Title: A UV/EFT Correspondence for Cosmology

Speakers: Scott Melville

Series: Quantum Gravity

Date: February 22, 2023 - 11:00 AM

URL: https://pirsa.org/23020060

Abstract: Experimental searches for new fundamental physics are increasingly adopting an Effective Field Theory (EFT) approach, in which the phenomenological effects of the underlying high-energy (UV) physics are parametrised by a series of EFT coefficients that can be readily compared with data.
While pragmatically useful, this begs the question: what UV information can be extracted from our measurements of these EFT coefficients?
In this talk, I will describe how scattering amplitudes techniques ("sum rules") can establish precise connections between EFT coefficients and the underlying UV physics.
In particular, I will focus on recent progress in applying these techniques in cosmology, where they have been used to connect our large-scale measurements of dark energy, gravitational waves and the CMB with properties of the underlying UV completion.

Zoom Link: https://pitp.zoom.us/j/99740767444?pwd=OTMxWlVDYitSTXdKdmlFRWxhdGl1dz09

UV/EFT Correspondence for Cosmology

Scott Melville

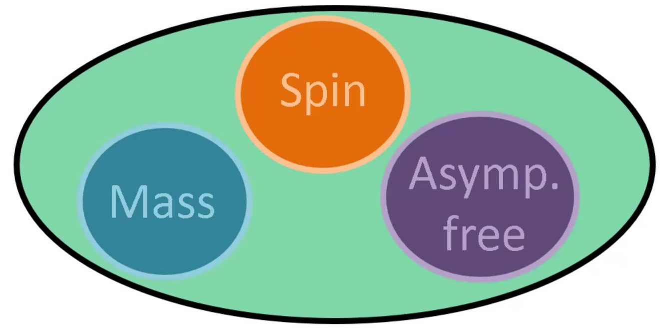
22 Feb 2023

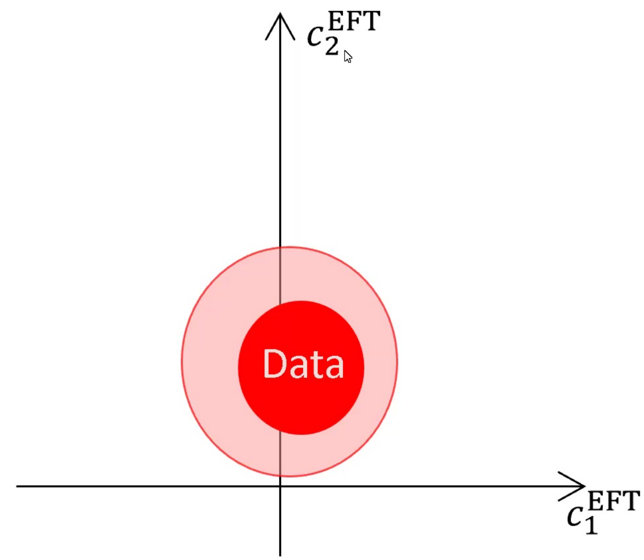
UNIVERSITY OF CAMBRIDGE

EPSRC
Engineering and Physical Sciences Research Council

Science & Technology Facilities Council

Properties of under
high-energy (UV) ph





Low-energy (EFT) parameter space

UV/EFT Correspondence for Cosmology

Scott Melville

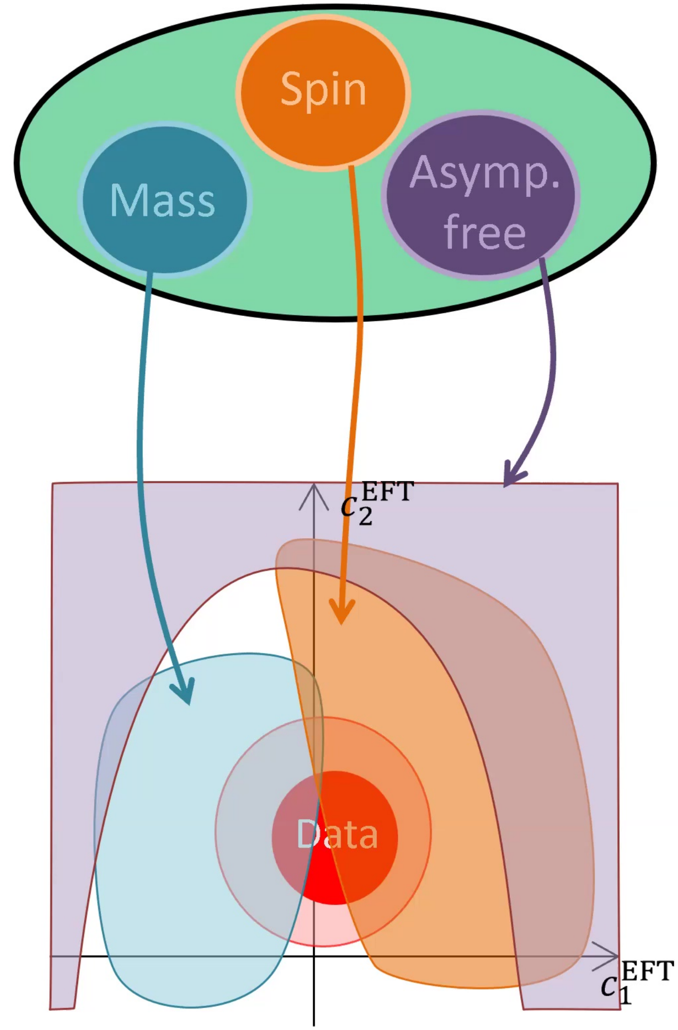
22 Feb 2023

UNIVERSITY OF CAMBRIDGE

EPSRC
Engineering and Physical Sciences Research Council

Science & Technology Facilities Council

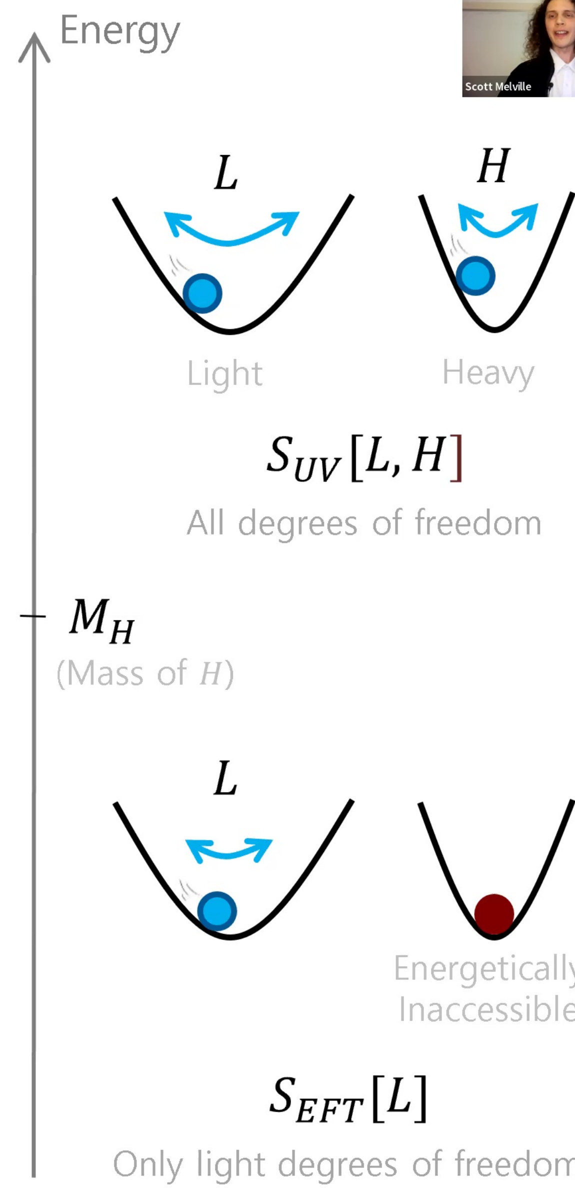
Properties of under
high-energy (UV) ph



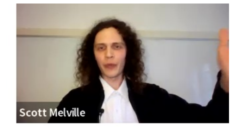
Low-energy (EFT) parameter space

Introduction to EFTs

- Effective Field Theories (EFTs) use a derivative expansion to capture the effects of high-energy fields which cannot be directly produced.
- Schematically, $$e^{iS_{\mathrm{EFT}}[L]} = \int \mathcal{D}H \; e^{iS_{\mathrm{UV}}[L,H]}$$
$$\Rightarrow \quad \mathcal{L}_{\mathrm{EFT}}[L] = (\partial L)^2 - m^2 L^2 + \sum_{n,q} C^{\mathrm{EFT}}_{n,q} \partial^n L^q$$
- At low energies, can neglect interactions with many derivatives.



Introduction to EFTs



- Effective Field Theories (EFTs) use a derivative expansion to capture the effects of high-energy fields which cannot be directly produced.
- Schematically, $$e^{iS_{\rm EFT}[L]} = \int \mathcal{D}H \; e^{iS_{\rm UV}[L,H]}$$

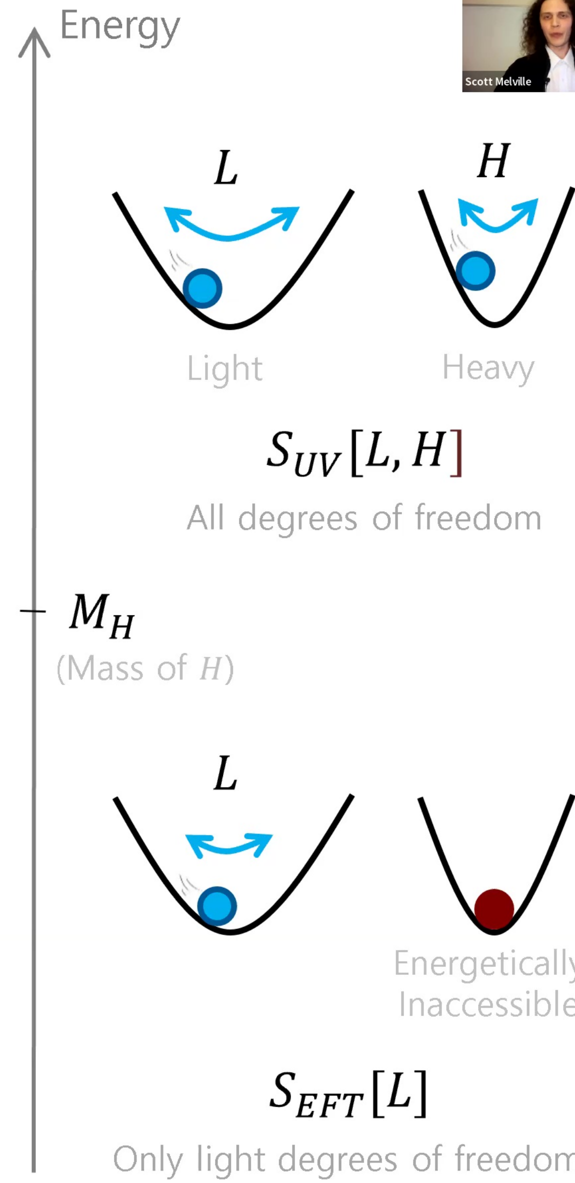
$$\Rightarrow \quad \mathcal{L}_{\rm EFT}[L] = (\partial L)^2 - m^2 L^2 + \sum_{n,q} C_{n,q}^{\rm EFT} \partial^n L^q$$

- At low energies, can neglect interactions with many derivatives.

- In principle, the EFT coefficients are determined by the underlying heavy fields which have been averaged over.
- In practice, do not know this heavy physics *a priori*, so $\mathcal{L}_{\rm EFT}[L]$ is used as a model-independent template to fit/analyse data. (i.e. we treat the $C_{n,q}^{EFT}$ as free parameters).

- For example, $$S_{\rm SMEFT} = S_{\rm SM} + \sum_j C_j^{\rm EFT} \mathcal{O}_j[\text{SM fields}]$$

$$S_{\rm GREFT} = S_{\rm GR}[g_{\mu\nu}] + C_1^{\rm EFT}\, \delta S_1[g_{\mu\nu}] + C_2^{\rm EFT} \delta S_2[g_{\mu\nu}] + \cdots$$



Introduction to EFTs

- Effective Field Theories (EFTs) use a derivative expansion to capture the effects of high-energy fields which cannot be directly produced.
- Schematically, $e^{iS_{\mathrm{EFT}}[L]} = \int \mathcal{D}H \; e^{iS_{\mathrm{UV}}[L,H]}$

$$\Rightarrow \quad \mathcal{L}_{\mathrm{EFT}}[L] = (\partial L)^2 - m^2 L^2 + \sum_{n,q} C_{n,q}^{\mathrm{EFT}} \partial^n L^q$$

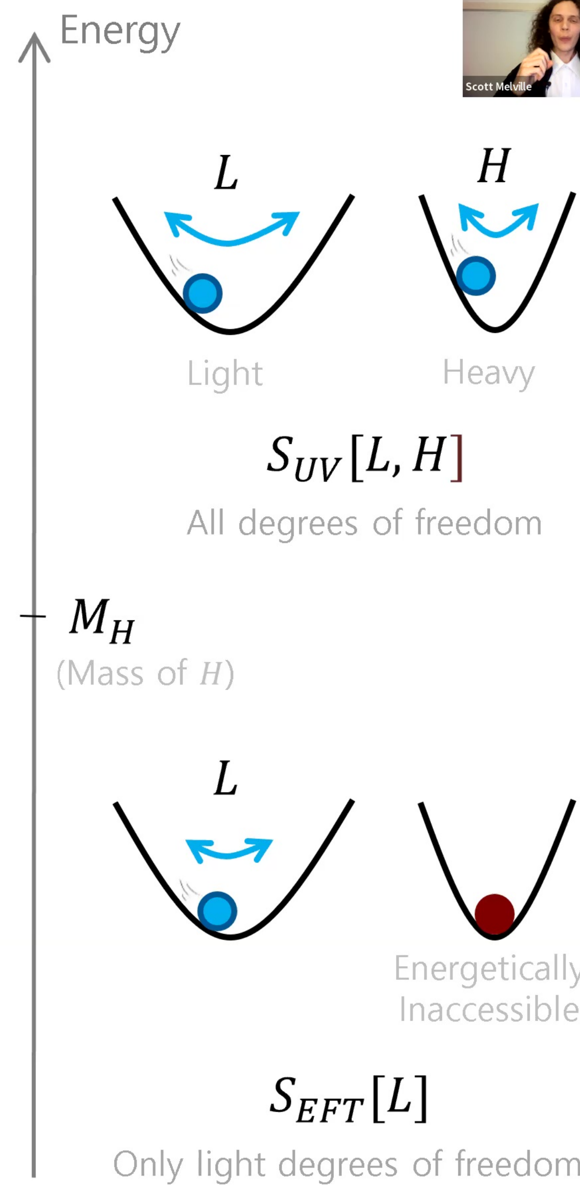
- At low energies, can neglect interactions with many derivatives.

- In principle, the EFT coefficients are determined by the underlying heavy fields which have been averaged over.
- In practice, do not know this heavy physics *a priori*, so $\mathcal{L}_{\mathrm{EFT}}[L]$ is used as a model-independent template to fit/analyse data. (i.e. we treat the $C_{n,q}^{EFT}$ as free parameters).

- For example, $S_{\mathrm{SMEFT}} = S_{\mathrm{SM}} + \sum_j C_j^{\mathrm{EFT}} \mathcal{O}_j[\mathrm{SM\ fields}]$

$$S_{\mathrm{GREFT}} = S_{\mathrm{GR}}[g_{\mu\nu}] + C_1^{\mathrm{EFT}} \, \delta S_1[g_{\mu\nu}] + C_2^{\mathrm{EFT}} \delta S_2[g_{\mu\nu}] + \cdots$$

Suppose we measure one or more EFT coefficients.
What can we learn about the underlying high-energy fields?



Outline

UV/EFT Relations from Scattering Amplitudes

A Dark Energy Example

An Inflationary Example

Beyond Scattering Amplitudes

Scattering Amplitudes

- The probability amplitude to transition between 2-particle states,

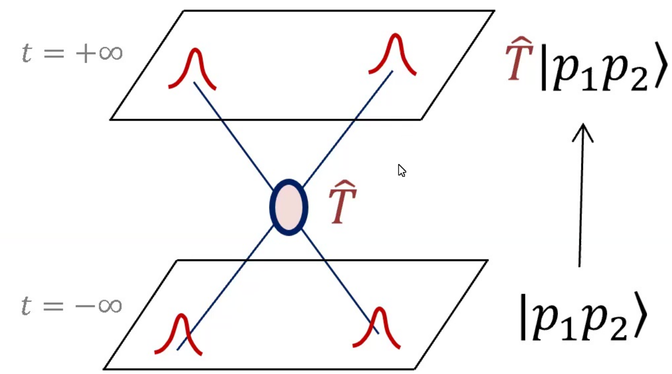
$$\langle p_3 p_4 | \hat{T} | p_1 p_2 \rangle = A(s,t)\, \delta^4(p_1 + p_2 - p_3 - p_4)$$

depends on only two Lorentz-invariant variables,

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2$$

- In terms of A, the EFT derivative expansion is a Taylor series in $\{s, t\}$.

$$A_{\text{EFT}}(s,t) = \text{light physics} + \sum_{a,b} c_{ab}^{\text{EFT}} \frac{s^a t^b}{M^{2a+2b}}$$



Scattering Amplitudes

- The probability amplitude to transition between 2-particle states,

$$\langle p_3 p_4 | \hat{T} | p_1 p_2 \rangle = A(s,t)\, \delta^4(p_1 + p_2 - p_3 - p_4)$$

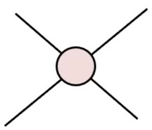
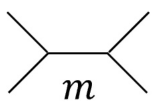
depends on only two Lorentz-invariant variables,

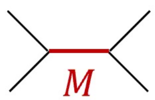
$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2$$

- In terms of A, the EFT derivative expansion is a Taylor series in $\{s,t\}$.

$$A_{\text{EFT}}(s,t) = \text{light physics} + \sum_{a,b} c_{ab}^{\text{EFT}} \frac{s^a t^b}{M^{2a+2b}}$$

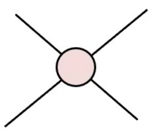
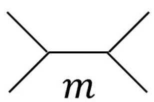
- *Simple example*: for tree-level exchange,

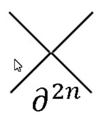
S_{UV}  $=$ 

$+$ 

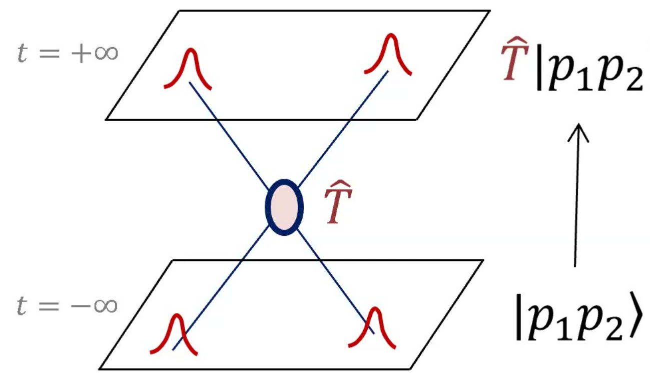
$= \frac{1}{m^2 - s} + \frac{1}{M^2 - s}$

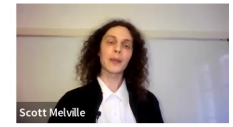
$\downarrow$

S_{EFT}  $=$ 

$+ \sum_n$ 

$= \frac{1}{m^2 - s} + \sum_n \frac{s^n}{M^{2n+2}}$





Scattering Amplitudes

- The probability amplitude to transition between 2-particle states,

$$\langle p_3 p_4 | \hat{T} | p_1 p_2 \rangle = A(s,t)\, \delta^4(p_1 + p_2 - p_3 - p_4)$$

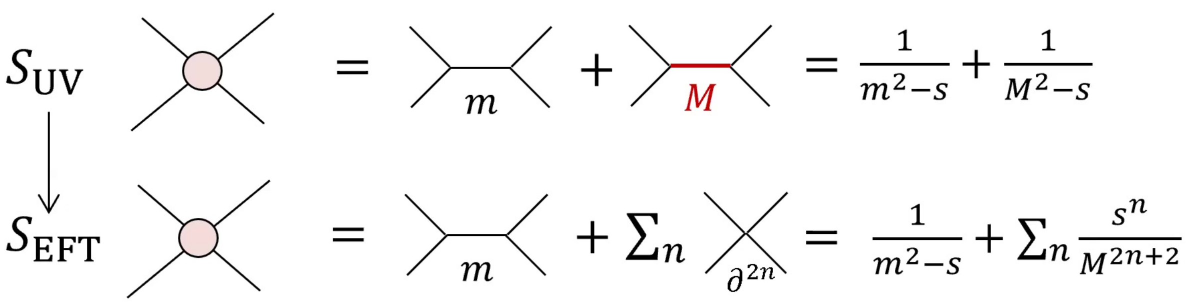
depends on only two Lorentz-invariant variables,

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2$$

- In terms of A, the EFT derivative expansion is a Taylor series in $\{s, t\}$.

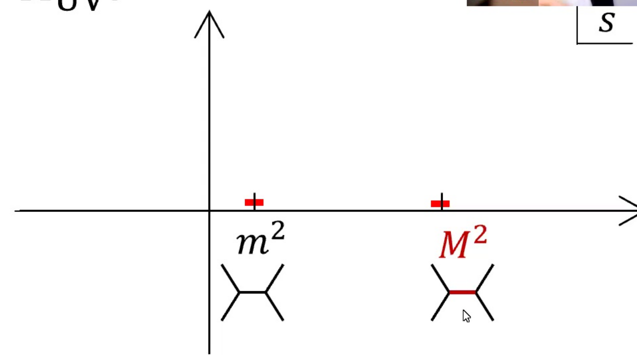
$$A_{\text{EFT}}(s,t) = \text{light physics} + \sum_{a,b} c_{ab}^{\text{EFT}} \frac{s^a t^b}{M^{2a+2b}}$$

- *Simple example*: for tree-level exchange,

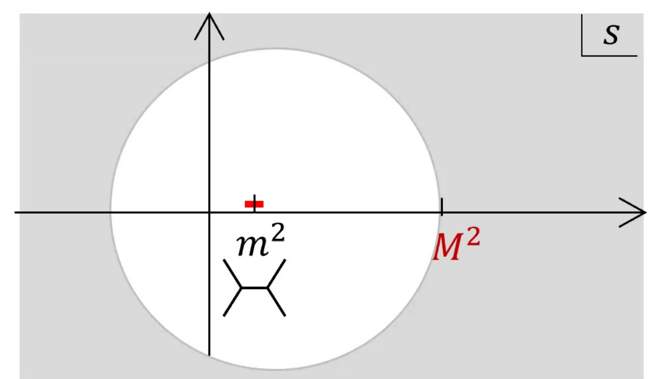


the EFT expansion replaces a pole at $s = M^2$ with infinite series which has finite radius of convergence at $s = M^2$.

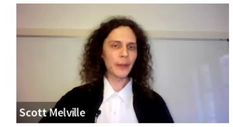
A_{UV}:



A_{EFT}:



Scattering Amplitudes



- *In general*: loops produce branch cuts as well as poles,

$$A_{\text{UV}}(s,t) = \text{light poles/cuts } + \text{heavy poles/cuts}$$

$$A_{\text{EFT}}(s,t) = \text{light poles/cuts} + \sum_{a,b} c_{ab}^{\text{EFT}} \frac{s^a t^b}{M^{2a+2b}}$$

Key observation #1:

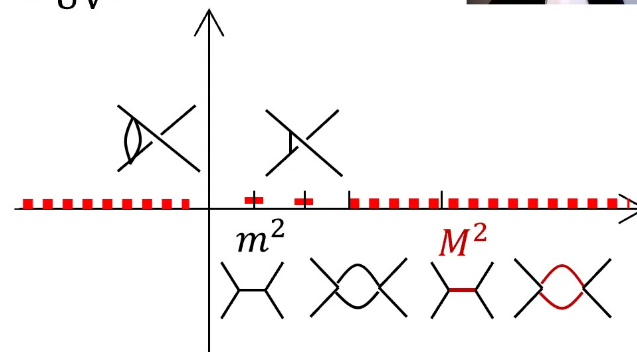
$$\boxed{\text{Causality} \;\; \Rightarrow \;\; A(s,t) \text{ is analytic for } \operatorname{Im} s \neq 0}$$

Proven rigorously for a massive scalar field. [Bremerman, Bros, Hepp, Kallen, Lehmann, Mandelstam, …, 1960s]

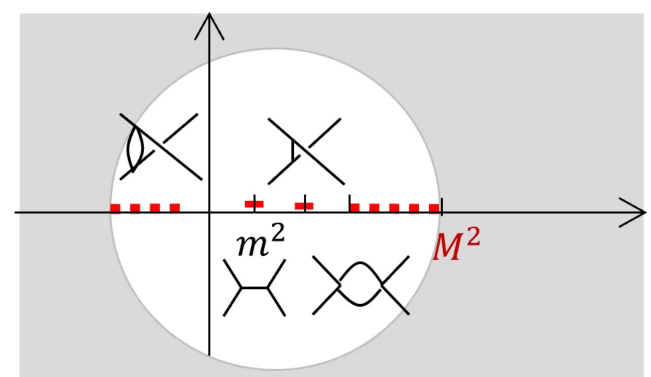
Beyond scalars, *assume* analytic structure is same as in perturbation theory,

Pert. th. $\Rightarrow$ $A(s,t)$ is analytic for all $\operatorname{Im} s \neq 0$ [Eden++, "*The Analytic S-Matrix*"]

A_{UV}:



A_{EFT}:



Scattering Amplitudes

- *In general*: loops produce branch cuts as well as poles,

$$A_{\text{UV}}(s,t) = \text{light poles/cuts} + \text{heavy poles/cuts}$$

$$A_{\text{EFT}}(s,t) = \text{light poles/cuts} + \sum_{a,b} c_{ab}^{\text{EFT}} \frac{s^a t^b}{M^{2a+2b}}$$

Key observation #1:

$$\boxed{\text{Causality} \;\Rightarrow\; A(s,t) \text{ is analytic for } \mathrm{Im}\, s \neq 0}$$

Proven rigorously for a massive scalar field. [Bremerman, Bros, Hepp, Kallen, Lehmann, Mandelstam, ..., 1960s]

Beyond scalars, *assume* analytic structure is same as in perturbation theory,

Pert. th. $\Rightarrow$ $A(s,t)$ is analytic for all $\mathrm{Im}\, s \neq 0$ [Eden++, "*The Analytic S-Matrix*"]

- Since EFT coefficients can be extracted via contour integration,

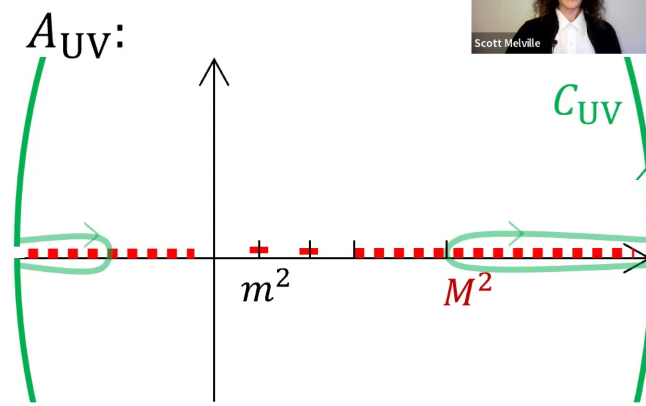
$$c_{ab}^{EFT} = \partial_t^b \int_{C_{\text{EFT}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{n+1}} \Big|_{t=0}$$

Cauchy's theorem gives rise to "sum rules" that connect UV/EFT:

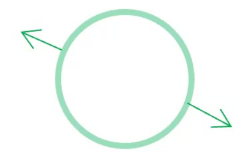
$$\boxed{c_{ab}^{EFT} = \partial_t^b \int_{C_{\text{UV}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{a+1}} \Big|_{t=0}}$$

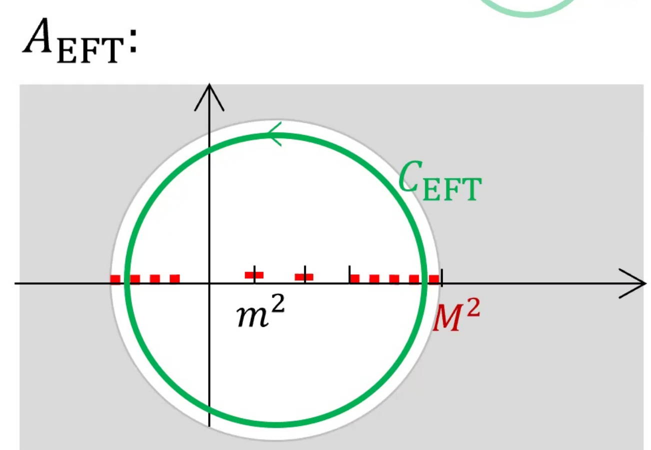
[Froissart, Martin, Weinberg, ..., >1960s]

[Pham, Truong, Pennington, ..., >1985]



|| Cauchy's theorem





Positivity Bounds

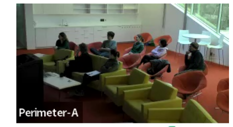
- The modern twist on these old ideas is to write the UV contour integral as an average over high-energy states.
- Explicitly: introduce $P, \mathcal{J}$ (translations/rotations in scattering plane) and define the average over 2-particle states:

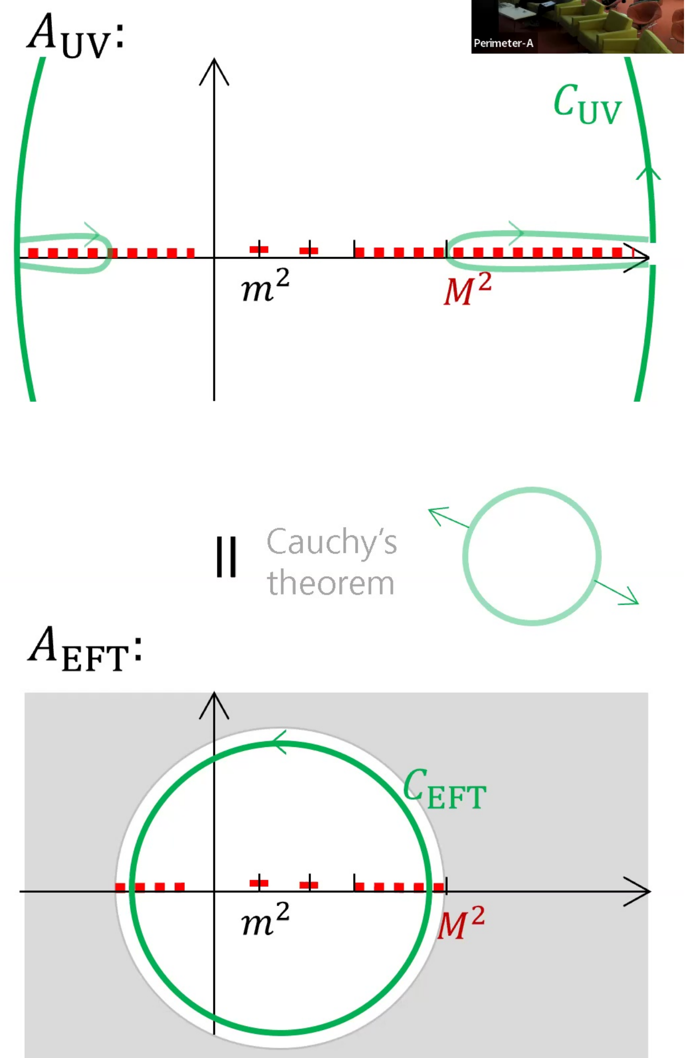
$$\left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2 > M^2} = \int_{M^2}^{\infty} \frac{ds}{s} \left\langle p_1 p_2 \left| \hat{T}^\dagger \, \frac{\mathcal{J}^{2b}}{P^{2a}} \, \hat{T} \right| p_1 p_2 \right\rangle$$

Key observation #2:

Unitary time evolution $\Rightarrow$ $\partial_t^b \int_{C_{\mathrm{UV}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{a+1}} \Big|_{t=0} \propto \left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2 > M^2}$

(see e.g. [Davighi+SM+You, 2108.06334])





Scattering Amplitudes

- *In general*: loops produce branch cuts as well as poles,

$$A_{\mathrm{UV}}(s,t) = \text{light poles/cuts } + \text{heavy poles/cuts}$$

$$A_{\mathrm{EFT}}(s,t) = \text{light poles/cuts} + \textstyle\sum_{a,b} c_{ab}^{\mathrm{EFT}} \frac{s^a t^b}{M^{2a+2b}}$$

Key observation #1:

$$\boxed{\text{Causality } \Rightarrow \ A(s,t) \text{ is analytic for } \mathrm{Im}\, s \neq 0}$$

Proven rigorously for a massive scalar field. [Bremerman, Bros, Hepp, Kallen, Lehmann, Mandelstam, ..., 1960s]

Beyond scalars, *assume* analytic structure is same as in perturbation theory,

Pert. th. $\Rightarrow$ $A(s,t)$ is analytic for all $\mathrm{Im}\, s \neq 0$ [Eden++, "*The Analytic S-Matrix*"]

- Since EFT coefficients can be extracted via contour integration,

$$c_{ab}^{EFT} = \partial_t^b \int_{C_{\mathrm{EFT}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{n+1}} \Big|_{t=0}$$

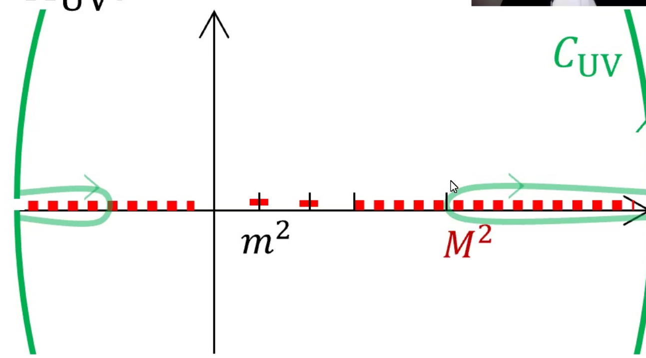
Cauchy's theorem gives rise to "sum rules" that connect UV/EFT:

$$\boxed{c_{ab}^{EFT} = \partial_t^b \int_{C_{\mathrm{UV}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{a+1}} \Big|_{t=0}}$$

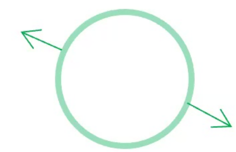
[Froissart, Martin, Weinberg, ..., >1960s]

[Pham, Truong, Pennington, ..., >1985]

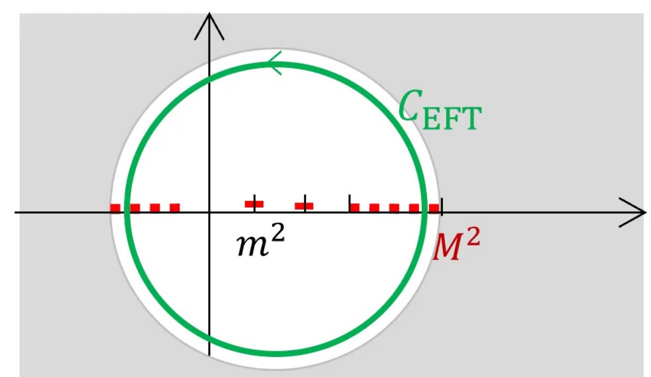
A_{UV}:



|| Cauchy's theorem



A_{EFT}:



Positivity Bounds

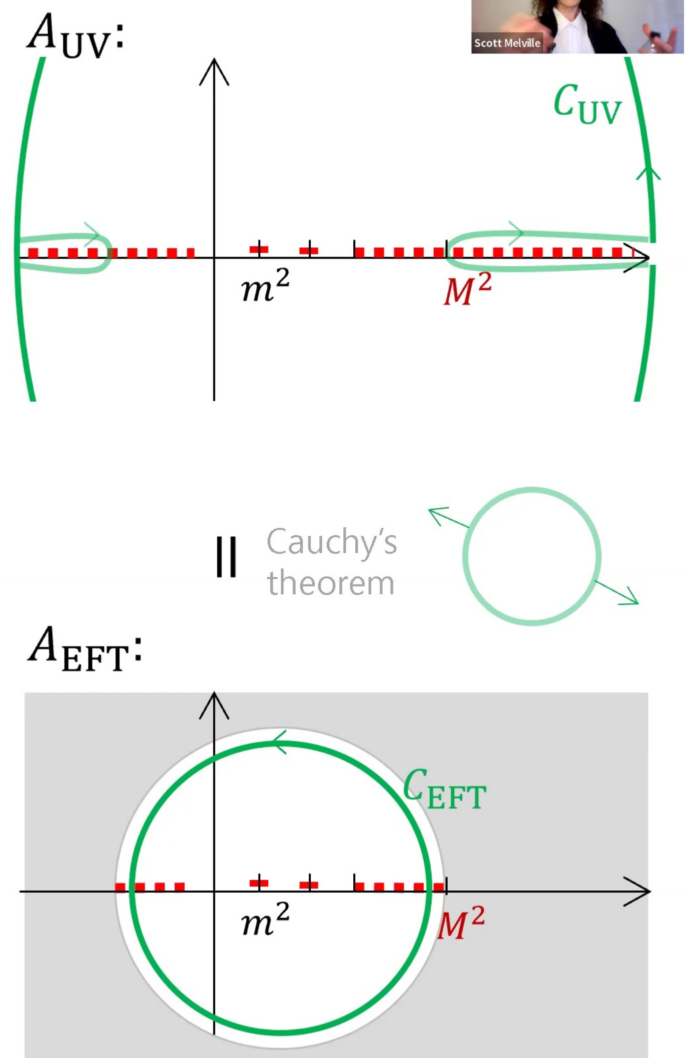
- The modern twist on these old ideas is to write the UV contour integral as an average over high-energy states.
- Explicitly: introduce $P, \mathcal{J}$ (translations/rotations in scattering plane) and define the average over 2-particle states:

$$\left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2 > M^2} = \int_{M^2}^{\infty} \frac{ds}{s} \left\langle p_1 p_2 \left| \hat{T}^\dagger \, \frac{\mathcal{J}^{2b}}{P^{2a}} \, \hat{T} \right| p_1 p_2 \right\rangle$$

Key observation #2:

Unitary time evolution $\Rightarrow$ $\partial_t^b \int_{C_{\mathrm{UV}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{a+1}} \Big|_{t=0} \propto \left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2 > M^2}$

(see e.g. [Davighi+SM+You, 2108.06334])



Positivity Bounds

- The modern twist on these old ideas is to write the UV contour integral as an average over high-energy states.
- Explicitly: introduce $P, \mathcal{J}$ (translations/rotations in scattering plane) and define the average over 2-particle states:

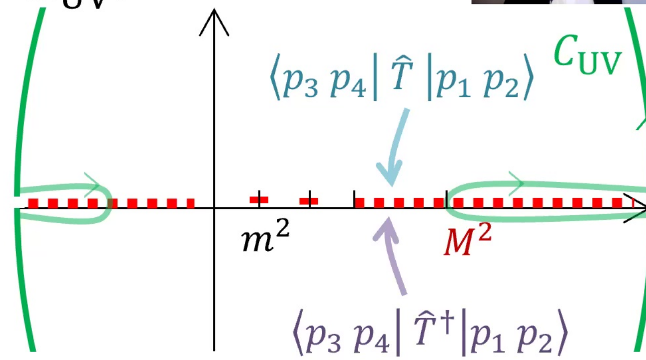
$$\left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2 > M^2} = \int_{M^2}^{\infty} \frac{ds}{s} \left\langle p_1 p_2 \left| \hat{T}^\dagger \, \frac{\mathcal{J}^{2b}}{P^{2a}} \, \hat{T} \right| p_1 p_2 \right\rangle$$

Key observation #2:

Unitary time evolution $\Rightarrow$ $\partial_t^b \int_{C_{\text{UV}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{a+1}} \Big|_{t=0} \propto \left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2 > M^2}$

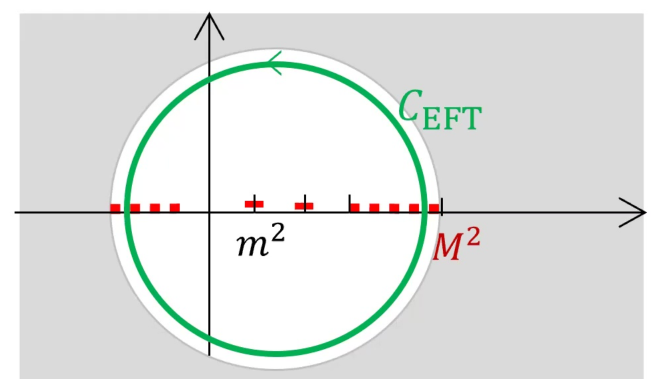
(see e.g. [Davighi+SM+You, 2108.06334])

A_{UV}:



$(\hat{1} + i\,\hat{T}$ Unitary $\Rightarrow$ $\hat{T} - \hat{T}^\dagger = i\,\hat{T}^\dagger \hat{T})$

A_{EFT}:



Positivity Bounds

- The modern twist on these old ideas is to write the UV contour integral as an average over high-energy states.
- Explicitly: introduce $P, \mathcal{J}$ (translations/rotations in scattering plane) and define the average over 2-particle states:

$$\left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2>M^2} = \int_{M^2}^{\infty} \frac{ds}{s} \left\langle p_1 p_2 \left| \hat{T}^\dagger \frac{\mathcal{J}^{2b}}{P^{2a}} \hat{T} \right| p_1 p_2 \right\rangle$$

Key observation #2:

Unitary time evolution $\Rightarrow$ $\partial_t^b \int_{C_{\text{UV}}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{a+1}} \Big|_{t=0} \propto \left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2>M^2}$

(see e.g. [Davighi+SM+You, 2108.06334])

- Causal + unitary UV physics implies that c_{ab}^{EFT} obey positivity bounds,

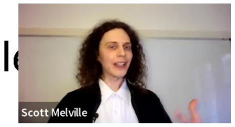
$$\frac{c_{20}^{\text{EFT}}}{M^4} = \left\langle \frac{1}{P^4} \right\rangle_{P^2>M^2} > 0$$

$$\frac{c_{40}^{\text{EFT}} - c_{20}^{\text{EFT}}}{M^8} = \left\langle \frac{P^4 - M^4}{P^8} \right\rangle_{P^2>M^2} > 0$$

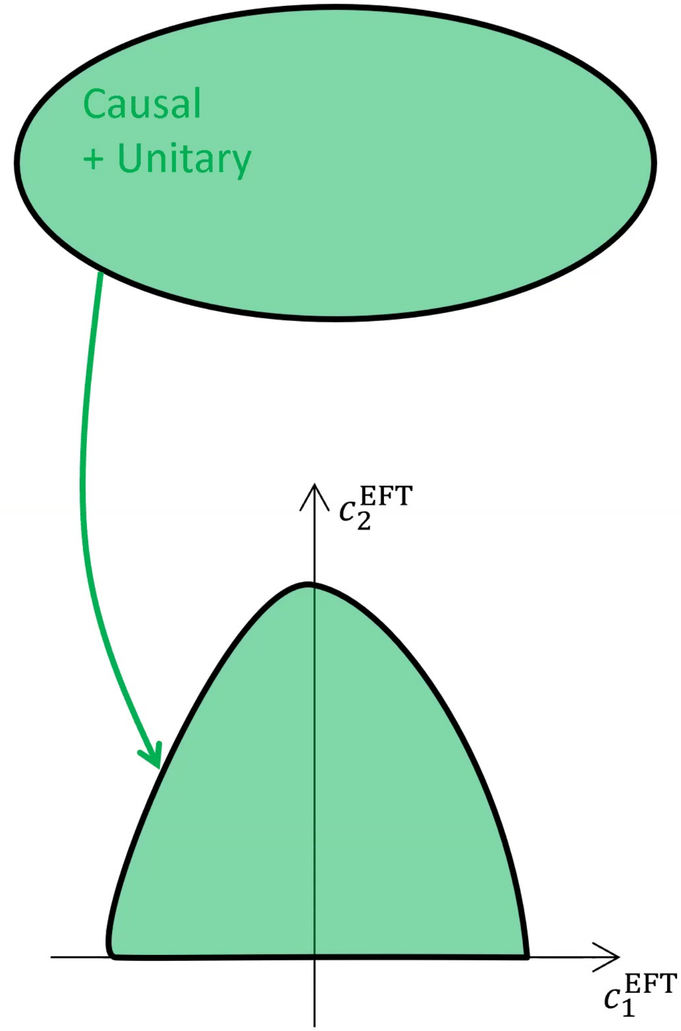
$$\frac{c_{21}^{\text{EFT}} + 3c_{20}^{\text{EFT}}}{M^6} = \left\langle \frac{\mathcal{J}^2 + 3(P^2 - M^2)}{P^6} \right\rangle_{P^2>M^2} > 0$$

[Adams++, 2006]
[de Rham+SM+Tolley+Zhou, 2017]
[Bellazzini++, 2021]
[Tolley+Zhou, 2021]
[Caron-Huot++, 2021]
⋮

- Many recent developments (e.g. Cauchy-Schwartz, null constraints, SDPB)



Space of possible UV theories



Low-energy (EFT) parameter space

Positivity Bounds

- The modern twist on these old ideas is to write the UV contour integral as an average over high-energy states.
- Explicitly: introduce $P, \mathcal{J}$ (translations/rotations in scattering plane) and define the average over 2-particle states:

$$\left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2>M^2} = \int_{M^2}^{\infty} \frac{ds}{s} \left\langle p_1 p_2 \left| \hat{T}^\dagger \, \frac{\mathcal{J}^{2b}}{P^{2a}} \, \hat{T} \right| p_1 p_2 \right\rangle$$

Key observation #2:

Unitary time evolution $\Rightarrow$ $\partial_t^b \int_{C_{\rm UV}} \frac{ds}{2\pi i} \frac{A(s,t)}{s^{a+1}} \Big|_{t=0} \propto \left\langle \frac{\mathcal{J}^{2b}}{P^{2a}} \right\rangle_{P^2>M^2}$

(see e.g. [Davighi+SM+You, 2108.06334])

- Causal + unitary UV physics implies that $c_{ab}^{\rm EFT}$ obey positivity bounds,

$$\frac{c_{20}^{\rm EFT}}{M^4} = \left\langle \frac{1}{P^4} \right\rangle_{P^2>M^2} > 0$$

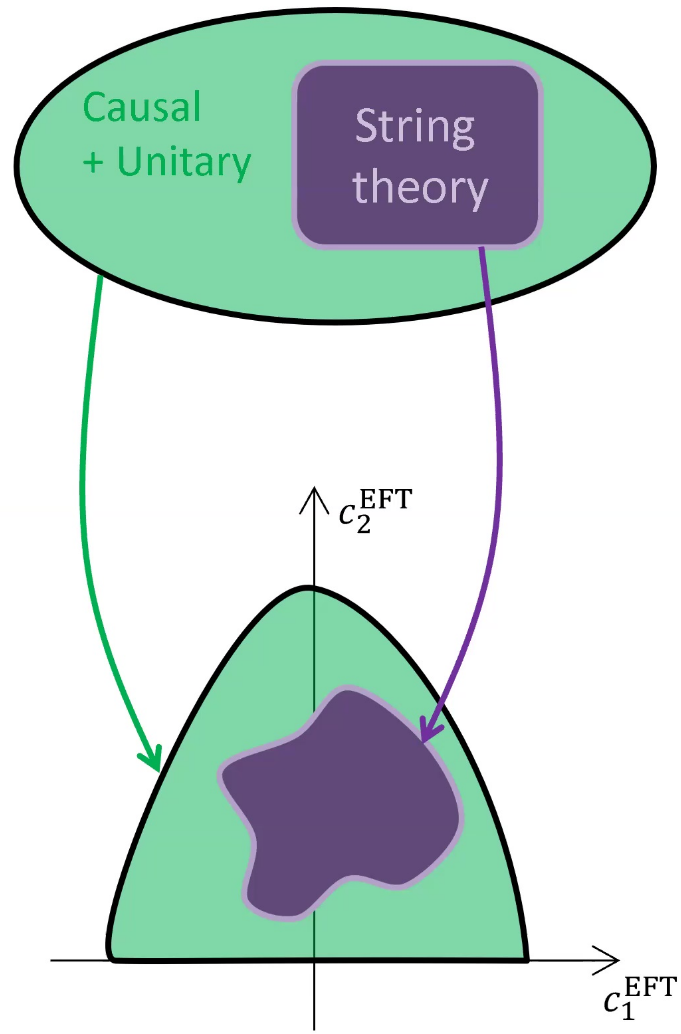
$$\frac{c_{40}^{\rm EFT} - c_{20}^{\rm EFT}}{M^8} = \left\langle \frac{P^4 - M^4}{P^8} \right\rangle_{P^2>M^2} > 0$$

$$\frac{c_{21}^{\rm EFT} + 3 c_{20}^{\rm EFT}}{M^6} = \left\langle \frac{\mathcal{J}^2 + 3(P^2 - M^2)}{P^6} \right\rangle_{P^2>M^2} > 0$$

[Adams++, 2006]
[de Rham+SM+Tolley+Zhou, 2017]
[Bellazzini++, 2021]
[Tolley+Zhou, 2021]
[Caron-Huot++, 2021]
⋮

- Many recent developments (e.g. Cauchy-Schwartz, null constraints, SDPB)

Space of possible UV theories

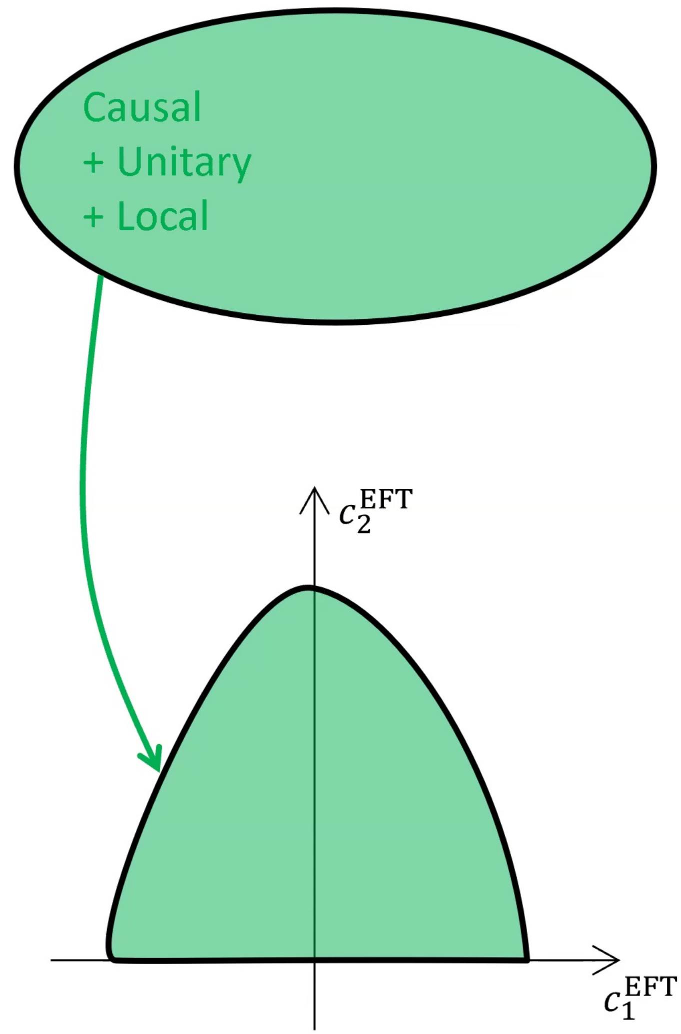


Low-energy (EFT) parameter space

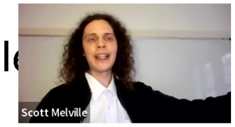
UV/EFT Correspondence

- Locality ensures that $\left\langle \frac{1}{P^{2n}} \right\rangle_{P^2 > M^2}$ converges for $n \geq 2$. [Froissart, 1960s]
- In asymp. free UV models ($\lim_{s\to\infty} A(s,t) = 0$), also converges for $n = 1$.

Space of possible UV theories



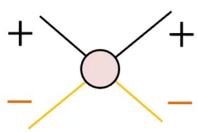
Low-energy (EFT) parameter space



UV/EFT Correspondence

- Locality ensures that $\left\langle \frac{1}{P^{2n}} \right\rangle_{P^2>M^2}$ converges for $n \geq 2$. [Froissart, 1960s]
- In asymp. free UV models ($\lim_{s\to\infty} A(s,t) = 0$), also converges for $n = 1$.

- Can probe this by scattering distinguishable fermions at low energies,

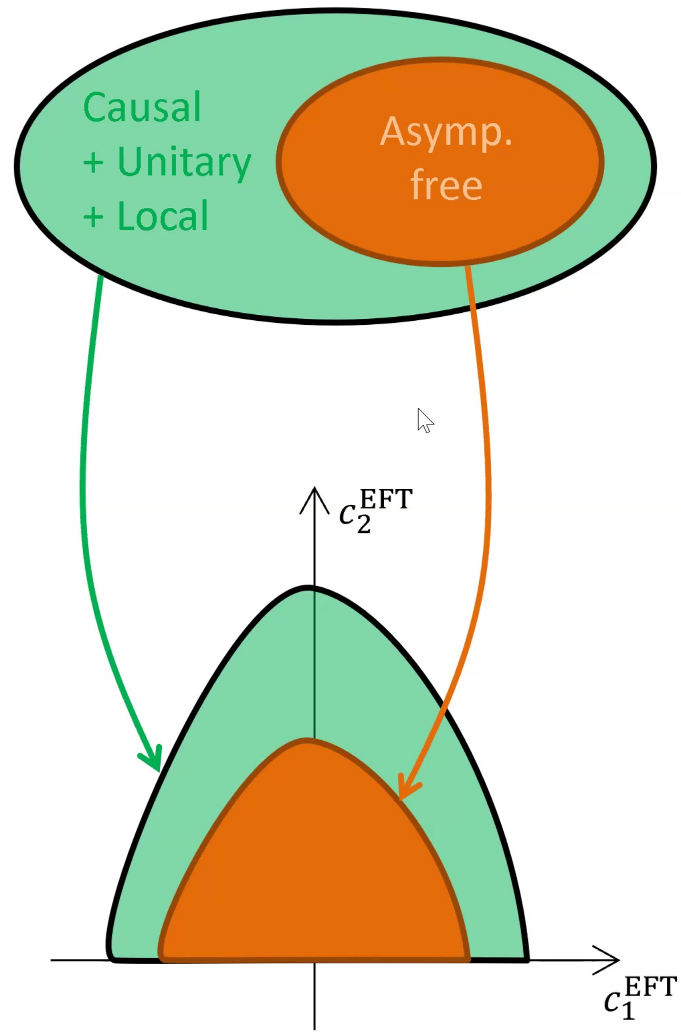


$$= c_{10}^{\text{EFT}} s + c_{01}^{\text{EFT}} t \quad \text{has} \quad c_{01}^{\text{EFT}} = \left\langle \frac{\mathcal{J}^2}{P^2} \right\rangle_{\substack{P^2>M^2 \\ \mathcal{J}\geq 0}} + \left\langle \frac{\mathcal{J}^2-1}{P^2} \right\rangle_{\substack{P^2>M^2 \\ \mathcal{J}\geq 1}}$$

Asymptotic freedom in the UV $\Rightarrow$ $c_{01}^{\text{EFT}} > 0$ in the EFT

[Davighi+SM+You, 2108.06334]

Space of possible UV theories

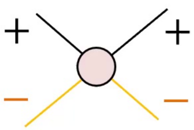


Low-energy (EFT) parameter space

UV/EFT Correspondence

- Locality ensures that $\left\langle \frac{1}{P^{2n}} \right\rangle_{P^2 > M^2}$ converges for $n \geq 2$. [Froissart, 1960s]
- In asymp. free UV models ($\lim_{s \to \infty} A(s,t) = 0$), also converges for $n = 1$.

- Can probe this by scattering distinguishable fermions at low energies,

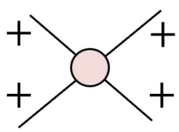


$$= c_{10}^{\text{EFT}} s + c_{01}^{\text{EFT}} t \quad \text{has} \quad c_{01}^{\text{EFT}} = \left\langle \frac{\mathcal{J}^2}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 0}} + \left\langle \frac{\mathcal{J}^2 - 1}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 1}}$$

Asymptotic freedom in the UV $\Rightarrow$ $c_{01}^{\text{EFT}} > 0$ in the EFT

[Davighi+SM+You, 2108.06334]

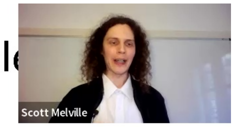
- Can also probe spin of UV states which mediate this scattering,



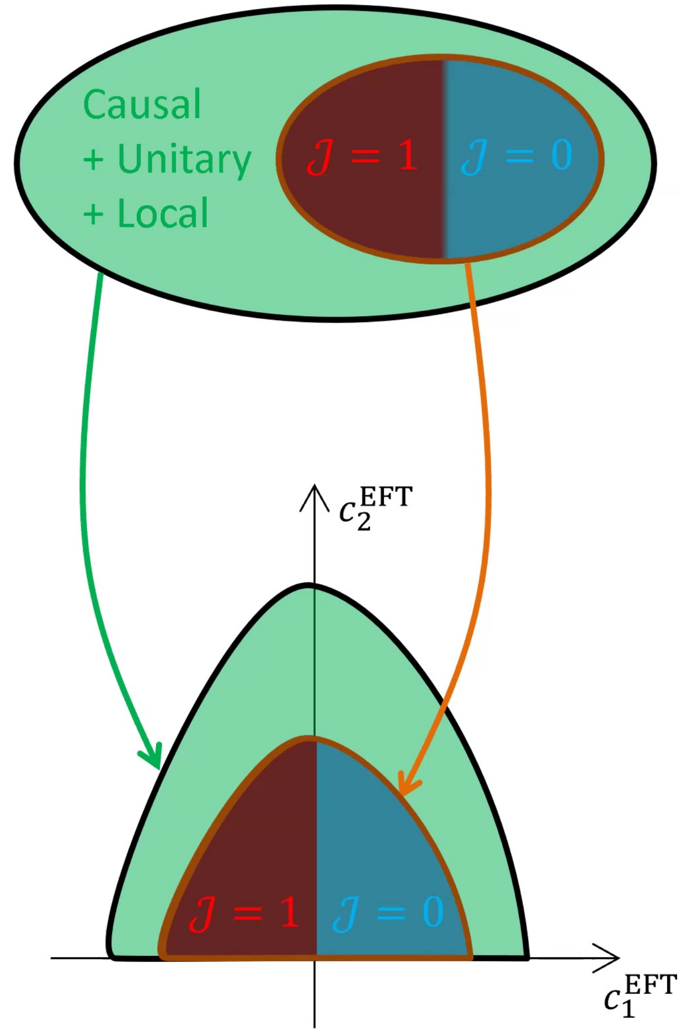
$$= c_1^{\text{EFT}} (s + t) \quad \text{has} \quad c_1^{\text{EFT}} = \left\langle \frac{1}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} = 0}} - \left\langle \frac{1}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} = 1}}$$

$\mathcal{J} = 0$ (or 1) dominates UV $\Rightarrow$ $c_1^{\text{EFT}} > 0$ (or < 0) in the EFT

[Remmen+Rodd, 2022]



Space of possible UV theories

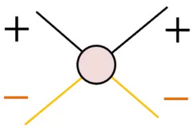


Low-energy (EFT) parameter space

UV/EFT Correspondence

- Locality ensures that $\left\langle \frac{1}{P^{2n}} \right\rangle_{P^2 > M^2}$ converges for $n \geq 2$. [Froissart, 1960s]
- In asymp. free UV models ($\lim_{s\to\infty} A(s,t) = 0$), also converges for $n = 1$.

- Can probe this by scattering distinguishable fermions at low energies,

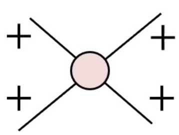


$$= c_{10}^{\text{EFT}} s + c_{01}^{\text{EFT}} t \quad \text{has} \quad c_{01}^{\text{EFT}} = \left\langle \frac{\mathcal{J}^2}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 0}} + \left\langle \frac{\mathcal{J}^2 - 1}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 1}}$$

Asymptotic freedom in the UV $\Rightarrow$ $c_{01}^{\text{EFT}} > 0$ in the EFT

[Davighi+SM+You, 2108.06334]

- Can also probe spin of UV states which mediate this scattering,



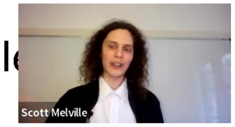
$$= c_1^{\text{EFT}}(s+t) \quad \text{has} \quad c_1^{\text{EFT}} = \left\langle \frac{1}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} = 0}} - \left\langle \frac{1}{P^2} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} = 1}}$$

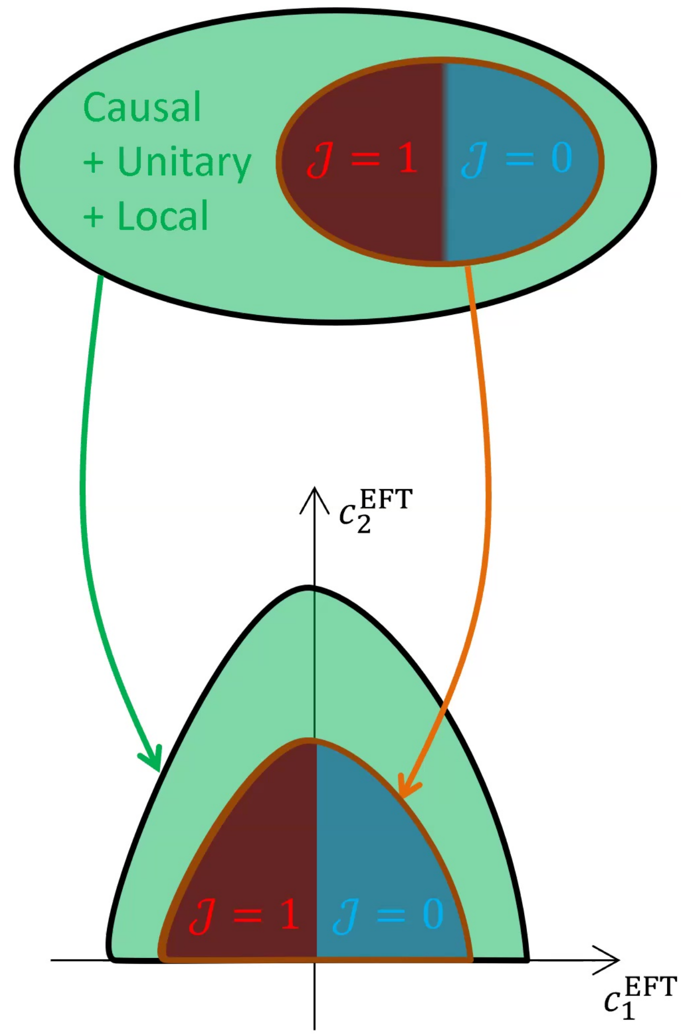
$\mathcal{J} = 0$ (or 1) dominates UV $\Rightarrow$ $c_1^{\text{EFT}} > 0$ (or < 0) in the EFT

[Remmen+Rodd, 2022]

> *Conclusion*: Sign of EFT coefficients can probe properties such as the mass/spin/high-energy growth of the underlying UV theory!

Space of possible UV theories





Low-energy (EFT) parameter space

A Dark Energy Example

The Problem with Cosmology

- The previous UV/EFT relations all required time translation invariance:

$$\langle \mathcal{O} \rangle = \int {}_{\text{future}}\langle p_1 p_2 | \hat{T}^\dagger \, \mathcal{O} \, \hat{T} | p_1 p_2 \rangle_{\text{past}} \quad \Rightarrow \quad \text{Need } |\Omega\rangle_{\text{past}} = |\Omega\rangle_{\text{future}} \text{ else not positive.}$$

- How can they be applied in cosmology, where the background breaks this symmetry?

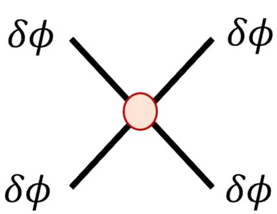
- **Simplest solution**: consider an S_{EFT} with both Minkowski and cosmological solutions.

$$\boxed{S_{\text{EFT}}[\phi] = S_{\text{GR}} + c_1 \delta S_1[\phi] + c_2 \delta S_2[\phi] + \cdots}$$

$\phi = 0 + \delta\phi$

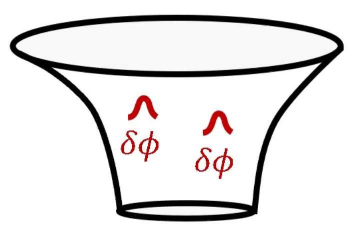
$\phi = \phi_{\text{cosmo}}(t) + \delta\phi$

We can compute A on Minkowski spacetime



$\Rightarrow$ Positivity bounds on the c_i

We can observe $\delta\phi$ on an expanding spacetime



$\Rightarrow$ Observational constraints on c_i

A Dark Energy Example

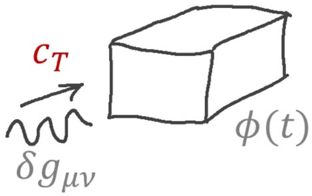
- Consider the scalar-tensor EFT ("quartic Horndeski"), $\left(X = -\frac{1}{2}(\nabla_\mu \phi)^2\right)$

$$\mathcal{L}_{\text{EFT}} = G_4(X)R + P(X) + G_4{}'(X)\left(\left(\nabla_\mu \nabla_\nu \phi\right)^2 - \left(\nabla_\mu \nabla^\mu \phi\right)^2\right)$$

[Horndeski, 1974]

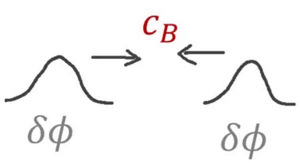
- ϕ describes dark energy, and modifies ΛCDM in two key ways:

(i)



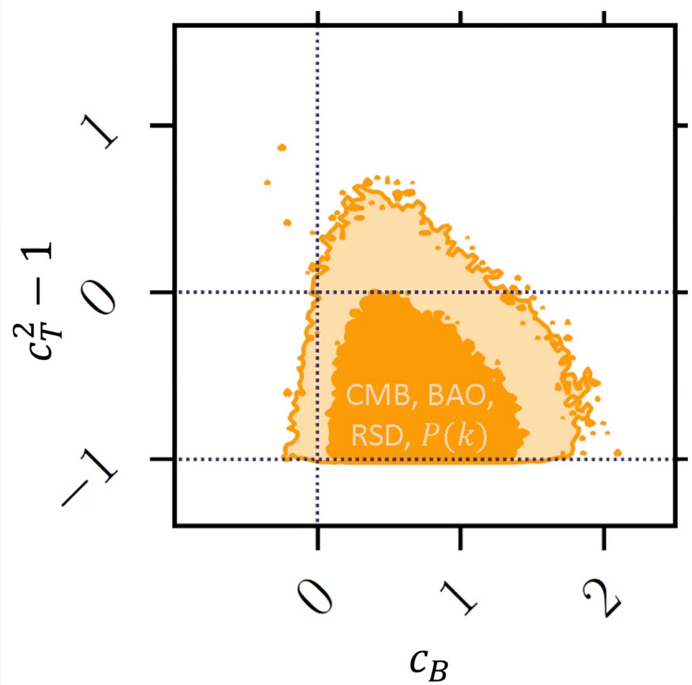
$G_4{}'$ determines c_T
(low-energy GW speed)

(ii)



$G_4{}''$ determines c_B
(DE clustering)

[Bellini+Sawicki 2014]



A Dark Energy Example

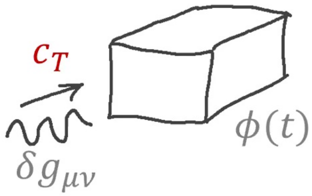
- Consider the scalar-tensor EFT (“quartic Horndeski”), $\left(X = -\frac{1}{2}(\nabla_\mu \phi)^2\right)$

$$\mathcal{L}_{\mathrm{EFT}} = G_4(X)R + P(X) + G_4{}'(X)\left(\left(\nabla_\mu \nabla_\nu \phi\right)^2 - \left(\nabla_\mu \nabla^\mu \phi\right)^2\right)$$

[Horndeski, 1974]

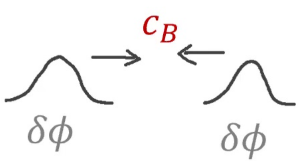
- ϕ describes dark energy, and modifies ΛCDM in two key ways:

(i)



$G_4{}'$ determines c_T (low-energy GW speed)

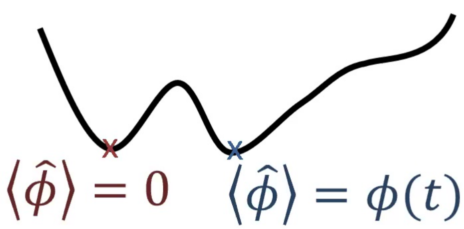
(ii)



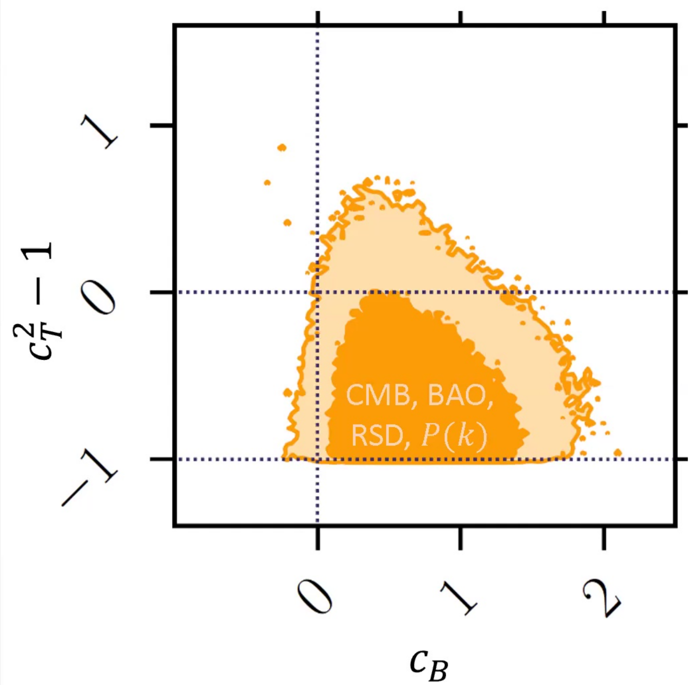
$G_4{}''$ determines c_B (DE clustering)

[Bellini+Sawicki 2014]

- $P(X)$ determines allowed vacua for ϕ, and typically* allows for both flat and cosmological solutions.

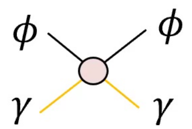
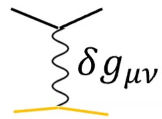


*with some notable exceptions, e.g. “ghost condensate” models

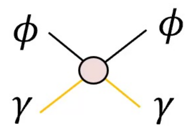


A Dark Energy Example

- Scattering about $\langle\phi\rangle = 0$ gives Lorentz-invariant amplitudes,

 $=$  $+\cdots \quad = G_4'\, s^2 + \cdots$

- Comparing this with pheno on cosmological $\langle\phi\rangle = \phi(t)$,

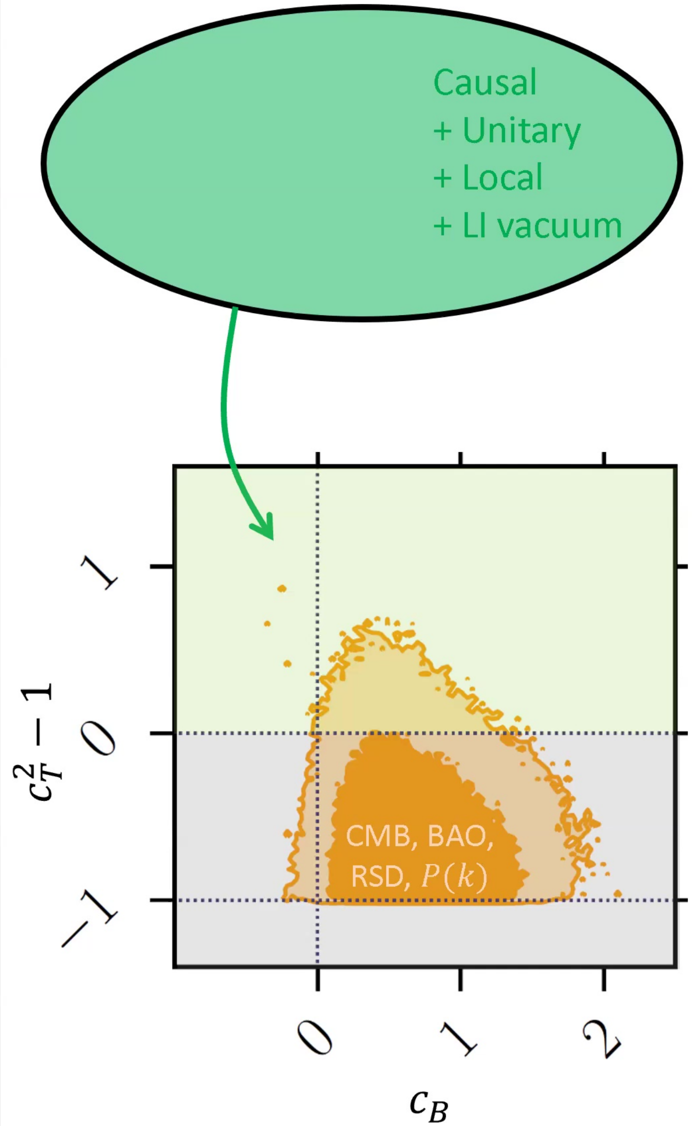
$$c_T^2 - 1 \;\propto G_4' + \mathcal{O}(XG_4'') = \partial_s^2$$  $$= \left\langle \frac{1}{P^4} \right\rangle_{P^2 > M^2}$$

- So in any UV model which is causal + unitary + local + has LI vacuum,

$$\boxed{c_T^2 - 1 \;\propto \left\langle \frac{1}{P^4} \right\rangle_{P^2 > M^2} > 0}$$

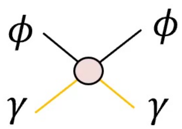
[de Rham+SM+Noller, 2103.06855]

Space of possible UV theories

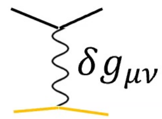


A Dark Energy Example

- Scattering about $\langle \phi \rangle = 0$ gives Lorentz-invariant amplitudes,



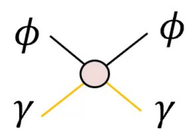
$=$



$+ \cdots \quad = G_4' \, s^2 + \cdots$

- Comparing this with pheno on cosmological $\langle \phi \rangle = \phi(t)$,

$c_T^2 - 1 \; \propto G_4' + \mathcal{O}(X G_4'') = \partial_s^2$



$= \left\langle \frac{1}{P^4} \right\rangle_{P^2 > M^2}$

- So in any UV model which is causal + unitary + local + has LI vacuum,

$$\boxed{c_T^2 - 1 \; \propto \left\langle \frac{1}{P^4} \right\rangle_{P^2 > M^2} > 0}$$

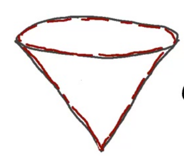
[de Rham+SM+Noller, 2103.06855]

- Low-energy GWs must propagate superluminally!

Note: LIGO/Virgo measures c_T at high $\omega \sim 100\text{Hz} \approx M$

[de Rham+SM 1806.09417]

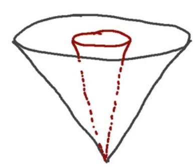
S_{UV}



$c_T = c_{\text{Matter}} = 1$

$\downarrow$

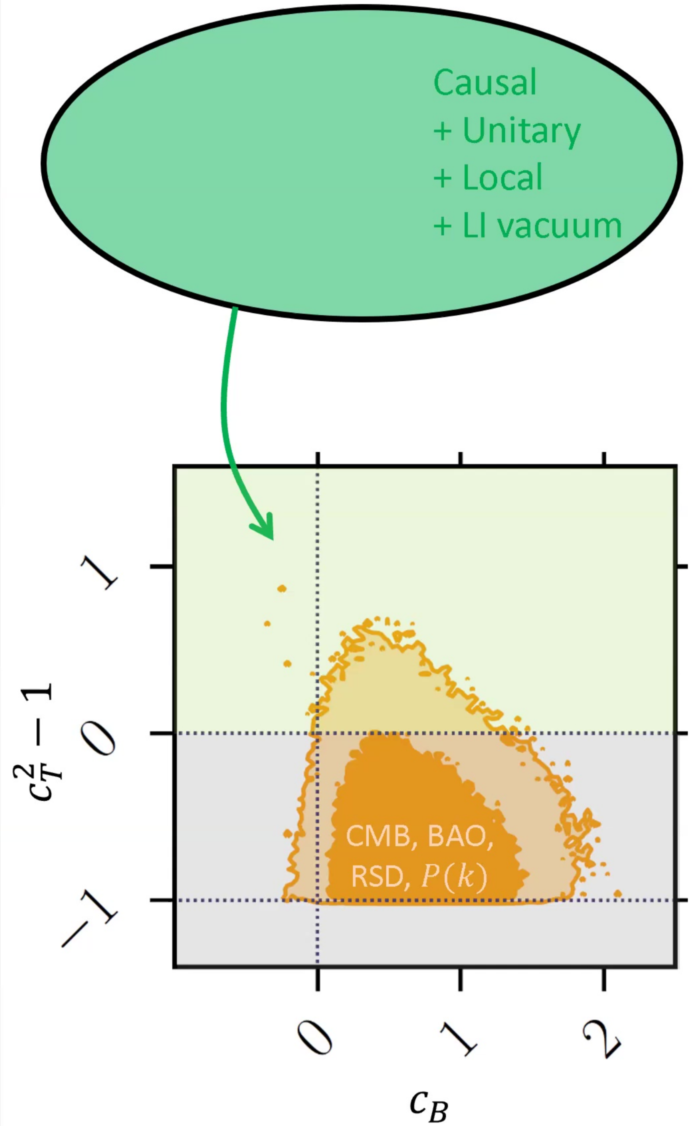
S_{EFT}

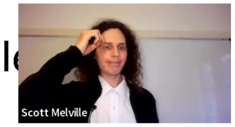


$c_T > c_{\text{Matter}} = 1$

[de Rham+Tolley, 2021]

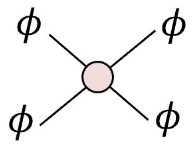
Space of possible UV theories



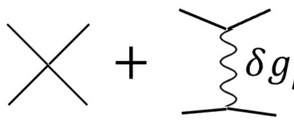


A Dark Energy Example

- Scattering about $\langle\phi\rangle = 0$ gives Lorentz-invariant amplitudes,



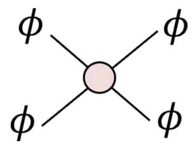
$$= \quad$$



$$+ \ \delta g_{\mu\nu} + \cdots = -\left(G_4'' + \frac{G_4'^2}{G_4} \right) s^2 t + \cdots$$

- Comparing this with pheno on cosmological $\langle\phi\rangle = \phi(t)$,

$$\frac{c_T^2 - 1}{c_T^2} - \frac{1}{2}c_B \propto -\left(G_4'' + \frac{G_4'^2}{G_4} \right) = \partial_s^2 \partial_t$$



$$= \left\langle \frac{\mathcal{J}^2}{P^6} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 0}}$$

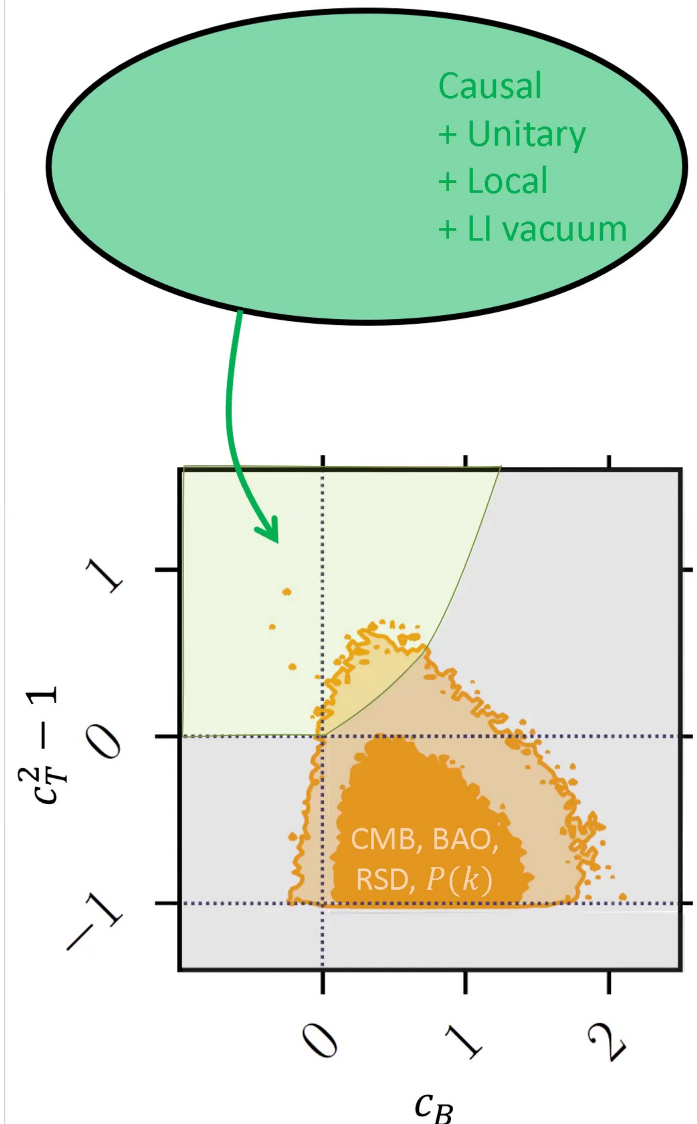
- So in any UV model which is causal + unitary + local + has LI vacuum,

$$\boxed{\begin{aligned} c_T^2 - 1 &\propto \left\langle \frac{1}{P^4} \right\rangle_{P^2 > M^2} > 0 \\ \frac{c_T^2 - 1}{c_T^2} - \frac{1}{2}c_B &\propto \left\langle \frac{\mathcal{J}^2}{P^6} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 0}} > 0 \end{aligned}}$$

[de Rham+SM+Noller, 2103.06855]

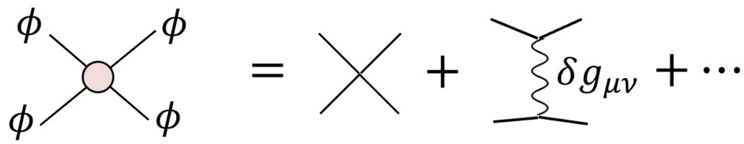
[SM+Noller, 1904.05874]

Space of possible UV theories



A Dark Energy Example

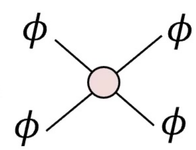
- Scattering about $\langle\phi\rangle = 0$ gives Lorentz-invariant amplitudes,



$$= -\left(G_4'' + \frac{{G_4'}^2}{G_4}\right) s^2 t + \cdots$$

- Comparing this with pheno on cosmological $\langle\phi\rangle = \phi(t)$,

$$\frac{c_T^2-1}{c_T^2} - \frac{1}{2}c_B \propto -\left(G_4'' + \frac{{G_4'}^2}{G_4}\right) = \partial_s^2 \partial_t$$



$$= \left\langle \frac{\mathcal{J}^2}{P^6} \right\rangle_{\substack{P^2>M^2 \\ \mathcal{J}\geq 0}}$$

- So in any UV model which is causal + unitary + local + has LI vacuum,

$$c_T^2 - 1 \propto \left\langle \frac{1}{P^4} \right\rangle_{P^2>M^2} > 0$$

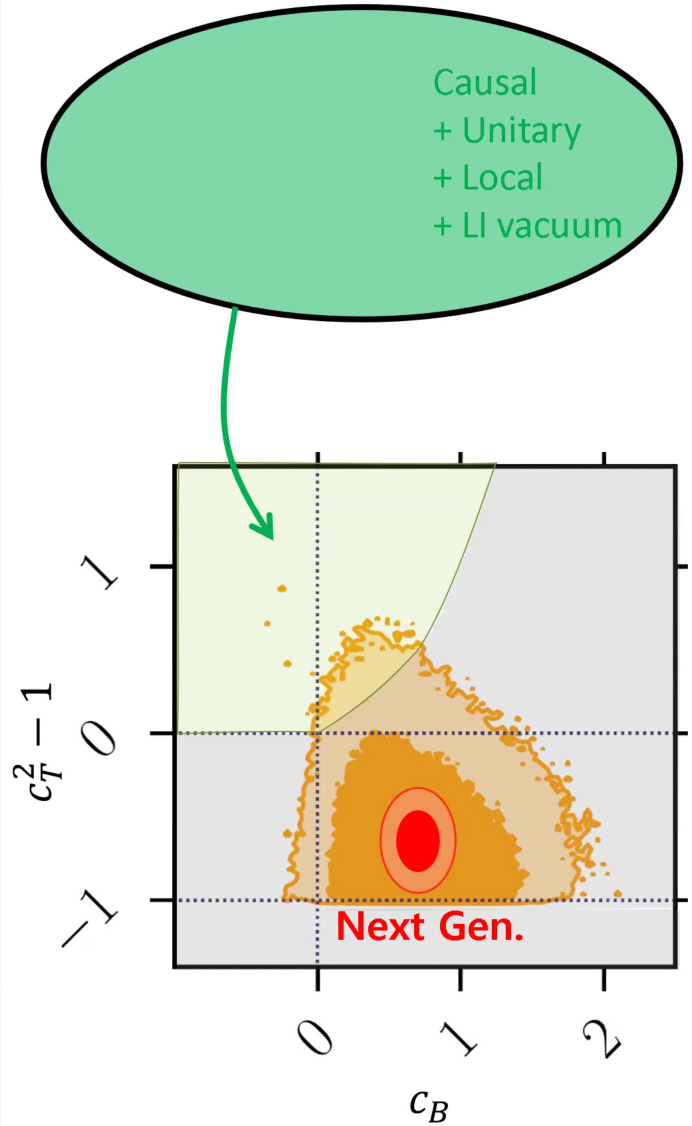
[de Rham+SM+Noller, 2103.06855]

$$\frac{c_T^2-1}{c_T^2} - \frac{1}{2}c_B \propto \left\langle \frac{\mathcal{J}^2}{P^6} \right\rangle_{\substack{P^2>M^2 \\ \mathcal{J}\geq 0}} > 0$$

[SM+Noller, 1904.05874]

- Next generation experiments forecast $\mathcal{O}(10)$ improvements.

Space of possible UV theories



A Dark Energy Example

- Scattering about $\langle \phi \rangle = 0$ gives Lorentz-invariant amplitudes,

$$\phi\phi \to \phi\phi \;=\; \times \;+\; \delta g_{\mu\nu} + \cdots = -\left(G_4'' + \frac{G_4'^2}{G_4} \right) s^2 t + \cdots$$

- Comparing this with pheno on cosmological $\langle \phi \rangle = \phi(t)$,

$$\frac{c_T^2 - 1}{c_T^2} - \frac{1}{2} c_B \propto -\left(G_4'' + \frac{G_4'^2}{G_4} \right) = \partial_s^2 \partial_t \, (\phi\phi \to \phi\phi) = \left\langle \frac{\mathcal{J}^2}{P^6} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 0}}$$

- So in any UV model which is causal + unitary + local + has LI vacuum,

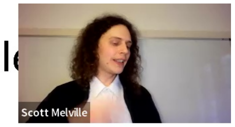
$$c_T^2 - 1 \propto \left\langle \frac{1}{P^4} \right\rangle_{P^2 > M^2} > 0$$

[de Rham+SM+Noller, 2103.06855]

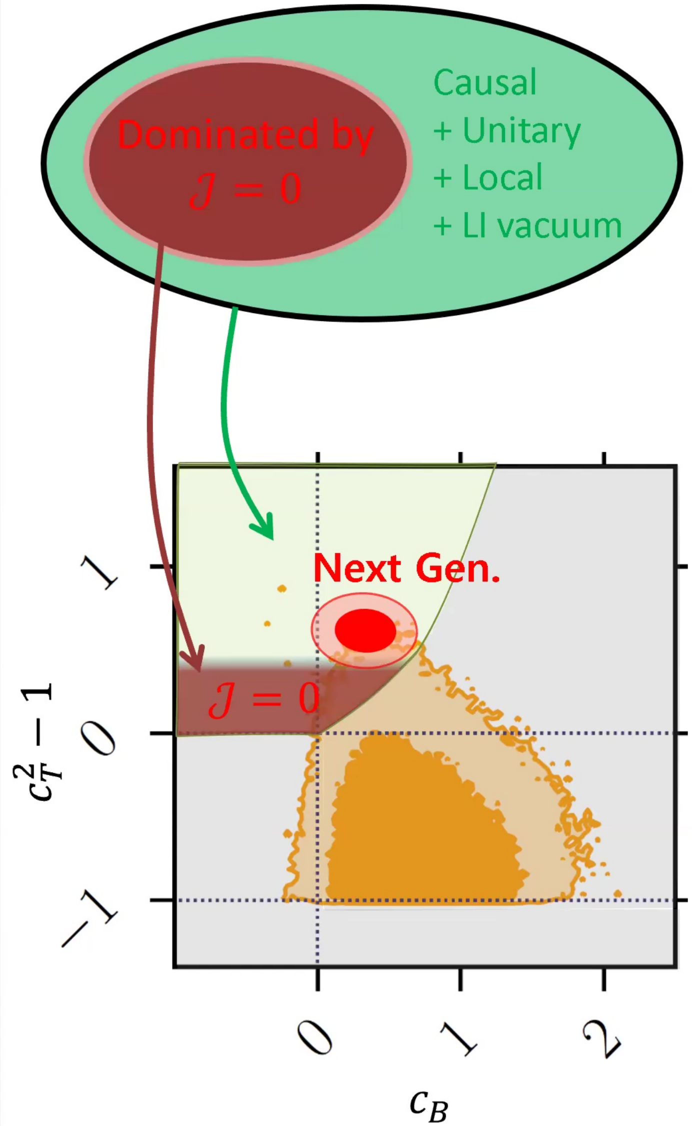
$$\frac{c_T^2 - 1}{c_T^2} - \frac{1}{2} c_B \propto \left\langle \frac{\mathcal{J}^2}{P^6} \right\rangle_{\substack{P^2 > M^2 \\ \mathcal{J} \geq 0}} > 0$$

[SM+Noller, 1904.05874]

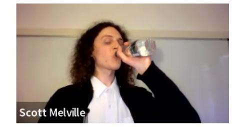
- Next generation experiments forecast $\mathcal{O}(10)$ improvements.
- Since size of $c_T^2 - 1$ is linked to the dominant $\mathcal{J}$ in these averages, these experiments can now give valuable info about the UV physics!



Space of possible UV theories



An Inflationary Example



Subhorizon Scattering

- On inflationary (quasi-de Sitter) spacetimes, boosts and time translation symmetry are broken at very different energy scales:

$$\Lambda_{\text{time trans}} \ll \Lambda_{\text{boosts}}$$

[Grall+SM, 2005.02366]

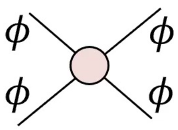
- In "subhorizon" regime, only boosts are broken,

Subhorizon $\Leftrightarrow$ $\Lambda_{\text{time trans}} \ll \omega \ll \Lambda_{\text{boosts}}$

- Propagation of particles in then determined by an effective metric,

$$\mathcal{L}_{\text{free}} = \dot{\phi}^2 - c_s^2 (\partial_i \phi)^2 =: \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

and scattering amplitudes now depend on 3 additional variables,

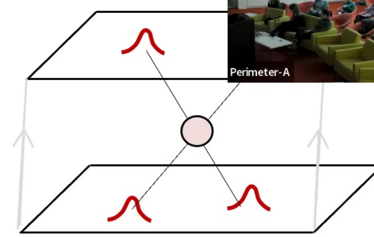


$= A(\tilde{s}, \tilde{t}, \omega_1, \omega_2, \omega_3)$ where $\tilde{s} = \tilde{g}^{\mu\nu}(p_1+p_2)_\mu (p_1+p_2)_\nu$ (and $\omega_4 = \omega_1 + \omega_2 - \omega_3$)

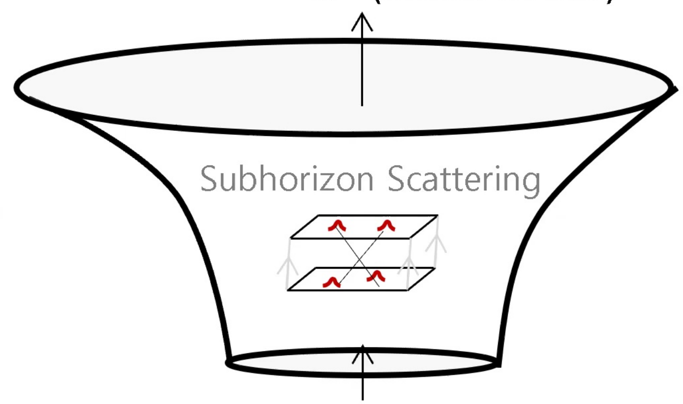
$\omega \gg \Lambda_{\text{boosts}}$

$\Downarrow$

$A(s,t)$



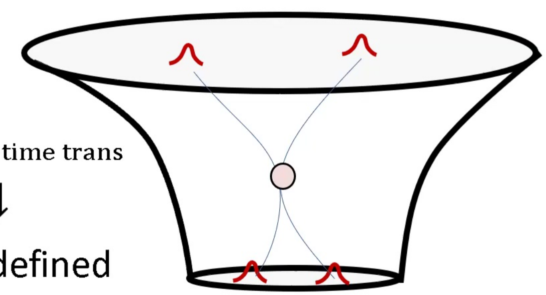
n^μ (breaks boosts)



$\omega \ll \Lambda_{\text{time trans}}$

$\Downarrow$

A ill-defined



Subhorizon Scattering

- On inflationary (quasi-de Sitter) spacetimes, boosts and time translation symmetry are broken at very different energy scales:

$$\Lambda_{\text{time trans}} \ll \Lambda_{\text{boosts}}$$

[Grall+SM, 2005.02366]

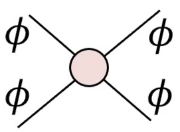
- In “subhorizon” regime, only boosts are broken,

$$\text{Subhorizon} \quad \Leftrightarrow \quad \Lambda_{\text{time trans}} \ll \omega \ll \Lambda_{\text{boosts}}$$

- Propagation of particles in then determined by an effective metric,

$$\mathcal{L}_{\text{free}} = \dot{\phi}^2 - c_s^2(\partial_i\phi)^2 =: \tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

and scattering amplitudes now depend on 3 additional variables,



$$= A(\tilde{s}, \tilde{t}, \omega_1, \omega_2, \omega_3) \quad \text{where} \quad \tilde{s} = \tilde{g}^{\mu\nu}(p_1+p_2)_\mu(p_1+p_2)_\nu$$

(and $\omega_4 = \omega_1 + \omega_2 - \omega_3$)

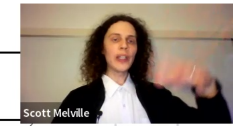
- The sum rules are analogous to Lorentz-invariant scattering:

Causality $\Rightarrow$ A is analytic for $\text{Im}\,\tilde{s} > 0$ and $c_s > 1$

Pert. th. $\Rightarrow$ A is analytic for $\text{Im}\,\tilde{s} \neq 0$ and all c_s

[Grall+SM 2102.05683]

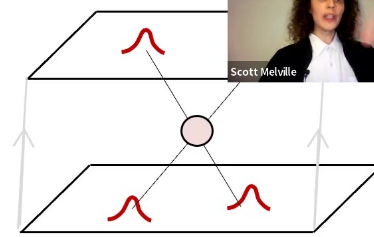
Cauchy thm $\Rightarrow$ $\boxed{c_n^{\text{EFT}} = \int_{C_{\text{UV}}} \frac{d\tilde{s}}{2\pi i}\frac{A}{\tilde{s}^{n+1}} = \left\langle \frac{1}{(\tilde{g}_{\mu\nu}P^\mu P^\nu)^n} \right\rangle_{\tilde{g}_{\mu\nu}P^\mu P^\nu > M^2}}$

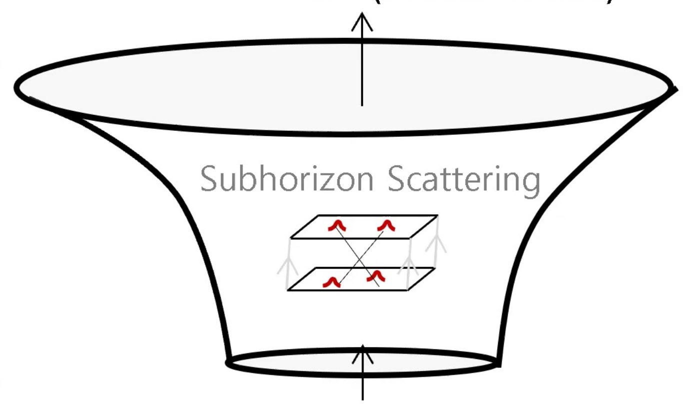


$\omega \gg \Lambda_{\text{boosts}}$

$\Downarrow$

$A(s,t)$

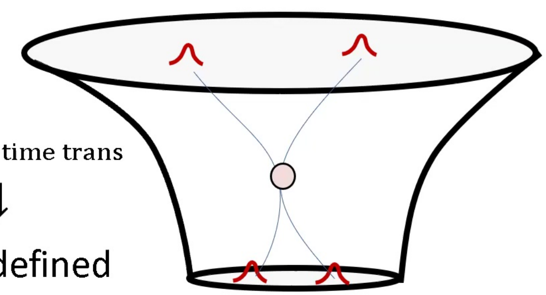


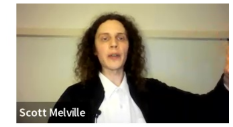


$\omega \ll \Lambda_{\text{time trans}}$

$\Downarrow$

A ill-defined





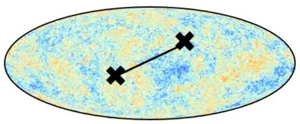
An Inflationary Example

[Cheung++ 2007]

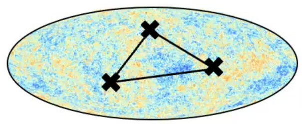
- EFT for single-field inflation contains a scalar $\pi(x)$,

$$\mathcal{L}_{\text{EFT}} = f_\pi^4 \left(\dot{\pi}^2 - c_s^2 (\partial_i \pi)^2 + \alpha_1 \, \dot{\pi}^3 + \beta_1 \, \dot{\pi}^4 + \begin{array}{c}\text{terms fixed}\\ \text{by symmetry}\end{array} \right)$$

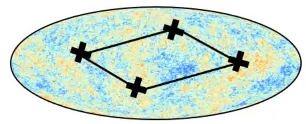
- The EFT coefficients control primordial CMB correlations,



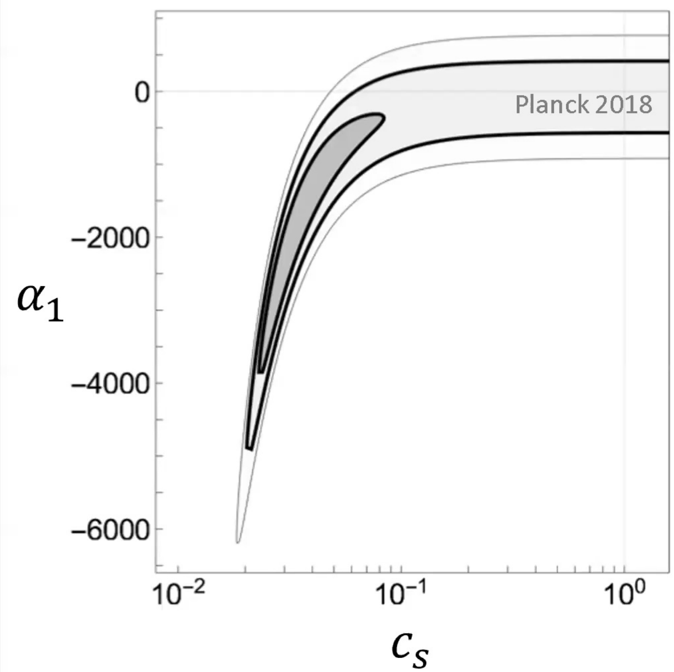
$f_\pi \approx 60H$



α_1, c_s



β_1, α_1, c_s



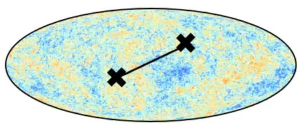
An Inflationary Example

[Cheung++ 2007]

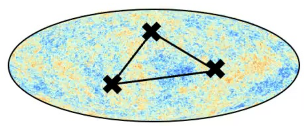
- EFT for single-field inflation contains a scalar $\pi(x)$,

$$\mathcal{L}_{\text{EFT}} = f_\pi^4 \left(\dot{\pi}^2 - c_s^2 (\partial_i \pi)^2 + \alpha_1 \dot{\pi}^3 + \beta_1 \dot{\pi}^4 + \text{terms fixed by symmetry} \right)$$

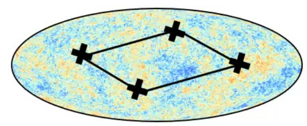
- The EFT coefficients control primordial CMB correlations,



$f_\pi \approx 60H$



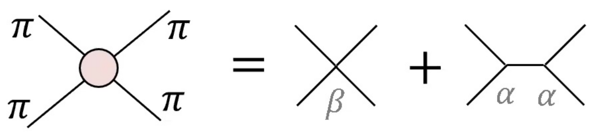
α_1, c_s



β_1, α_1, c_s

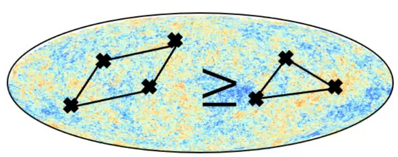
[Grall+SM, 2102.05683]
(also [Baumann++, 1502.07304])

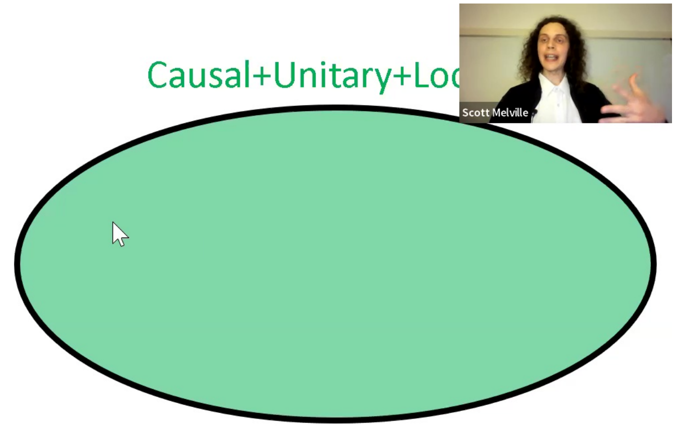
- The tree-level scattering amplitude,

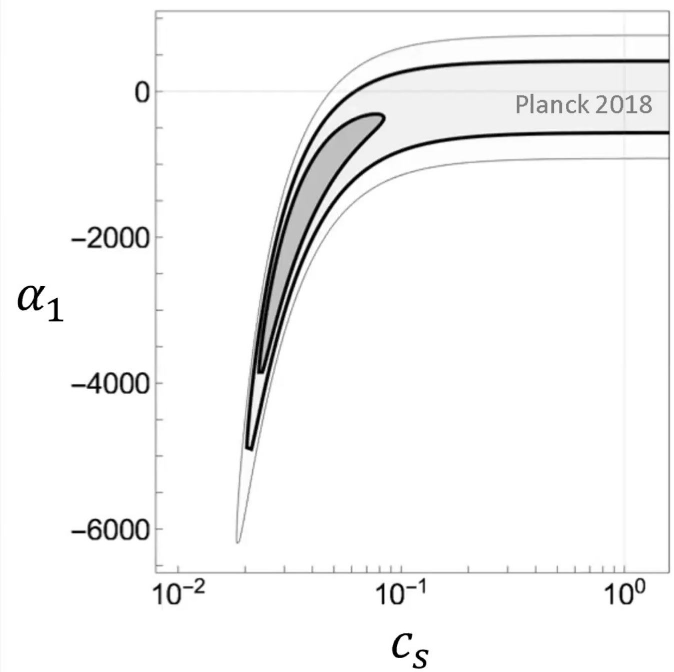


places a positivity bound on the CMB 4-point correlation,

$$\beta_1 - \frac{3}{2}\alpha_1^2 + 2\alpha_1 + \frac{1}{3}\frac{1-c_s^2}{c_s^4} = \left\langle \frac{1}{\left(\tilde{g}_{\mu\nu}P^\mu P^\nu\right)^2} \right\rangle_{\tilde{g}_{\mu\nu}P^\mu P^\nu > M^2} > 0$$







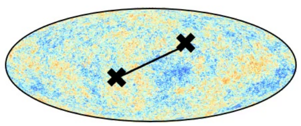
An Inflationary Example

[Cheung++ 2007]

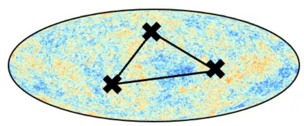
- EFT for single-field inflation contains a scalar $\pi(x)$,

$$\mathcal{L}_{\text{EFT}} = f_\pi^4 \left(\dot{\pi}^2 - c_s^2 (\partial_i \pi)^2 + \alpha_1 \dot{\pi}^3 + \beta_1 \dot{\pi}^4 + \text{terms fixed by symmetry} \right)$$

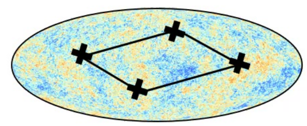
- The EFT coefficients control primordial CMB correlations,



$f_\pi \approx 60H$



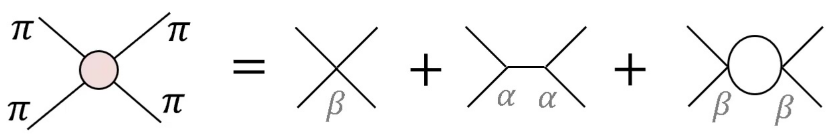
α_1, c_s



β_1, α_1, c_s

[Grall+SM, 2102.05683]
(also [Baumann++, 1502.07304])

- The one-loop scattering amplitude,



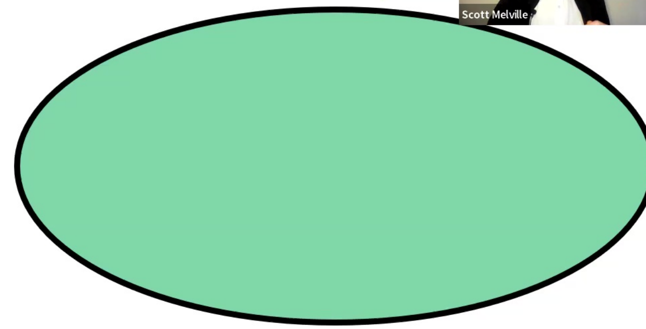
places a positivity bound on the CMB 3-point correlation!

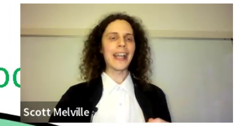
$$\beta_1 - \tfrac{3}{2}\alpha_1^2 + 2\alpha_1 + \tfrac{1}{3}\tfrac{1-c_s^2}{c_s^4} - \frac{M^4}{\pi^2 f_\pi^4}\beta_1^2 = \left\langle \frac{1}{\left(\tilde{g}_{\mu\nu}P^\mu P^\nu\right)^2} \right\rangle_{\tilde{g}_{\mu\nu}P^\mu P^\nu > M^2} > 0$$

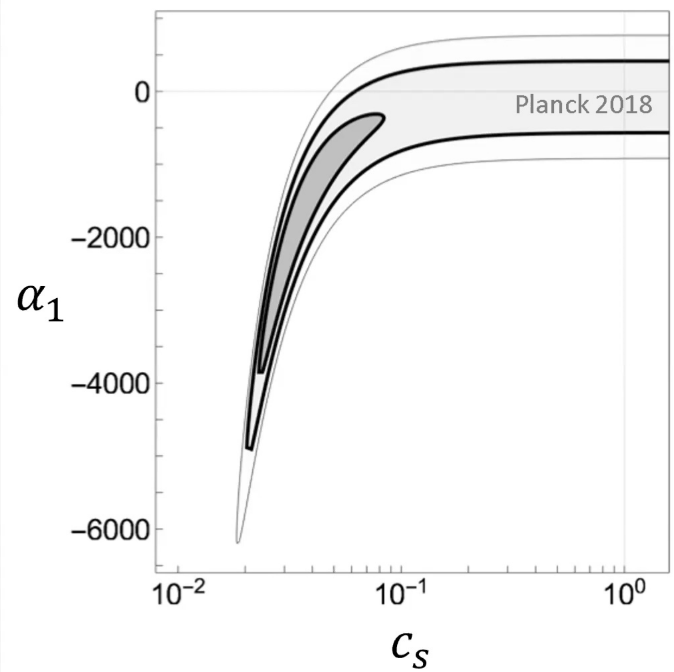
$$\boxed{\Rightarrow \quad \frac{M^4}{f_\pi^4}\left(1 - c_s^2 + \tfrac{3}{2}\alpha_1 c_s^2\right) \geq -30\pi^2 c_s^4}$$

Connects EFT coefficients to mass of UV physics

Causal+Unitary+Loc







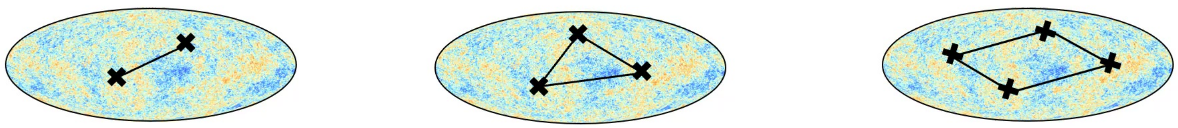
An Inflationary Example

[Cheung++ 2007]

- EFT for single-field inflation contains a scalar $\pi(x)$,

$$\mathcal{L}_{\text{EFT}} = f_\pi^4 \left(\dot{\pi}^2 - c_s^2 (\partial_i \pi)^2 + \alpha_1 \dot{\pi}^3 + \beta_1 \dot{\pi}^4 + \text{terms fixed by symmetry} \right)$$

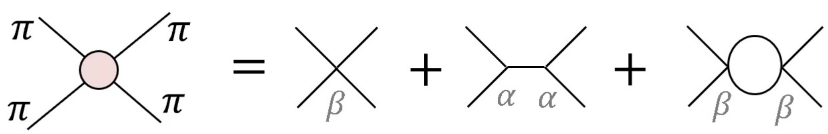
- The EFT coefficients control primordial CMB correlations,



$f_\pi \approx 60H$ α_1, c_s β_1, α_1, c_s

[Grall+SM, 2102.05683]
(also [Baumann++, 1502.07304])

- The one-loop scattering amplitude,

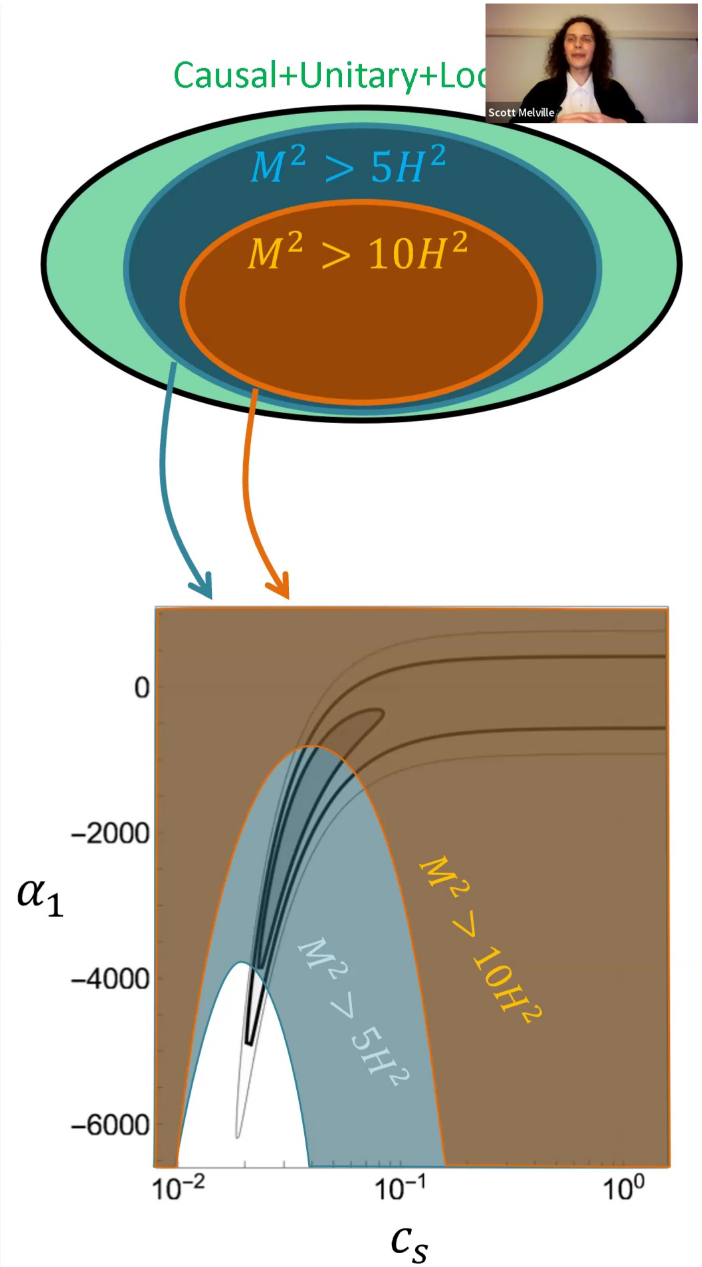


places a positivity bound on the CMB 3-point correlation!

$$\beta_1 - \frac{3}{2}\alpha_1^2 + 2\alpha_1 + \frac{1}{3}\frac{1-c_s^2}{c_s^4} - \frac{M^4}{\pi^2 f_\pi^4}\beta_1^2 = \left\langle \frac{1}{\left(\tilde{g}_{\mu\nu}P^\mu P^\nu\right)^2} \right\rangle_{\tilde{g}_{\mu\nu}P^\mu P^\nu > M^2} > 0$$

$$\boxed{\Rightarrow \quad \frac{M^4}{f_\pi^4}\left(1 - c_s^2 + \frac{3}{2}\alpha_1 c_s^2\right) \geq -30\pi^2 c_s^4}$$

Connects EFT coefficients to mass of UV physics



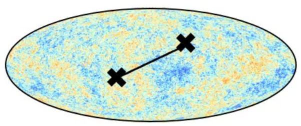
An Inflationary Example

[Cheung++ 2007]

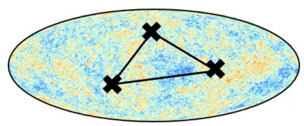
- EFT for single-field inflation contains a scalar $\pi(x)$,

$$\mathcal{L}_{\text{EFT}} = f_\pi^4 \left(\dot{\pi}^2 - c_s^2 (\partial_i \pi)^2 + \alpha_1 \dot{\pi}^3 + \beta_1 \dot{\pi}^4 + \begin{array}{c}\text{terms fixed}\\ \text{by symmetry}\end{array} \right)$$

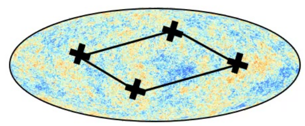
- The EFT coefficients control primordial CMB correlations,



$f_\pi \approx 60H$



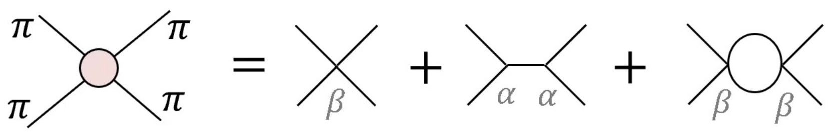
α_1, c_s



β_1, α_1, c_s

[Grall+SM, 2102.05683]
(also [Baumann++, 1502.07304])

- The one-loop scattering amplitude,

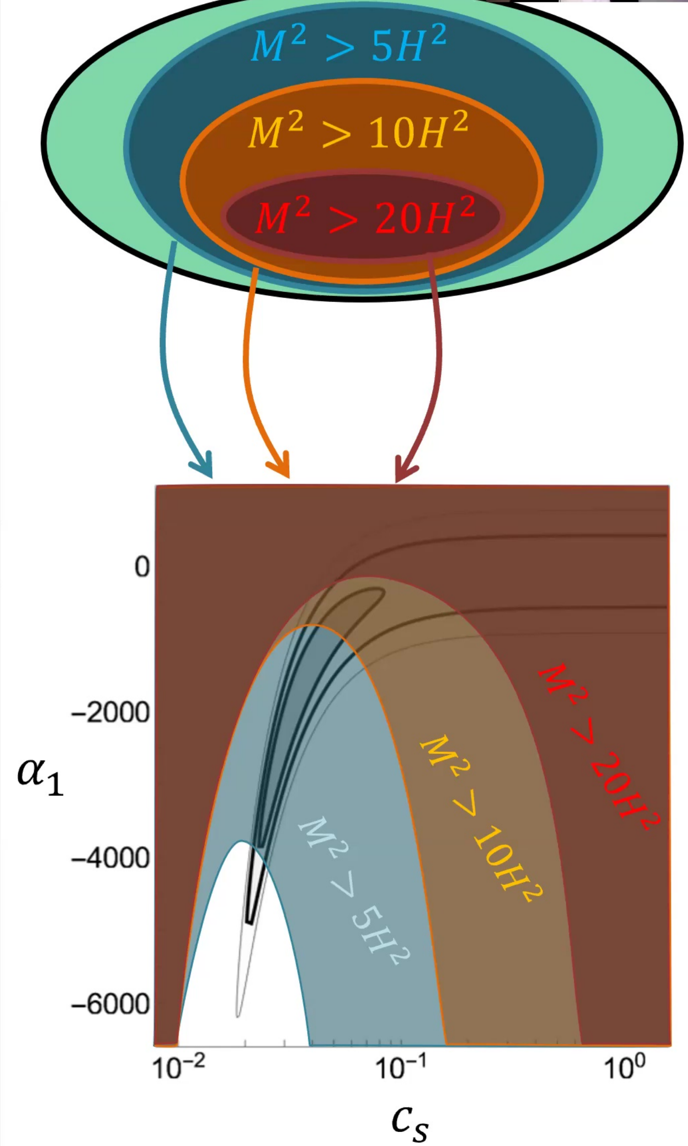


places a positivity bound on the CMB 3-point correlation!

$$\beta_1 - \frac{3}{2}\alpha_1^2 + 2\alpha_1 + \frac{1}{3}\frac{1-c_s^2}{c_s^4} - \frac{M^4}{\pi^2 f_\pi^4}\beta_1^2 = \left\langle \frac{1}{\left(\tilde{g}_{\mu\nu}P^\mu P^\nu\right)^2} \right\rangle_{\tilde{g}_{\mu\nu}P^\mu P^\nu > M^2} > 0$$

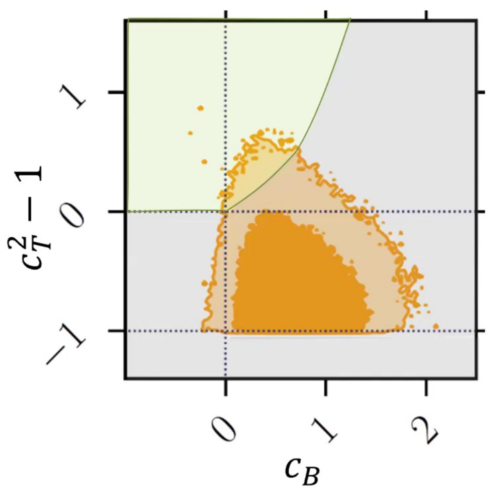
$$\boxed{\Rightarrow \quad \frac{M^4}{f_\pi^4}\left(1 - c_s^2 + \frac{3}{2}\alpha_1 c_s^2\right) \geq -30\pi^2 c_s^4}$$

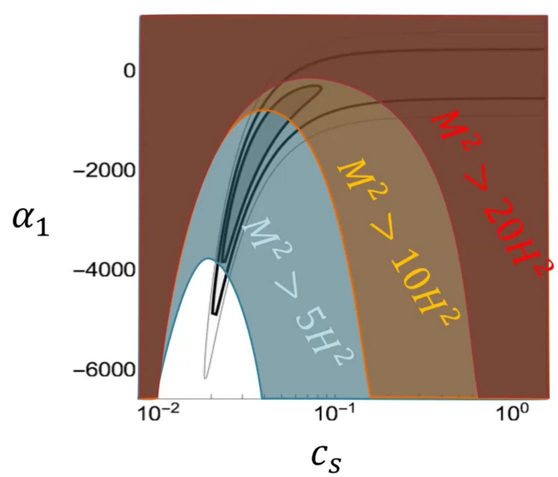
Connects EFT coefficients to mass of UV physics



Summary

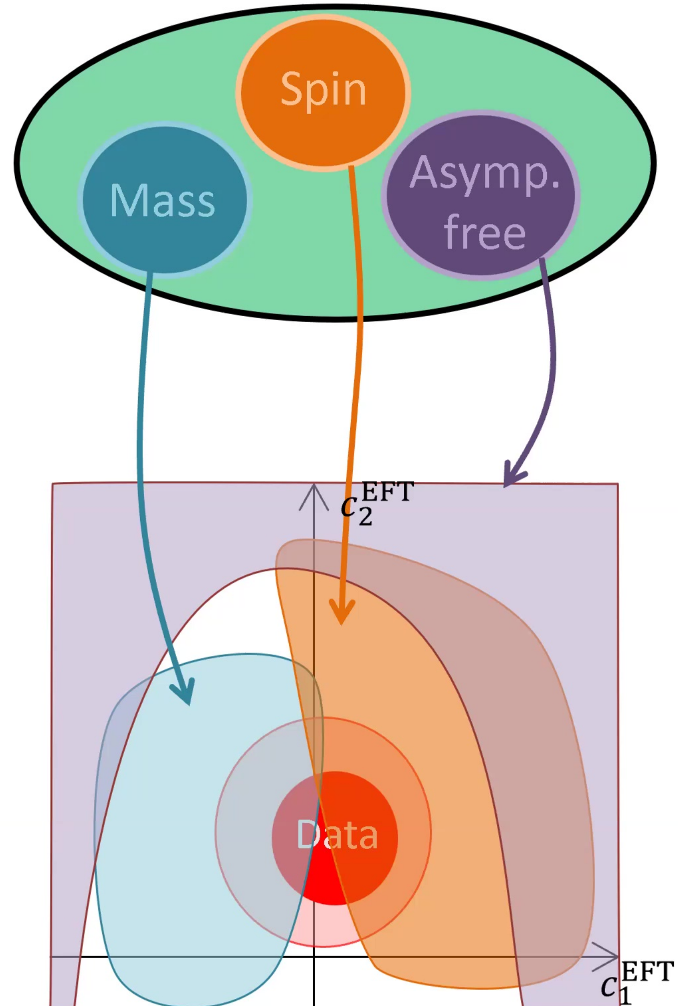
- EFTs increasingly used to analyze experimental data.
- Amplitudes can connect EFT coefficients with properties of the underlying UV physics (e.g. mass/spin/high-energy growth)
- These techniques are very powerful on Minkowski spacetime, and we are beginning to find ways to apply them more broadly.
- For instance,
  - scattering around different vacua $\Rightarrow$ dark energy bounds
  - scattering on subhorizon scales $\Rightarrow$ inflation bounds





- Still many open questions to explore…

Properties of underl[…] high-energy (UV) ph[…]



Low-energy (EFT) parameter space