Title: Formation of Primordial black hole during the QCD phase-transition

Speakers: Ilia Musco

Series: Strong Gravity

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Abstract: The formation of Primordial black holes is naturally enhanced during the quark-hadron phase transition, because of the softening of the equation of state: at a scale between 1 and 3 solar masses, the threshold is reduced of about 10% with a corresponding abundance of primordial black significantly increased by more than 100 times. Performing detailed numerical simulation we have computed the modified mass spectrum for such black holes. Making then a confutation with the LVK phenomenological models describing the GWTC-3 catalog, it is shown that a sub-population of primordial black black holes formed in the solar mass range is compatible with the current observational constraint and could explain some of the interesting sources emitting gravitational waves detected by LIGO/VIRGO in the black hole mass gap, such as GW190814, and other light events.

Zoom Link:https://pitp.zoom.us/j/94805744664?pwd=Z3oyd05YWVlwenI5NFV6bFZOYUR0dz09

Primordial Black Hole formation during the QCD phase transition

Ilia Musco

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Online seminar - Perimeter Institute

16 February 2023

HORIZON 2020 G.A. n. 754496

Introduction: a very brief overview

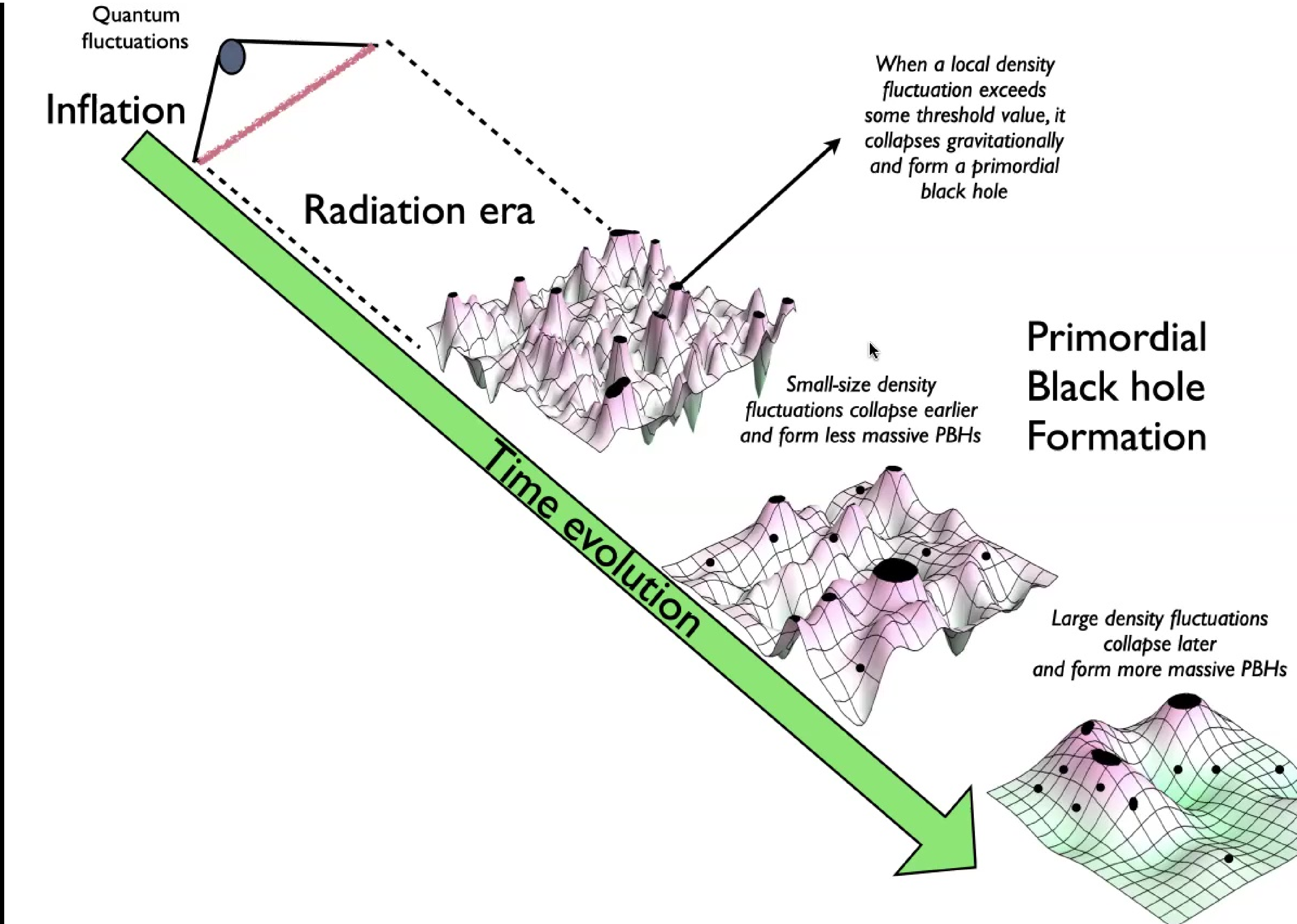
- **Primordial Black Holes (PBHs)** [**Zeldovich & Novikov** (1967), **Hawking** (1971)] could form from the collapse of cosmological perturbation during the radiation dominated era.

$$p = \frac{\rho}{3}$$

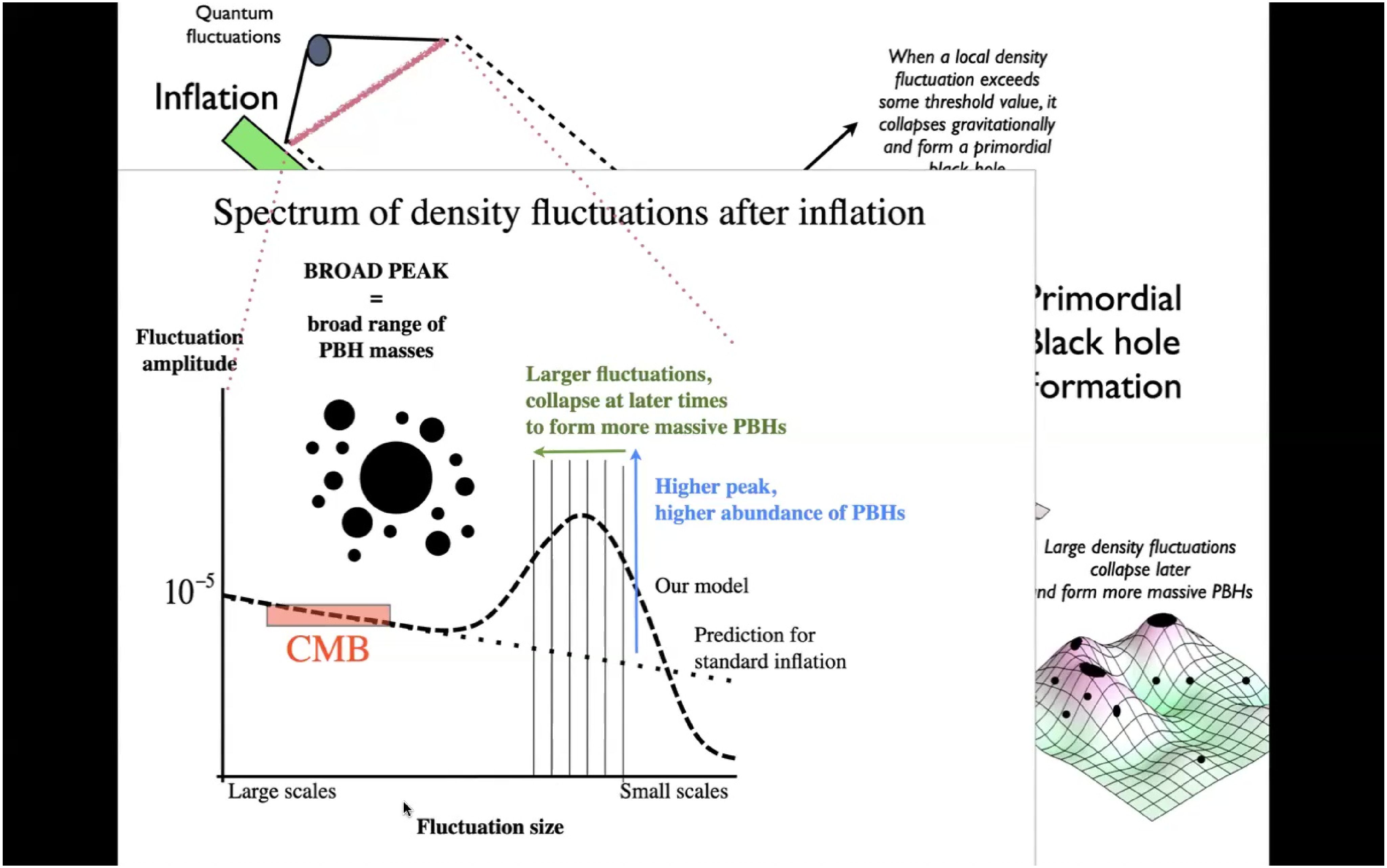
- **PBHs could span a large wide range of masses and if not evaporated** [BH evaporation **Hawking** (1974)]: PBHs with $M > 10^{15}\,g$ are interesting candidates for dark matter, intermediate mass black holes and the seeds of supermassive black holes.

- **Numerical hydrodynamical simulations in spherical symmetry** of a cosmological perturbation, characterized by an amplitude δ, have shown:

  - $\delta > \delta_c \;\Rightarrow\;$ PBH formation
  - $\delta < \delta_c \;\Rightarrow\;$ perturbation bounce
  - $\delta_c \sim c_s^2 \equiv \frac{\partial p}{\partial \rho}$ **(Carr 1975)**



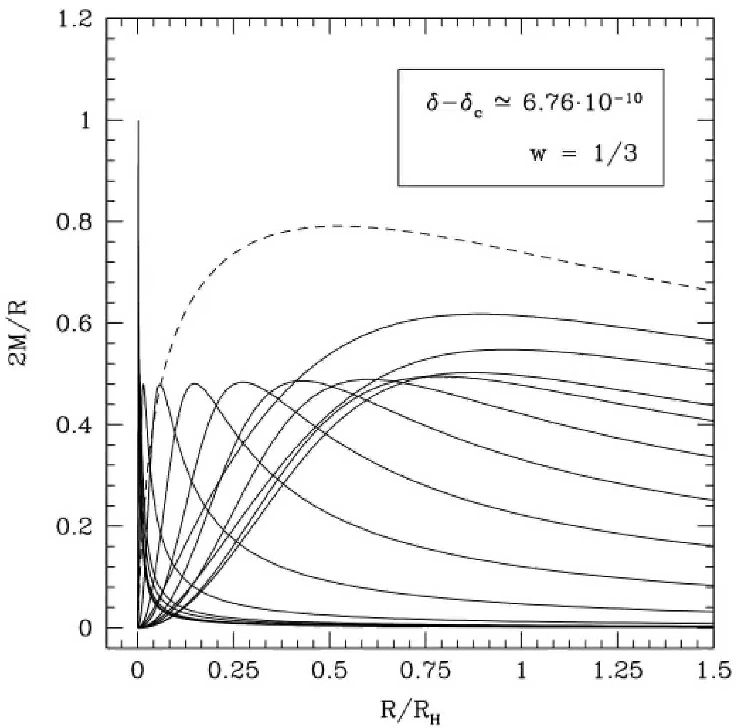
Quantum fluctuations
Inflation
Radiation era
When a local density fluctuation exceeds some threshold value, it collapses gravitationally and form a primordial black hole
Primordial Black hole Formation
Small-size density fluctuations collapse earlier and form less massive PBHs
Time evolution
Large density fluctuations collapse later and form more massive PBHs

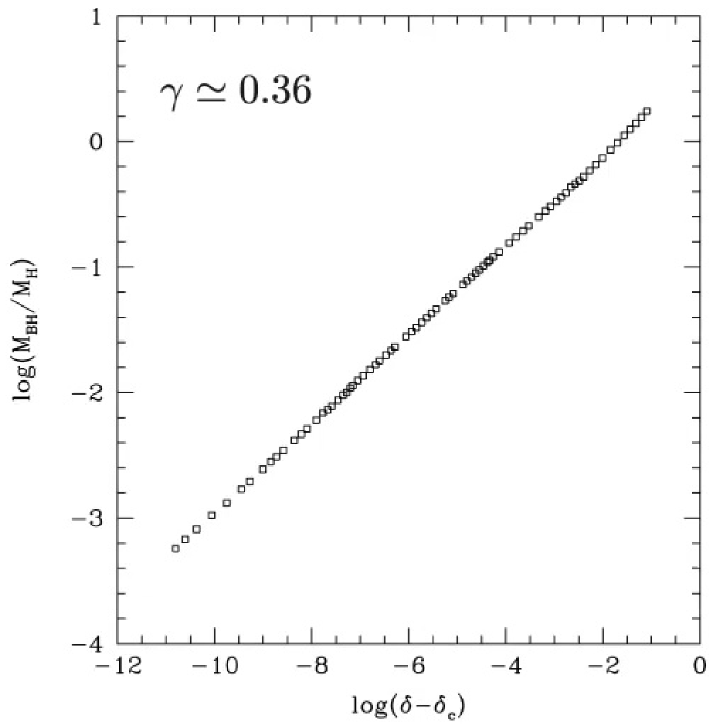

Quantum fluctuations
Inflation
When a local density fluctuation exceeds some threshold value, it collapses gravitationally and form a primordial
Spectrum of density fluctuations after inflation
BROAD PEAK
=
broad range of PBH masses
Fluctuation amplitude
Larger fluctuations, collapse at later times to form more massive PBHs
Higher peak, higher abundance of PBHs
Our model
Prediction for standard inflation
10^{-5}
CMB
Large scales
Small scales
Fluctuation size
rimordial
lack hole
ormation
Large density fluctuations collapse later
nd form more massive PBHs

Numerical Results: PBH formation / mass spectrum

$$R(r,t) = 2M(r,t)$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^{\gamma} M_H$$





IM, J. Miller, Polnarev, Rezzolla - CQG (2005 - 2013)

$\mathcal{K}, \delta_c$ − shape dependent

Initial conditions: curvature profile

- The asymptotic metric ($t \longrightarrow 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t) e^{2\zeta(r)} \left[dr^2 + r^2 d\Omega^2 \right]$$

- In the "linear regime" of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the **quasi-homogeneous / gradient expansion approach**.

$$\frac{\delta\rho}{\rho_b} = -\left(\frac{1}{aH}\right)^2 \frac{4}{9} \left[\nabla^2 \zeta(r) + \frac{1}{2} \left(\nabla \zeta(r)\right)^2 \right] e^{-2\zeta(r)}$$

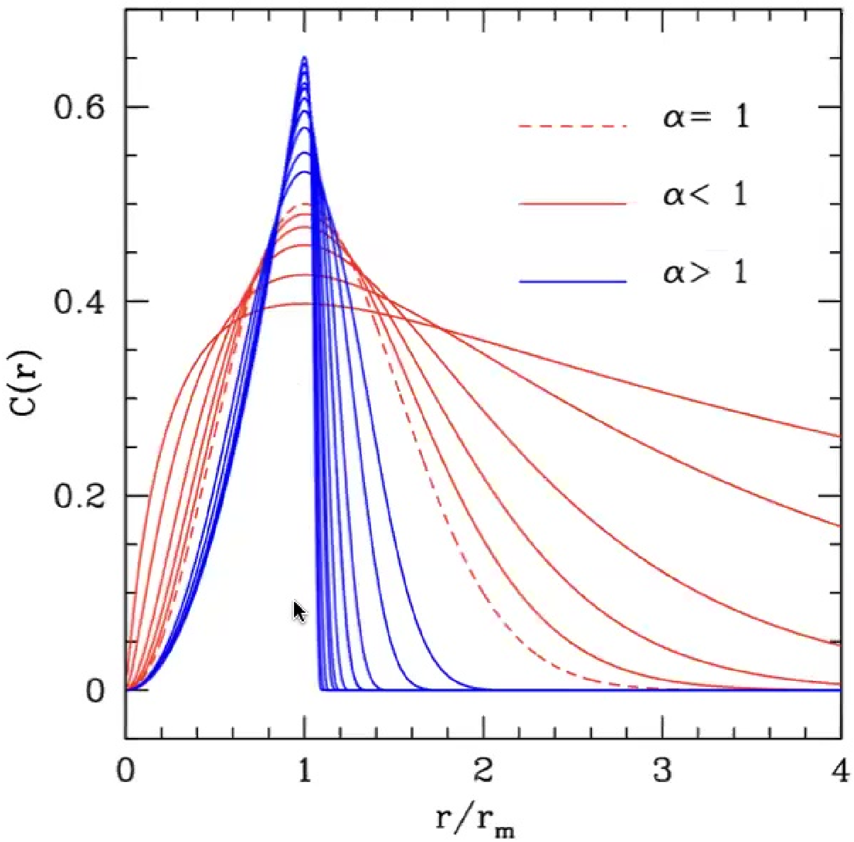
- The perturbation amplitude δ is measured by the peak of the compaction function, corresponding to the excess of mass of the over density.

$$\mathcal{C}(r) := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = -\frac{4}{3} \tilde{r} \zeta'(r) \left[1 + \frac{1}{2} \tilde{r} \zeta'(r) \right] \quad \Rightarrow \quad \delta = \delta_G \left[1 - \frac{3}{8} \delta_G \right]$$

Shape parameter

I. Musco - PRD (2019)

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$



$$\alpha \equiv -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)} = \frac{\alpha_G}{\left(1 - \frac{1}{2}\Phi_m\right)\left(1 - \Phi_m\right)}$$

$$\boxed{0.4 \le \delta_c(\alpha) \le \frac{2}{3}}$$

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

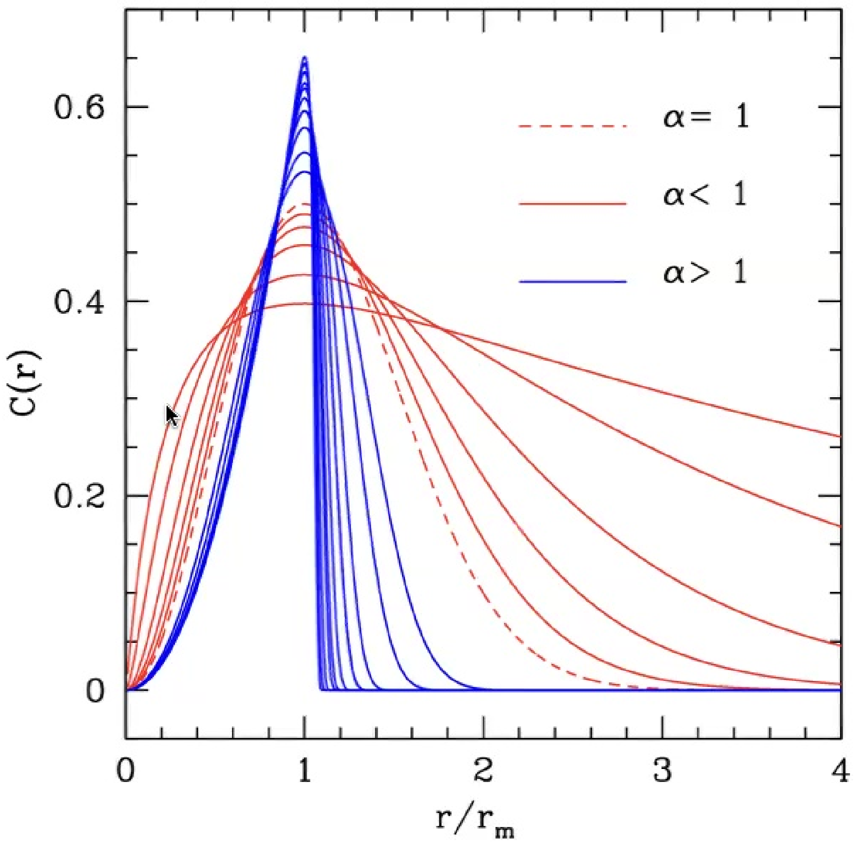
IM, De Luca, Franciolini, Riotto - PRD (2021)

Shape parameter

I. Musco - PRD (2019)

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

$$\delta(r_m, t_H) = 3\frac{\delta\rho}{\rho_b}(r_m, t_H)$$



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IM, De Luca, Franciolini, Riotto - PRD (2021)

PBH Abundance (Peak Theory)

C.Germani, IM - PRL (2019)

- **PDF of δ follows a Gaussian distribution:** $P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$

$$\sigma^2 = \langle \delta^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k} (k r_m)^4 T^2(k, r_m) W^2(k, r_m) \mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left(\frac{k_*}{a_m H_m}\right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \qquad \nu_c \equiv \frac{\delta_c}{\sigma}$$

- **If** $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$
- **Narrow peak:** $\frac{k_*}{\sigma} \gg 1 \;\Rightarrow\; \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$
- **Broad peak:** $\frac{k_*}{\sigma} \ll 1 \;\Rightarrow\; \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \;\Rightarrow\; \mathcal{P}_0 \sim 3 \times 10^{-3}$
- **Non linear effects:** $\delta = \delta_G \left[1 - \frac{3}{8}\delta_G\right] \quad \Rightarrow \quad 1.5 \lesssim \frac{\mathcal{P}_{0_{NL}}}{\mathcal{P}_{0_L}} = \frac{16\left(1 - \sqrt{1 - \frac{3}{2}\delta_c}\right)^2}{9\delta_c^2} \lesssim 4$

S.Young, IM, C.Byrnes JCAP (2019)

De Luca, Franciolini, Kehagias, Peloso, Riotto and Unal (2019)

PBH threshold prescription

Curvature power spectrum $\mathcal{P}_\zeta$

$\Downarrow$

Characteristic overdensity scale $k_* \hat{r}_m$

$\Downarrow$

Characteristic shape parameter α

$\Downarrow$

Threshold δ_c

IM, De Luca, Franciolini, Riotto - PRD (2021)

1. **The power spectrum of the curvature perturbation**: take the primordial power spectrum $\mathcal{P}_\zeta$ of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function $T(k,\eta)$

$$P_\zeta(k,\eta) = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)T^2(k,\eta).$$

2. **The comoving length scale** $\hat{r}_m$ of the perturbation is related to the characteristic scale k_* of the power spectrum P_ζ. Compute the value of $k_*\hat{r}_m$ by solving the following integral equation

$$\int dk k^2 \left[(k^2\hat{r}_m^2 - 1)\frac{\sin(k\hat{r}_m)}{k\hat{r}_m} + \cos(k\hat{r}_m)\right] P_\zeta(k,\eta) = 0\,.$$

3. **The shape parameter:** compute the corresponding shape parameter α of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha)\left[1+F(\alpha)\right]\alpha = -\frac{1}{2}\left[1+\hat{r}_m\frac{\int dk k^4 \cos(k\hat{r}_m)P_\zeta(k,\eta)}{\int dk k^3 \sin(k\hat{r}_m)P_\zeta(k,\eta)}\right]$$

$$F(\alpha) = \sqrt{1-\frac{2}{5}e^{-1/\alpha}\frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right)-\Gamma\left(\frac{5}{2\alpha},\frac{1}{\alpha}\right)}}\,.$$

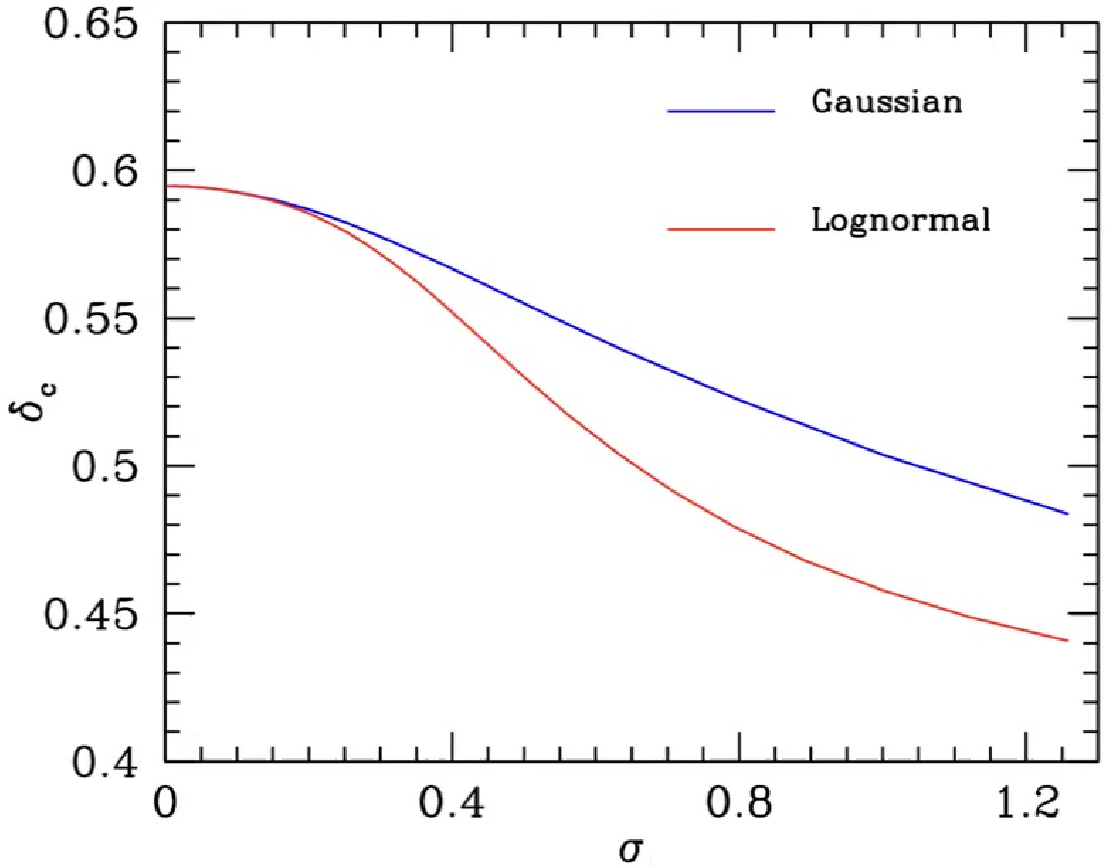
4. **The threshold** δ_c: compute the threshold as function of α, fitting the numerical simulations, at *superhorizon scales*, making a linear extrapolation at horizon crossing ($aHr_m = 1$).

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

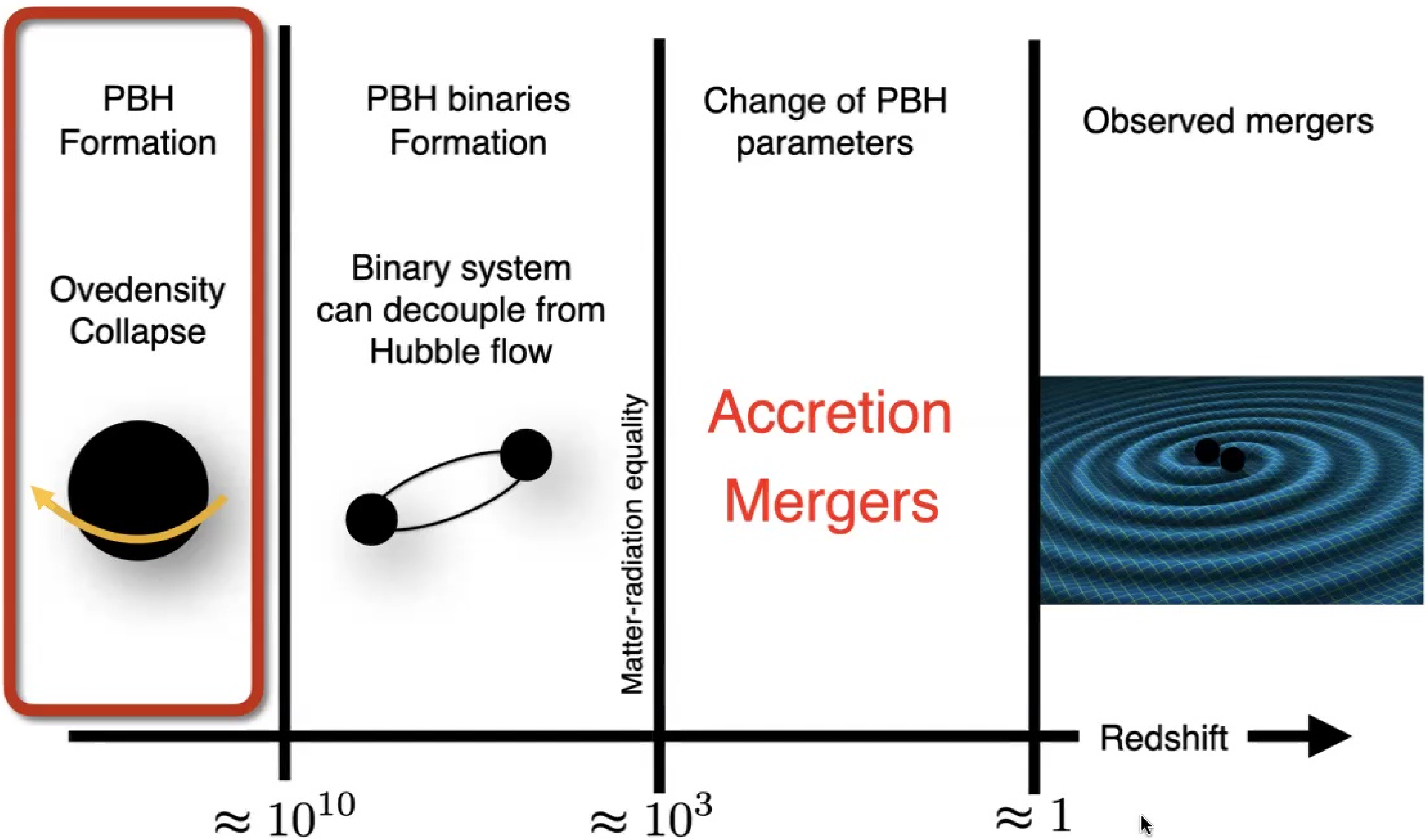
Power Spectrum:

Gaussian: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-(k-k_*)^2/2\sigma^2\right]$

Lognormal: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-\ln^2(k/k_*)/2\sigma^2\right]$



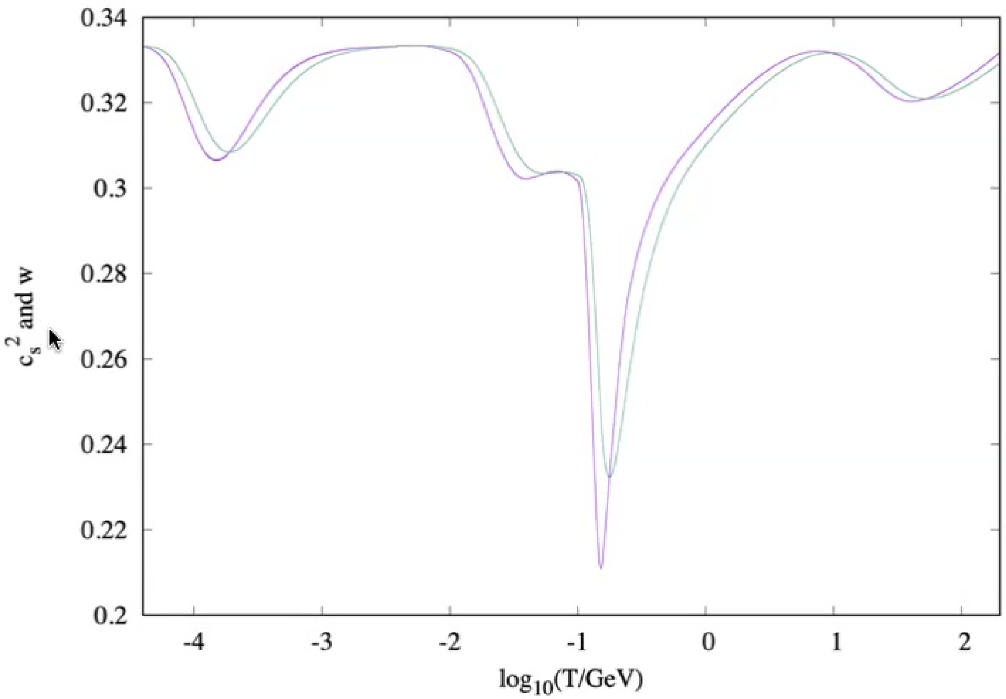
PBH evolution



Equation of State of the Early Universe

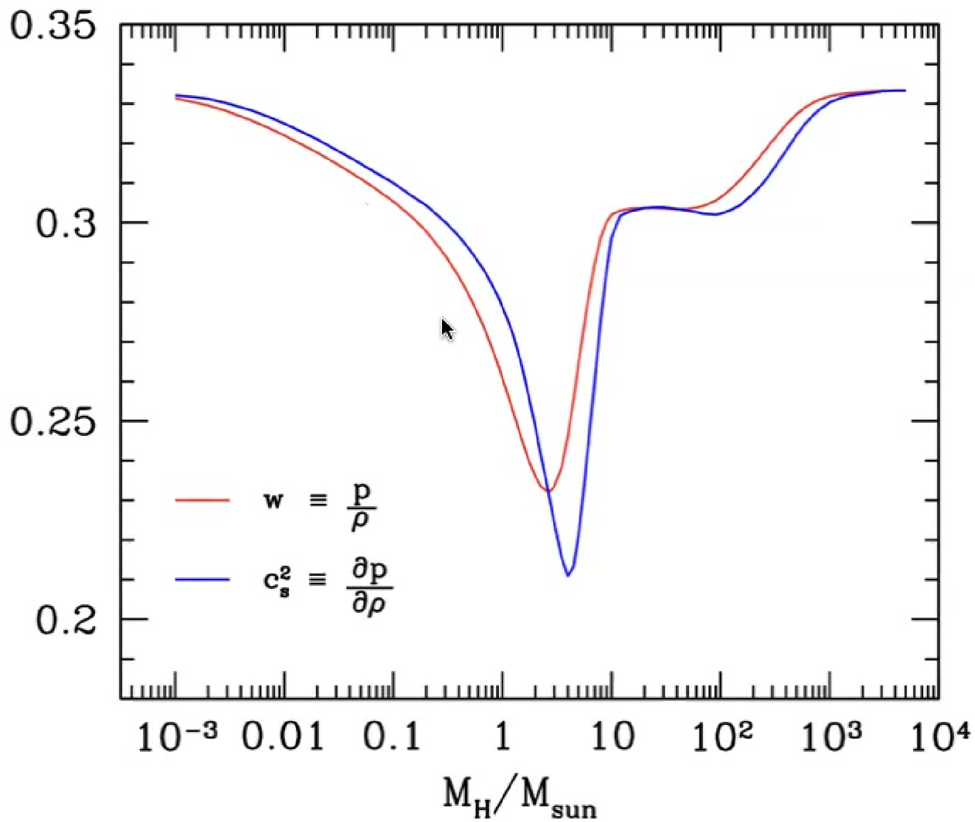
The easy Universe goes through 3 main transitions before matter-radiation equality:

- Electroweak phase-transition
- **QCD phase- transition** (crossover)
- Nucelosynthesis (e+e- annihilation)



QCD Phase-transition

- Significant softening of the equation of state (lattice QCD simulations)
- Introducing an intrinsic scale



$$\rho = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$$

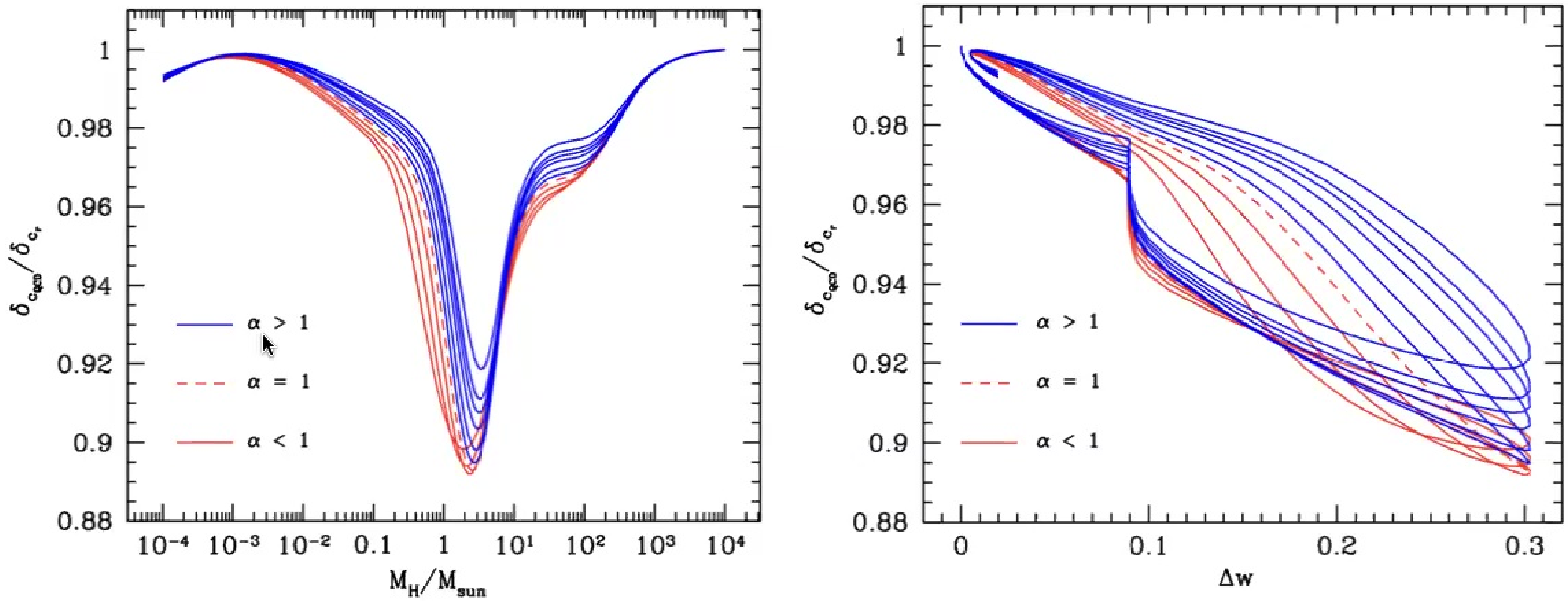
$$s = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3$$

$$p = sT - \rho = w(T)\rho$$

$$w(T) = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1$$

$$c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\rm eff} + Th'_{\rm eff})}{3(4g_{\rm eff} + Tg'_{\rm eff})} - 1$$

PBH Threshold during the QCD



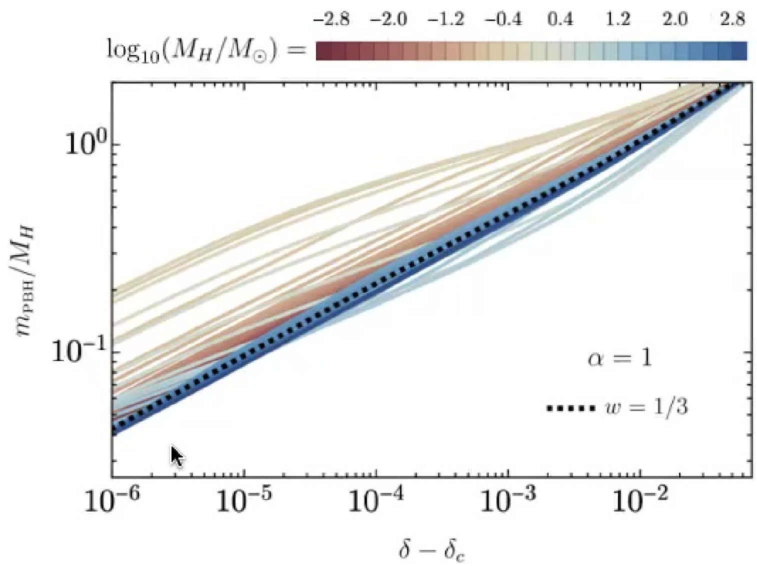
Depending on the shape, the threshold for PBH formation during the QCD phase transition is reduced about 10% around the minimum of ***w(T)***.

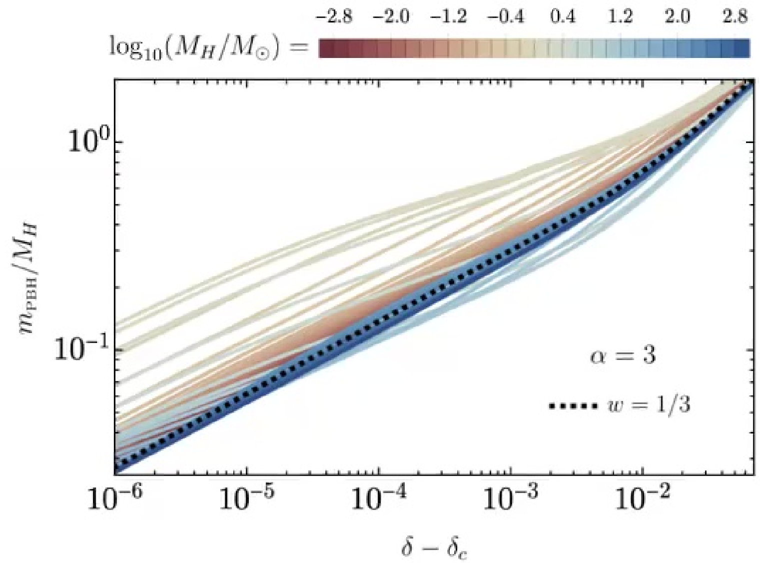
Significant enhancement of PBH formation around the solar mass scale: abundance increased of about O(3) with respect radiation!

$$\Delta w = \frac{1/3 - w}{1/3} = 1 - 3w$$

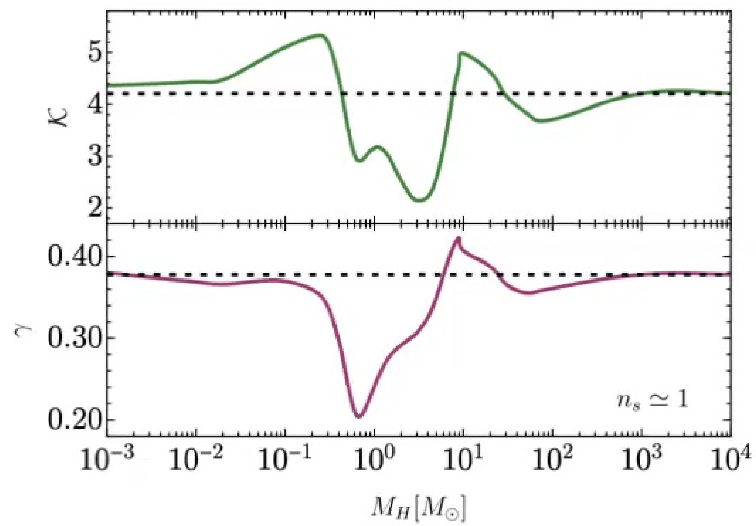
IM, K. Jedamzik, Sam Young - in progress…

PBH Mass Spectrum





$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$



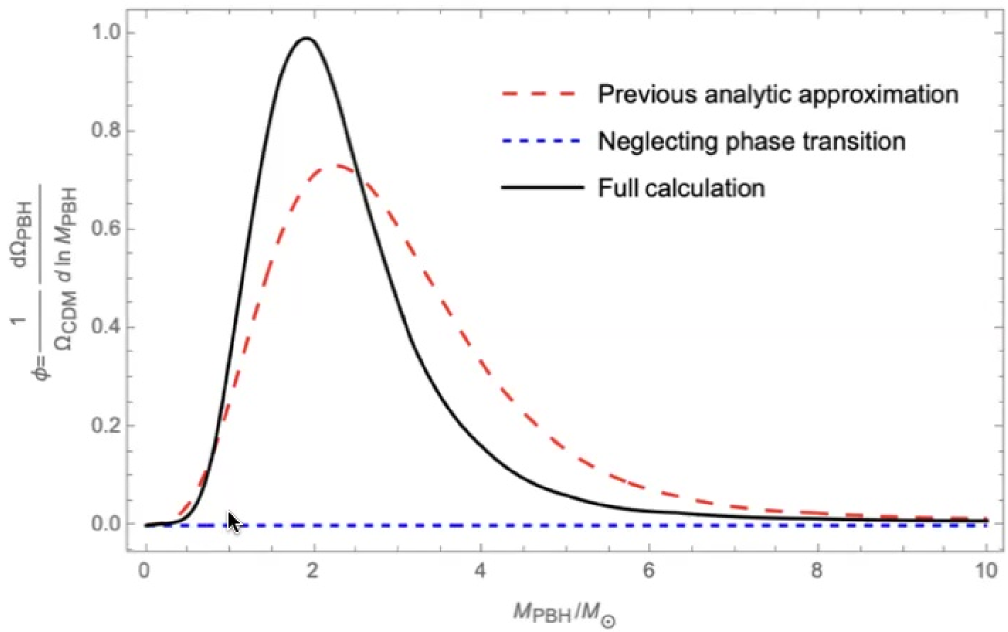
$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$

IM, K. Jedamzik, Sam Young **- in progress…**

PBH mass distribution - QCD

Mass Function ψ(mPBH): fraction of PBHs with mass in the infinitesimal interval of mPBH

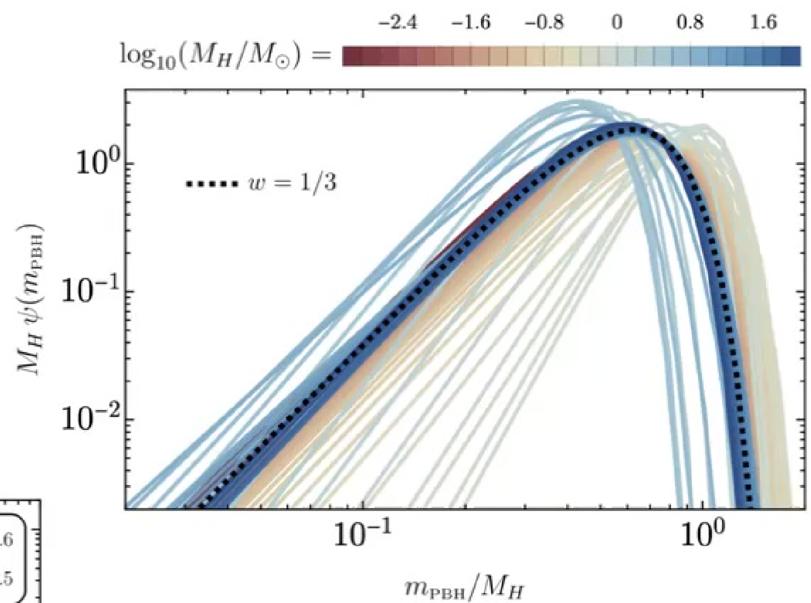
$$\psi(M_{\rm PBH}) = \frac{1}{\Omega_{\rm PBH}} \frac{{\rm d}\Omega_{\rm PBH}}{{\rm d}M_{\rm PBH}}$$

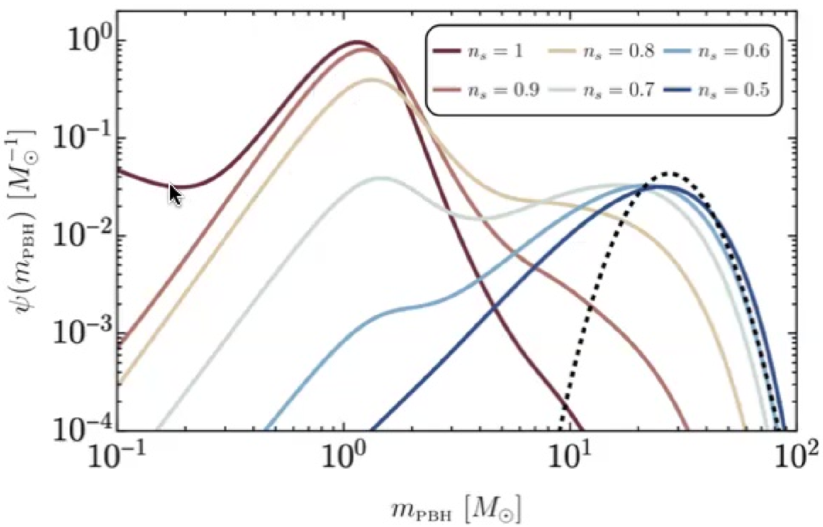


- The main effect is given by the modification of the threshold.
- The modified mass spectrum prides a pile up of PBHs on smaller masses.

PBH mass distribution

Integrating the full mass distribution, we see the enhancement of the QCD depending on the tilt of the power spectrum.





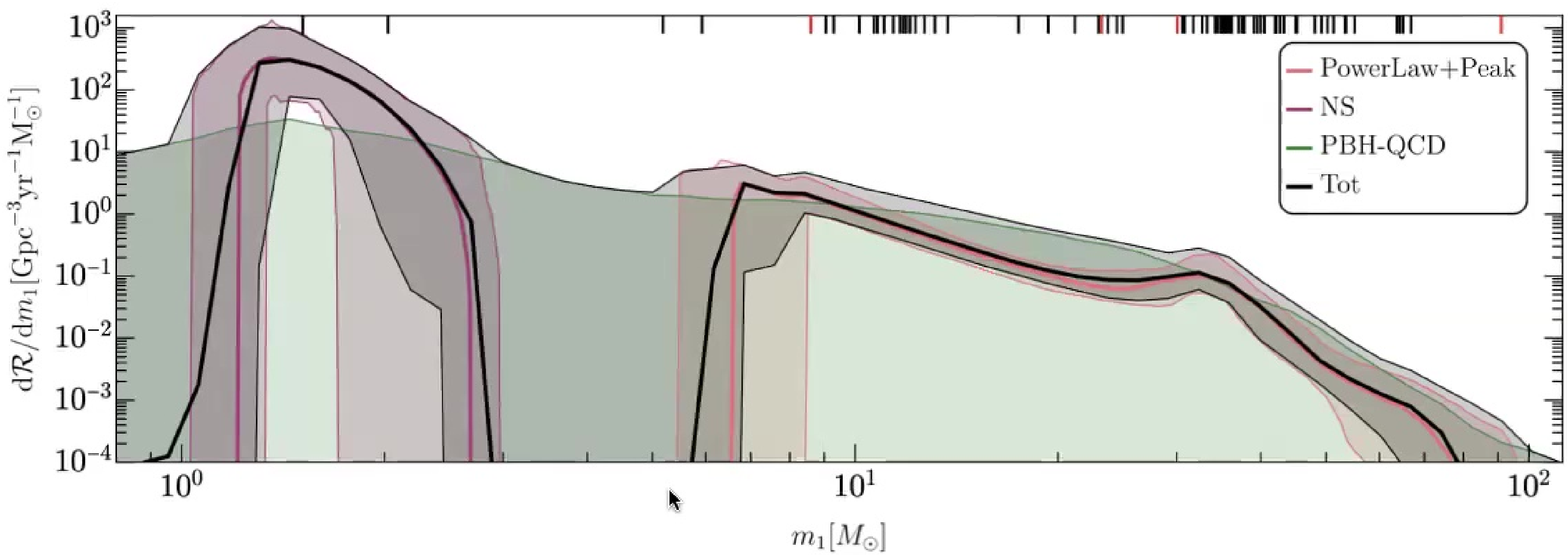
$$\int \mathrm{d}m_{\rm PBH}\ \psi(m_{\rm PBH}) = 1$$

G. Franciolini, IM, P.Pani, A urbano - PRD (2022)

GWs from PBH mergers

G. Franciolini, IM, P.Pani, A urbano - PRD (2022)

- Making **Bayesian inference analysis** we found that a <u>sub-population of PBHs is compatible with the LVK catalog</u>.
- PBHs give a natural explanation for the events in with BH mass gap: in particular GW190814 falling within the lower mass gap (predictions for O4 and O5).



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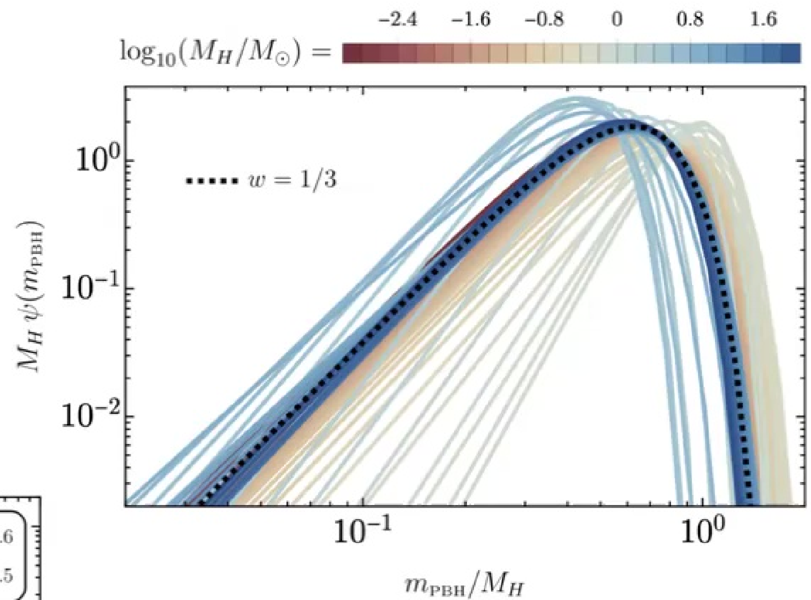

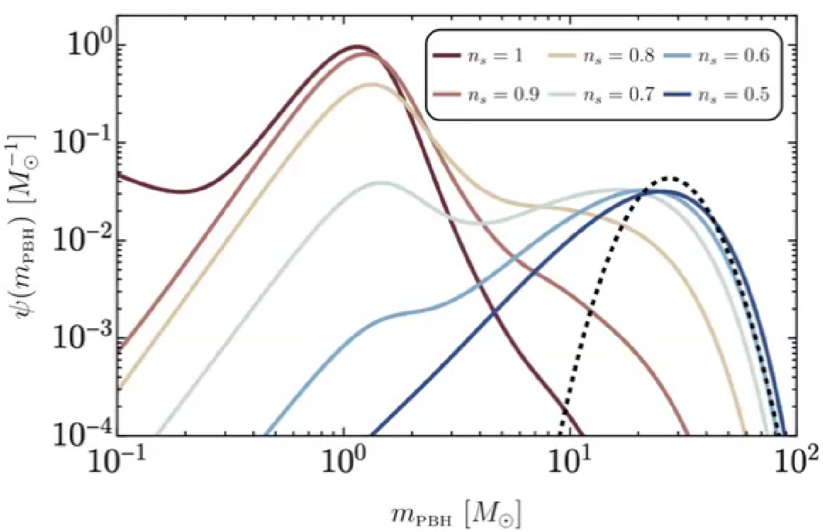

$$\int dm_{PBH}\ \psi(m_{PBH}) = 1$$

G. Franciolini, IM, P.Pani, A urbano - PRD (2022)

GWs from PBH mergers

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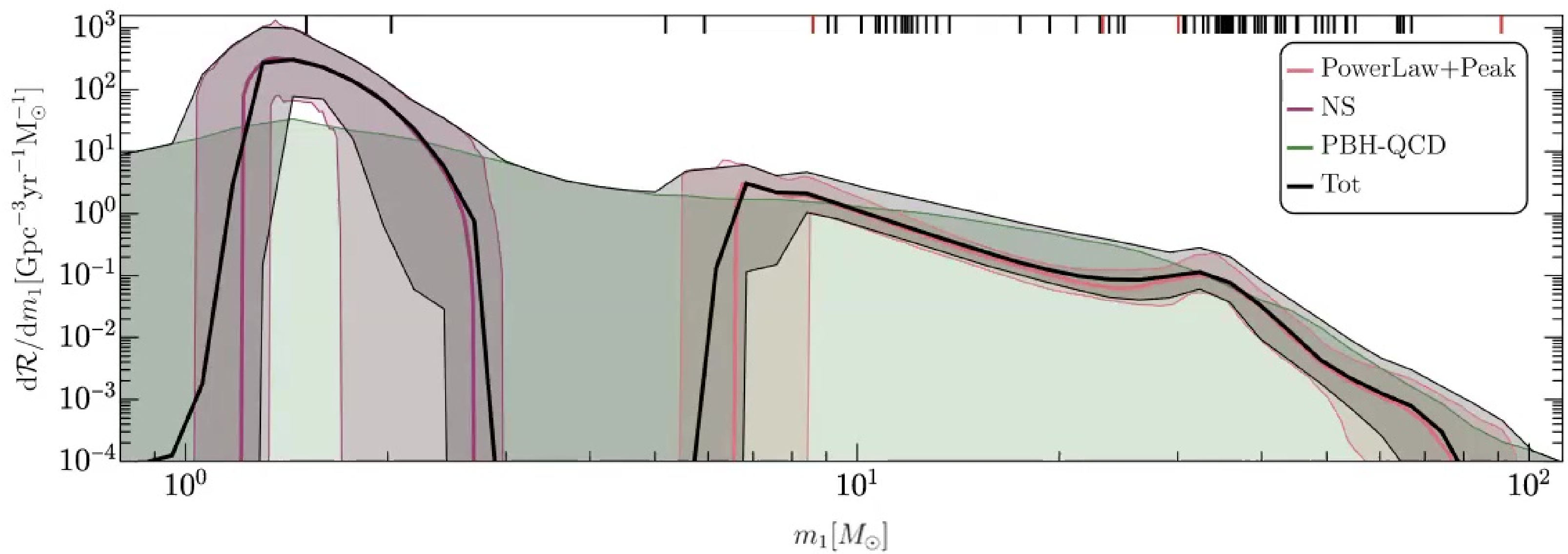
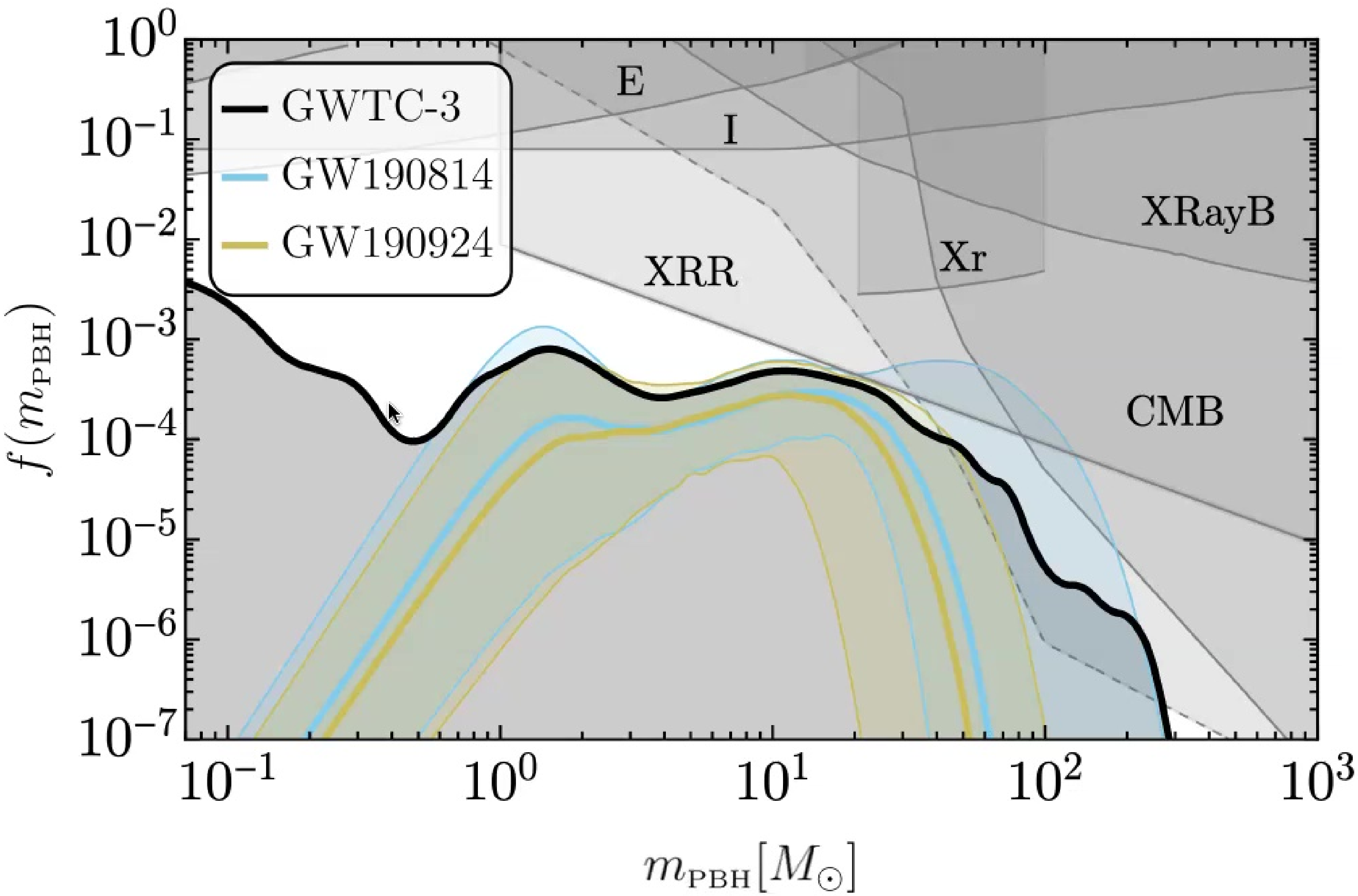


PBH constraints

G. Franciolini, IM, P.Pani, A urbano - arXiv:2209.05959



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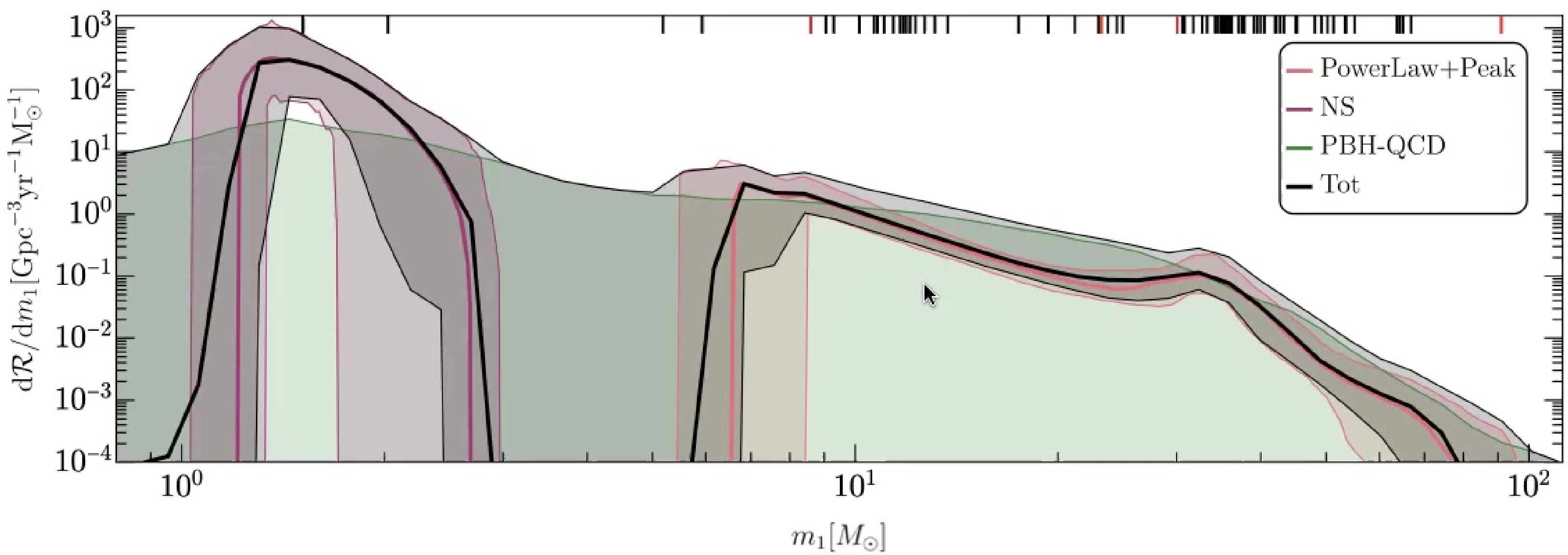
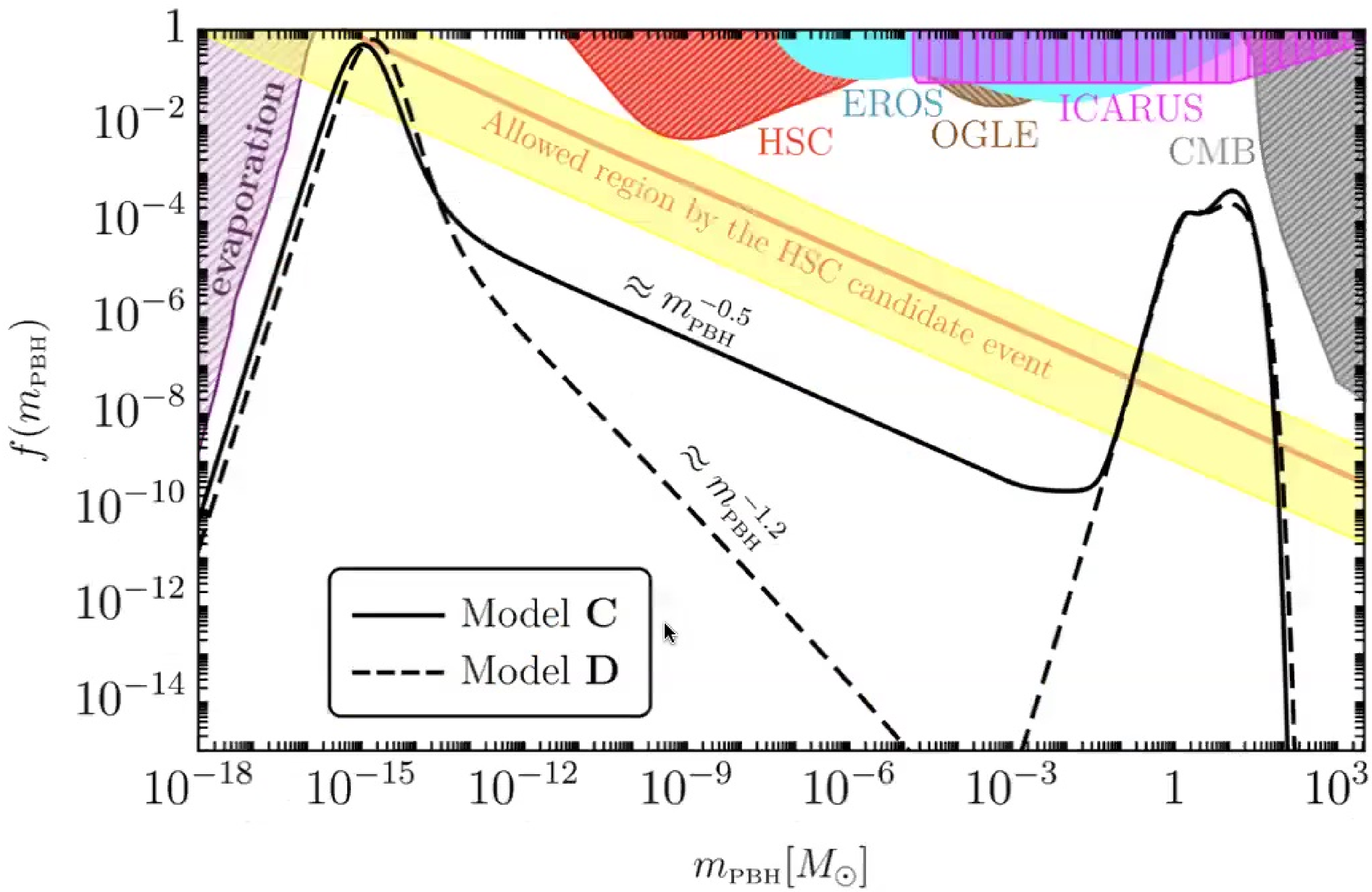


PBH and Dark Matter



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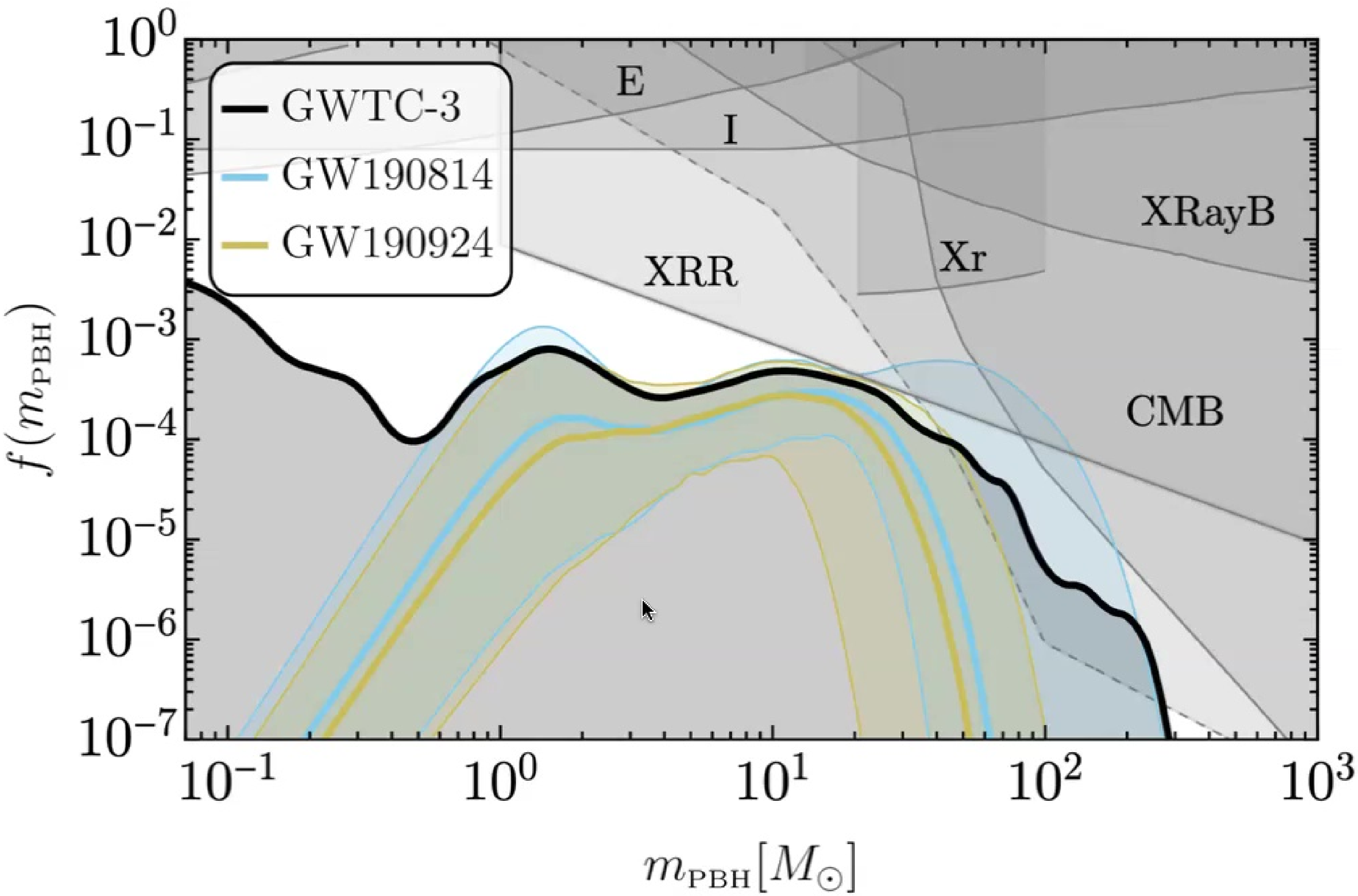

Conclusions

- The **non linear threshold for PBH** and **the mass spectrum** could be fully computed from the **shape of the power spectrum of cosmological perturbations**, making relativistic numerical simulations.

- A softening of the equation of state **(QCD) significantly enhances the formation of PBHs**, with a **mass distribution peaked between 1 and 2 solar masses** (the range of heavy NSs and light BHs).

- This could gives **a sub-population of BH mergers compatible with the LVK catalog**, explaining mass gap events as **GW190814**.

- Our analysis predicts a **constraint on the abundance of DM in PBHs formed during the QCD (up to 0.1%)**, compatible with the current observational constraints.

- **A large enough feature of the power spectrum could account for all dark matter in PBHs in the asteroidal mass range (USR inflation models)**.