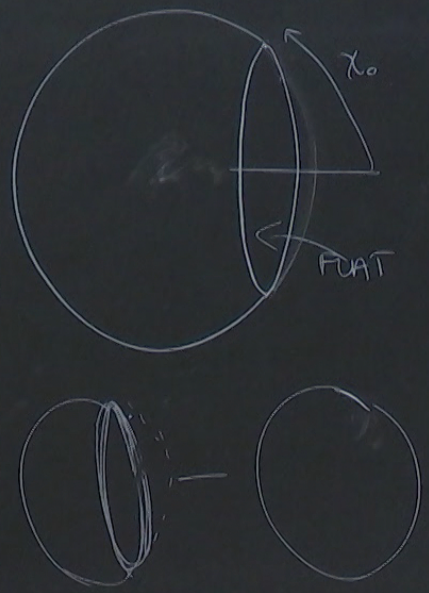
EUCLIDEAN



LORENTZ

$$\frac{(1-\cos\chi_0)}{\ell \sin\chi_0} = 4\pi G\sigma$$

To find achon calculate I_{INST}
& subtract background



$$B = I_{INST} - I_{BACK} = -\frac{1}{8\pi G}\int_{\partial M} \Delta K\sqrt{h} + \int_{WALL} \sigma\sqrt{h} + \frac{1}{16\pi G}\int_{\chi\leq\chi_0} \sqrt{g}(R-2\Lambda)$$

$\partial M \uparrow 12\pi G\sigma$; 2Λ

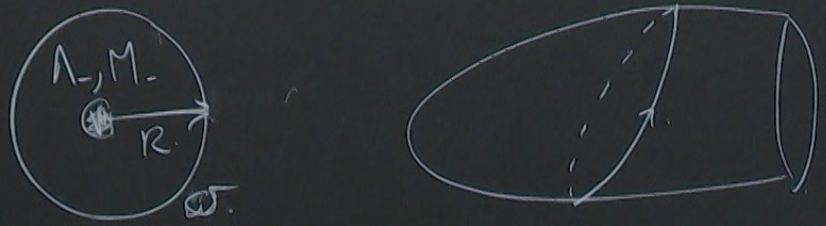
$$= -\frac{\sigma}{2}\cdot 2\pi^2 R^3 + \frac{3}{8\pi G\ell^2}\cdot 2\pi^2\ell^4\int_0^{\chi_0}\sin^3\chi\, d\chi$$

$$= -\frac{\pi^2 R^3(1-\cos\chi_0)}{4\pi G R} + \frac{3\pi\ell^2}{4G}\left[\frac{2}{3} - \cos\chi_0 + \frac{1}{3}\cos^3\chi_0\right]$$

$$= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)^2 \longrightarrow \frac{\pi\ell^2}{G}\frac{(4\pi G\sigma\ell)^4}{[1+(4\pi G\sigma\ell)^2]^2}$$

+ BLACK HOLE $R(\tau)$

Λ_+, M_+



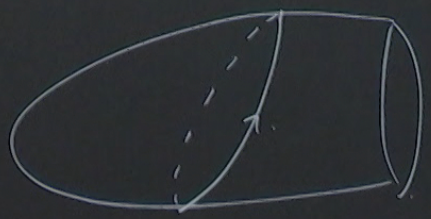
BEFORE:

BH, M_+, Λ_+

$= \frac{\pi \ell}{4G}(1-\cos\chi_0)^2$ —

HOLE $R(\tau)$

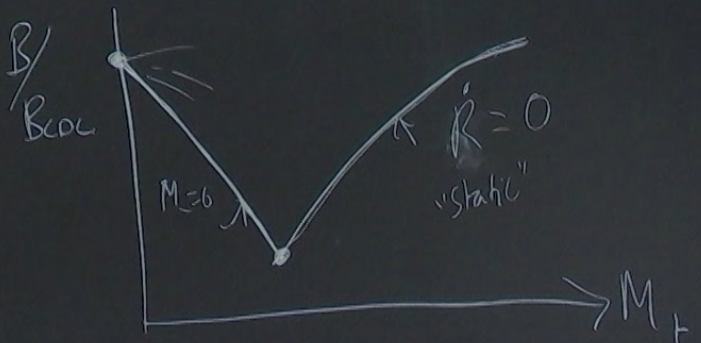
M_+



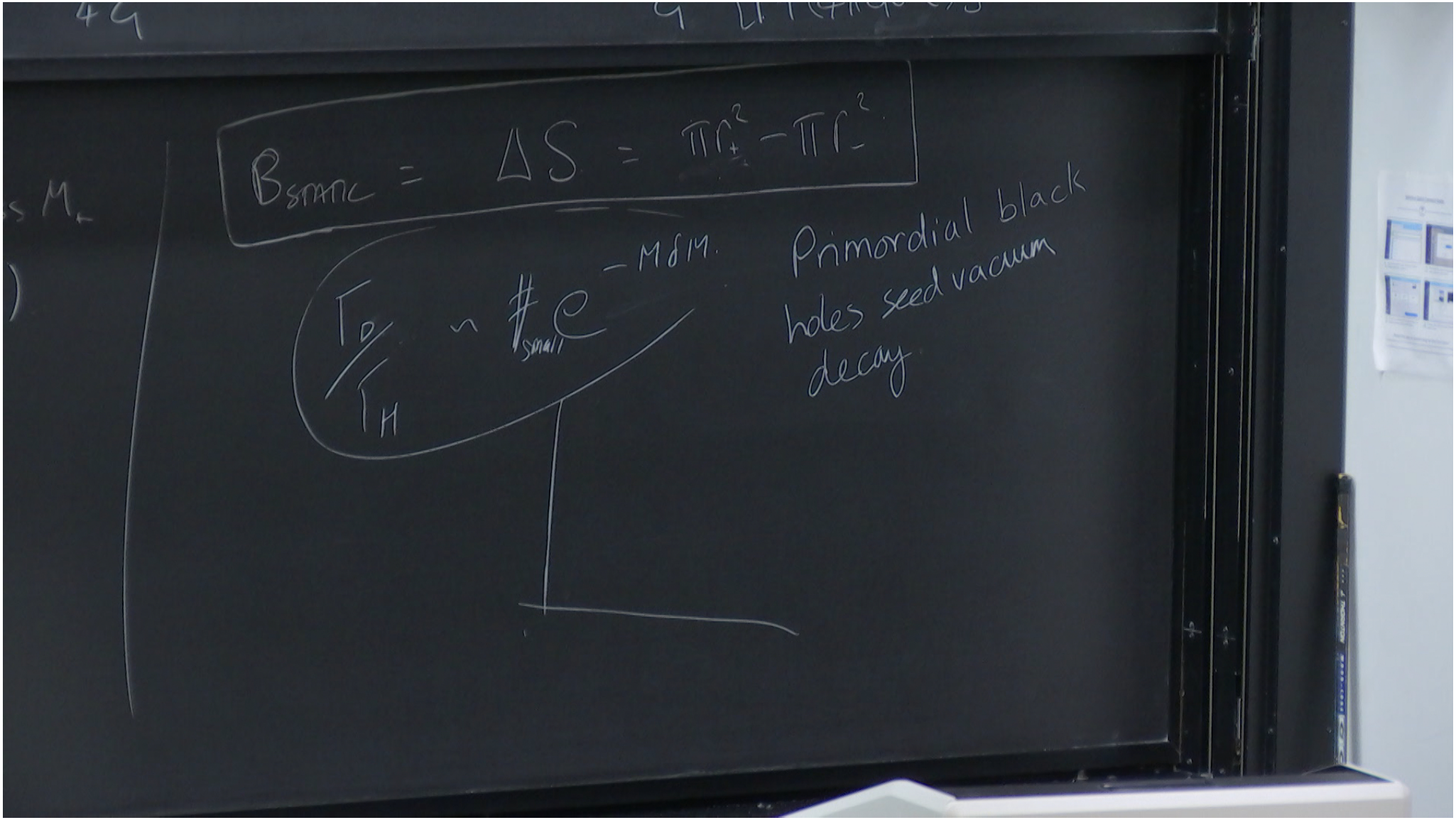
Instantons with seed mass M_+

remnant M_-, $B(M_+, M_-, \Lambda_+, \Lambda_-)$

range of M_- for an M_+



$B_{static} = \Delta$


$B_{static} = \Delta S = \pi r_+^2 - \pi r_-^2$
$\frac{\Gamma_D}{\Gamma_H} \sim \#_{small} e^{-M\delta M}$
Primordial black holes seed vacuum decay