Title: A simple parameter can switch between different weak-noiseâ€“induced phenomena in neurons

Speakers: Marius Yamakou

Series: Machine Learning Initiative

Date: February 21, 2023 - 1:00 PM

URL: https://pirsa.org/23020062

Abstract: This talk will consider a stochastic multiple-timescale dynamical system modeling a biological neuron. With this model, we will separately uncover the mechanisms underlying two different ways biological neurons encode information with stochastic perturbations: self-induced stochastic resonance (SISR) and inverse stochastic resonance (ISR). We will then show that in the same weak noise limit, SISR and ISR are related through the relative geometric positioning (and stability) of the fixed point and the generic folded singularity of the model's critical manifold. This mathematical result could explain the experimental observation where neurons with identical morphological features sometimes encode different information with identical synaptic input. Finally, if time permits, we shall discuss the plausible applications of this result in neuro-biologically inspired machine learning algorithms, particularly reservoir computing based on liquid-state machines.

Zoom link: https://pitp.zoom.us/j/94345141890?pwd=aTRFM3M0a0xCOEM3aXZjY2hFYzVrQT09

A simple parameter can switch between different weak-noise–induced phenomena in neurons

Marius E. Yamakou[1,2] and **Jürgen Jost**[2]

[1]Department of Mathematics & Data Science, University of Erlangen-Nürnberg, Germany
[2]Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

Seminar talk at

Perimeter Institute for Theoretical Physics, Waterloo, Canada

21st February 2023

INTRODUCTION

Resonance Phenomena in Biological Neurons:

- Stochastic resonance (SR), Longtin (1993)
- Coherence resonance (CR), Pikovsky & Kurth (1997)
- (Noise) Vibrational resonance (VR), Landa & Mclintock (2000)
- Self-induced stochastic resonance (SISR), Muratov et al (2005)
- Diversity induced resonance (DIR), Tessone et al. (2006)
- Inverse stochastic resonance (ISR), Jost et al (2009)
- Quenched resonance (QR), Kuehn (2017)
- Recurrence resonance (RS), Krauss et al (2019)
- Inverse chaotic resonance (ICR), Yu et al (2022)

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-Transition mechanism between SR and ISR. —> ***Commun. Nonlinear Sci. Numer. Simul. 82, 105024 (2020)***

The Goal:

- Establish the transition mechanism between SISR and ISR.
- Build physics-informed (with SR, CR, VR, QR, RS, ICR, SISR or ISR) reservoir computing based on liquid-state machines algorithm.

Outline

- Stochastic slow-fast FitzHugh-Nagumo neuron
- Self-induced Stochastic Resonance (SISR)
- Inverse Stochastic Resonance (ISR)
- Summary and conclusion

The Stochastic FitzHugh-Nagumo Neuron Model

$$\begin{cases} \frac{dv_\tau}{d\tau} &= \varepsilon^{-1} f(v_\tau, w_\tau) + \frac{\sigma}{\sqrt{\varepsilon}} \frac{dW_\tau}{d\tau} \\ dw_\tau &= g(v_\tau, w_\tau) d\tau \end{cases} \overset{\varepsilon t = \tau}{\longleftrightarrow} \begin{cases} \frac{dv_t}{dt} &= f(v_t, w_t) + \sigma \frac{dW_t}{dt} \\ dw_t &= \varepsilon g(v_t, w_t) dt \end{cases}$$

where the deterministic velocity vector fields are given by the polynomials

$$\begin{cases} f(v, w) &= -av + (a+1)v^2 + ev^3 + fw, \\ g(v, w) &= d + bv - cw. \end{cases} \quad (1)$$

- $v \in \mathbb{R}$ is a fast variable and $w \in \mathbb{R}$ is a slow variable because the singular parameter is such that $0 < \varepsilon := \tau/t \ll 1$.
- $(-1 \leq a \leq 1, b > 0, c > 0, d \geq 0, e < 0, f < 0, 0 < \varepsilon \ll 1) \in \mathbb{R}^7$

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- $\sigma \geq 0$ is the amplitude of the Gaussian White noise with: $\langle W(t) \rangle = 0$, $\langle W(t), W(t') \rangle = \delta(t - t')$

Singular limits of the FitzHugh-Nagumo model

Definition

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fast subsystem — slow subsystem

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- $\mathcal{M}_0 := \{f = 0\} =$ critical manifold $=$ equil. of fast subsystem.
- $\mathcal{M}_0$ is normally hyperbolic if $(d_v f)(p) \neq 0$, $\forall p \in \mathcal{M}_0$.
- $\mathcal{M}_0$ is attracting if $(d_v f)(p) < 0$, repelling if $(d_v f)(p) > 0$.

Singular limits of the FitzHugh-Nagumo model

Theorem

Suppose $\mathcal{M}_0$ is a compact normally hyperbolic submanifold (possibly with boundary) of the critical manifold $\mathcal{M}'_0$ of a slow-fast dynamical system, $\exists \varepsilon_0 > 0$ s.t. $\forall \varepsilon \in (0, \varepsilon_0]$:

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- *(F1) $\exists$ a locally invariant adiabatic manifold $\mathcal{M}_\varepsilon$ diffeomorphic to $\mathcal{M}_0$.*
- *(F2) $\mathcal{M}_\varepsilon$ has a Hausdorff distance $\mathcal{O}(\varepsilon)$ from $\mathcal{M}_0$.*
- *(F3) The flow on $\mathcal{M}_\varepsilon$ converges to the slow flow on $\mathcal{M}_0$ as $\varepsilon \to 0$.*
- *(F4)$\mathcal{M}_\varepsilon$ is C^r-smooth for any $r < \infty$ (as long as $f, g \in C^\infty$).*
- *(F5) $\mathcal{M}_\varepsilon$ is normally hyperbolic, same stability properties wrt the fast variables as $\mathcal{M}_0$.*
- *(F6) $\mathcal{M}_\varepsilon$ is usually not unique. Manifolds satisfying (F1)-(F5) lie at distance $\mathcal{O}(e^- K/\varepsilon)$ from each other, $K > 0$, $K = \mathcal{O}(1)$.*

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- *(f7) Same conclusions hold for stable/unstable manifolds of $\mathcal{M}_0$.*

Self-Induced Stochastic Resonance (SISR)

Question 1: Can noise induce a limit cycle solution when the bifurcation parameter c is bounded away from its singular Hopf bifurcation value c_{sh}?

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- $\mathcal{M}_0$ is of saddle-type if $\ldots \leq re(\lambda_{j_k}) < 0 < re(\lambda_{j_{k+1}}) \leq \ldots$
- a fold point s: $(d_v f)(s) = 0$, $\mathcal{M}_0$ looses hyperbolicity at s.

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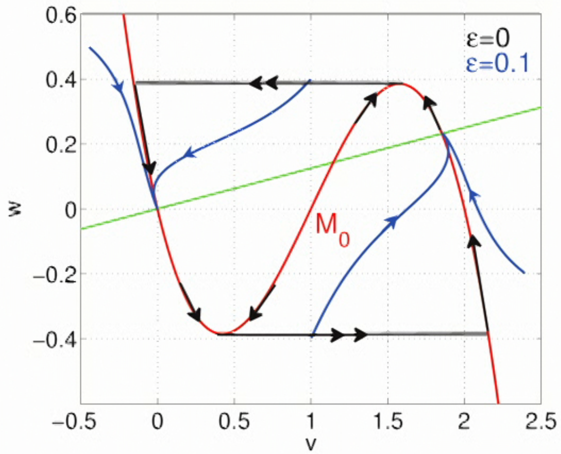
$$\text{invariant sets :} \begin{cases} \mathcal{M}_0 := \left\{(v,w) \in \mathbb{R}^2 : f(v,w) = 0\right\} \\ (v_e, w_e) := \left\{(v,w) \in \mathbb{R}^2 : f(v,w) = g(v,w) = 0\right\} \\ \mathcal{M}_0 \ni (v_-, w_-) := \left\{(v,w) \in \mathbb{R}^2 : \partial_v f(v,w) = 0\right\} \end{cases}$$

- $\mathcal{M}_0$ is stable if $\partial_v f(v,w) = -3v^2 + 2(a+1)v - a < 0$
- (v_e, w_e) is stable if $\mathrm{tr}J < 0$, $\det J > 0 \Leftrightarrow -3v_e^2 + 2(a+1)v_e - a < 0$



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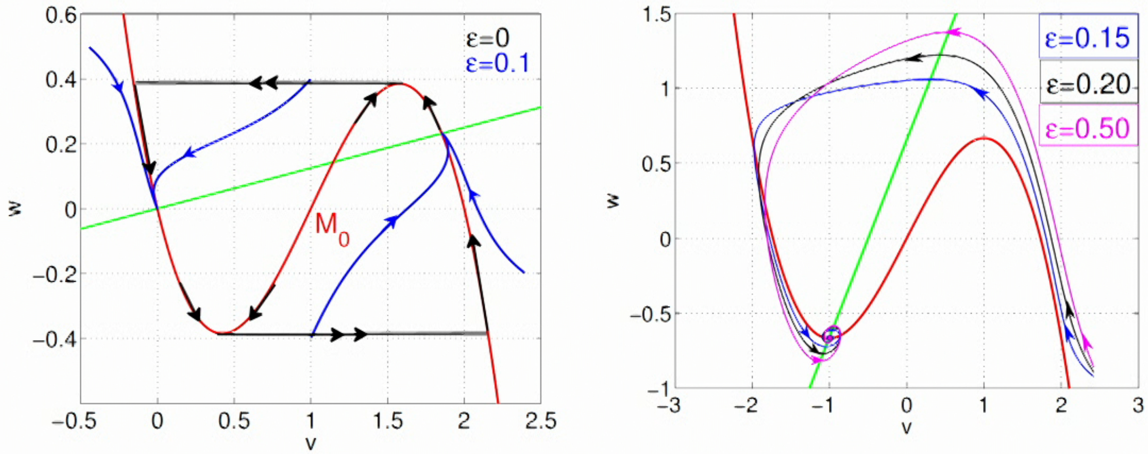
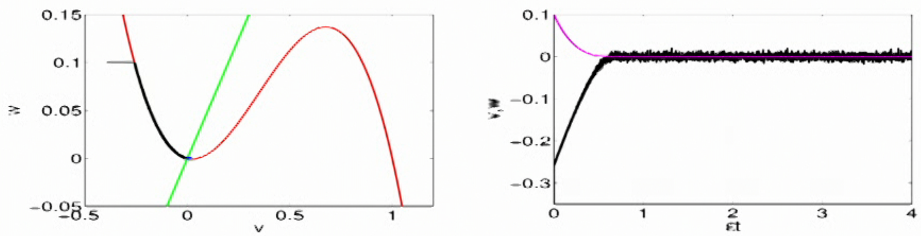


Figure: $a = 1$, $v_- = -1.0$, $v_e = -1.003988$, $|c - c_{sh}| > 0$, $c_{sh} = 0.7499$

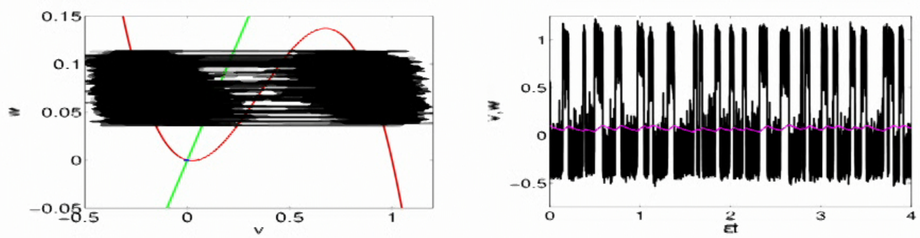
Deterministic and stochastic time-scales of trajectories

- Deterministic time-scale is $\mathcal{O}(\varepsilon^{-1})$, slow in the singular limit $\varepsilon \to 0$.
- Stochastic time-scales from FPE are $E_{r_\pm}(w) = \mathcal{O}\Big[\exp\Big(\frac{2\triangle U_\pm(w)}{\sigma^2}\Big)\Big]$.

$$\mathcal{O}(\varepsilon^{-1}) < \mathcal{O}\Big[\exp\Big(\frac{2\triangle U_\pm(w)}{\sigma^2}\Big)\Big]$$



$$\mathcal{O}(\varepsilon^{-1}) > \mathcal{O}\Big[\exp\Big(\frac{2\triangle U_\pm(w)}{\sigma^2}\Big)\Big]$$



Asymptotic matching of time-scales (Answer to Question 1)

$\mathcal{O}(\varepsilon^{-1}) = \mathcal{O}\left[\exp\left(\frac{2\triangle U_{\pm}(w_{\pm})}{\sigma^2}\right)\right]$, so that we obtain the following conditions:

$$\begin{cases} \lim\limits_{(\varepsilon,\sigma)\to(0,0)} \frac{1}{2}\sigma^2 \log_e(\varepsilon^{-1}) \in \left(\triangle U_{-}(w_e), \frac{3}{4}\right) \\ \lim\limits_{(\varepsilon,\sigma)\to(0,0)} \frac{1}{2}\sigma^2 \log_e(\varepsilon^{-1}) = \mathcal{O}(1) \Leftrightarrow w_e < w_{-},\ w_{-} \text{ is unique as } \triangle U_{-}(w) \nearrow [-\frac{2}{3}, 0] \\ a > 0\ (a \ll 0) \Rightarrow v_e < v_{-}\ (v_e > v_{-}),\ \ |c - c_{sh}| > 0. \end{cases}$$

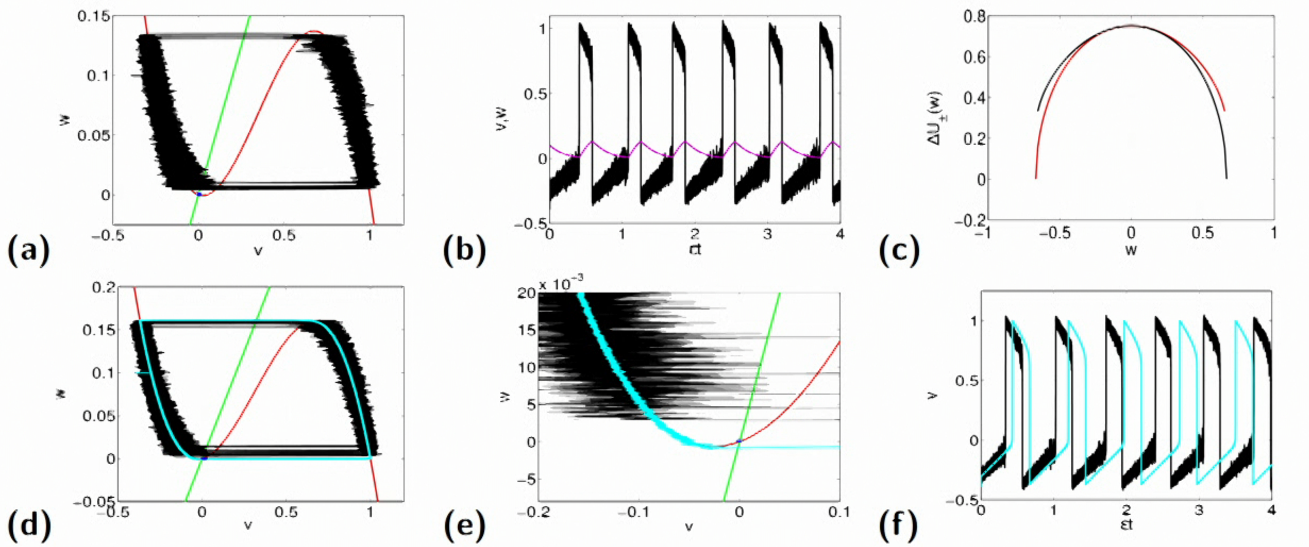
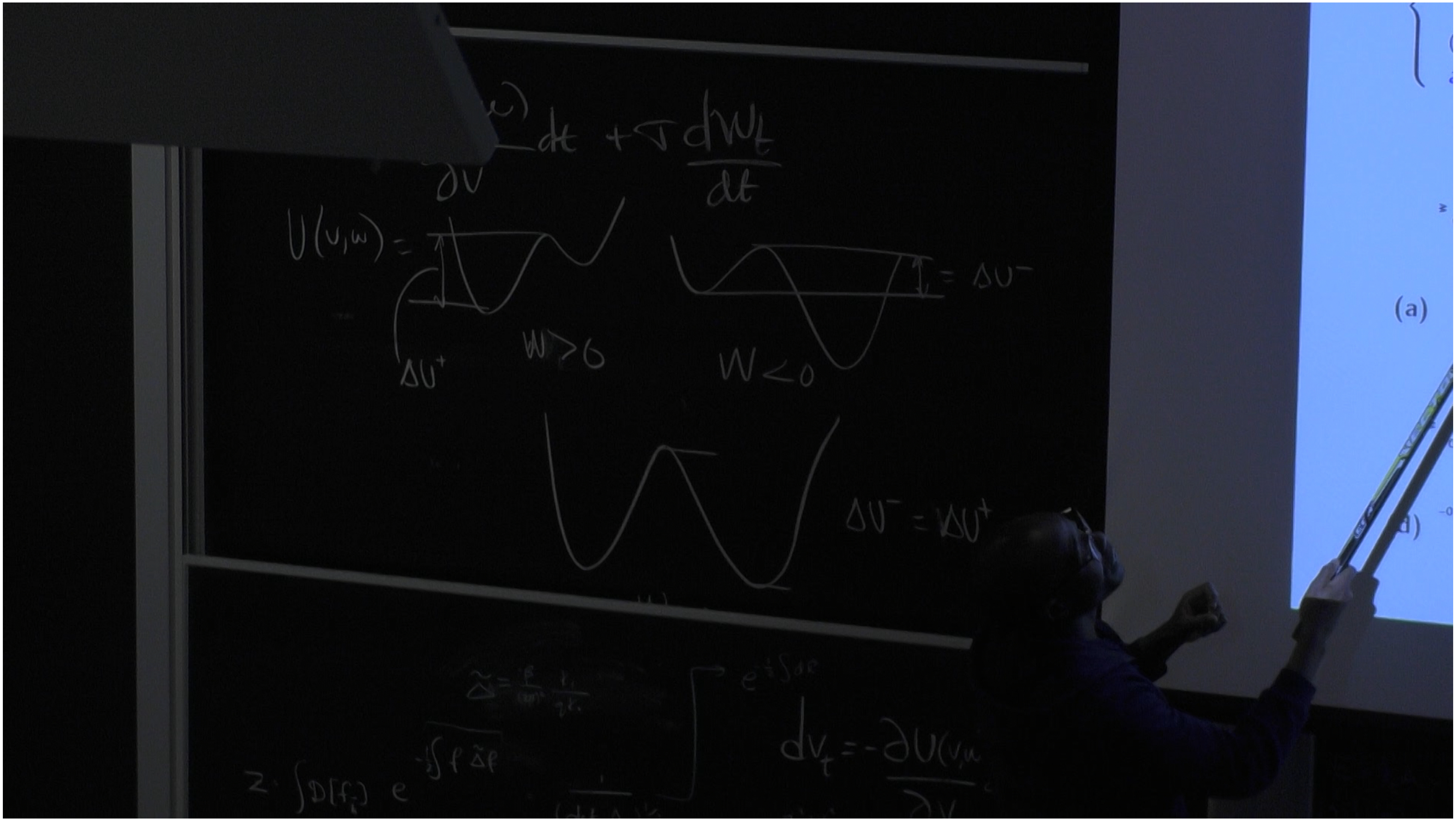


Figure: Self-induce stochastic resonance (SISR)


dt + σ dW_t / dt
U(v,w) =
ΔU⁺
W>0
W<0
= ΔU⁻
ΔU⁻ = ΔU⁺
dv_t = -∂U(v,w) / ∂v
(a)

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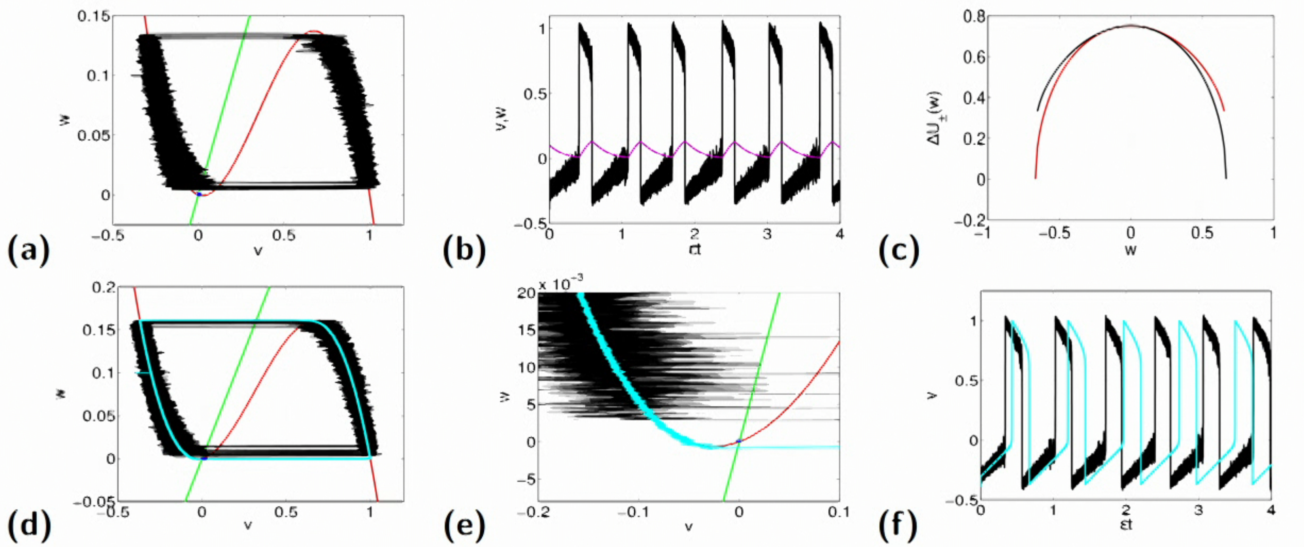
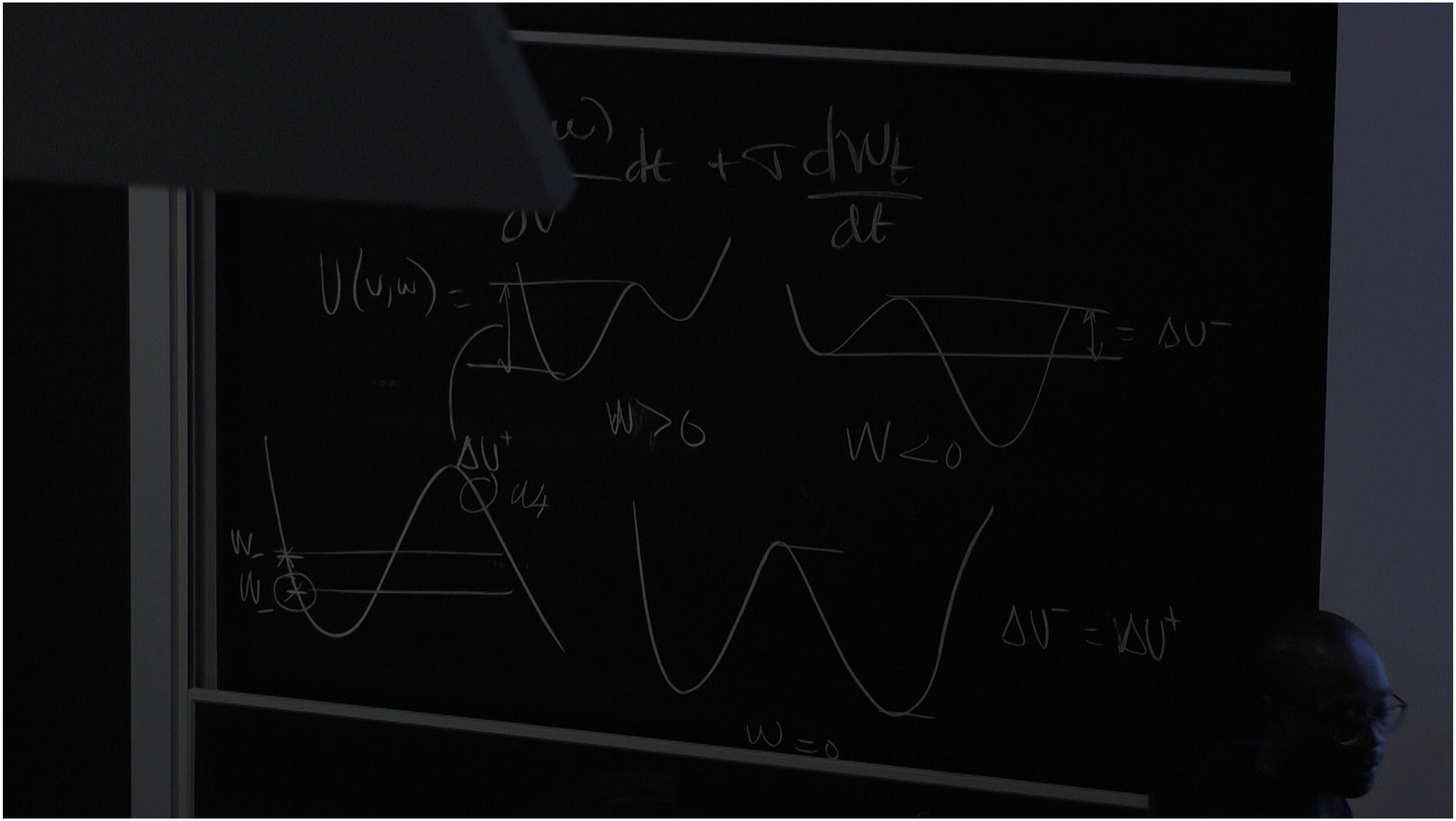


Figure: Self-induce stochastic resonance (SISR)


dt + σ dW_t
dt
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W>0
W<0
= ΔU⁻
ΔU⁺
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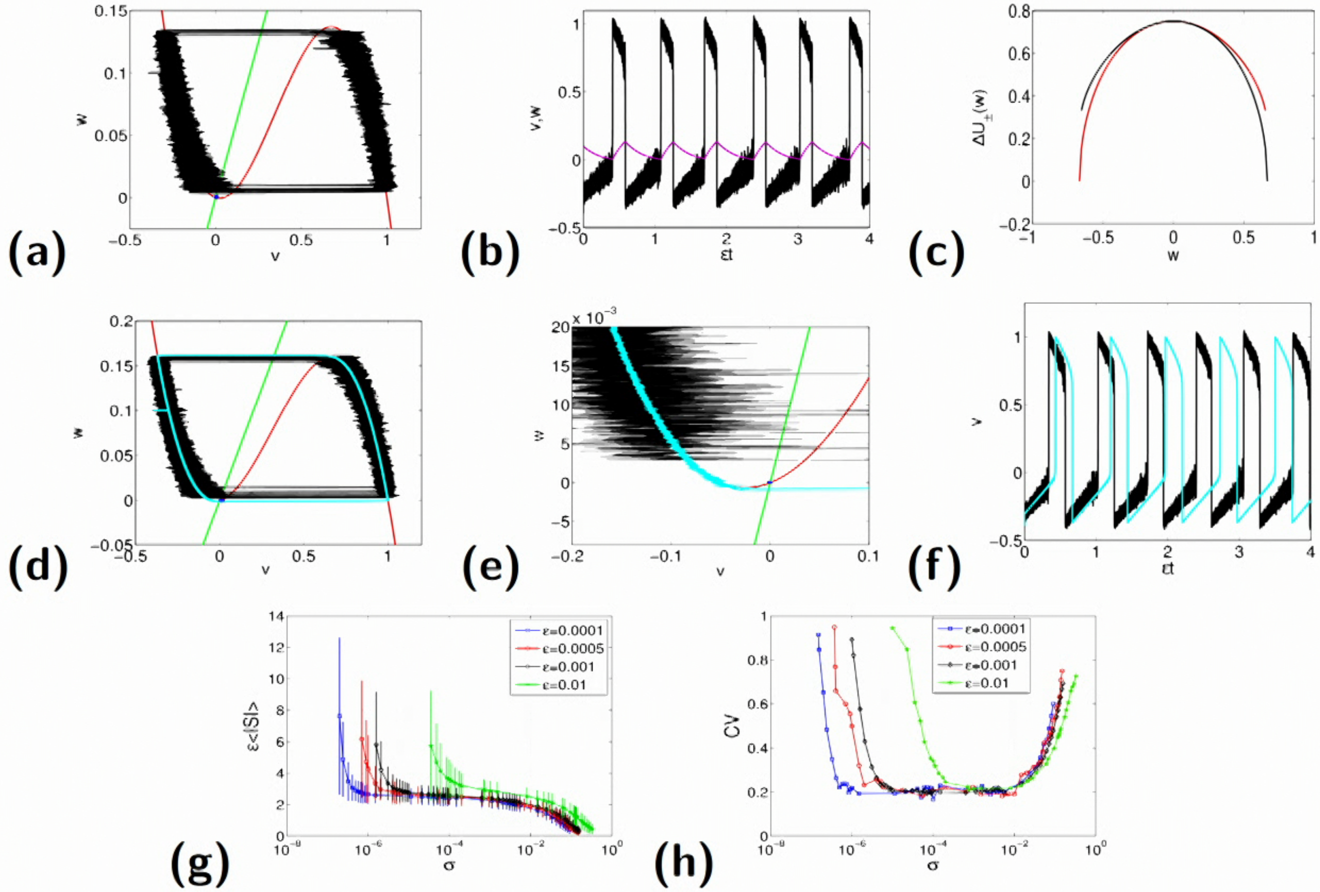
Asymptotic matching of time-scales (SISR).



M.E. Yamakou & J. Jost, Nonlinear Dynamics 93, 2121 (2018)

Detour: Diversity-induced resonance (DIR)

$$\begin{cases} \dfrac{dv_i}{dt} &= v_i(a_i - v_i)(v_i - 1) - w_i + K\sum_{j=1}^{N}(v_j - v_i) + \eta_i(t), \\ \dfrac{dw_i}{dt} &= \varepsilon(bv_i - cw_i). \end{cases} \quad (2)$$

where $a_i \sim \mathcal{N}(a_m, \sigma_d)$ is the diversity in the range $(0, 1+\sqrt{2})$.

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- T. Shibata and K. Kaneko, Europhys. Lett. 38, 417 (1997).
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- C. Zhou, J. Kurths, and B. Hu, Phys. Rev. Lett. 87, 098101 (2001).

Detour: Diversity-induced decoherence (DIDR)

$$\begin{cases} V_f < V_{\min}, \\ \lim_{(\sigma_n,\varepsilon)\to(0,0)} \left[\frac{\sigma_n^2}{2}\ln(\varepsilon^{-1})\right] \in \left(\Delta U^L(W_L^*), \Phi\right), \\ W_L^* > W_f, \\ \Delta U^L(W), \Delta U^R(W) \nearrow W \in [W_{min}, W_{max}]. \end{cases} \tag{5}$$

where

$$U(V,W) = \frac{V^4}{4} - \frac{(1+A)}{3}V^3 + \frac{(3M+A)}{2}V^2 - [W - M(1+A)]V.$$

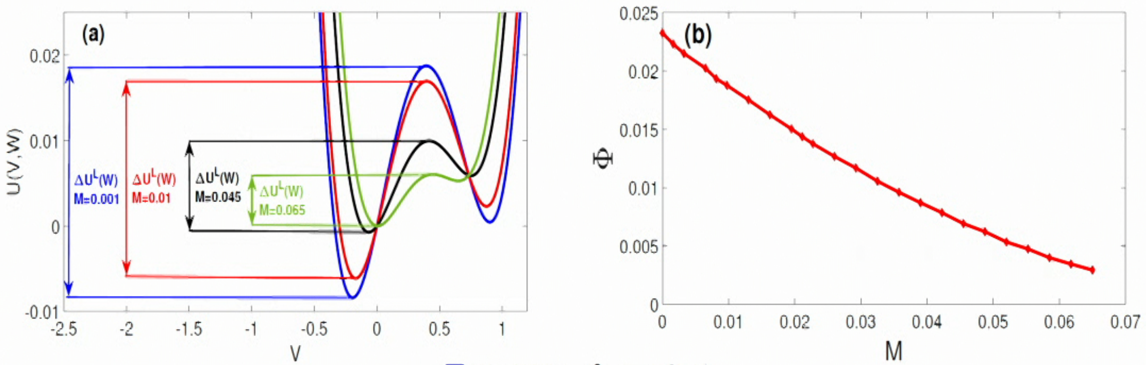

Figure: $A = 0.1$.

Detour: Diversity-induced decoherence (DIDR)

PHYSICAL REVIEW E **106**, L032401 (2022)

Letter

Diversity-induced decoherence

Marius E. Yamakou [1,*] Els Heinsalu,[2,†] Marco Patriarca [2,‡] and Stefano Scialla [2,3,§]

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[2] *National Institute of Chemical Physics and Biophysics - Akadeemia tee 23, 12618 Tallinn, Estonia*

[3] *Department of Engineering, Università Campus Bio-Medico di Roma - Via Á. del Portillo 21, 00128 Rome, Italy*

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SISR vs. CR

The conditions necessary for Coherence Resonance (CR) are:

$$\begin{cases} c - c_{sh} \leq \delta, \quad 0 < \delta \ll 1, \\ \sigma \ll 1, \\ \sigma \gtrsim (1 - w_e^2 - \varepsilon c)^{3/2}. \end{cases}$$

These are completely different from those of SISR.

$$\begin{cases} \lim\limits_{(\varepsilon,\sigma)\to(0,0)} \frac{1}{2}\sigma^2 \log_e(\varepsilon^{-1}) \in \left(\triangle U_-(w_e), \frac{3}{4}\right) \\ \frac{\sigma^2}{2} \log_e(\varepsilon^{-1}) = \mathcal{O}(1) \Leftrightarrow w_e < w_-, \; w_- \text{ is unique: } \triangle U_-(w) \nearrow [-\frac{2}{3}, 0] \\ |c - c_{sh}| > 0 \Longrightarrow v_e < v_- \end{cases}$$

Detour: Some other results on the control of CR and SISR in multiplex networks

PHYSICAL REVIEW E **100**, 022313 (2019)

Control of coherence resonance by self-induced stochastic resonance in a multiplex neural network

Marius E. Yamakou[1,*] and Jürgen Jost[1,2,†]
[1]*Max-Planck-Institut für Mathematik in den Naturwissenschaften, Inselstraße 22, 04103 Leipzig, Germany*
[2]*Santa Fe Institute for the Sciences of Complexity, Santa Fe, New Mexico 87501, USA*

(Received 11 May 2019; published 20 August 2019)

Detour: Some other results on the control of CR and SISR in adaptive multiplex networks

frontiers | Frontiers in Physics

ORIGINAL RESEARCH
published: 04 July 2022
doi: 10.3389/fphy.2022.909365

Check for updates

Optimal Resonances in Multiplex Neural Networks Driven by an STDP Learning Rule

Marius E. Yamakou[1,2], Tat Dat Tran[3,2] and Jürgen Jost[2,4]*

[1]Department of Data Science, Friedrich-Alexander-Universität Erlangen-Nürnberg, Erlangen, Germany, [2]Max-Planck-Institut für Mathematik in den Naturwissenschaften, Leipzig, Germany, [3]Fakultät für Mathematik und Informatik, Universität Leipzig, Leipzig, Germany, [4]Santa Fe Institute for the Sciences of Complexity, Santa Fe, NM, United States

Inverse Stochastic Resonance (ISR)

Question 2: Can the same weak noise limit inhibit the limit cycle solution?

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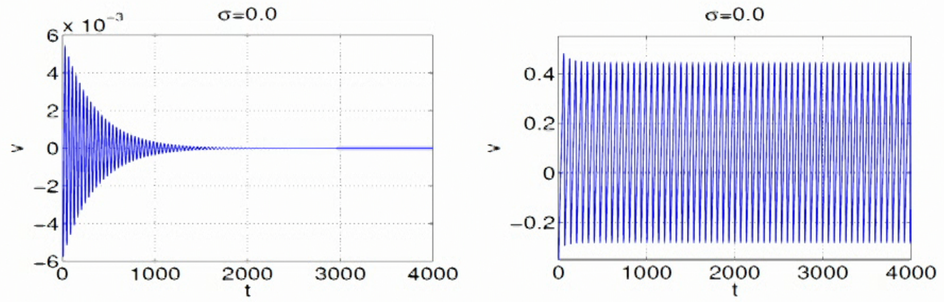
Answer: Yes! Noise can destroy and modulate the limit cycle solution.



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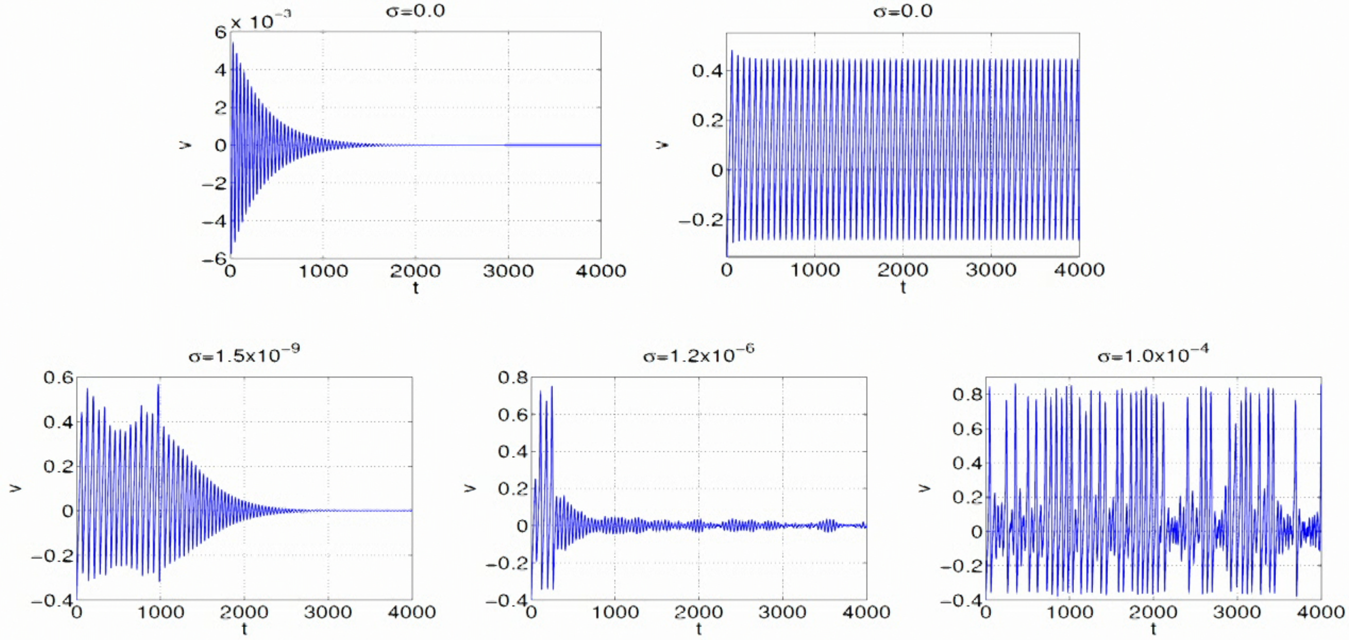

Inverse stochastic resonance (ISR)

First numerical observation of ISR (2009)

Naturwissenschaften (2009) 96:1091–1097
DOI 10.1007/s00114-009-0570-5

ORIGINAL PAPER

Inhibition of rhythmic neural spiking by noise: the occurrence of a minimum in activity with increasing noise

Boris S. Gutkin · Jürgen Jost · Henry C. Tuckwell

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First experimental observation of ISR (2016)

PLOS | COMPUTATIONAL BIOLOGY

RESEARCH ARTICLE

Inverse Stochastic Resonance in Cerebellar Purkinje Cells

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Mathematical analysis was still lacking!

Deterministic bifurcation analysis and bistability

- In singular limit $\varepsilon = 0$, $\mathcal{M}_0 : w = -v^3 + (a+1)v^2 - av$.
- $\mathcal{M}_0$ looses normal hyperbolicity at fold points located at: $v_\pm = \frac{a+1}{3} \pm \frac{1}{3}\sqrt{a^2 - a + 1}$.
- $\frac{dw}{d\tau} > 0$ on $v_0^*(w) = \mathcal{M}_0 \cap \{v_- \leq v \leq v_+\} \Rightarrow$ instability.
- $\frac{dw}{d\tau} < 0$ on $v_\pm^*(w) = \mathcal{M}_0 \cap \{\pm v > \pm v_\pm\} \Rightarrow$ stability.

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- $\frac{dw}{d\tau} < 0$ on $v_\pm^*(w) = \mathcal{M}_0 \cap \{\pm v > \pm v_\pm\} \Rightarrow$ stability.
- Note that $v_- < 0 \iff a < 0$.
- The fixed point $(v_0, w_0) = (0, 0)$ is unique $\iff \frac{(a-1)^2}{4} < \frac{b}{c}$.
- (v_0, w_0) is stable when $-\frac{a}{\varepsilon} < c$ (and $a > -\frac{b}{c}$), i.e., when a is not too negative, and in the limit $\varepsilon \to 0$ only for $a \geq 0$.
- When $c^2 < \frac{b}{\varepsilon}$ and $3\varepsilon c \leq a^2 - a + 1$, we get a Hopf bifurcation at $v_{hp} = \frac{a+1}{3} - \sqrt{\frac{(a+1)^2}{9} - \frac{a+\varepsilon c}{3}}$.

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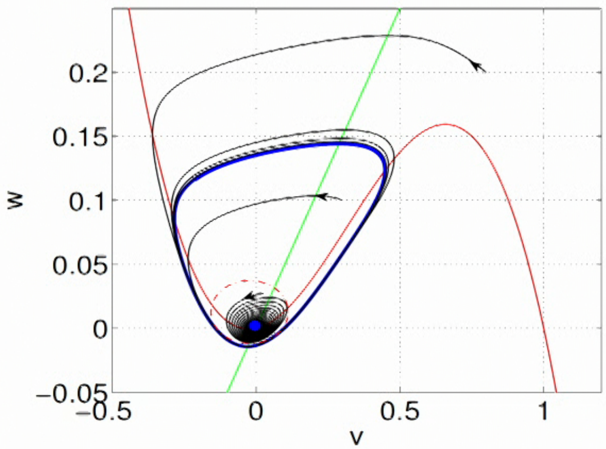
- For $a < 0$, $v_- < v_0 = 0$, and this Hopf bifurcation occurs at $v_0 = 0$ for the value $\varepsilon_{hp} = -\frac{a}{c}$.
- Thus, v_0 loses its stability via Hopf bifurcation when ε_{hp} decreases.

Deterministic bifurcation analysis and bistability

- Hence, we have a stable fixed point (v_0, w_0) and an unforced stable limit cycle $[\bar{v}(t), \bar{w}(t)]$ as long as $-\frac{a}{c} < \varepsilon$.
- That is, $v_- < v_0 < -\frac{a}{c} = \varepsilon_{hp} \iff$ bistability
- E.g., for $a = -0.05$, $b = 1.0$, and $c = 2.0$, $v_- = -0.25305 < v_0 = 0 < -\frac{a}{c} = \varepsilon_{hp} = 0.025 > 0$.



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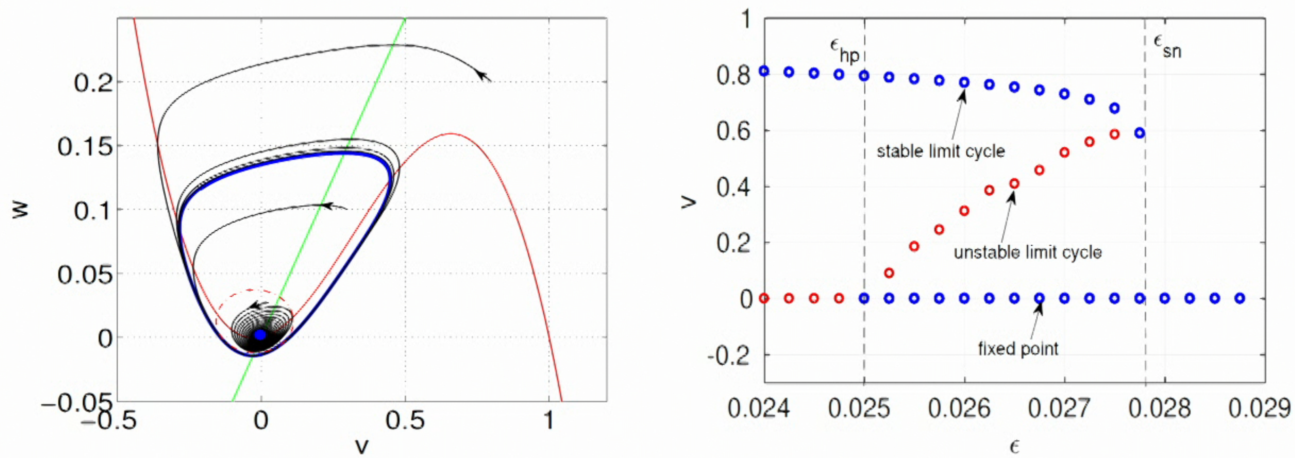


Figure: Phase portrait and corresponding bifurcation diagram

Numerical results: Inverse stochastic resonance (ISR)

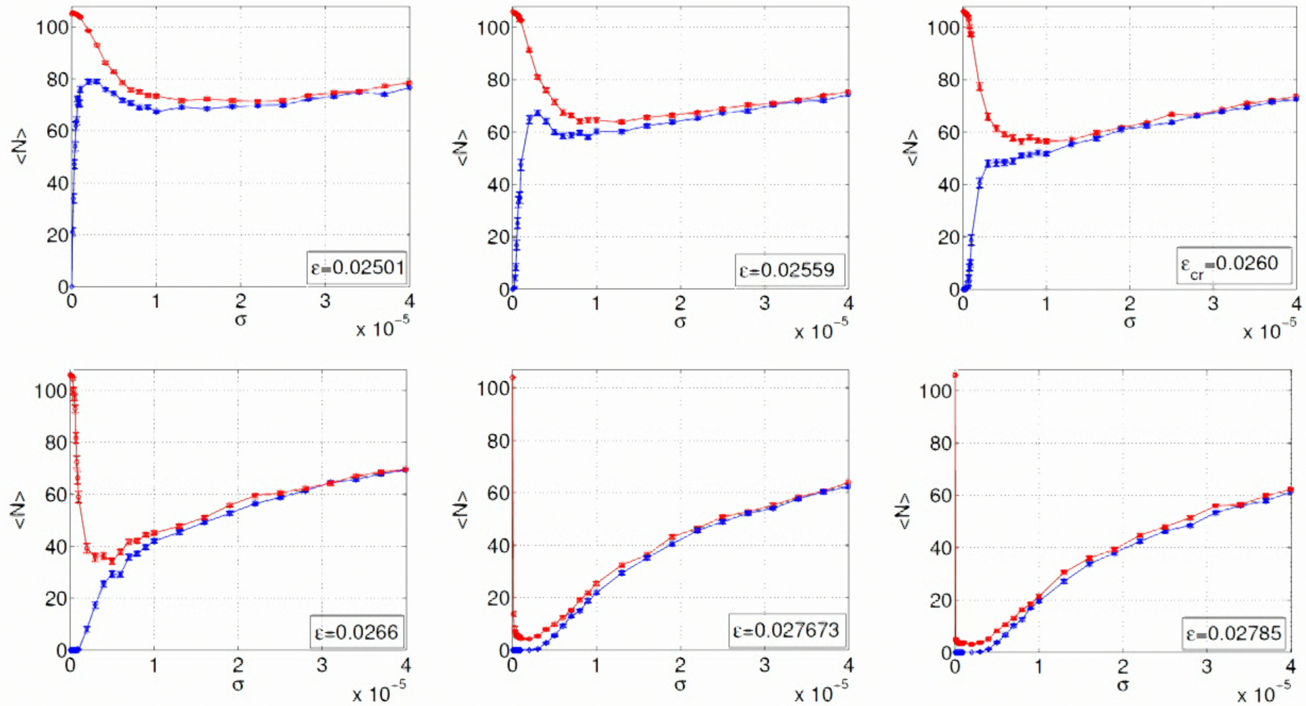


Figure: $\langle N \rangle$ vs. σ for 200 trials for 7500 units of time interval, for $\varepsilon \in (\varepsilon_{hp}, \varepsilon_{sn})$. ISR always occurs when $\big(v(0), w(0)\big) \in \mathcal{B}\big[\bar{v}(t), \bar{w}(t)\big]$, see all the (red curves). For $\big(v(0), w(0)\big) \in \mathcal{B}(v_0, w_0)$ (blue curves), ISR *only* occurs if $\varepsilon \in (0.025, 0.0260)$.

Stochastic sensitivity analysis and the Mahalanobis metric

- The first exit time of a random orbit in $\mathcal{B}(v_0, w_0)$ and $\mathcal{B}\left[\bar{v}(t), \bar{w}(t)\right]$ can be obtained from the probability density $P\left\{\left[v(t), w(t)\right]\right\}$ of the corresponding Kolmogorov-Fokker-Planck equation.
- Asymptotics based on the quasi-potential function Ψ can be used to approximate $P\left\{\left[v(t), w(t)\right]\right\}$: $\Psi = -\lim_{\sigma \to 0} \sigma^2 \log P\left\{\left[v(t), w(t)\right], \sigma\right\}$, which is a solution of the corresponding Hamilton-Jacobi equation.
- The first exit time of random trajectories in a basin of attraction depends on two factors:
  - On the geometry of the basins of attraction: $\mathcal{B}(v_0, w_0)$, $\mathcal{B}\left[\bar{v}(t), \bar{w}(t)\right]$.
  - On the sensitivity of the attractor to random perturbations.
- A quadratic form of Ψ gives a Gaussian approximations of $P_{g1,2}$ of the $P\left\{\left[v(t), w(t)\right]\right\}$ in the vicinity of the attractors as:

Stochastic sensitivity analysis and the Mahalanobis metric

$$\begin{cases} P_{g1} = \frac{1}{Z_1} \exp\left[-\frac{1}{2\sigma^2} \begin{pmatrix} v(t) - v_0 \\ w(t) - w_0 \end{pmatrix}^\top \Omega_{ij}^{-1} \begin{pmatrix} v(t) - v_0 \\ w(t) - w_0 \end{pmatrix} \right], \\ P_{g2} = \frac{1}{Z_2} \exp\left[-\frac{1}{2\sigma^2} \begin{pmatrix} v(t) - \bar{v}(t) \\ w(t) - \bar{w}(t) \end{pmatrix}^\top \Theta_{ij}^{-1}(t) \begin{pmatrix} v(t) - \bar{v}(t) \\ w(t) - \bar{w}(t) \end{pmatrix} \right]. \end{cases}$$

where

- $\left[v(t), w(t)\right]$ are the coordinates of the separatrix between the basins.
- covariance matrices: Ω_{ij} and $\Theta_{ij}(t)$ are the sensitivity matrices.
- The Mahalanobis distances of attractors from separatrix are given as:

$$\begin{cases} D_m(fp) = \sqrt{\begin{pmatrix} v(t) - v_0 \\ w(t) - w_0 \end{pmatrix}^\top \Omega_{ij}^{-1} \begin{pmatrix} v(t) - v_0 \\ w(t) - w_0 \end{pmatrix}}, \\ D_m(lc) = \sqrt{\begin{pmatrix} v(t) - \bar{v}(t) \\ w(t) - \bar{w}(t) \end{pmatrix}^\top \Theta_{ij}^{-1}(t) \begin{pmatrix} v(t) - \bar{v}(t) \\ w(t) - \bar{w}(t) \end{pmatrix}}. \end{cases}$$

Stochastic sensitivity functions and the Mahalanobis metric

- The maximum eigenvalues λ_{max} of Ω_{ij} and $\Theta_{ij}(t)$ are the stochastic sensitivity functions of the attractors.
- The direction of the corresponding eigenvectors give the direction in which escape from a basin of attraction is most probable.
- For an exponentially stable attractors, the largest Lyapunov exponent is 0 and the others are negative.

$$\begin{cases} J_{ij}\Omega_{ij} + \Omega_{ij}J_{ij}^{\top} + G_{ij} = \mathbf{0}, \\ \dfrac{d\Theta_{ij}(t)}{dt} = J_{ij}(t)\Theta_{ij}(t) + \Theta_{ij}(t)J_{ij}(t)^{\top} + P_{ij}(t)G_{ij}P_{ij}(t) \end{cases}$$

$$\begin{cases} \Theta_{ij}(0) = \Theta_{ij}(T), \\ \Theta_{ij}(t)\begin{pmatrix} f\left[\bar{v}(t), \bar{w}(t)\right] \\ g\left[\bar{v}(t), \bar{w}(t)\right] \end{pmatrix} \equiv 0, \end{cases}$$

where J_{ij}, G_{ij}, T, $P_{ij} = P_{ji}$, $\Theta_{ij}(t)$ is singular.

Stochastic sensitivity functions and the Mahalanobis metric

For 2-D systems, $\Theta_{ij}(t)$ can also be written in the form:
(See M.E. Yamakou & J. Jost, Biological Cybernetics 112, 445 (2018) for details.)

$$\begin{cases} \Theta_{ij}(t) = \mu(t)P_{ij}(t), \\ \Theta_{ij}(t) = \frac{1}{\mu(t)}P_{ij}(t), \end{cases}$$

$$D_m\Big\{\big[v(t), w(t)\big]; \big[\bar{v}(t), \bar{w}(t)\big]\Big\} = \frac{\left\| \begin{pmatrix} v(t) - \bar{v}(t) \\ w(t) - \bar{w}(t) \end{pmatrix} \right\|}{\sqrt{\mu(t)}}.$$

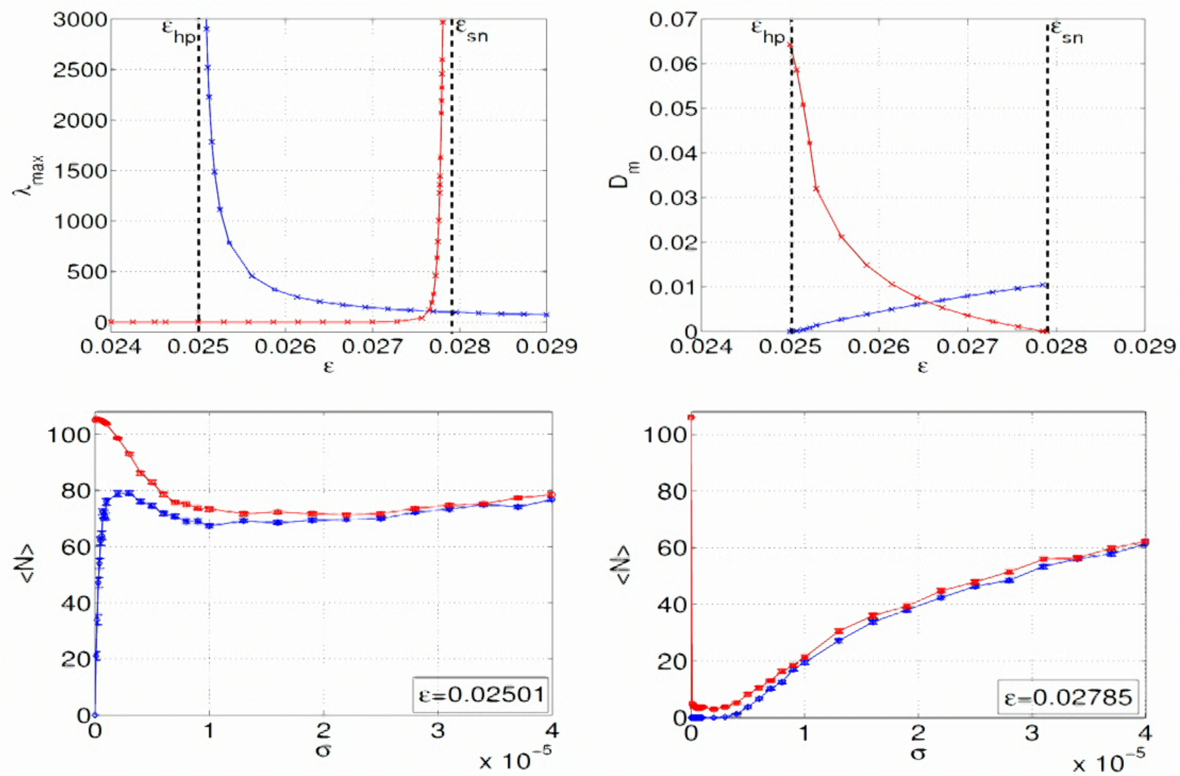
Here, $\mu(t) = \mu(t+T) > 0$ is the unique solution of the boundary problem

$$\begin{cases} d\mu = \alpha(t)\mu(t)dt + \beta(t)dt, \\ \mu(0) = \mu(T), \end{cases}$$

with T-periodic coefficients

$$\begin{cases} \alpha(t) = q(t)^\top \big[J(t)^\top + J(t)\big] q(t), \\ \beta(t) = q(t)^\top G_{ij} q(t), \\ q(t) \perp (f(v,w), g(v,w))^\top. \end{cases}$$

Stochastic sensitivity functions, Mahalanobis distances of attractors and inverse stochastic resonance



M.E. Yamakou & J. Jost, Biological Cybernetics 112, 445 (2018)

Summary and Conclusion

Question: Why will physiologically identical neuron under similar synaptic noise perturbations encode very different information, i.e, SISR or ISR?

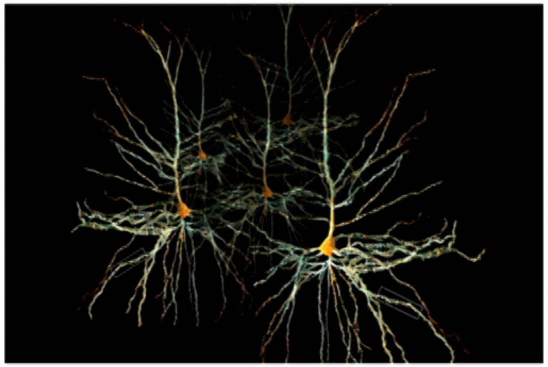
Answer: Because of simple parameter switch leading to a change in the voltage of the quiescent state.

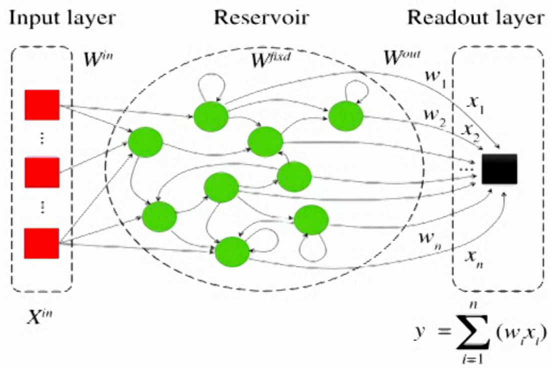
$$\begin{cases} SISR \\ \sigma \to 0 \\ \varepsilon \to 0 \\ a > 0 \; (a \ll 0) \Rightarrow v_0 < v_- \; (v_0 > v_-) \\ |c - c_{sh}| > 0 \\ \frac{1}{2}\sigma^2 \log_e(\varepsilon^{-1}) \in \left(\triangle U_-(w_e), \frac{3}{4}\right) \\ \frac{1}{2}\sigma^2 \log_e(\varepsilon^{-1}) = \mathcal{O}(1) \Rightarrow w_e < w_- \end{cases} \longleftrightarrow \begin{cases} ISR \\ \sigma \to 0 \\ \varepsilon > 0 \\ -1 \ll a < 0 \Rightarrow (v_0 > v_-) \\ v_- < v_0 < -\frac{a}{c} = \varepsilon_{hp} \\ \big(v(0), w(0)\big) \in \mathcal{B}\left[\bar{v}(t), \bar{w}(t)\right] \\ \big(v(0), w(0)\big) \in \mathcal{B}(v_0, w_0), \\ \varepsilon \in (0.025, 0.026) \end{cases}$$

Open problems and future research

- Mathematical analysis of SISR and ISR in isolated neuron with non-Gaussian noise, in particular
  - skewed Lévy noise
  - Ornstein-Uhlenbeck process (colored noise)
  - Poisson dichotomous noise

- Determine whether SISR and ISR can improve the reservoir algorithm computing based on liquid-state machines. (Collaboration with Estelle Inack)





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