Title: Quantum entropy thermalization

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Series: Quantum Matter

Date: February 21, 2023 - 3:30 PM

URL: https://pirsa.org/23020059

Abstract: In an isolated quantum many-body system undergoing unitary evolution, the entropy of a subsystem (smaller than half the system size) thermalizes if at long times, it is to leading order equal to the thermodynamic entropy of the subsystem at the same energy. We prove entropy thermalization for a nearly integrable Sachdev-Ye-Kitaev model initialized in a pure product state. The model is obtained by adding random all-to-all 4-body interactions as a perturbation to a random free-fermion model. In this model, there is a regime of "thermalization without eigenstate thermalization." Thus, the eigenstate thermalization hypothesis is not a necessary condition for thermalization. Joint work with Aram W. Harrow

Zoom Link: https://pitp.zoom.us/j/91710478120?pwd=OVRDOStOSkdIVG9mcGJqMWJlU1FRdz09

Quantum entropy thermalization

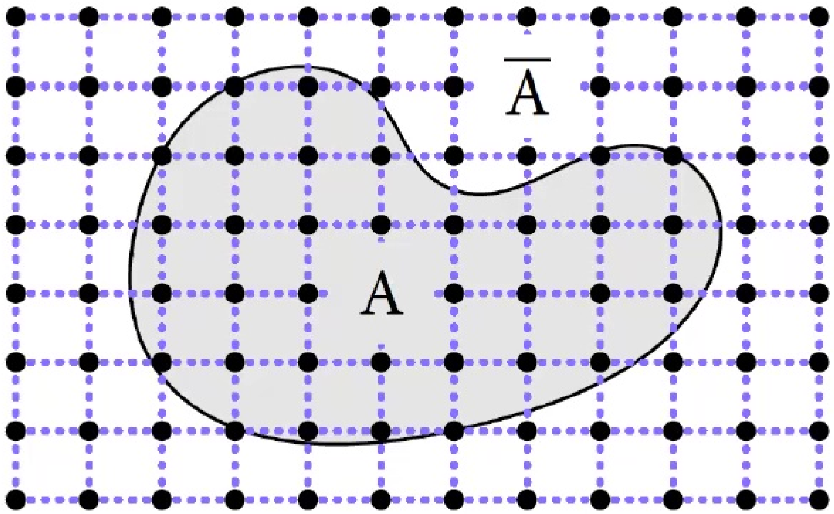
Yichen Huang (Harvard)

February 21, 2023

arXiv:2023.10165, 2022.09826, joint work with Aram W. Harrow

Thermalization

- Quantum thermalization:
- $\rho(t) \to e^{-\beta H}/Z$



- In isolated system
- View $\bar{A}$ as bath of A
- Does $\rho_A(t)$ thermalize?

Thermalization

Ergodic hypothesis $\rightarrow$

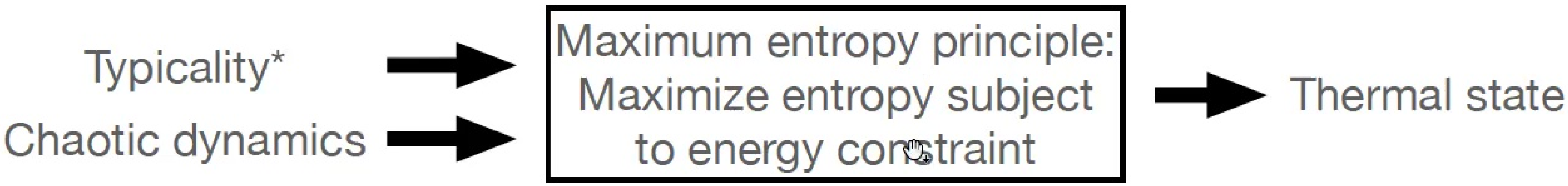
Principle of equal a prior probabilities $\rightarrow$

$$e^{-\beta H}$$

$\uparrow$?

Axioms of quantum mechanics

Entropy thermalization



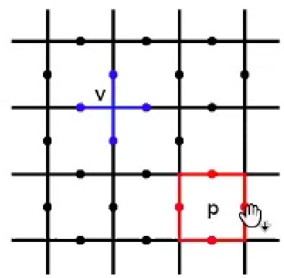
- Definition (entropy thermalization): The entropy of subsystem A thermalizes if after long-time evolution, it is to leading order (in the size of A) equal to the thermodynamic entropy of A at the same energy.
- Do not assume A is small
- Conjecture: Quantum chaos $\Longrightarrow$ entropy thermalization
- *Lloyd, PhD thesis, 1998; Goldstein et al., PRL 96, 050403, 2006; Popescu et al., Nature Physics 2, 754, 2006.

Entanglement thermalization

- Entanglement entropy of $|\psi\rangle$:
- $$S(\psi_A) := -\operatorname{tr}(\psi_A \ln \psi_A), \quad \psi_A := \operatorname{tr}_{\bar{A}} |\psi\rangle\langle\psi|$$
- Entropy thermalization = entanglement thermalization*
- *Zhang et al., PRE 91, 062128, 2015

Model

- Quantum phase transition: Transverse field Ising chain
- $$H_{\text{Ising}} = \sum_i \sigma_i^z \sigma_{i+1}^z + h\sigma_i^x$$
- Topological order: Toric code



- We prove entropy thermalization in a nearly integrable Sachdev-Ye-Kitaev (SYK) model.

Sachdev-Ye-Kitaev model

- Complex SYKq model:* random all-to-all q-body interactions between fermions.
- $$H_{\mathrm{SYK2}} = \sum_{i,j=1}^{N} h_{ij} a_i^\dagger a_j + \mathrm{h.c.}, \quad H_{\mathrm{SYK4}} = \sum_{i,j,k,l=1}^{N} J_{ijkl} a_i^\dagger a_j^\dagger a_k a_l + \mathrm{h.c.}$$
- All coefficients ~ complex standard normal random variable
- H_{SYK2} is integrable (free fermions); H_{SYK4} is chaotic. Both conserve fermion number.
- *Sachdev and Ye, PRL 70, 3339, 1993; Kitaev, 2015

Nearly integrable SYK model

- ϵ_1, ϵ_2 infinitesimal; fermion number operator: $Q = \sum_i a_i^\dagger a_i$
- Our model: $H = Q + \epsilon_1 H_{\mathrm{SYK2}} + \epsilon_1 \epsilon_2 H_{\mathrm{SYK4}}$
- It conserves fermion number and is nearly integrable.

Initial state

- Initial state: $\underbrace{\bullet \quad \bullet \quad \cdots \quad \bullet \quad \cdots \quad \bullet}_{n \text{ filled modes}} \quad \underbrace{\circ \quad \circ \quad \cdots \quad \circ}_{N-n \text{ vacant modes}}$

- Why product state? Simple, natural, experimentally easy, zero initial entropy

Thermodynamic entropy

- ϵ_1, ϵ_2 infinitesimal $\implies H = Q + \epsilon_1 H_{\text{SYK2}} + \epsilon_1 \epsilon_2 H_{\text{SYK4}}$ and $Q = \sum_i a_i^\dagger a_i$ have the same thermodynamic entropy
- For a state with n fermions, the thermodynamic entropy density is the binary entropy function
- $H_b(\nu) := -\nu \ln \nu - (1-\nu)\ln(1-\nu)$
- of the filling fraction $\nu := n/N$

Results

- L: subsystem size; N: system size. C's in different cells are different positive constants.

	$\nu = \frac{1}{2}$ or $\frac{L}{N} \neq \frac{1}{2}$	$\nu \neq \frac{1}{2}$ and $\frac{L}{N} = \frac{1}{2}$
upper bound	$H_b(\nu)L - C$	$H_b(\nu)L - C\sqrt{L}$
lower bound	$H_b(\nu)L - C\ln L$	$H_b(\nu)L - C\sqrt{L}\ln L$
thermal	$H_b(\nu)L$	$H_b(\nu)L$
random*	$H_b(\nu)L - C$	$H_b(\nu)L - C\sqrt{L}$

- *Bianchi et al., PRX Quantum 3, 030201, 2022.

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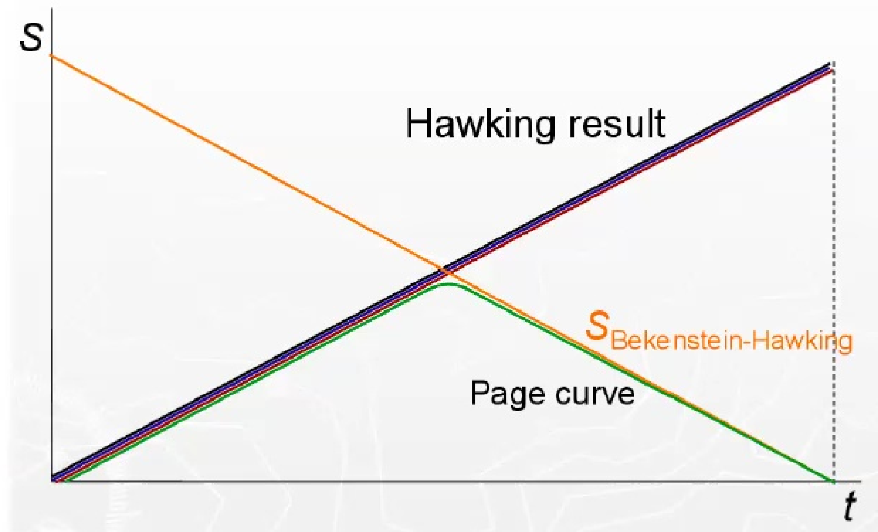
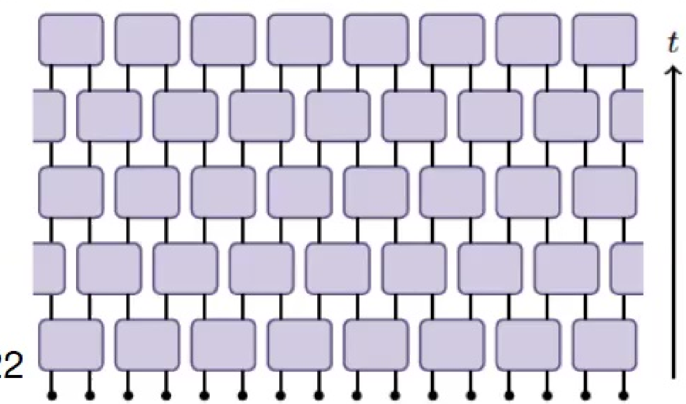
Page curve



- Page curve: entanglement between black hole and its Hawking radiation (picture from KCL black hole information group)
- Random state ➡ Page curve*
- Random quantum circuits ➡ Page curve**
- We establish Page curve from evolution of time-independent Hamiltonian
- *Page, PRL 91, 1291, 1993. **Colter et al., PRA 105, 022416, 2022



Quantum chaos and integrability breaking

- Real systems may be described by
- $H = H_{\text{intgrable}} + \epsilon H_{\text{perturbation}}, \quad \epsilon \ll 1$

  ↑
- Singular effects, makes the model non-integrable
- In finite-size systems, how large ϵ leads to chaotic behavior?

Entropy thermalization & integrability breaking

- $H = H_{\text{intgrable}} + \epsilon_1 \epsilon_2 H_{\text{SYK4}}, \quad H_{\text{intgrable}} = Q + \epsilon_1 H_{\text{SYK2}}$
- $H_{\text{intgrable}}$ does not lead to entropy thermalization
- infinitesimal integrability-breaking perturbation
  ➡ chaotic entanglement dynamics at long times
- independent of the microscopic details of the perturbation

Eigenstate thermalization hypothesis (ETH)

- If the properties in the first row are close to thermal for states in the first column, then we say

	Reduced density matrix	Subsystem entropy
Time-evolved states	Thermalization	Entropy thermalization
Eigenstates	ETH*	ETH for entropy

- Under mild additional assumptions, ETH implies thermalization
- Thermalization implies entropy thermalization
- *Deutsch, PRA 43, 2046, 1991; Srednicki, PRE 50, 888, 1994; Rigol et al., Nature 452, 854, 2008.

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Experiment

STATISTICAL PHYSICS

Quantum thermalization through entanglement in an isolated many-body system

Adam M. Kaufman, M. Eric Tai, Alexander Lukin, Matthew Rispoli, Robert Schittko, Philipp M. Preiss, Markus Greiner*

Statistical mechanics relies on the maximization of entropy in a system at thermal equilibrium. However, an isolated quantum many-body system initialized in a pure state remains pure during Schrödinger evolution, and in this sense it has static, zero entropy. We experimentally studied the emergence of statistical mechanics in a quantum state and observed the fundamental role of quantum entanglement in facilitating this emergence. Microscopy of an evolving quantum system indicates that the full quantum state remains pure, whereas thermalization occurs on a local scale. We directly measured entanglement entropy, which assumes the role of the thermal entropy in thermalization. The entanglement creates local entropy that validates the use of statistical physics for local observables. Our measurements are consistent with the eigenstate thermalization hypothesis.

Summary

- We prove entropy thermalization for a nearly integrable Sachdev-Ye-Kitaev model.
- First proof of entropy thermalization in a particular quantum system
- Thermalization without eigenstate thermalization