

Title: Back to Heterotic Strings on ALE Spaces

Speakers: Michele Del Zotto

Series: Quantum Fields and Strings

Date: February 21, 2023 - 2:00 PM

URL: <https://pirsa.org/23020058>

Abstract: I will revisit the physics of Heterotic Strings on four-dimensional ADE singularities in light of recent progress in the context of higher global symmetries combined with our improved understanding of 6d SCFTs over the past decade. In particular, on ADE singularities the fractional heterotic little string instantons enjoy a continuous $U(1)$ group symmetry that one can exploit to constrain possible networks of T-dualities. These predictions are confirmed exploiting duality with F-theory, and also shed light on the question of the geometric engineering limit of heterotic strings on ALE spaces. Based on joint works with Kantaro Ohmori, Paul-Konstantin Öhlmann and Muyang Liu.

Zoom Link: <https://pitp.zoom.us/j/92559165562?pwd=R3NLRktuWFYyKzFNQmhFeS9sQ2tQZz09>

Back to Heterotic Strings on ALE spaces

Perimeter Institute QF&S Seminars, 2023-02-21

Michele Del Zotto - Uppsala University



Simons Collaboration on Global Categorical Symmetries

BASED ON:

[1] 1511.05565 w/ BHARDWAD, HECKMAN, RUDELIVS
MORRISON, VAFA

[2] 2009.03489 w/ OHMORI

[3] 2209.10551 w/ LIU, OEHLMANN

[4] 2212.05331 w/ " , "

[5] 2303.xxxxxx w/ " , "

[6] WORK IN PROGRESS w/ " , " , BRAUN

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[6] WORK IN PROGRESS w/ " , " , BRAUN

COMMERCIALS

- **NORDITA** CATEGORICAL ASPECTS OF SYMMETRIES PROGRAM

CONFERENCE AUG 22-25, 2023

- GCS CONFERENCE + GRADUATE SCHOOL

LES DIABLERET | AUG 27-SEPT 8, 2023

DETAILS ON: **SCGCS.BERKELEY.EDU**

OVER THE PAST DECADE LOTS OF
PROGRESS IN OUR UNDERSTANDING OF

GENERALIZATION
OF SYMMETRY

&

STRUCTURE/
CLASSIFICATION
OF HIGHER
DIMENSIONAL
THEORIES

TODAY: WE COMBINE
THESE TO REVISIT
A WELL-KNOWN BEAUTIFUL STORY
BUILDING ON THE INTERPLAY OF STRINGS
AND GEOMETRY

T DUALITY OF HETEROTIC ALE INSTANTONS

$$\text{Het}_{E_8 \times E_8} \xleftrightarrow{T} \text{Het}_{\text{Spin}(32)/\mathbb{Z}_2}$$

HAVE GAUGE DOFS

→ HAVE INSTANTONS

$\cdot$
 $\mathbb{R}^4$

$\sim$
 $\mathbb{R}^5$

$\sim$
 $\mathbb{R}^6$

...

6-dimensional
object

$\mathbb{R}^{10}$

///

Heterotic
NS5 brane

T DUALITY OF HETEROTIC ALE INSTANTONS

$$\begin{array}{ccc}
 \text{Het } E_8 \times E_8 & \xleftrightarrow{T} & \text{Het } \text{Spin}(32)/\mathbb{Z}_2 \\
 \text{Heterotic} & & \text{Heterotic} \\
 \text{N NS5 brane} & & \text{NS5 brane N} \\
 E_8 \times E_8 \supseteq \text{SO}(16)^2 \subseteq \text{Spin}(32)/\mathbb{Z}_2
 \end{array}$$

REMARK [SEIBERG 96]: ONE CAN DECOUPLE GRAVITY FROM NS5 BRANES AND IS LEFT WITH A 6D THEORY WITH T-DUALITY, A HAGEDORN DENSITY OF STATES $\rho \sim E^2 e^{\beta_H E}$, AND NO GRAVITON

T DUALITY OF HETEROTIC ALE INSTANTONS

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REMARK [SEIBERG 96]: ONE CAN DECOUPLE GRAVITY FROM NSS BRANES AND IS LEFT WITH A 6D THEORY WITH T-DUALITY, A HAGEDORN DENSITY OF STATES $\rho \sim E^2 e^{\beta_H E}$, AND NO GRAVITON: **WOW!**

T DUALITY OF HETEROTIC ALE INSTANTONS

$$\begin{array}{ccc}
 \text{Het } E_8 \times E_8 & \xleftrightarrow{T} & \text{Het } \text{Spin}(32)/\mathbb{Z}_2 \\
 \text{Heterotic} & \text{LITTLE} & \text{Heterotic} \\
 \text{NS5 brane} & \text{STRING} & \text{NS5 brane} \\
 \text{N} & \text{THEORY} & \text{N} \\
 E_8 \times E_8 & \supseteq \text{SO}(16)^2 & \subseteq \text{Spin}(32)/\mathbb{Z}_2
 \end{array}$$

REMARK [SEIBERG 96]:

LITTLE STRING
TENSION

↑ E
6D LST
6D SCFT

T DUALITY OF HETEROTIC ALE INSTANTONS

$$\begin{array}{ccc}
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 \text{Heterotic} & \text{LITTLE} & \text{Heterotic} \\
 \text{NS5 brane} & \text{STRING} & \text{NS5 brane} \\
 \text{N} & \text{THEORY} & \text{N} \\
 E_8 \times E_8 & \supseteq \text{SO}(16)^2 & \subseteq \text{Spin}(32)/\mathbb{Z}_2 \\
 \text{ON } \mathbb{C}^2/\Gamma & & \text{ON } \mathbb{C}^2/\Gamma
 \end{array}$$

T DUALITY OF HETEROTIC ALE INSTANTONS

$$\begin{array}{ccc}
 \text{Het } E_8 \times E_8 & \xleftrightarrow{T} & \text{Het } \text{Spin}(32)/\mathbb{Z}_2 \\
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 \text{NS5 brane} & \text{STRING} & \text{NS5 brane} \\
 \text{N} & \text{THEORY} & \text{N} \\
 E_8 \times E_8 & \geq \text{SO}(16)^2 & \leq \text{Spin}(32)/\mathbb{Z}_2 \\
 \text{ON } \mathbb{C}^2/\Gamma & \text{INSTANTONS} & \text{ON } \mathbb{C}^2/\Gamma \\
 & \text{ARE LABELED} & \\
 & \text{BY CHOICE OF} & \\
 & \text{FLAT CONNECTION} & \\
 & \text{AT INFINITY} &
 \end{array}$$

T DUALITY OF HETEROTIC ALE INSTANTONS

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 \text{Het } E_8 \times E_8 & \xleftrightarrow{T} & \text{Het } \text{Spin}(32)/\mathbb{Z}_2 \\
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 E_8 \times E_8 & \geq \text{SO}(16)^2 & \leq \text{Spin}(32)/\mathbb{Z}_2 \\
 \text{ON } \mathbb{C}^2/\Gamma & \text{INSTANTONS} & \text{ON } \mathbb{C}^2/\Gamma \\
 \mu_1: \Gamma \rightarrow E_8 & \text{ARE LABELED} & \\
 \mu_2: \Gamma \rightarrow E_8 & \text{BY CHOICE OF} & \\
 & \text{FLAT CONNECTION} & \\
 & \text{AT INFINITY} & \\
 & & \lambda: \Gamma \rightarrow \frac{\text{Spin}(32)}{\mathbb{Z}_2}
 \end{array}$$

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 \text{Het } E_8 \times E_8 & \xleftrightarrow{T} & \text{Het } Spin(32)/\mathbb{Z}_2 \\
 \text{Heterotic} & \text{LITTLE} & \text{Heterotic} \\
 \text{NS5 brane} & \text{STRING} & \text{NS5 brane} \\
 \text{N} & \text{THEORY} & \text{N} \\
 E_8 \times E_8 & \supseteq SO(16)^2 & \subseteq Spin(32)/\mathbb{Z}_2 \\
 \text{ON } \mathbb{C}^2/\Gamma & \text{INSTANTONS} & \text{ON } \mathbb{C}^2/\Gamma \\
 \mu_1: \Gamma \rightarrow E_8 & \text{ARE LABELED} & \lambda: \Gamma \rightarrow Spin(32) \\
 \mu_2: \Gamma \rightarrow E_8 & \text{BY CHOICE OF} & \sim \\
 K_N(\Gamma; \mu_1, \mu_2) & \text{FLAT CONNECTION} & \tilde{K}_N(\Gamma; \lambda) \\
 & \text{AT INFINITY} & \mathbb{Z}_2
 \end{array}$$

T DUALITY OF HETEROTIC ALE INSTANTONS

$$\begin{array}{ccc}
 \text{Het } E_8 \times E_8 & \xleftrightarrow{T} & \text{Het } Spin(32)/\mathbb{Z}_2 \\
 \text{Heterotic} & & \text{Heterotic} \\
 \text{N NS5 brane} & & \text{NS5 brane N}
 \end{array}$$

INTRILIGATOR, BLUM 97:
 FULL ANALYSIS OF THIS CASE
 ON $\mathbb{C}^2/\Gamma$ ORBIFOLD

$$\begin{array}{l}
 \text{ON } \mathbb{C}^2/\Gamma \\
 \mu_1: \Gamma \rightarrow E_8 \\
 \mu_2: \Gamma \rightarrow E_8
 \end{array}$$

$$K_N(\Gamma; \mu_1, \mu_2)$$

$$\begin{array}{l}
 \lambda: \Gamma \rightarrow \underline{Spin(32)} \\
 \tilde{K}_N(\Gamma; \lambda) \quad \mathbb{Z}_2
 \end{array}$$

$$\xleftrightarrow{T}$$

T DUALITY OF HETEROTIC ALE INSTANTONS

$$\text{Het } E_8 \times E_8 \xleftrightarrow{T} \text{Het } \text{Spin}(32)/\mathbb{Z}_2$$

Heterotic
 N NS5 brane

ASPINWALL-MORRISON 97

FULL ANALYSIS OF $M_1, M_2 = \text{id}$

ON $\mathbb{C}^2/\Gamma$

$$M_1: \Gamma \rightarrow E_8$$

$$M_2: \Gamma \rightarrow E_8$$

$$K_N(\Gamma; M_1, M_2)$$

Heterotic
 N NS5 brane

INTRILIGATOR, BLUM 97:

FULL ANALYSIS OF THIS CASE

ON $\mathbb{C}^2/\Gamma$ ORBIFOLD

$$\lambda: \Gamma \rightarrow \text{Spin}(32)$$

$$\tilde{K}_N(\Gamma; \lambda) \mathbb{Z}_2$$

$$\xleftrightarrow{T}$$

T DUALITY OF HETEROTIC ALE INSTANTONS

Het $E_8 \times E_8$ $\xleftrightarrow{T}$

Het $Spin(32)/\mathbb{Z}_2$

Heterotic

Heterotic

N NS5 brane

NS5 brane N

ASPINWALL-MORRISON 97

INTRILIGATOR, BLUM 97:

FULL ANALYSIS OF $M_1, M_2 = id$

FULL ANALYSIS OF THIS CASE

ON $\mathbb{C}^2/\Gamma$

ON $\mathbb{C}^2/\Gamma$ ORBIFOLD

$M_1: \Gamma \rightarrow E_8$

$M_2: \Gamma \rightarrow E_8$

STILL
OPEN!

$\lambda: \Gamma \rightarrow Spin(32)$

$K_N(\Gamma; M_1, M_2)$

$\tilde{K}_N(\Gamma; \lambda) \mathbb{Z}_2$

$\xleftrightarrow{T}$

BACK TO HETEROTIC STRINGS ON ALE SPACES

① DETERMINE THE THEORIES $K_N(\mu_1, \mu_2)$ [3]

→ DONE VIA 6D CONFORMAL MATTER

D2, HECKMAN, TOMASIELLO, VAFA 14
HECKMAN, MORRISON, RUDELUS, VAFA 15
FREY, RUDELUS 17

② ESTABLISH CRITERIA TO MATCH THEM WITH $\tilde{K}_N(1)$

→ MATCHING OF 5D COULOMB BRANCHES [3]

MATCHING OF 2-GROUP STRUCTURE CONSTANTS

D2, OHMORI 20

T DUALITY OF HETEROTIC ALE INSTANTONS

③ EXPLICIT CHECK VIA F-THEORY [4]

→ GEOMETRIZATION OF LST T-DUALITY

BHARDWAJ, D7, HECKMAN, MORRISON, Vafa IS

④ BOOTSTRAP GEOMETRIC ENGINEERING
LIMIT OF HETEROTIC STRINGS ON ALE [5]

$$\text{Het}_{E_8 \times E_8} / \mathbb{C}^2 / \Gamma \quad \mu_1, \mu_2 \equiv K_0(\Gamma, \mu_1, \mu_2)$$

$$\text{Het}_{\text{Spin}(32)/2\mathbb{Z}_2} / \mathbb{C}^2 / \Gamma \quad \lambda \equiv \tilde{K}_0(\Gamma, \lambda)$$

T DUALITY OF HETEROTIC ALE INSTANTONS

⑤ FOUND MANY EXAMPLES OF MULTIPLE EXOTIC DUALITIES FOR NON-GEOMETRIC HETEROTIC LSTs VIA F-THEORY [4, 6]

$$\begin{array}{ccc}
 E_8^2 \times SU(3) & \text{---} & Spin(32)/\mathbb{Z}_2 \times SU(3) \\
 \swarrow & \text{"T-Hexality"} & \searrow \\
 E_6^3 / \mathbb{Z}_3 & & U(18) / \mathbb{Z}_3 \\
 \swarrow & & \searrow \\
 (Spin(14) \times SU(12)) / \mathbb{Z}_4 & \text{---} & (Spin(20) \times E_7) / \mathbb{Z}_2 \times U(1)
 \end{array}$$

GENERALIZATION OF SYMMETRIES

MANTRA: KAPUSTIN THORNGREN 13
GAIOTTO KAPUSTIN SEIBERG WILLET 14

SYMMETRIES IN QFT ARE ENCODED BY
SUBSECTOR OF (QUASI) TOPOLOGICAL
OPERATORS WITH SUPPORTS OF VARIOUS
CODIMENSIONS

4

GENERALIZATION OF SYMMETRIES

MANTRA #2:

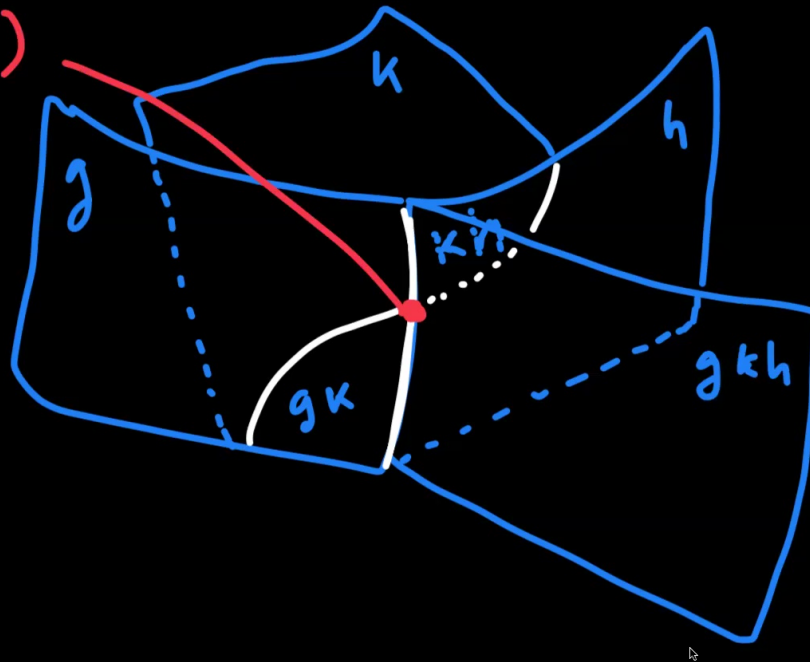
FUSION $\leftrightarrow$ GENERALIZES GROUP LAW

CAN INVOLVE TOPOLOGICAL
OPERATORS OF DIFFERENT
CODIMENSION

GENERALIZATION OF SYMMETRIES

SIMPLEST EXAMPLE: HIGHER GROUPS

$\beta(gkh)$

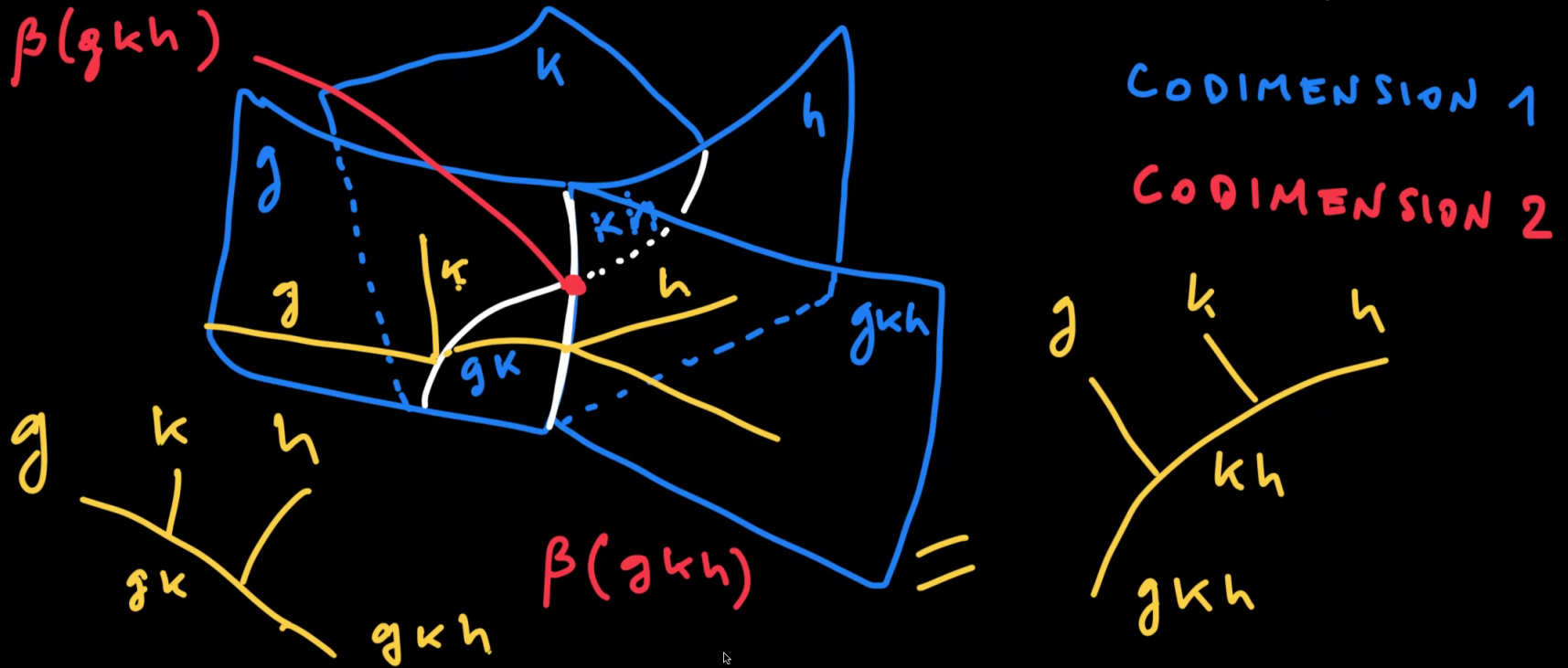


CODIMENSION 1

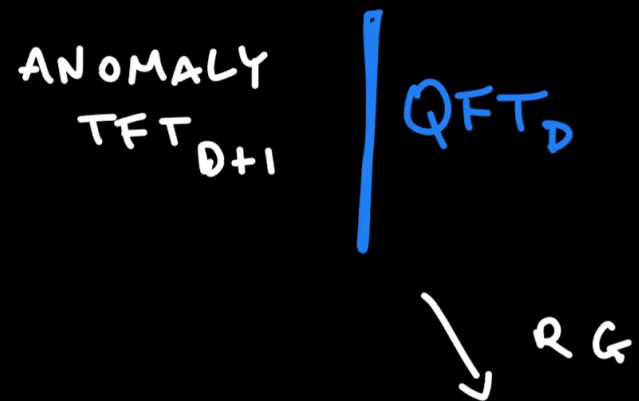
CODIMENSION 2

GENERALIZATION OF SYMMETRIES

SIMPLEST EXAMPLE: HIGHER GROUPS



GENERALIZATION OF SYMMETRIES



INTERESTING
CONSTRAINTS
ON DYNAMICS



GENERALIZATION OF SYMMETRIES

ANOMALY
 TFT_{D+1}

SYMMETRY
 TFT_{D+1}

QFT_D

APRUŽIĆ, BONETTI, GARCÍA ETXEBARRIA;
HOSEINI, SCHÄFER-JAMEKI 21

$\searrow$ RG

INTERESTING
CONSTRAINTS
ON DYNAMICS

ANOMALY
 TFT_{D+1}

SYMMETRY
 TFT_{D+1}

QFT_D

GAOTTO, KULP 20 ; DZ, GARCÍA ETXEBARRIA 22

GENERALIZATION OF SYMMETRIES

ANOMALY
 TFT_{D+1}

SYMMETRY
 TFT_{D+1}

QFT_D

DUAL

INTERESTING
CONSTRAINTS
ON DYNAMICS

ANOMALY
 TFT_{D+1}

SYMMETRY
 TFT_{D+1}

$\sim$
 QFT_D

GENERALIZATION OF SYMMETRIES

E.G.

2-GROUP STRUCTURE
MUST MATCH AMONG
DUAL THEORIES

INTERESTING
CONSTRAINTS
ON DYNAMICS

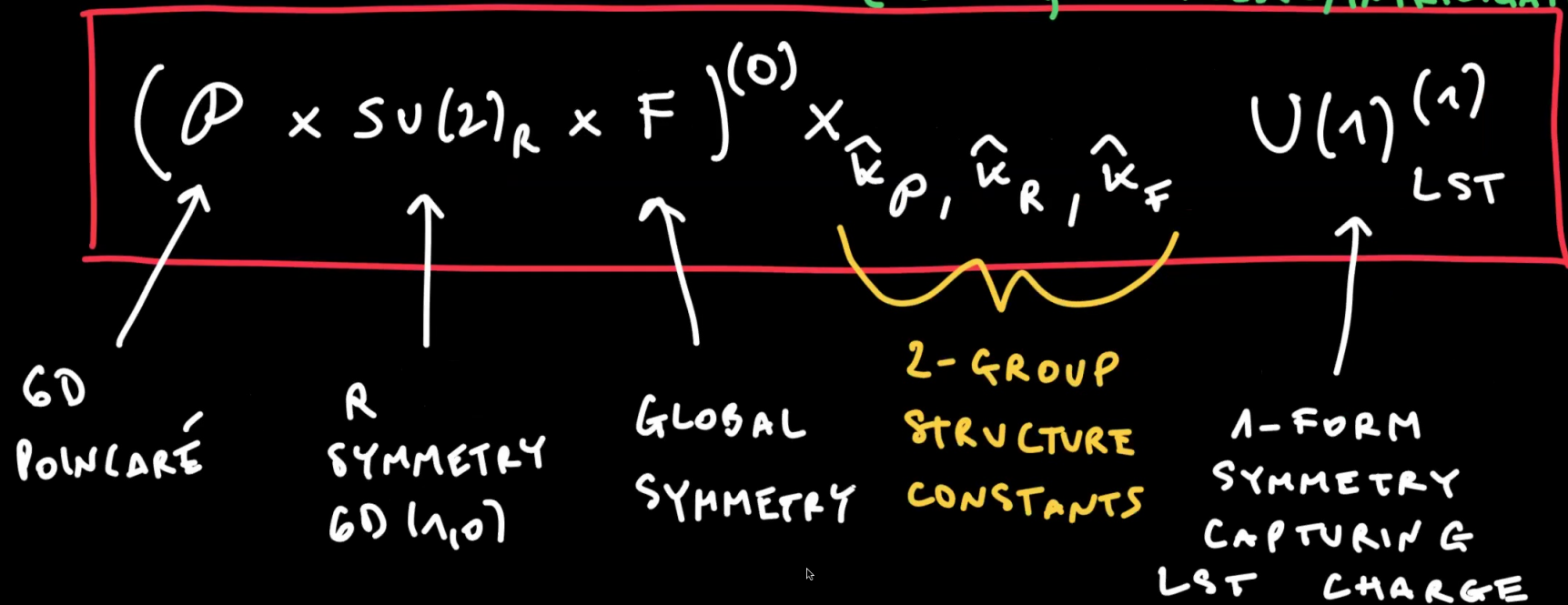
DZ, OHMORI 20

LEE, OHMORI, TACHIKAWA 21

2-GROUP SYMMETRY OF LITTLE STRINGS

6D LSTs HAVE A^{CONTINUOUS} 2-GROUP SYMMETRY:

[CORDOVA, DUMITRESCU, INTRILIGATOR]
20

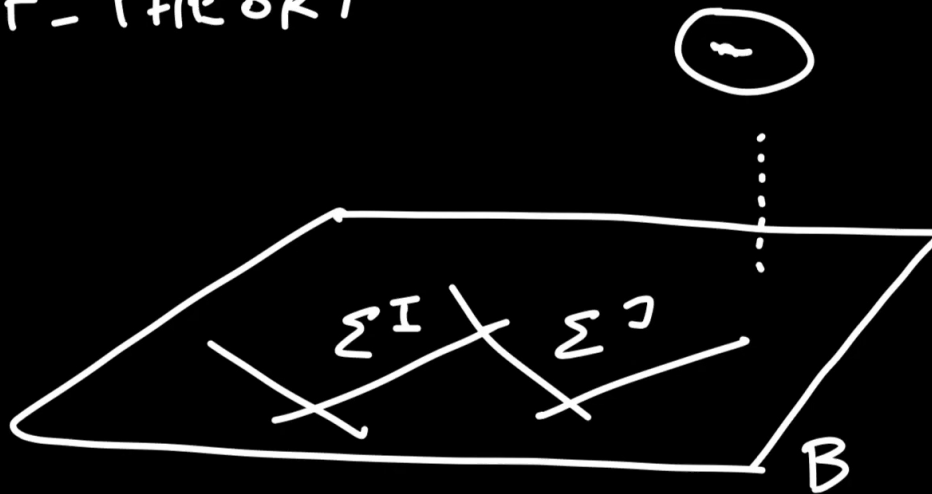


2-GROUP SYMMETRY OF LITTLE STRINGS (2)

6D SCFTS $\longleftrightarrow \eta^{IJ} > 0$

F-THEORY

DIRAC PAIRING BETWEEN
BPS STRINGS



$$\eta^{IJ} = - \Sigma^I \cdot \Sigma^J$$

2-GROUP SYMMETRY OF LITTLE STRINGS [2]

6D SCFTS



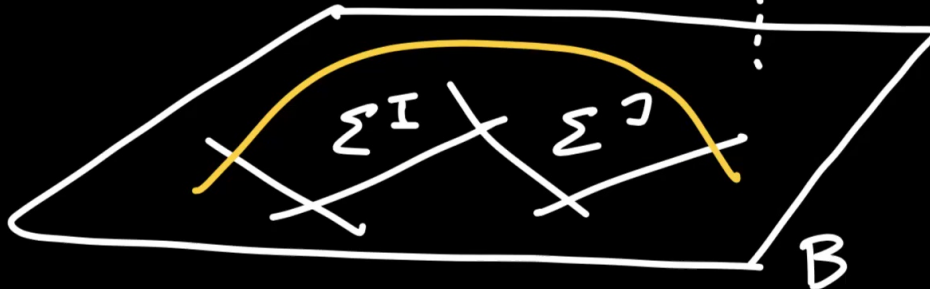
$$\eta^{IJ} > 0$$

6D LST



$$\eta^{IJ} \geq 0$$

F-THEORY



$$\eta^{IJ} = - \Sigma^I \cdot \Sigma^J$$

AFFINIZATION

$$\exists! \boxed{\eta^{IJ} N_J = 0}$$

η^{IJ} CONTROLS TENSOR BRANCH KINETIC
 TERM : ZERO EIGENVALUE $\equiv$ BACKGROUND
 2-FORM FIELD $\Rightarrow$ $\boxed{U(1)^{(1)}_{LST}}$

GREEN-SCHWARZ MECHANISM $\Rightarrow$ HIGHER
 GROUP STRUCTURE

BAEZ, SATI, SCHREIBER $\sim$ 05, 07

$$\int_6 b_I^{(2)} \eta^{IJ} X_J^{(4)} \rightarrow I_8^{GS} = \eta^{IJ} X_I^{(4)} X_J^{(4)}$$

FOR LSTs :

$$H_I = d b_I^{(2)}$$

$$d H_{LST}^{(3)} = \sum_I N^I d H_I^{(3)}$$

$$= \hat{\kappa}_R c_2(R) - \frac{\hat{\kappa}_\rho}{4} p_1(TM_6)$$

$$- \frac{1}{4} \hat{\kappa}_{F,A} \text{Tr} F_A^2$$

$$d H_I^{(3)} = h_{g_I}^v c_2(R) + \frac{1}{4} (\gamma^{II} - 2) p_1(TM_6) + \frac{1}{4} \gamma^{IA} \text{Tr} F_A^2$$

ALLOWS TO EXPLICITLY COMPUTE:

$$\hat{K}_{F,A} = - \sum_{I=1}^{n+1} N_I \gamma^{IA}$$

$$\hat{K}_R = \sum_{I=1}^{n+1} N_I h^{\nu}_{g_I}$$

$$\hat{K}_\theta = - \sum_{I=1}^{n+1} N_I (\gamma^{II} - 2)$$

GEOMETRIZATION OF T-DUALITY [1]

$$LST_1: F / T^2 \rightarrow X \rightarrow B$$

ASPINWALL
MORRISON 97

↓ Vafa 96 $F/S^1 = M$

$$LST_1 / S^1: M / X$$

5D KK theory

FLOPS $\leftrightarrow$ CHAMBER
PHASE

STRUCTURE OF COULOMB

$$M / \tilde{X}: \tilde{T}^2 \rightarrow \tilde{X} \rightarrow \tilde{B}$$

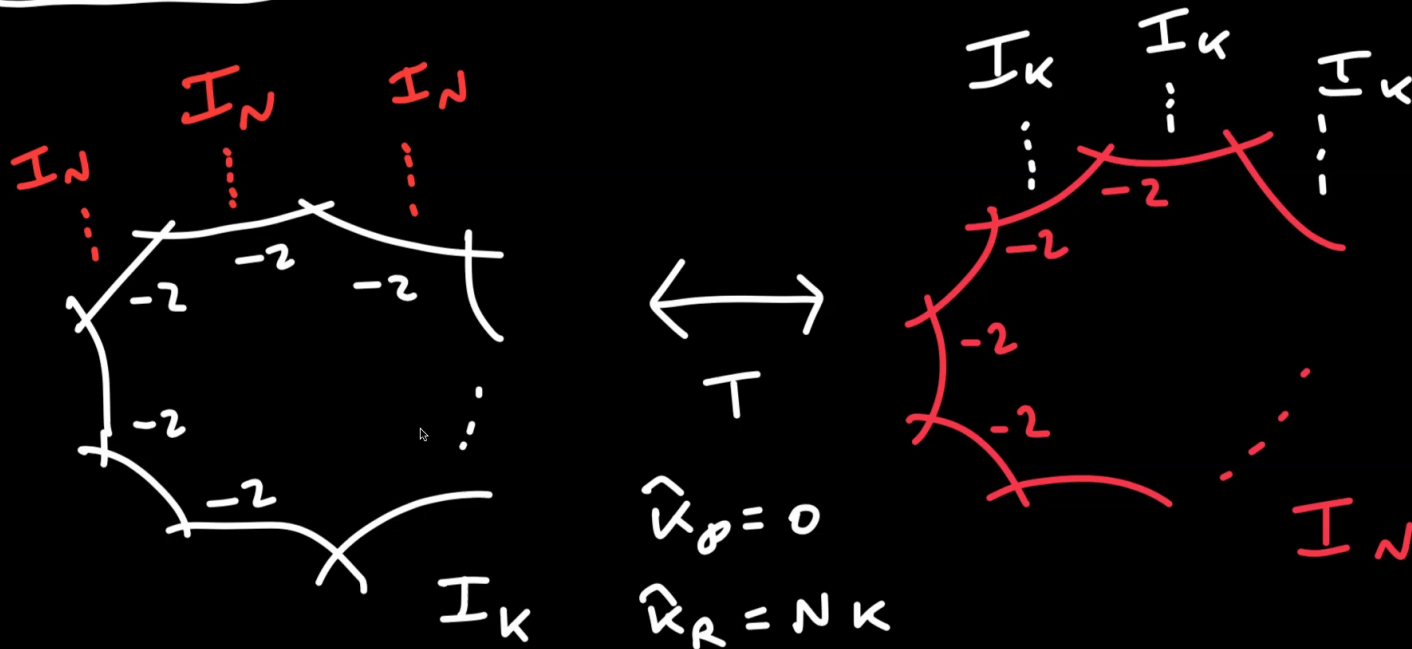
INEQUIVALENT

$$LST_2: F / \tilde{T}^2 \rightarrow \tilde{X} \rightarrow \tilde{B}$$

← NEW
UPLIFT

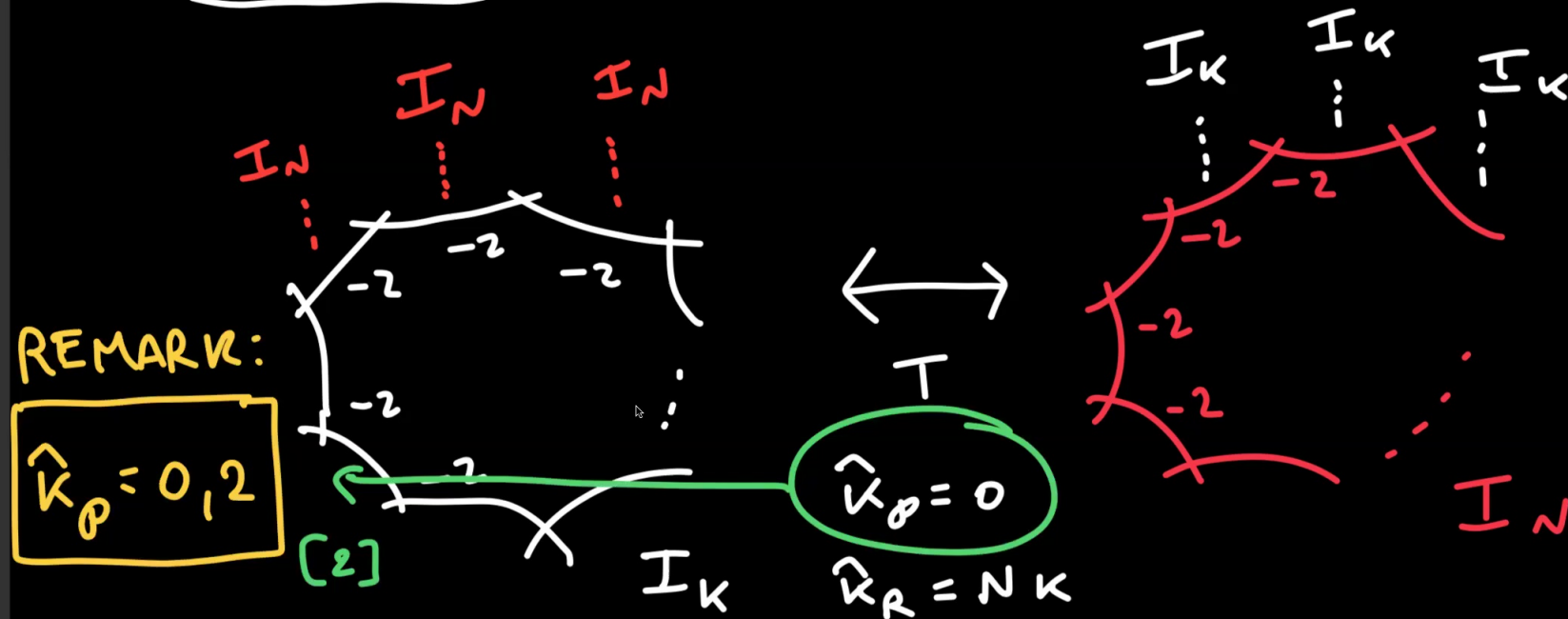
GEOMETRIZATION OF T-DUALITY [1]

EXAMPLE: $N^I = (1, 1, \dots, 1)$



GEOMETRIZATION OF T-DUALITY [1]

EXAMPLE: $N^I = (1, 1, \dots, 1)$





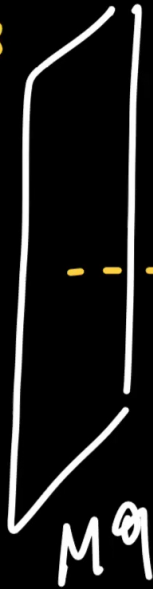
✓
HOŘAVA
WITTEN
95-96

$$M / (S^1 / \mathbb{Z}_2)$$

$$= \text{Het } E_8 \times E_8$$

BACK TO HETEROTIC ALE INSTANTONS

$$\mu_1: \Gamma \rightarrow E_8$$



M^2

$$F_i = C(\mu_i(\Gamma), E_8)$$



N MS

$$\mu_2: \Gamma \rightarrow E_8$$



M^2

HORAVA
WITTEN
95-96

$$\mathbb{C}^2 / \Gamma$$

$$\equiv T(\mu_1, \Gamma) \quad \text{RUDELIVS
FREY 17}$$

BACK TO HETEROTIC ALE INSTANTONS

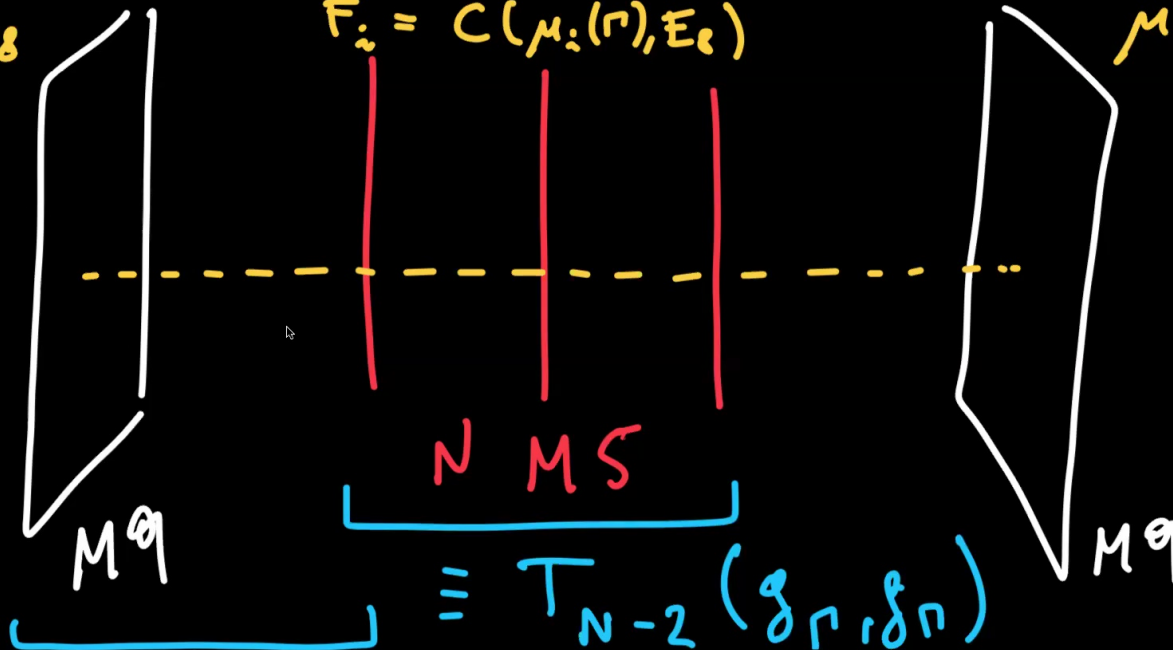
$$\mu_1: \Gamma \rightarrow E_8$$

$$F_i = C(\mu_i(\Gamma), E_8)$$

$$\mu_2: \Gamma \rightarrow E_8$$

HOŘAVA
WITTEN
95-96

$$\mathbb{C}^2 / \Gamma$$



$$\equiv T(\mu_1, \Gamma)$$

RUELIVUS
FREY 17

CONFORMAL
MATTER

BACK TO HETEROTIC ALE INSTANTONS

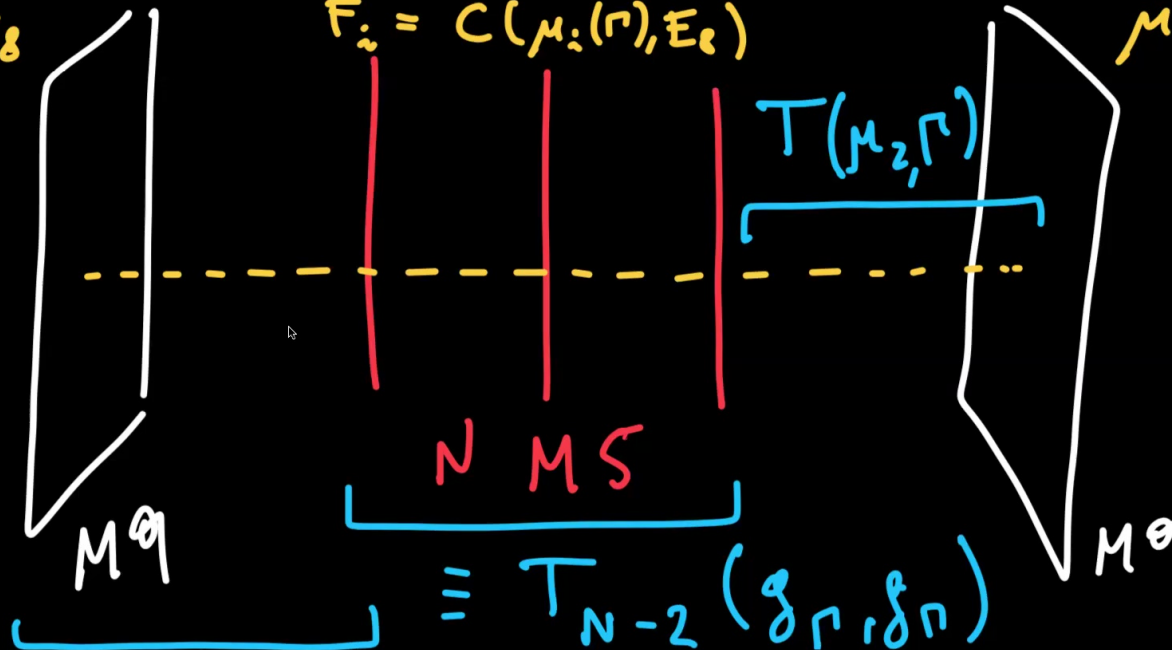
$$\mu_1: \Gamma \rightarrow E_8$$

$$F_i = C(\mu_i(\Gamma), E_8)$$

$$\mu_2: \Gamma \rightarrow E_8$$

HORAVA
WITTEN
95-96

$$\mathbb{C}^2 / \Gamma$$



$$\equiv T(\mu_1, \Gamma)$$

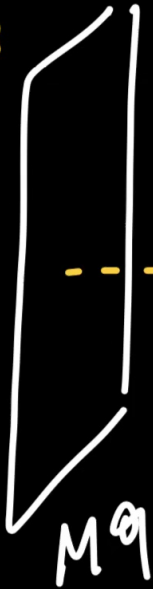
RUELIGS
FREY 17

CONFORMAL
MATTER

ALL
KNOWN
6D
SCFTs!

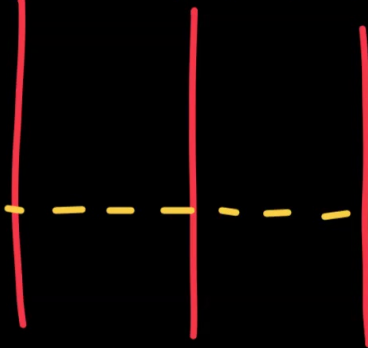
BACK TO HETEROTIC ALE INSTANTONS

$\mu_1: \Gamma \rightarrow E_8$



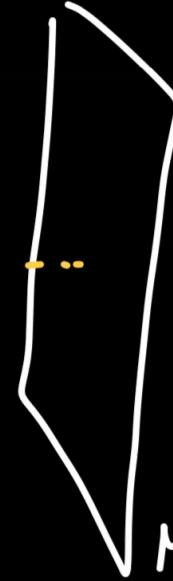
M^2

$F_2 = C(\mu_2(\Gamma), E_8)$



$N \text{ M5}$

$\mu_2: \Gamma \rightarrow E_8$



M^2

HORAVA
WITTEN
95-96

$\mathbb{C}^2 / \Gamma$

$$\equiv T(\mu_1, \Gamma) \xrightarrow{g_\Gamma} T_{N-2}(g_\Gamma, g_\Gamma) \xrightarrow{g_\Gamma} T(\mu_2, \Gamma)$$

BACK TO HETEROTIC ALE INSTANTONS

$$T(\mu_1, g_r) \xrightarrow{g_r} T_{N-2}(g_r, g_r) \xrightarrow{g_r} T(\mu_2, g_r)$$

FULLY
CLASSIFIED

FUSION
(6D GENERALIZATION)
OF GAUGING

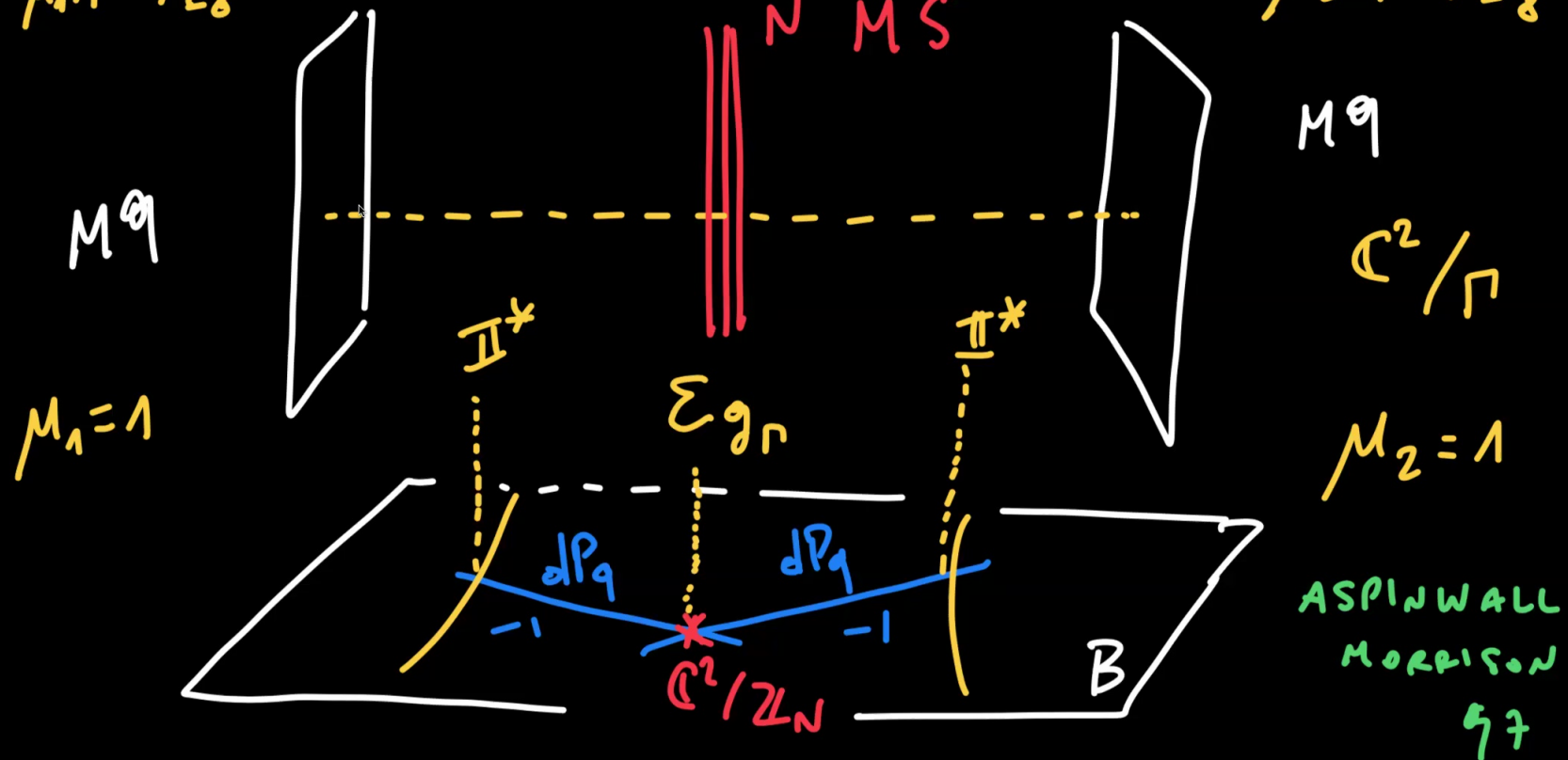
(g_r, g_r)
CONFORMAL MATTER

6D ORBI-INSTANTON
SCFTs : $[F_1] \bullet [g_r]$

GEOMETRIZE HETEROTIC ALE INSTANTONS

$$\mu_1: \Gamma \rightarrow E_8$$

$$\mu_2: \Gamma \rightarrow E_8$$



GEOMETRIZE HETEROTIC ALE INSTANTONS

DIFFERENT ELLIPTIC $K3$

FRIEDMAN
MORGAN
WITTEN

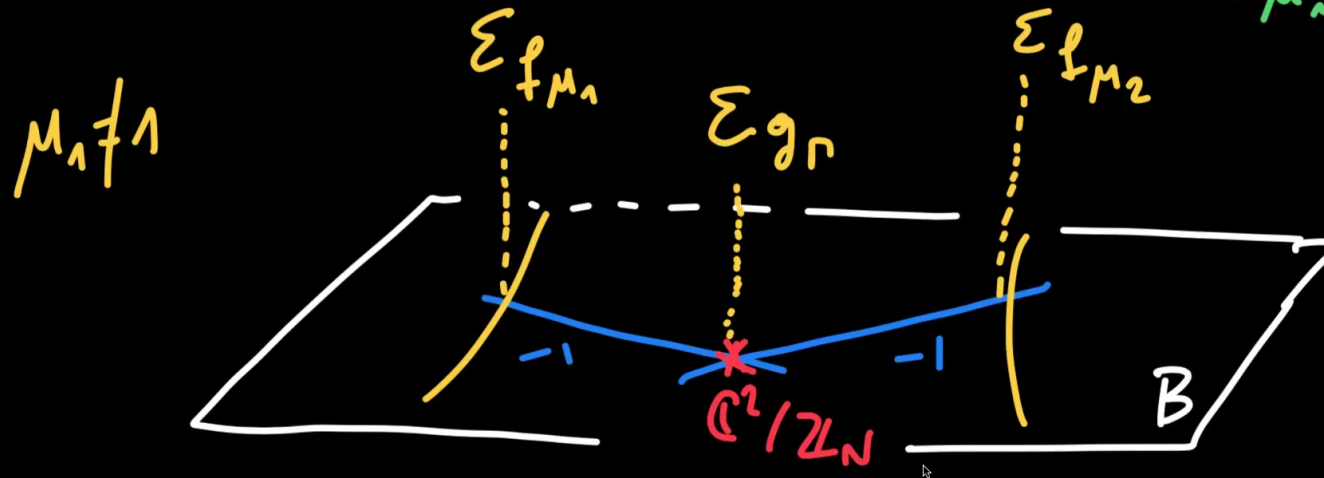
$$H_2(K3) \simeq U^{\oplus 3} \oplus E_8^{\oplus 2} \simeq \text{Pic} \oplus \text{Trasc}$$

↑
CHOICES
OF M_i

$$\mathbb{C}^2/\Gamma$$

$$M_2 \neq 1$$

[4]



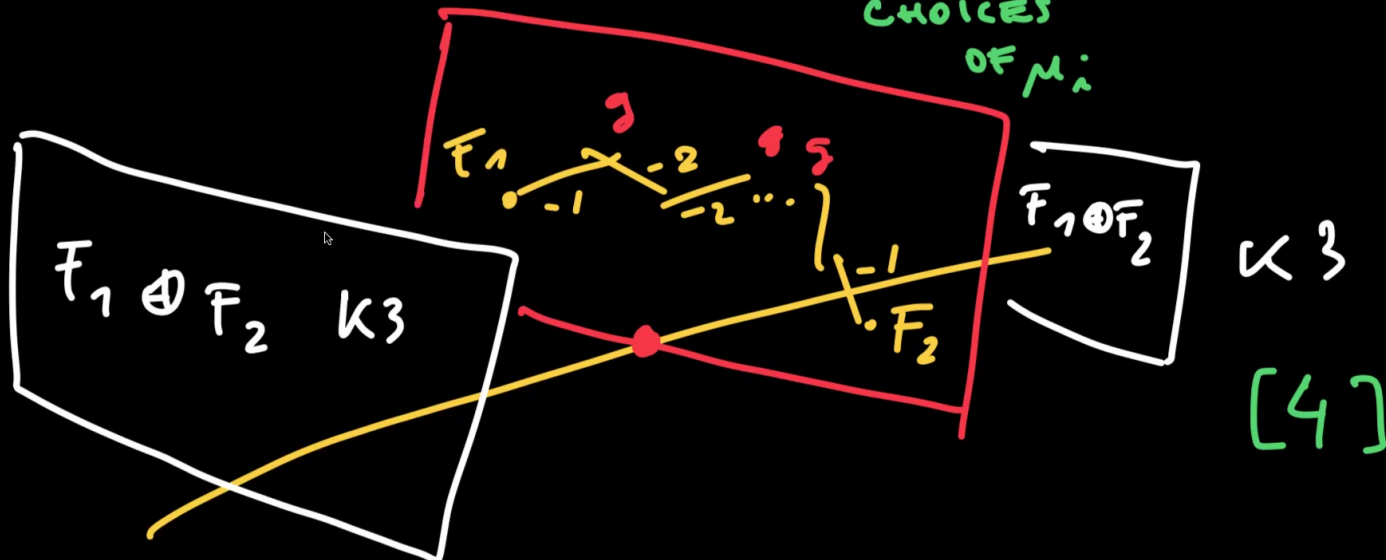
GEOMETRIZE HETEROTIC ALE INSTANTONS

DIFFERENT ELLIPTIC $K3$

FRIEDMAN
MORGAN
WITTEN

$$H_2(K3) \simeq U^{\oplus 3} \oplus E_8^{\oplus 2} \simeq \text{Pic} \oplus \text{Trasc}$$

↑
CHOICES
OF M_i



GEOMETRIZE HETEROTIC ALE INSTANTONS

DIFFERENT ELLIPTIC $K3$

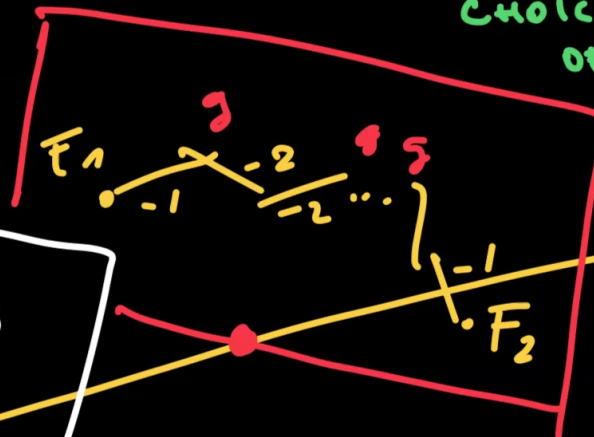
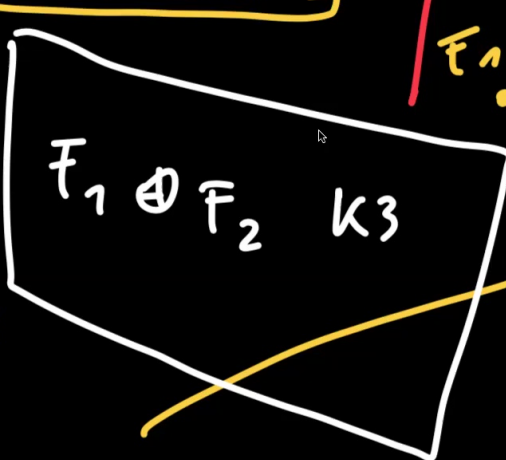
FRIEDMAN
MORGAN
WITTEN

$$H_2(K3) \simeq U^{\oplus 3} \oplus E_8^{\oplus 2} \simeq \text{Pic} \oplus \text{Trasc}$$

↑
CHOICES
OF M_i

INEQUIVALENT
ELLIPTIC STRUCTURES
ON $K3$

GIVES
T
DUALITY!

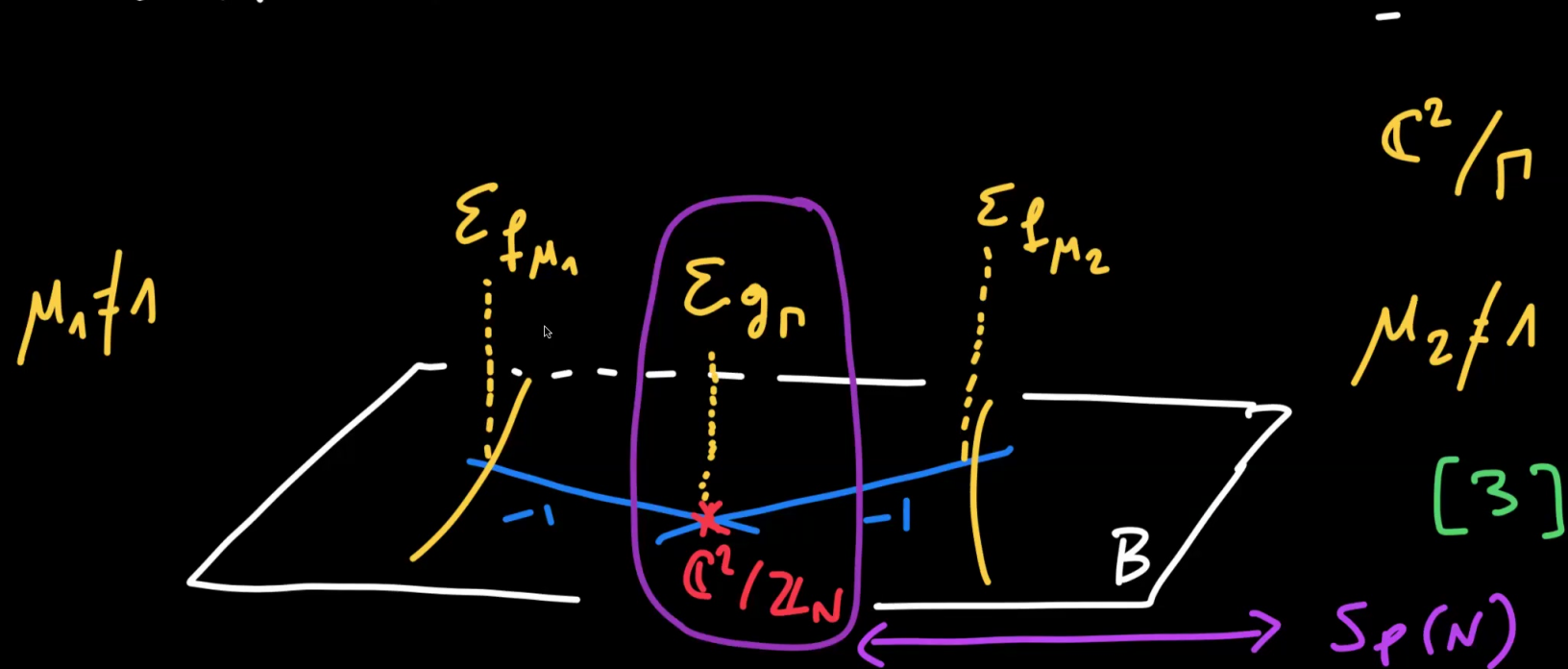


[4]

REFINED KVLIKOV STYLE
MODEL FOR THE CY GEOMETRY

GEOMETRIZE HETEROTIC ALE INSTANTONS

EXAMPLE: FIBER / BASE SWAP



EXAMPLE: LSTs w/ $SO(16)^2$ SYMMETRY ON $\mathbb{C}^2 / \mathbb{Z}_k$



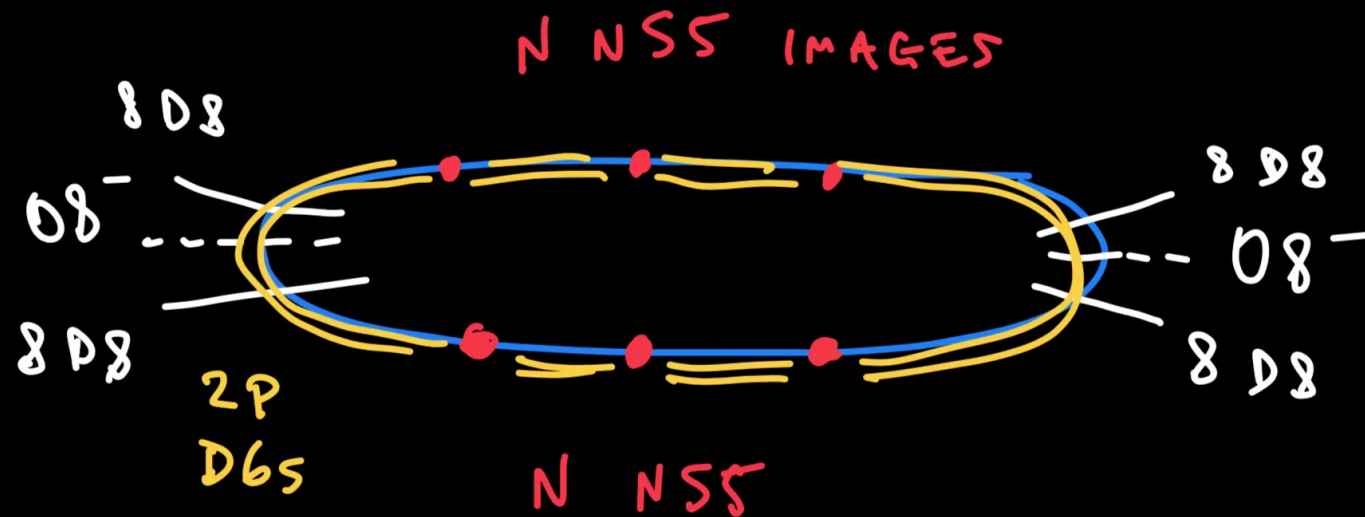
$k = 2p$: $2' + \dots + 2'$ p times $\Rightarrow SO(16)$

$$[SO(16)] \begin{array}{cccccc} & SP(p) & SU(2p) & & SU(2p) & SP(p) \\ 1 & 2 & \dots & 2 & 1 & \end{array} [SO(16)]$$

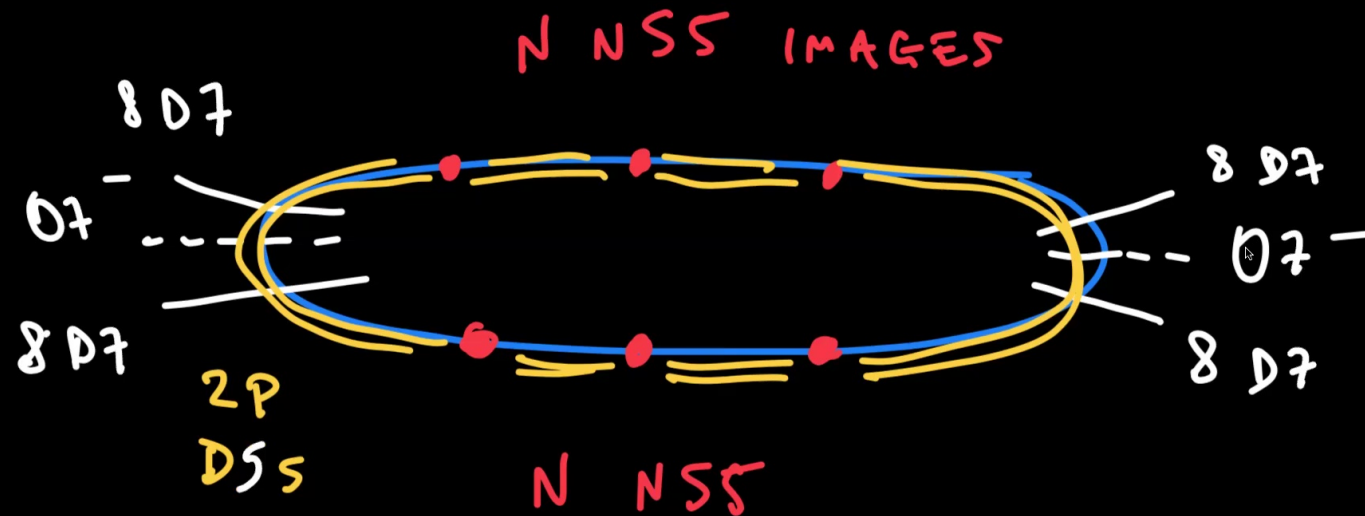
$\underbrace{\hspace{10em}}_{N+1}$

$$\hat{K}_Q = 2 \quad \hat{K}_R = 2p + (N-1)2p = 2Np$$

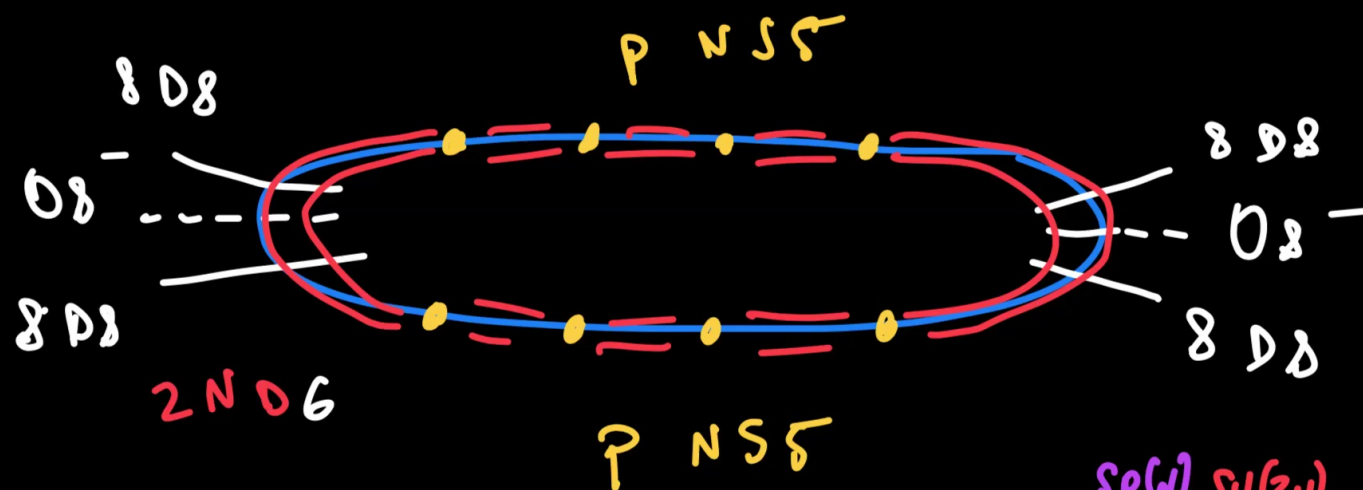
HAS DUAL DESCRIPTION IN TYPE I



HAS DUAL DESCRIPTION IN TYPE I/
ON A CIRCLE $\rightarrow$ TYPE II B



BACK TO TYPE I'



$$\hat{k}_P = 2 \quad \hat{k}_R = 2N + (P-1)2N = 2NP$$

$$\begin{matrix} sp(N) & su(2N) \dots & sp(N) \\ \left[so(16) \right] & \underbrace{1 \quad 2 \quad 2 \dots 2 \quad 1}_{P+1} & \left[so(16) \right] \end{matrix}$$

EXAMPLE: LSTs w/ $SO(16)^2$ SYMMETRY ON $\mathbb{C}^2 / \mathbb{Z}_k$



$k = 2p$: $2' + \dots + 2'$ p times $\Rightarrow SO(16)$

$$[SO(16)] \begin{array}{ccccc} & SP(p) & SU(2p) & & SU(2p) & SP(p) \\ 1 & 2 & \dots & 2 & 1 \end{array} [SO(16)]$$

$N + 1$

$$\hat{K}_Q = 2 \quad \hat{K}_R = 2p + (N-1)2p = 2Np$$

IN [3] A SYSTEMATIC STUDY IS
PRESENTED FOR THE $\mathbb{C}^2/\mathbb{Z}_k$ CASE
COMPARING TO GEOMETRY.

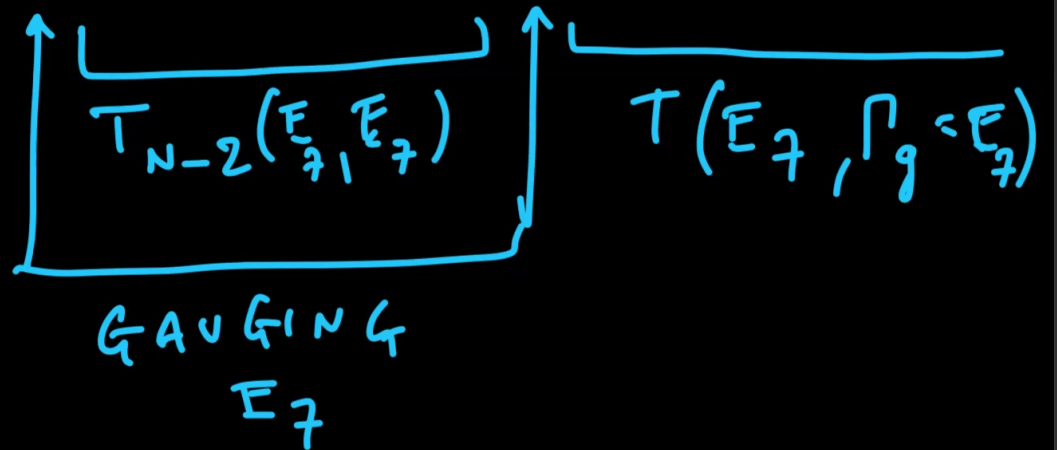
OUR ANALYSIS CAN BE EXTENDED TO
 $\mathbb{C}^2/\Gamma_{D_k}$ ORBIFOLDS STRAIGHTFORWARDLY
BY INTRODUCING D6 PLANES IN TYPE I'

IT IS FOR $\mathbb{C}^2/\Gamma_{E_{6,7,8}}$ THAT F-THEORY
IS CRUCIAL

EXAMPLE: $K_N(E_6, E_7; \Gamma_g = E_7)$

$\overset{e_7}{E_6}] 13151321 \overset{e_7}{7} 12321 \overset{e_7}{7} 12321 \dots \overset{e_7}{7} 12321 \overset{e_7}{7} 12321 \overset{e_7}{7} 123151321 [\overset{e_7}{E_7}$

$T(E_6, \Gamma_g = E_7)$



HERE WE ARE USING THE FOLLOWING
 "NON HIGGSABLE" CLUSTERS MORRISON-TAYLOR 12

$$\begin{array}{ccccc}
 SU(3) & & P_4 & & e_8 \\
 3 & & 5 & & (12) \\
 & & & & 2 \quad 2 \quad 3 \\
 & & & & \frac{1}{2}(2, 7+1)
 \end{array}
 \quad
 \begin{array}{ccccc}
 SU(2) & & g_2 & & SU(2) & & g_2 \\
 & & & & 2 & & 3 \\
 & & & & \frac{1}{2}(2, 7+1)
 \end{array}$$

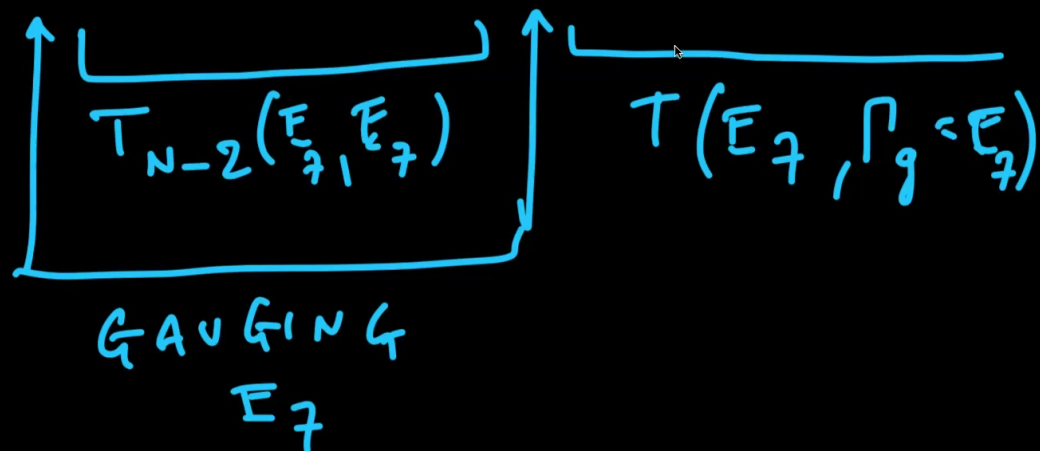
AS WELL AS

$$\begin{array}{ccc}
 e_8 & & e_8 \\
 (11) & = & 12 \\
 & & 1
 \end{array}$$

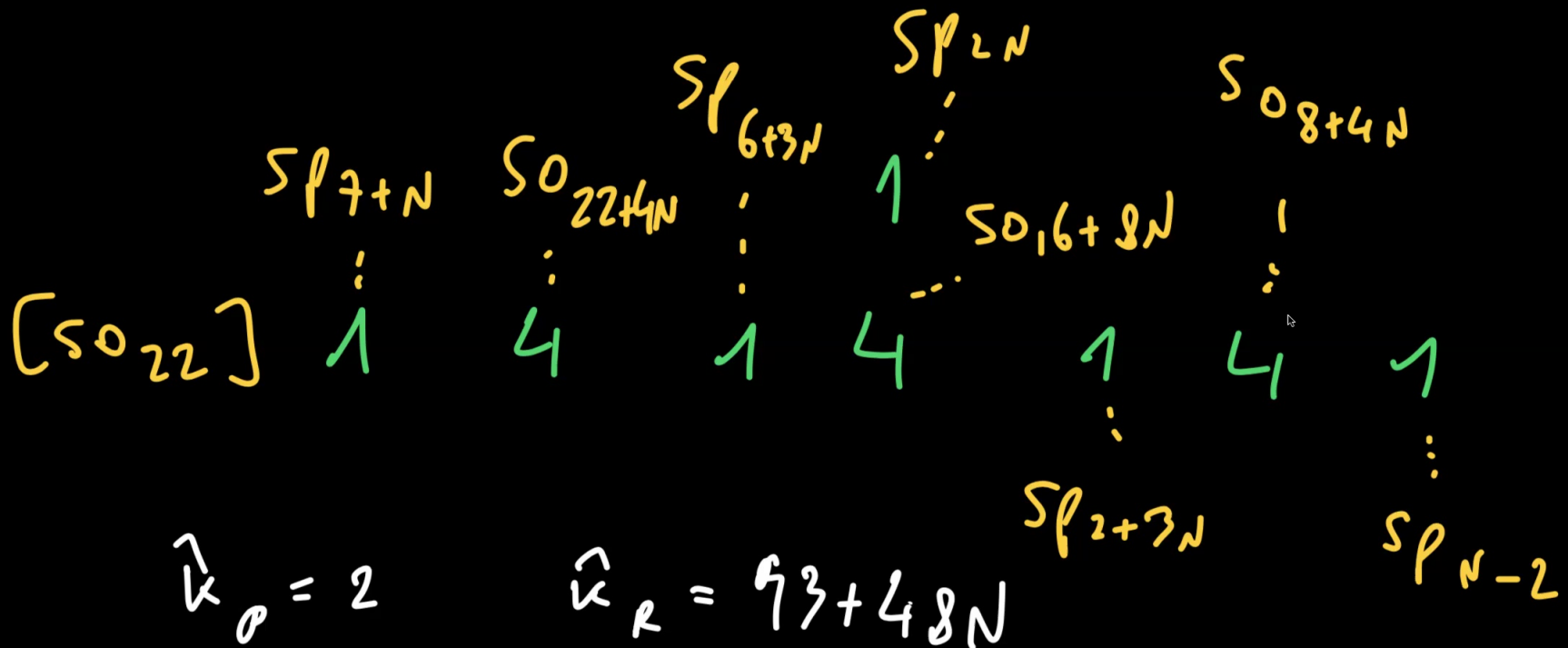
EXAMPLE: $K_N(E_6, E_7; \Gamma_g = E_7)$

E_6 $\overset{e_7}{1} \overset{e_7}{3} \overset{e_7}{5} \overset{e_7}{1} \overset{e_7}{3} \overset{e_7}{2} \overset{e_7}{1} \overset{e_7}{7} \overset{e_7}{1} \overset{e_7}{2} \overset{e_7}{3} \overset{e_7}{2} \overset{e_7}{1} \dots \overset{e_7}{7} \overset{e_7}{1} \overset{e_7}{2} \overset{e_7}{3} \overset{e_7}{2} \overset{e_7}{1} \overset{e_7}{7} \overset{e_7}{1} \overset{e_7}{2} \overset{e_7}{3} \overset{e_7}{2} \overset{e_7}{1} \overset{e_7}{7} \overset{e_7}{1} \overset{e_7}{2} \overset{e_7}{3} \overset{e_7}{5} \overset{e_7}{1} \overset{e_7}{3} \overset{e_7}{2} \overset{e_7}{1} [E_7]$

$T(E_6, \Gamma_g = E_7)$



IS T-DUAL TO



$$\hat{k}_0 = 2$$

$$\hat{k}_R = 93 + 48N$$

$$d_{CB} = 43 + 18N$$

IN THE PAPER [3] ALL POSSIBLE EXAMPLES
WITH EXCEPTIONAL SIMPLY-LACED SYMMETRY
ON EXCEPTIONAL SINGULARITIES ARE
ANALYZED IN DETAIL, BOTH FROM THE
GEOMETRIC AND THE FIELD THEORETIC
PERSPECTIVES.

ONE REMARK IS THAT FOR E_6 SINGULARITIES
WE OBTAIN A FOLDED BASE

1 4 1 2 2

IS T-DUAL TO

$$\begin{array}{ccccc}
 & \text{Sp}_{7+N} & \text{SO}_{22+4N} & \text{SU}_{8+4N} & \\
 & i & i & i & i \\
 [\text{SO}_{22}] & 1 & 4 & 1 & 2 \\
 & & [\text{Sp}_1] & [\text{SO}_2] & \text{SU}_{4+2N} \\
 & & & & i
 \end{array}$$

$$\hat{k}_\theta = 2$$

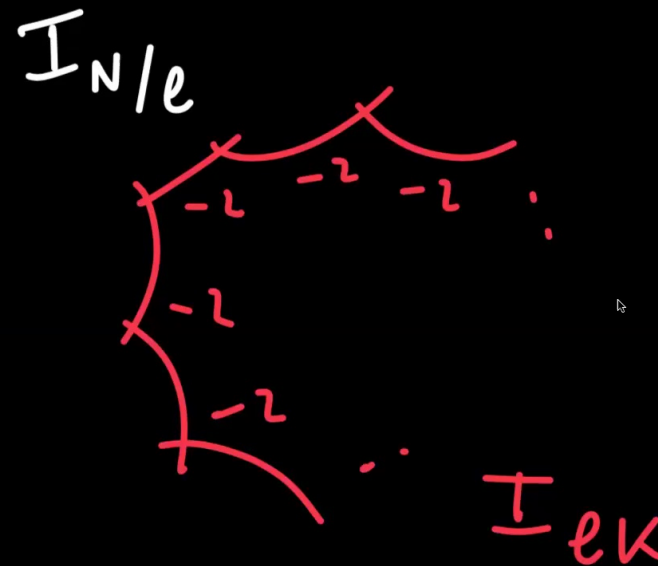
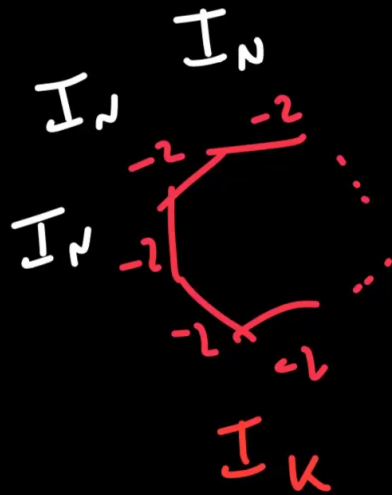
$$\hat{k}_R = 69 + 24N$$

IN SOME CASES MORE THAN 2 THEORIES
HAVE MATCHING $\hat{K}_L, \hat{K}_\theta, \hat{K}_F$ AND SD
COULOMB BRANCHES,

IN THOSE CASES WE CONJECTURE THESE
ARE ALL T-DUAL ONE ANOTHER,
GENERALIZING A REMARK BY HOHENEGLER,
IQBAL, LEE.

WE CAN EXPLICITLY CHECK THIS FOR SOME
EXAMPLES.

E.G. WHEN $N = ke$



AND THE STRUCTURE
CONSTANTS MATCH

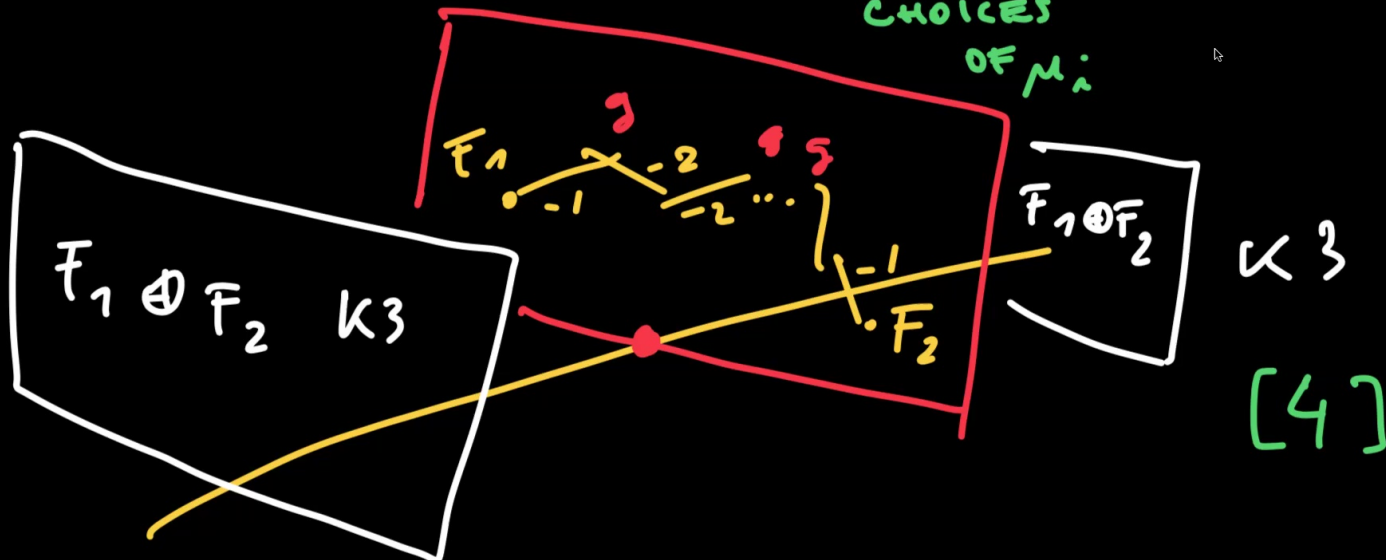
GEOMETRIZE HETEROTIC ALE INSTANTONS

DIFFERENT ELLIPTIC $K3$

FRIEDMAN
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WITTEN

$$H_2(K3) \simeq U^{\oplus 3} \oplus E_8^{\oplus 2} \simeq \text{Pic} \oplus \text{Trasc}$$

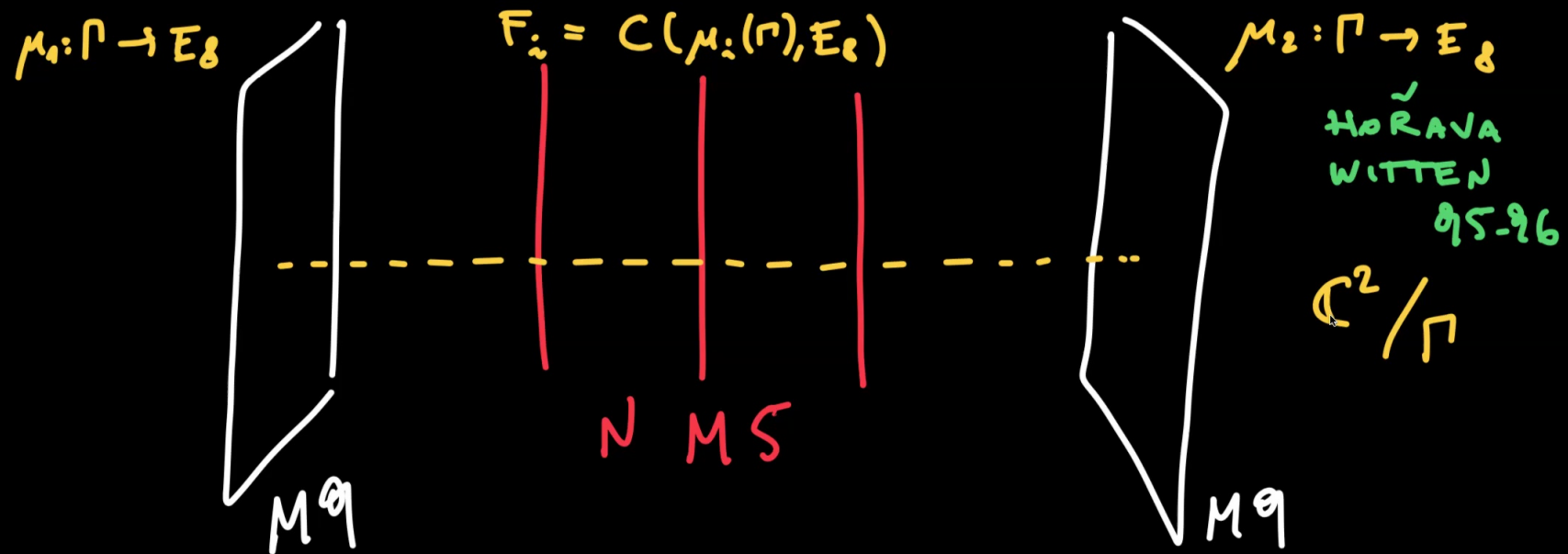
↑
CHOICES
OF M_i



EXPLOITING GENERALIZED KULIKOV MODEL
 FOUND MANY EXAMPLES OF MULTIPLE
 EXOTIC DUALITIES FOR NON-GEOMETRIC
 HETEROTIC LSTs VIA F-THEORY [4, 6]

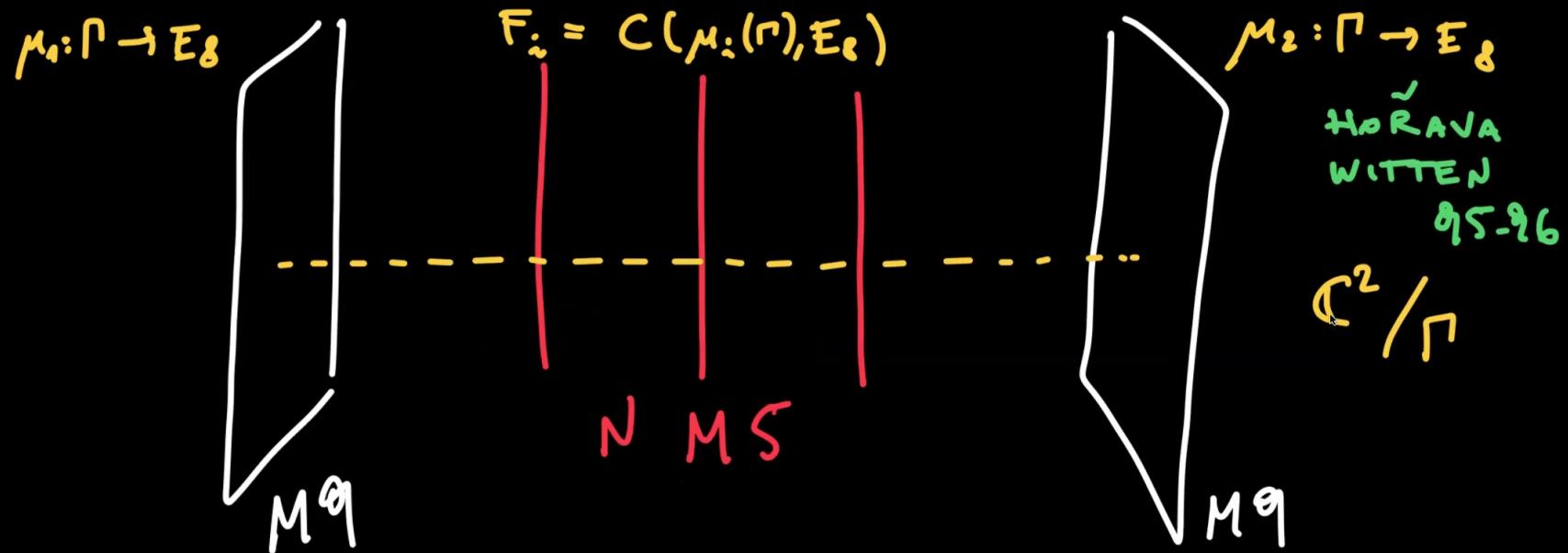
$$\begin{array}{ccc}
 E_8^2 \times SU(3) & \text{---} & Spin(32)/\mathbb{Z}_2 \times SU(3) \\
 \swarrow & \text{"T-Hexality"} & \searrow \\
 E_6^3 / \mathbb{Z}_3 & & U(18) / \mathbb{Z}_3 \\
 \swarrow & & \searrow \\
 (Spin(14) \times SU(12)) / \mathbb{Z}_4 & \text{---} & (Spin(20) \times E_7) / \mathbb{Z}_2 \times U(1)
 \end{array}$$

GEOMETRIC ENGINEERING LIMIT BOOTSTRAP:



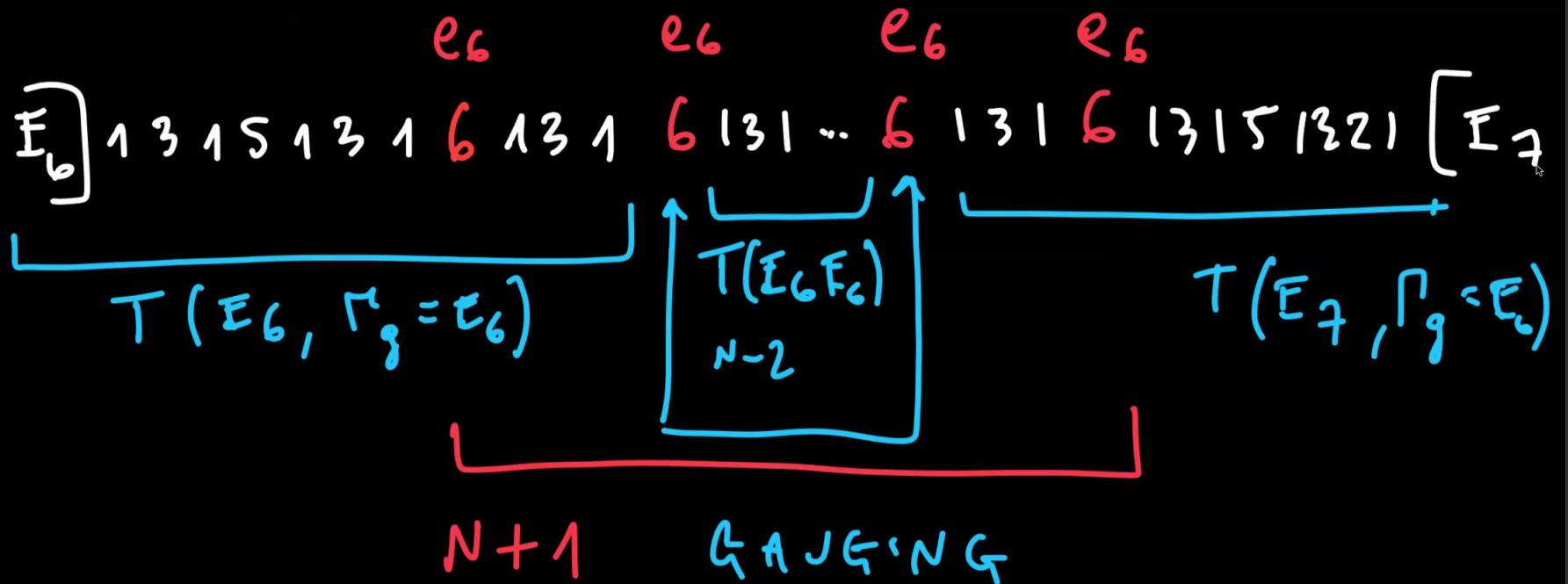
$$\equiv T(\mu_1, \Gamma) \xrightarrow{g_\Gamma} T_{N-2}(g_\Gamma, g_\Gamma) \xrightarrow{g_\Gamma} T(\mu_2, \Gamma)$$

GEOMETRIC ENGINEERING LIMIT BOOTSTRAP:



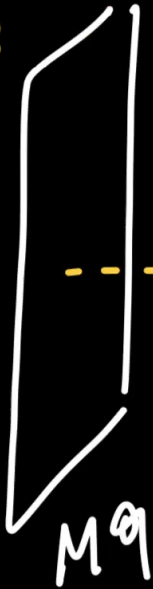
$$\equiv T(\mu_1, \Gamma) \xrightarrow{g_\Gamma} T_{N-2}(g_\Gamma, g_\Gamma) \xrightarrow{g_\Gamma} T(\mu_2, \Gamma)$$

EXAMPLE: $K_N(E_6, E_7; \Gamma_g = E_6)$



GEOMETRIC ENGINEERING LIMIT BOOTSTRAP:

$$\mu_1: \Gamma \rightarrow E_8$$

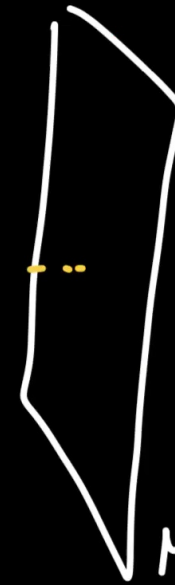


$$F_i = C(\mu_i(\Gamma), E_8)$$

REMOVE
NSS



$$\mu_2: \Gamma \rightarrow E_8$$



✓
HOŘAVA
WITTEN
95-96

$$C^2/\Gamma$$

$$\equiv T(\mu_1, \Gamma) \xrightarrow{g_\Gamma} \text{FUSE TOGETHER} \xrightarrow{g_\Gamma} T(\mu_2, \Gamma)$$

CONCLUSIONS / OUTLOOK

- 2-GROUPS MATCHING ALLOWS CHARTING T-DUALITY OF HETEROTIC ALE INSTANTONS
- W.I.P. TO EXTEND THIS TO TWISTED T-DUALITIES / GENUS ONE FIBRATIONS
MORE GENERAL EXAMPLES

IF $T^2 \rightarrow X \xrightarrow{\pi} B$ IS A GENUS ONE
FIBRATION $\mathcal{I}(X) \neq X$, BECAUSE π HAS
MULTISECTIONS.

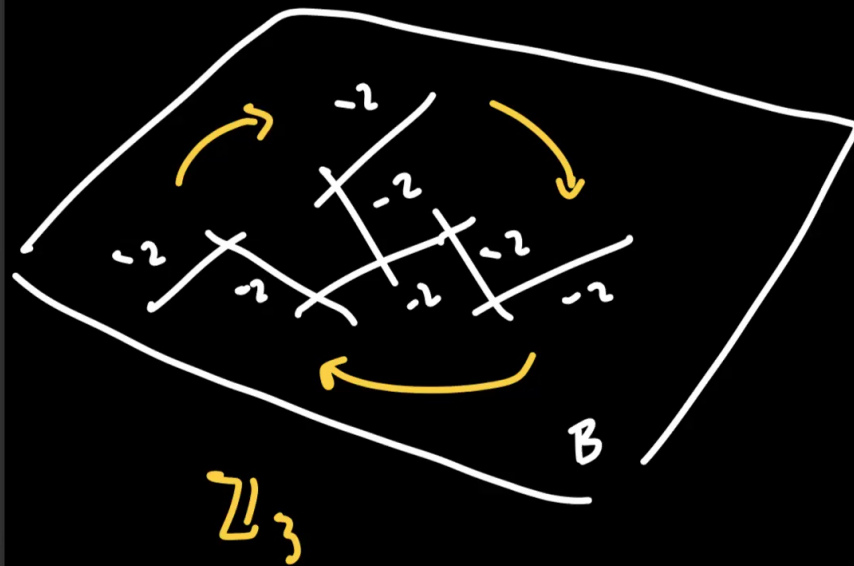
6D LST: $\mathbb{F}/\mathcal{I}(X)$

↓ ON S^1 TWISTED BY DISCRETE SYMMETRY

5D TWISTED KK THEORY M/X

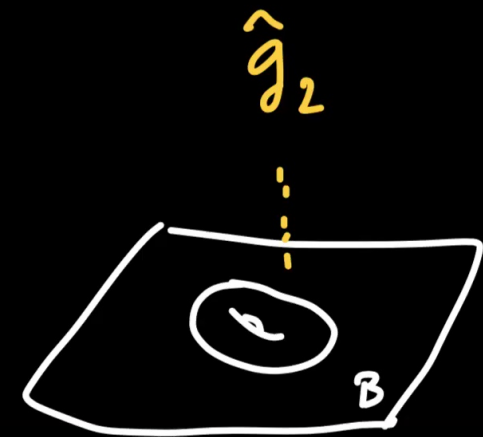
GENERALIZATION OF T-DUALITY

$(2,0)$ 6D LST e_6



τ/U_3

$(1,1)$ 6D
LST of g_2



SUITABLE GENERALIZATION OF MATCHING D_2, D_4, D_6, D_8
120

THANK YOU FOR YOUR
ATTENTION !