Title: TBA

Speakers: Hong Liu

Series: Quantum Fields and Strings

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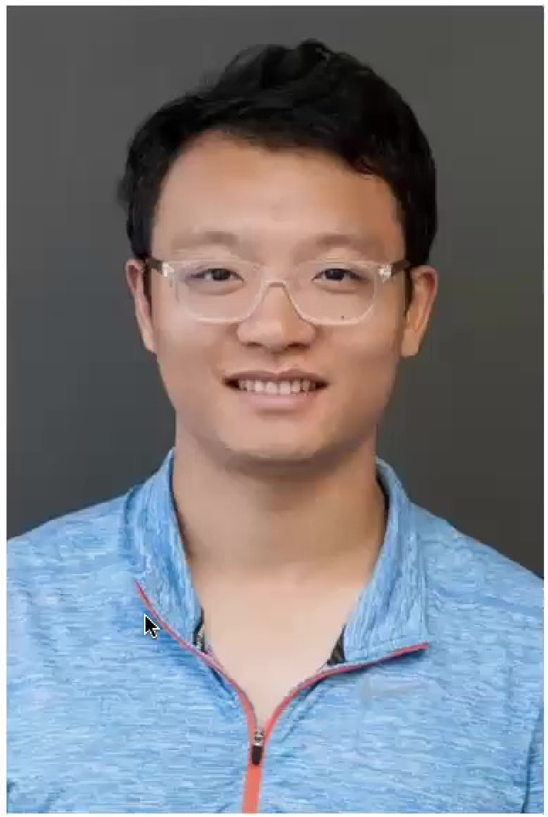
An effective field theory for non-maximal quantum chaos

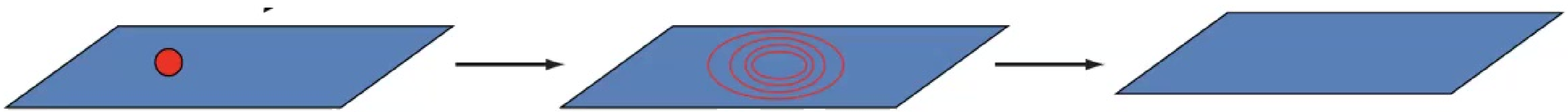
Hong Liu

Perimeter Institute
Feb. 17, 2023

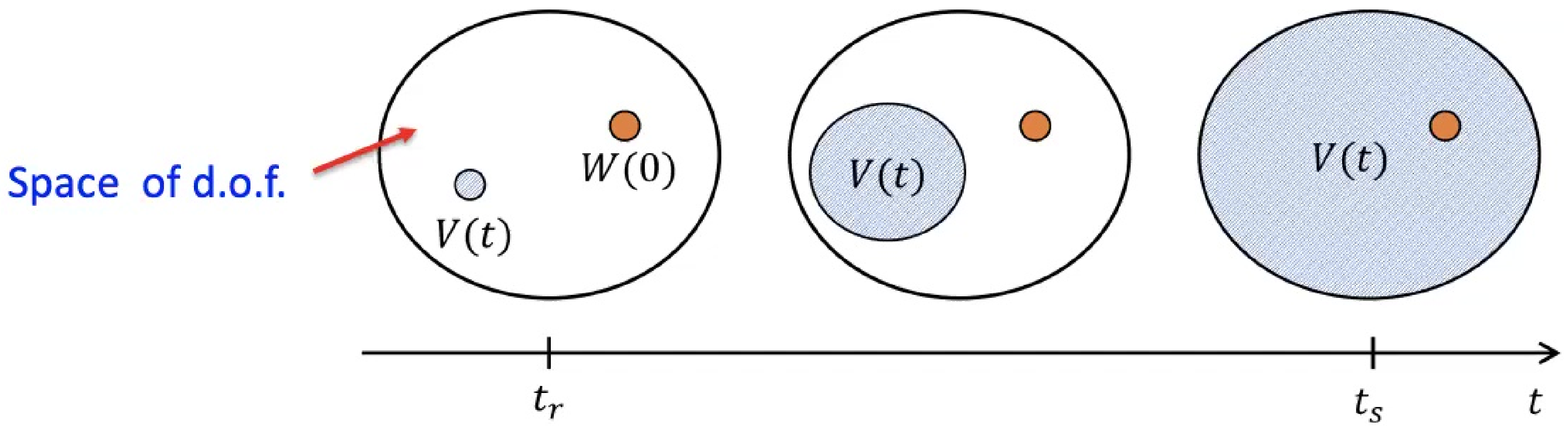
Based on work with Ping Gao, arXiv:2301.05256



Scrambling of quantum information



$$V(t) = e^{iHt} V(0) e^{-iHt}$$



V, W generic few-body operators

Scrambling and quantum chaos

OTOC:

Larkin and Ovchinnikov, 1969,
Shenker, Stanford, Kitaev 2013

$$F(t) = \langle W(0)V(t)W(0)V(t)\rangle_\beta = \langle \Psi_2(t)|\Psi_1(t)\rangle$$

$$|\Psi_1(t)\rangle \equiv W(t)V(0)|\Psi_\beta\rangle, \qquad |\Psi_2(t)\rangle \equiv V(0)W(t)|\Psi_\beta\rangle$$

$$F(t) = 1 - \frac{a}{N}e^{\lambda t} + \cdots$$

N : number of degrees of freedom

quantum Chaos bound:

$$\lambda \leq \frac{2\pi}{\beta}$$

Maldacena, Shenker, Stanford, 2015

TOCs:

$$\langle V(t)W(0)W(0)V(t)\rangle_\beta \sim O(1)$$

$$\langle W(0)V(t)V(t)W(0)\rangle_\beta \sim O(1)$$

Maximally chaotic systems

$$\lambda_{\max} = \frac{2\pi}{\beta}$$

1. Any finite T holographic systems in the classical gravity limit

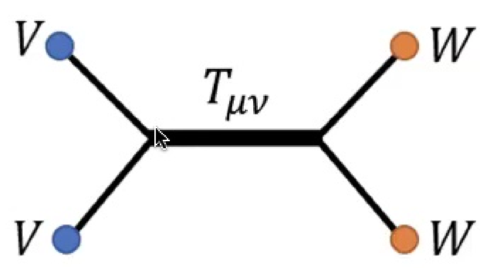
2. Sachdev-Ye-Kitaev (SYK) model and its variants in the low temperature limit

3. two-dimensional CFTs in the large central charge limit

……

A common theme:

leading term given by exchange of the stress tensor



Non-maximal chaotic systems

Non-maximal chaotic systems: $\lambda < \frac{2\pi}{\beta}$

Some solvable examples: Large-q SYK, holographic systems with stringy corrections, conformal Regge theory

$$1 - F(t) \sim \frac{1}{N} \sum_j e^{\frac{2\pi}{\beta}(j-1)t} \sim \frac{1}{N} e^{\lambda t}$$

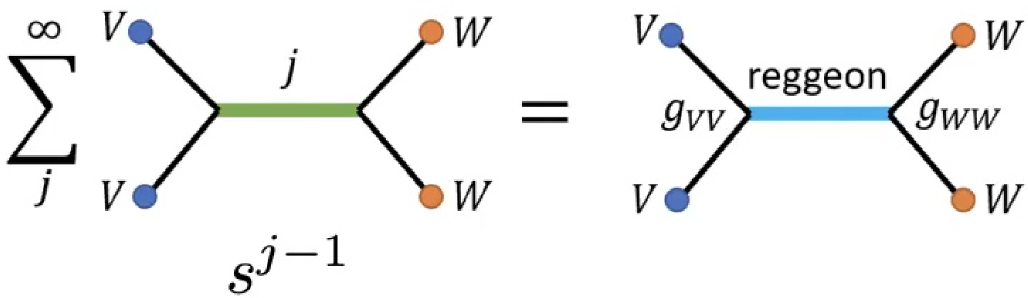
j: integers ("spin")

As if coming from exchange of a single operator of effective spin

$$j_{\text{eff}} = 1 + \lambda \frac{\beta}{2\pi}$$

Scattering in the Regge regime

$$V + W \to V + W \qquad s \to \infty, \quad \mathrm{t} \text{ fixed}$$



$$\mathcal{A}(s,\mathrm{t}) \propto g_{WW}(\mathrm{t})g_{VV}(\mathrm{t})s^{\alpha(\mathrm{t})-1}$$

In holographic systems, $F(t) = \langle W(0)V(t)W(0)V(t)\rangle_\beta$

coming from scattering of the corresponding bulk particles in the Regge Regime.

Shenker and Stanford, 2014

Basic Question

Can the exponential behavior of OTOCs be given an effective description in terms of exchanges of one or a small number of effective fields?

Exponential behavior of OTOCs

Absence of exponential behavior of TOCs

Can provide a universal description of general quantum chaotic systems, powerful handle on extracting universal properties.

Plan

1. Review of EFT for maximally chaotic systems

Blake, Lee, HL, 2018
Blake, HL, 2021

2. An effective field theory description of non-maximal chaos

Ping Gao, HL
arXiv: 2301.05256

EFT for maximal chaos

In this case, need an effective theory of the stress tensor at a finite T.

For $\Delta L, \Delta t \gg \beta$, hydrodynamics

But to capture the Lyapunov behavior $e^{\lambda t}$, need $\Delta t \sim \beta$

A recent formulation of hydrodynamics based on action principle and symmetries can be used to do this. Crossley, Glorioso, HL 2015-2016

Promote spacetime coordinates to dynamical variables:

$X^\mu(\sigma^a)$ σ^a : labels of fluid elements and internal clock

Effective theory of $X^\mu(\sigma^a)$ captures dynamics of the stress tensor

There is a systematic procedure to write down an action for $X^\mu(\sigma^a)$

$\Delta L, \Delta t \gg \beta$: local $\qquad$ $\Delta t \sim \beta$: non-local

Equilibrium: $\quad X^\mu = \sigma^a \delta^\mu_a$

More convenient:

$$X^\mu(\sigma^a) \to \sigma^a(x^\mu) = x^\mu \delta^a_\mu + \epsilon^a(x^\mu)$$

$$\langle \epsilon\epsilon \rangle_\beta \sim 1/N$$

Ingredients of EFT for maximal chaos

Blake, Lee and HL, 2018

1. Coupling of generic operator to the stress tensor:

$$V(X^\mu(\sigma^a)) \to V(x^\mu; \epsilon^a(x^\mu))$$
$$= V_0(x^\mu) + L_a[V_0\epsilon^a] + O(\epsilon^2)$$

$$\langle V_0(x)V_0(0)\rangle_\beta \equiv g_V(x) \qquad \langle V_0(x)W_0(0)\rangle_\beta = 0$$

$$L_t[V_0\epsilon^0] \equiv \sum_{m,n=0}^{\infty} c_{mn}\partial_t^m V_0(t)\partial_t^n \epsilon^0$$ (effective vertex)

V =

2. Both the action and the effective coupling between V_0 and σ^a are invariant under the following shift symmetry

$$\epsilon^0 \to \epsilon^0 + e^{\pm \lambda t}$$

This is interpreted as an emergent low energy gauge symmetry for maximally chaotic systems.

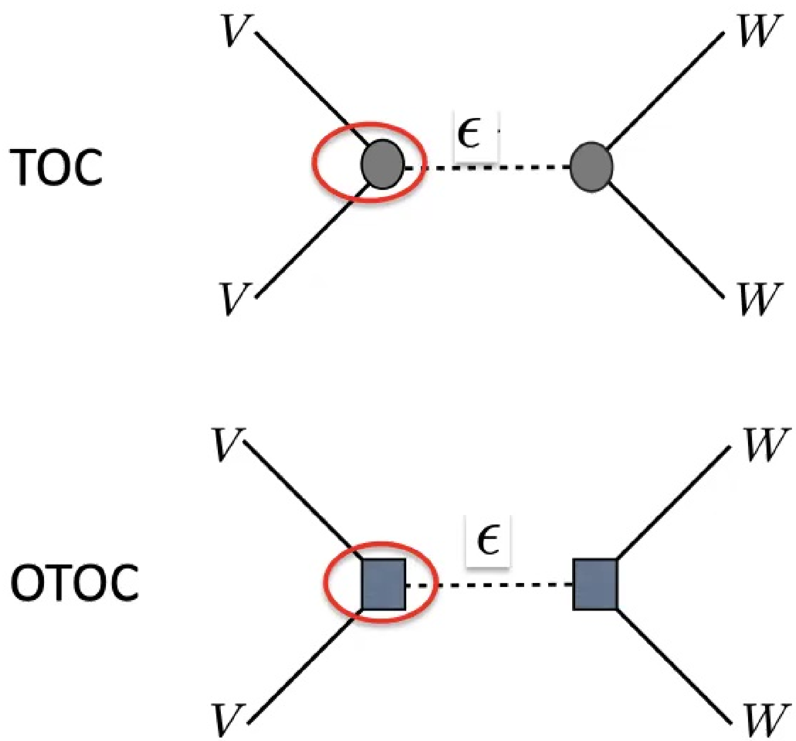
(In holographic systems, it arises from horizon symmetries, Knysh, HL, Pinzani-Fokeeva, to appear)

Naively the above ingredients can be formulated for any λ

fluctuation-dissipation relation of two-point functions of V
+ shift symmetry of the effective vertex

$$\Rightarrow \quad \lambda = \frac{2\pi}{\beta} = \lambda_{\max}$$

Four-point functions



No exponential growth

exponential growth

Consequence of the shift symmetry:
independent of details of the action and the effective vertex

2. Both the action and the effective coupling between V_0 and σ^a are invariant under the following shift symmetry

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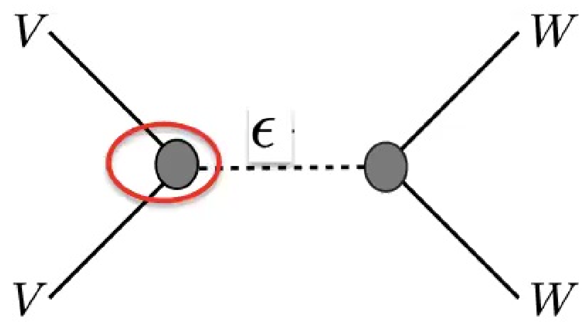
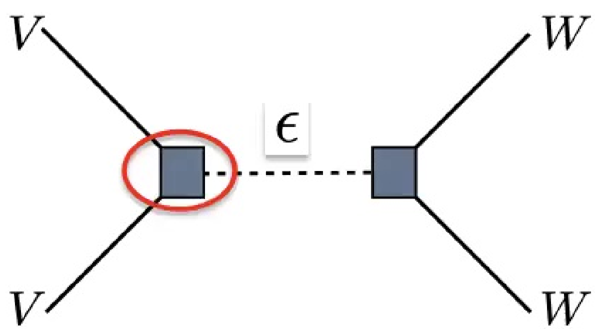
Four-point functions

TOC



No exponential growth

OTOC



exponential growth

EFT for non-maximal chaos

$$F(t) \sim \sum_j e^{\frac{2\pi}{\beta}(j-1)t} \sim e^{\lambda t}$$

An EFT description in terms of a small number of fields is different from the usual sense of an EFT.

There is no separation of scales

no clear candidate for effective fields

close analogue with the Regge scattering

Can we generalize the EFT for maximal chaos to non-maximal chaos case?

Can the exponential behavior be understood as some kind of symmetry ?

Setup

We will approach this problem "phenomenologically".

With $\lambda < \lambda_{\max}$ as an input parameter

Would like to write down a theory with a small number of fields and a shift symmetry

1. Whose exchanges lead to exponential behavior in OTOCs, but no exponential in TOC.

2. Satisfy all the KMS relations satisfied by 4-point functions and two-point functions

$$\mathcal{F}(t_1, t_2; t_3, t_4) = \langle \mathcal{T} W(t_1) W(t_2) V(t_3) V(t_4) \rangle_\beta$$

$\mathcal{T}$: operators ordered from left to right according to the ascending order of their corresponding $\Im t_i$

It turns out it is possible, and may be interpreted as a minimal generalization of the maximal chaos EFT.

The structure is a nevertheless bit intricate.

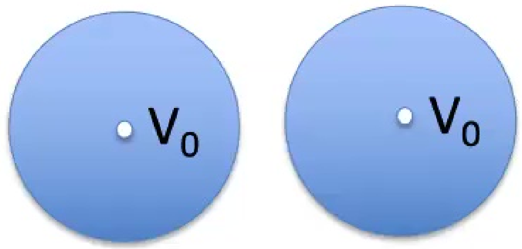
The structure was motivated from the solution of Choi, Mezei and Sarosi, 2019 of the large q-SYK model , but can be formulated in general terms, independent of any specific systems.

For simplicity, only consider (0+1)-dimension

General structure

Recall for maximal chaos:

$$V(t_1)V(t_2) =$$



$$= V_0(t_1)V_0(t_2) + L_{t_1}[V_0(t_1)\epsilon(t_1)]V_0(t_2) + V_0(t_1)L_{t_2}[V_0(t_2)\epsilon(t_2)] + O(\epsilon^2)$$

Non-maximal: introduce two fields $\phi_{1,2}$

$$V(t)V(t') = V_0(t)V_0(t') + \sum_{i=1}^{2} \mathcal{D}_V^{(i)}(t,t')\phi_i(t,t') + O(\phi^2)$$

$\phi_{1,2}$ now depend on both t and t’

We take $\phi_{1,2}$ to couple "mainly" to one of the V's

$$\bar{t} = \frac{t+t'}{2} \qquad (t_S, t_L) = (t, t'),\ \Im t_S < \Im t_L,$$

$$\phi_1(t,t') = \phi_1(\bar{t}; t_S) \qquad \phi_2(t,t') = \phi_2(\bar{t}; t_L)$$

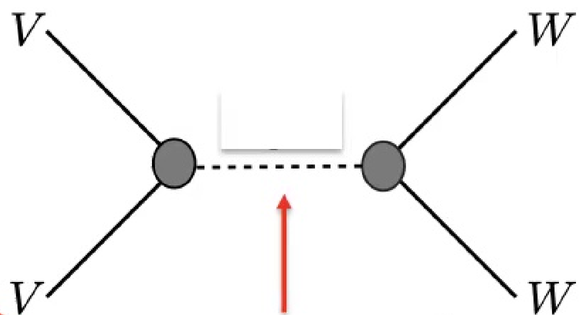
Both $\phi_{1,2}$ depend weakly on $\bar{t}$

$$\mathcal{D}_V^{(1)}(t,t')\phi_1(t,t') = V_0(t_L) L_{t_S}[V_0(t_S)\phi_1(\bar{t}; t_S)]$$

$$\mathcal{D}_V^{(2)}(t,t')\phi_2(t,t') = V_0(t_S) L_{t_L}[V_0(t_L)\phi_2(\bar{t}; t_L)]$$

$$L_t[V_0(t)\phi(\bar{t};t)] \equiv \sum_{m,n=0}^{\infty} c_{mn}\partial_t^m V_0(t)\partial_t^n \phi(\bar{t};t)$$

$$\langle \mathcal{T} W(t_1) W(t_2) V(t_3) V(t_4) \rangle_\beta =$$



$$\langle \hat{\mathcal{T}} \phi_s \phi_{s'} \rangle_{\rm EFT}, \;\; s, s' = 1, 2$$

KMS relations satisfied by the four-point functions require nontrivial "KMS" relations in the EFT of $\phi_{1,2}$ $(\beta = 2\pi)$

$$\langle \hat{\mathcal{T}} \phi_1(\bar{t}; t) \phi_i(\bar{t}'; t') \rangle \simeq \langle \hat{\mathcal{T}} \phi_2(\bar{t} + \pi i; t + 2\pi i) \phi_i(\bar{t}'; t') \rangle, \quad \Im t < \Im t'$$

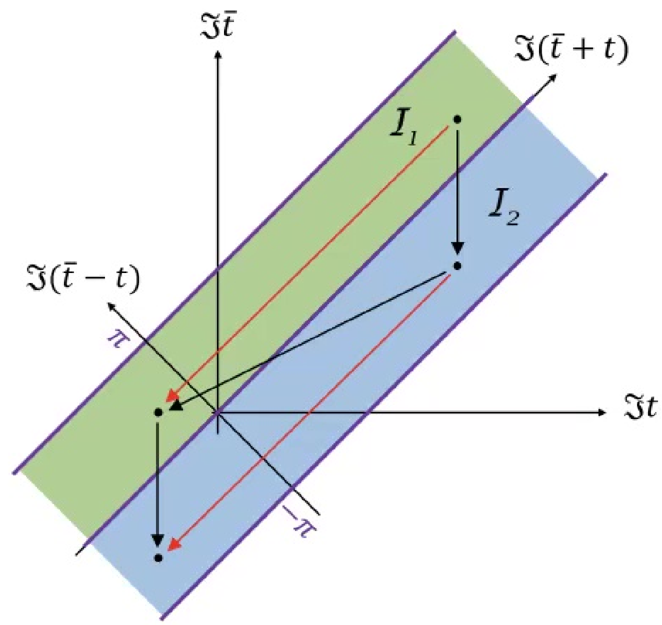
$$\langle \hat{\mathcal{T}} \phi_2(\bar{t}; t) \phi_i(\bar{t}'; t') \rangle \simeq \langle \hat{\mathcal{T}} \phi_1(\bar{t} + \pi i; t) \phi_i(\bar{t}'; t') \rangle$$

But we cannot directly write down an action for $\phi_{1,2}$

Two complications:

1. they are not defined in the same domain

2. The unusual KMS relations they need to satisfy



$$\eta_{\pm}(\bar{t};t) = \frac{1}{\sqrt{2}}(\phi_1(\bar{t};t-i\pi) \pm \phi_2(\bar{t};t))$$

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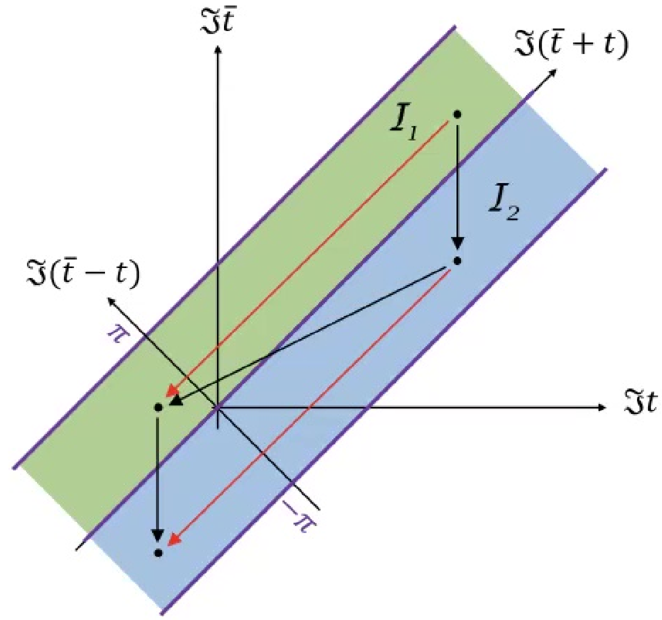

$$\eta_{\pm}(\bar{t};t) = \frac{1}{\sqrt{2}}(\phi_1(\bar{t};t-i\pi) \pm \phi_2(\bar{t};t))$$

$$\langle \hat{\mathcal{T}} \eta_s(\bar{t};t)\eta_{s'}(\bar{t}';t')\rangle = s\langle \hat{\mathcal{T}} \eta_s(\bar{t}+i\pi;t+i\pi)\eta_{s'}(\bar{t}';t')\rangle$$

$$s = +, -$$

So far the time variables $\bar{t}, t$ are complex. To write down an effective action we need to choose a real section in the complex $\bar{t}, t$ planes.

It is convenient to choose the section to be that of imaginary $\bar{t}$ and real t

$$\bar{t} = -i\bar{\tau} \qquad \eta_s(\bar{\tau}; t)$$

It can be viewed as a two-dimensional field theory with $\bar{\tau}$ being a "spatial" coordinate and (real) t being time.

The EFT is at a finite temperature with respect to t

η_s still do not have the conventional KMS relations

$$\eta_s(\bar{\tau};t) = \eta_{s,0}(\bar{\tau};t) + \eta_{s,+}(\bar{\tau};t) + \eta_{s,-}(\bar{\tau};t)$$

$$\eta_{s,p}(\bar{\tau}+\pi;t) = e^{2\pi i p/3}\eta_{s,p}(\bar{\tau};t), \quad p = 0, \pm$$

$\eta_{s,p}$ can now finally be treated as ordinary fields at a finite temperature (in terms of t)

$$S_{\mathrm{EFT}} = \sum_{s,p}\int_0^{\pi} d\bar{\tau}\int_{-\infty}^{\infty} dt\left[\eta^a_{s,-p}K^{ar}_{s,p}(\partial_{\bar{\tau}},\partial_t)\eta^r_{s,p} + \frac{1}{2}\eta^a_{s,-p}K^{aa}_{s,p}(\partial_{\bar{\tau}},\partial_t)\eta^a_{s,p}\right]$$

Shift symmetry

The action and the vertices are invariant under

$$\eta_-^r \to \eta_-^r + a_+ e^{\lambda t} + a_- e^{-\lambda t}$$

There is no exponential growth in the symmetric correlation functions of η_+

Exponential behavior of OTOCs

Absence of exponential behavior of TOCs

Independent of details of the action or effective vertices

General structure of OTOCs

Can be used to obtain the general form of OTOCs

$$F_4 = \alpha e^{\lambda(t_1+t_2-t_3-t_4+i\pi)/2} \frac{G^W_{even}(\lambda, t_{12}) G^V_{even}(-\lambda, t_{34})}{\cosh \frac{\lambda(t_{12}+i\pi)}{2} \cosh \frac{\lambda(t_{34}+i\pi)}{2}} + (\lambda \leftrightarrow -\lambda)$$

G^V_{even} constructed from $g_V(t)$ and the effective vertex

Agree with the results of Kitaev-Suh (2018) and Gu-Kitaev (2019)

Agree with the results obtained from large-q SYK, string scattering in holography and conformal Regge theory

Second term: a new prediction

Now need to expand $V(t)V(t')$ to all orders in $\phi_{1,2}$

$$V(t)V(t') = V_0(t)V_0(t') + \sum_{i=1}^{2} \mathcal{D}_V^{(i)}(t,t')\phi_i(t,t') + O(\phi^2)$$

$$V(t_1)V(t_2) = V_0(t_1)V_0(t_2) + \sum_{n=1,\{i_k\}} \mathcal{D}_{i_1 \cdots i_n} \phi_{i_1} \cdots \phi_{i_n}$$

We again require the effective vertex is invariant under the same shift symmetry

$$F = \int_0^\infty d\tilde{y} \int_0^\infty dy\, e^{-\alpha y \tilde{y} e^{\lambda(t_1+t_2-t_3-t_4+i\pi)/2}} h(t_{12}, y)\tilde{h}(t_{34}, \tilde{y})$$

Future perspectives

1. Higher dimensional systems

   Butterfly velocity

2. How do the effective fields couple to the stress tensor and conserved currents

   What becomes of pole skipping   Transports?

3. Physical nature of the effective fields and shift symmetry

   Stringy version of the horizon symmetry?

4. Potential new ideas for Reggeon field theory in QCD