Title: Reparametrization mode and chaos on the worldsheet

Speakers: Shota Komatsu

Series: Quantum Fields and Strings

Date: February 14, 2023 - 2:00 PM

URL: https://pirsa.org/23020048

Abstract: The path integral over reparametrization modes in one dimension played an important role in the duality between JT gravity and the SYK model. In this talk, I will explain that the reparametrization modes are important also in certain computations involving the string worldsheet with boundaries. A few cases in which it is expected to play a crucial role are the Wilson loop expectation value in confining string, open strings with massive endpoints, and the string dual to the half-BPS Wilson loop in N=4 supersymmetric Yang-Mills. After reviewing these cases briefly, I will focus on the last case and explain how to compute the correlation function on the BPS Wilson loop from the string worldsheet in the conformal gauge. In particular, I will show that the inclusion of the reparametrization modes is crucial for reproducing the answer obtained previously in the static gauge. I will then use the reparametrization mode path integral to study the four-point functions in the out-of-time-ordered configuration and obtain an exact answer in a double-scaling regime interpolating between the Lyapunov regime and the late-time exponential decay. Interestingly the result has exactly the same functional form as in JT gravity although the actions for the reparametrization modes are different.

Zoom link: https://pitp.zoom.us/j/99063427266?pwd=aG5iTlczNWhxdE9xNEZoVTlMSnVOQT09

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Reparametrization Talk

Reparametrization Mode & Chaos on the Worldsheet

Shota Komatsu

CERN

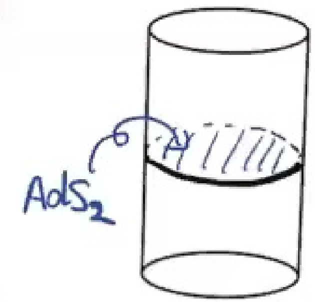
based on arXiv : 2212.14842

with Simone Giombi, Bendeguz Offertaler

A (intentionally) technical summary

- $\frac{1}{2}$ BPS Wilson loop in $\mathcal{N}=4$ SYM

  $\rightsquigarrow$ Dual to AdS_2 string $\subset AdS_5 \times S^5$



- Study correlation functions on the string

$$ — \Phi^{i_1} — \Phi^{i_2} — \Phi^{i_3} — \cdots = $$

- In conformal gauge, the reparametrization mode at the body of the worldsheet plays a key role.

- It allows us to compute exact OTOC in the double-scaling limit. (non-Schwarzian maximal chaos)

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Reparametrization Talk

Answer

Reparametrization modes are important also in other problems

- JT gravity / SYK model
- String worldsheet with boundaries

(½ BPS WL is perhaps the simplest of of such setups)

Plan

1. Why reparametrization modes?
2. Correlators on AdS_2 string
3. Comparison with Schwarzian
4. Chaos and double-scaling
5. Conclusion & Future

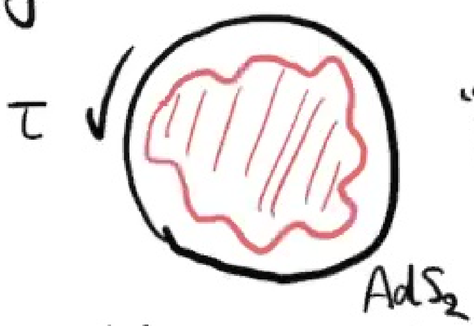
Why reparametrization ? I

- JT gravity + free matter

$$S_{JT} = -\frac{1}{16\pi G_N}\left[\int_M d^2\sigma \sqrt{h}\, \Phi (R+2) + 2\int d\theta \sqrt{h_{\theta\theta}}\, \Phi (K-1)\right]$$

- Φ e.o.m $\Rightarrow$ $R=-2$ $\rightsquigarrow$ locally AdS_2

- Boundary condition : Boundary length $\frac{2\pi}{\epsilon}$, $\Phi|_b = \frac{\Phi_0}{\epsilon}$



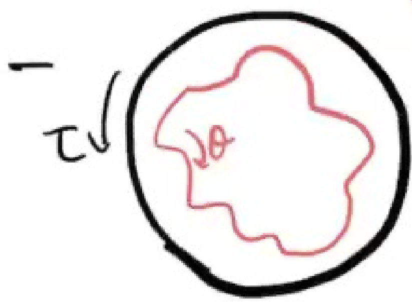
"phase space"

bdy coordinate

- Fix the bdy coordinate by $h_{\theta\theta} = \frac{1}{\epsilon^2}$

$\rightsquigarrow$ Shapes are specified by $\tau(\theta)$

Why reparametrization? I

- 

$$S_{JT} = -\frac{\Phi_0}{8\pi G_N}\int d\theta \left(\{\tau, \theta\} + \frac{1}{2}(\dot{\tau}(\theta))^2 \right)$$

Schwarzian Action

- Matter 2pt function $\displaystyle \int D\tau \, e^{-S_{JT}} \frac{\dot{\tau}(\theta)^{\Delta}\,\dot{\tau}(\theta')^{\Delta}}{\left(2\sin\left(\frac{\tau(\theta)-\tau(\theta')}{2}\right)\right)^{2\Delta}}$

- It arises also in the low-T limit of SYK

Why reparametrization ? II

– String ending on a fixed curve in spacetime [Nelson, Polchinski]

Boundary condition

$$X^\mu(\tau, 0) = x^\mu(\tau)$$

– In conformal gauge, we require $T - \bar{T} = 0 \,|_{\text{bdy}}$

$\rightsquigarrow$ Bdy condition needs to be inv. under $\tau \to f(\tau)$

– True for D-branes $X^\mu(\tau, 0) = 0$, but not for [Alvarez]

$X^\mu(\tau, 0) = x^\mu(\tau)$ $\left(\rightsquigarrow X^\mu(f(\tau), 0) \neq x^\mu(\tau) \right)$

– Only way to make it invariant is to integrate over $f(\tau)$ cf. [Makeenko, Olesen]

Why reparametrization? III

- String with massive endpoints

- Action: $\frac{1}{2}\int d\tau \left(e|\dot{x}|^2 + \frac{m^2}{e} \right) + S_{\text{Polchinski}}$ $\quad (e^{-1} \sim \sqrt{h_{\tau\tau}})$

  einbein

- Boundary action depends explicitly on $e(\tau)$

  ⇝ we need to integrate over $e(\tau)$

  = integration over the reparametrization

- Relevant for meson spectrum in
  - confining string
  - 2d QCD

Why reparametrization modes? IV

- $\frac{1}{2}$ BPS Wilson Loop in $\mathcal{N}=4$ SYM

  ⇝ dual to string in $AdS_2 \subset AdS_5 \times S^5$

$$\sim \sqrt{\frac{2}{\pi}}\, \lambda^{-3/4}\, e^{\sqrt{\lambda}} + \cdots$$

- The bdy condition: $z(\tau,0) = 0, \quad x^1(\tau,0) = \cos\tau, \quad \cdots$

  ⇝ fixed spacetime contour

- We expect the reparametrization modes are important in the conformal gauge.

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Reparametrization Talk

2. Correlators on AdS_2 string

Correlators on AdS_2 string

- AdS_2 string inside $AdS_5 \times S^5$ in static gauge

$(\tau, \sigma) = (x', z)$

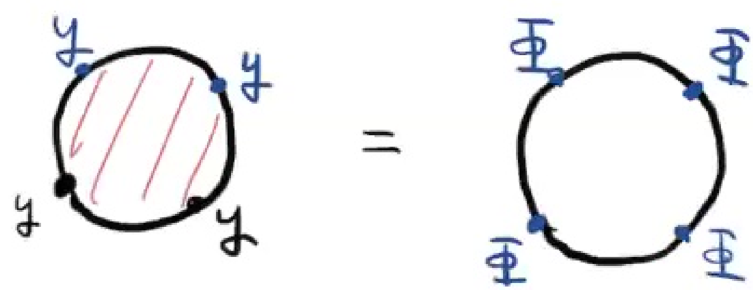
$\rightsquigarrow$ 3 massive scalar + 5 massless scalar + fermion

$\underset{AdS_5}{\uparrow}$ "x" $\underset{S^5}{\uparrow}$ "y"

- Preserves $SL(2,R) \longrightarrow$ dual to WL Defect CFT

[Giombi, Roiban Tseytlin]

- Correlation functions



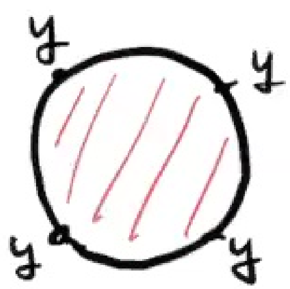
Necessity of reparametrization

- Toy model $AdS_2 \times S^1$

- Static gauge $S[y] = \int d^2\sigma \sqrt{\det\left[\mathbb{1} + \partial_\alpha y \partial_\beta y\right]}$

$$= \frac{1}{2}(\partial y)^2 - \frac{1}{8}(\partial y)^4 + \cdots$$

- Conformal gauge $S[y] = \int d^2\sigma\, \partial y \bar{\partial} y$ No interaction!

- 

$$\Big|_{\text{connected}} = \begin{cases} \neq 0 & \text{static gauge} \\ = 0 & \text{conformal gauge ??} \end{cases}$$

$\rightsquigarrow$ Reparametrization modes resolve the mismatch!

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Reparametrization Talk

(Semiclassical) Reparametrization Action

– Basic Strategy

- Specify parametrization of the curve $\begin{cases} z(0,t) = 0 \\ x(0,t) = \alpha(t) \\ y(0,t) = \tilde{y}(\alpha(t)) \end{cases}$

- Compute on-shell Polyakov action

$$S_{\text{on-shell}}[\alpha(t)]$$

- $\int \mathcal{D}\alpha(t) \quad e^{-S_{\text{on-shell}}}$

– Classical result $\quad S_{cl} = \min_{\alpha} S_{\text{on-shell}}[\alpha(t)] \rightsquigarrow \alpha(t) = t$

– we determined $S_{\text{on-shell}}[\alpha(t)]$ up to quadratic order around $\alpha(t) = t$

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Reparametrization Talk

(Semiclassical) Reparametrization Action

- y-part : e.o.m $\partial\bar\partial y = 0$, bdy $y(0,t) = \tilde{y}(\alpha(t))$

$$\rightsquigarrow \quad S_y^{\text{on-shell}} = -\int dt\,dt' \frac{\left(\tilde{y}(\alpha(t)) - \tilde{y}(\alpha(t'))\right)^2}{(t-t')^2}$$

$\downarrow$

complex analysis

- x, z-part : $\displaystyle\int \frac{\partial x \bar\partial x + \partial z \bar\partial z}{z^2}\, d^2\sigma$ $\rightsquigarrow$ nonlinear

$\rightsquigarrow$ Work perturbatively

$$\begin{cases} \alpha(t) = t + \epsilon(t) \\ x(t) = t + \xi(s,t) \\ z(t) = s + \zeta(s,t) \end{cases}$$

$\rightsquigarrow$ Impose e.o.m to write $\xi[\epsilon]$, $\zeta[\epsilon]$

$\rightsquigarrow$ Plug them in the action

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(Semiclassical) Reparametrization Action

– Final form

$$S^{\text{on-shell}}_{x,\bar z} = \int \underbrace{dt\, dt'}_{\text{worldsheet}} \frac{\left(\dot\epsilon(t) - \dot\epsilon(t')\right)^2}{(t-t')^2} \underset{\alpha(t) = t+\epsilon(t)}{=} \int dt\, dt' \frac{\left(\dot\alpha(t) - \dot\alpha(t')\right)^2}{(t-t')^2}$$

$$\underset{t(\alpha) = \alpha+\epsilon(\alpha)}{=} \int \underbrace{d\alpha\, d\alpha'}_{\text{bdy}} \frac{\left(\dot\epsilon(\alpha) - \dot\epsilon(\alpha')\right)^2}{(\alpha-\alpha')^2}$$

$$S_y^{\text{onshell}} = -\int dt\, dt' \frac{\left(\tilde y(\alpha(t)) - \tilde y(\alpha(t'))\right)^2}{(t-t')^2}$$

$$= -\int d\alpha\, d\alpha' \frac{\dot\alpha(t)\, \dot\alpha(t')}{(t-t')^2} \left(\tilde y(\alpha(t)) - \tilde y(\alpha(t'))\right)^2$$

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(Semiclassical) Reparametrization Action

- Final form

$$S^{\text{on-shell}}_{x,z} = \int \underbrace{dt\, dt'}_{\text{worldsheet}} \frac{\left(\dot\epsilon(t) - \dot\epsilon(t')\right)^2}{(t-t')^2} \overset{\alpha(t) = t + \epsilon(t)}{=} \int dt\, dt' \frac{\left(\dot\alpha(t) - \dot\alpha(t')\right)^2}{(t-t')^2}$$

$\alpha(t) = t + \epsilon(t)$

$$\overset{t(\alpha) = \alpha + \epsilon(\alpha)}{=} \int \underbrace{d\alpha\, d\alpha'}_{\text{bdy}} \frac{\left(\dot\epsilon(\alpha) - \dot\epsilon(\alpha')\right)^2}{(\alpha - \alpha')^2}$$

$$S^{\text{onshell}}_{y} = -\int dt\, dt' \frac{\left(\tilde y(\alpha(t)) - \tilde y(\alpha(t'))\right)^2}{(t-t')^2}$$

$$= -\int d\alpha\, d\alpha' \frac{\dot\alpha(t)\ \dot\alpha(t')}{(t-t')^2} \left(\tilde y(\alpha(t)) - \tilde y(\alpha(t'))\right)^2$$

Correlation Functions

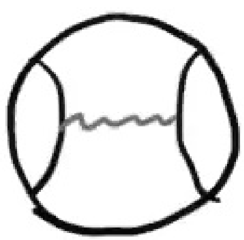
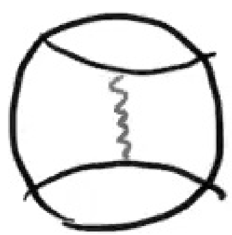
$$B(\theta_1,\theta_2) \equiv \frac{(1+\dot{\epsilon}(\theta_1))\,(1+\dot{\epsilon}(\theta_2))}{2\sin\left(\frac{1}{2}(\theta_1-\theta_2)\right)^2}$$

Here we have $\frac{1}{\sin}$ since we consider a circular loop

$$-\langle y(\theta_1)\, y(\theta_2)\, y(\theta_3)\, y(\theta_4)\rangle_{conn}$$

$$= \langle B(\theta_1,\theta_2)\, B(\theta_3,\theta_4)\rangle + \langle B(\theta_1,\theta_3)\, B(\theta_2,\theta_4)\rangle$$

$$+ \langle B(\theta_1,\theta_4)\, B(\theta_2,\theta_3)\rangle$$

 +  + 

= 

static gauge

(Semiclassical) Reparametrization Action

- Final form

$$S^{\text{on-shell}}_{x,z} = \int \underset{\text{worldsheet}}{dt\,dt'} \frac{(\dot{\epsilon}(t) - \dot{\epsilon}(t'))^2}{(t-t')^2} = \int dt\,dt' \frac{(\dot{\alpha}(t) - \dot{\alpha}(t'))^2}{(t-t')^2}$$

$\alpha(t) = t + \epsilon(t)$

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$$= \int \underset{\text{bdy}}{d\alpha\,d\alpha'} \frac{(\dot{\epsilon}(\alpha) - \dot{\epsilon}(\alpha'))^2}{(\alpha - \alpha')^2}$$

$t(\alpha) = \alpha + \epsilon(\alpha)$

$$S^{\text{onshell}}_{y} = -\int dt\,dt' \frac{(\tilde{y}(\alpha(t)) - \tilde{y}(\alpha(t')))^2}{(t-t')^2}$$

$$= -\int d\alpha\,d\alpha' \frac{\dot{\alpha}(t)\,\dot{\alpha}(t')}{(t-t')^2} (\tilde{y}(\alpha(t)) - \tilde{y}(\alpha(t')))^2$$

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Reparametrization Talk

Correlation Functions

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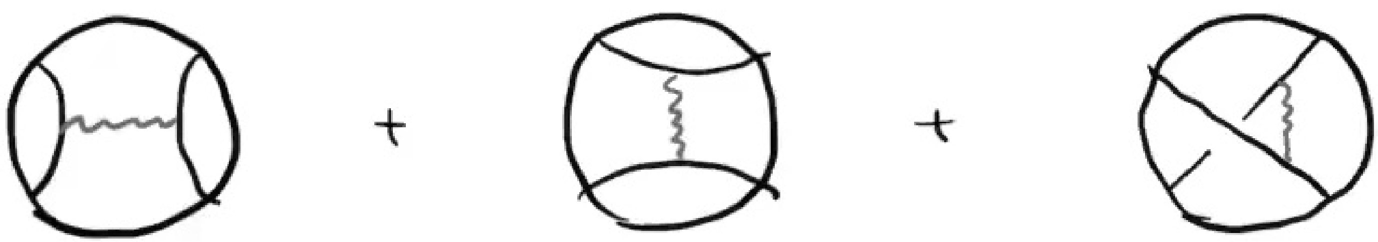

$=$  | static gauge

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Reparametrization Talk

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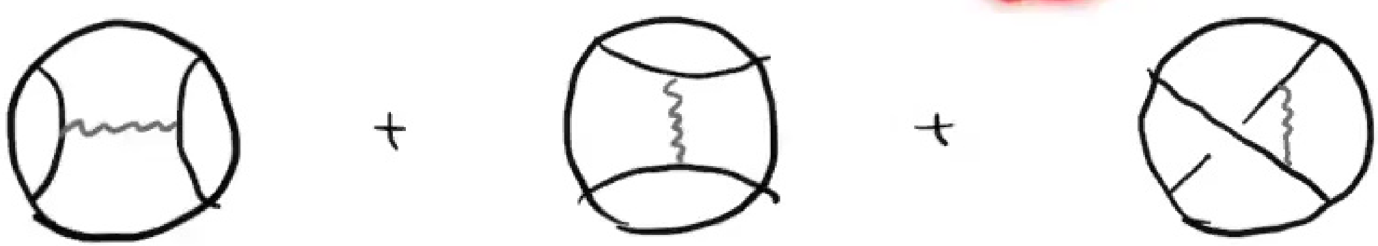

$=$  static gauge

Correlation Functions

$$\frac{\langle y_1\, y_2\, y_3\, y_4 \rangle\big|_{conn}}{\langle y_1\, y_2 \rangle \langle y_3\, y_4 \rangle} = -\Bigg[4 + \frac{2-\chi}{\chi}\log\left((1-\chi)^2\right) + \frac{\chi^2}{(1-\chi)^2}\left(4 + \frac{1+\chi}{1-\chi}\log \chi^2\right)$$

$$+ \chi^2\left(4 + (2\chi-1)\log\left(\frac{(1-\chi)^2}{\chi^2}\right)\right)\Bigg]$$

$$\chi = \frac{\sin\frac{\theta_{12}}{2}\,\sin\frac{\theta_{34}}{2}}{\sin\frac{\theta_{13}}{2}\,\sin\frac{\theta_{24}}{2}}$$

Symmetries of Schwarzian vs. AdS string

– Schwarzian

$$\int d\theta \left\{ \tan\frac{\tau}{2}, \theta \right\}$$

$$\sim \int d\theta \; \ddot{\epsilon}(\theta)^2 - \dot{\epsilon}(\theta)^2$$

local

– AdS_2 string

$$\int d\theta \, d\theta' \; \frac{(\dot{\epsilon}(\theta) - \dot{\epsilon}(\theta'))^2 - (\epsilon(\theta) - \epsilon(\theta'))^2}{\left(2\sin\frac{\theta-\theta'}{2}\right)^2}$$

non-local

– Left $SL(2,R)$: $\tau(\theta) \to f(\tau(\theta))$ $\quad f \in SL(2,R)$

⇝ Schwarzian gauge $SL(2,R)$ invariant

AdS string worldsheet $SL(2,R)$ invariant

– Right $SL(2,R)$: $\tau(\theta) \to \tau(f(\theta))$

⇝ Schwarzian physical $SL(2,R)$ broken

AdS string target-space $SL(2,R)$ invariant

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However, certain quantities

(Lyapunov exponent, double-scaled OTOC)

coincide ....

Non-Schwarzian Maximal Chaos

[Milekhin]

Thermal OTOC & Chaos

- Out-of-ordered thermal 4pnt function
  ⇝ diagnosis of chaos

$$\langle V\, W(t)\, V\, W(t) \rangle = 1 - G_N \cdot e^{\lambda t} \quad \leftarrow \langle [p,\, q(t)]^2 \rangle$$

$$\lambda \leq \frac{2\pi}{\beta} \quad \text{(bound on chaos)}$$

[Maldacena, Stanford, Shenker]

- Schwarzian & SYK | saturate the bound
  Einstein gravity |

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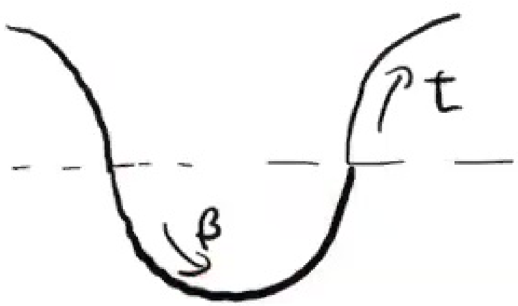
Reparametrization Talk

OTOC on WL

- to get OTOC, we perform analytic cont.



- In cross ratio, this corresponds to

$$\chi = \frac{2}{1 - i \sinh t} \quad \& \quad t \gg 1$$

- $\langle y_1 y_2 y_3 y_4 \rangle \sim 1 - \frac{\pi}{2} \frac{1}{\sqrt{\lambda}} e^{t}$: Maximal Chaos with $\beta = 2\pi$

Thermal OTOC & Chaos

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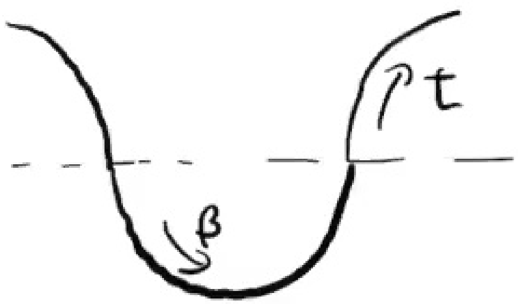

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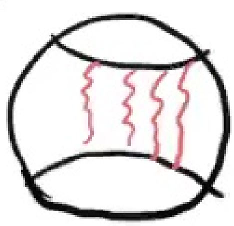
Double scaling limit

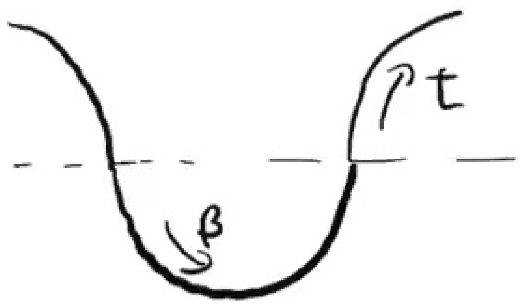
- In Schwarzian, there exists a double scaling limit in which only quadratic Lagrangian contributes
- Physically, it is eikonal resummation @ high energy



- Assume this is also the case in AdS_2 string
- $\lambda \to \infty$, $t \to \infty$ with $K = \dfrac{e^t}{16\sqrt{\lambda}}$ fixed

OTOC on WL

- to get OTOC, we perform analytic cont.



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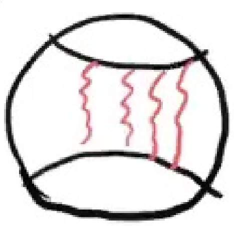

- Assume this is also the case in AdS_2 string
- $\lambda \to \infty$, $t \to \infty$ with $\kappa = \frac{e^t}{16\sqrt{\lambda}}$ fixed

Double scaling limit (rough sketch)

- $\langle (B(\theta_1,\theta_2))^\Delta (B(\theta_3,\theta_4))^\Delta \rangle$

$$B = \frac{(1+\dot{\epsilon}(\theta_1))(1+\dot{\epsilon}(\theta_2))}{\left(1+\frac{\epsilon_{12}}{x_{12}}\right)^2}$$

- $$(1+\dot{\epsilon}(\theta))^\Delta = \left(\frac{d}{d\alpha}\right)^\Delta e^{\alpha(1+\dot{\epsilon})},$$ exponentiation

$$\frac{1}{\left(1+\frac{\epsilon_{12}}{x_{12}}\right)^{2\Delta}} = \frac{1}{\Gamma(2\Delta)}\int_0^\infty dp\, p^{\Delta-1} e^{-p\left(1+\frac{\epsilon_{12}}{x_{12}}\right)}$$

- Perform the Gaussian integration

Double scaling limit (result)

$$\langle V W(t)\, V W(t) \rangle$$

$$= \frac{1}{K^{2\Delta_r}}\, U(2\Delta_r,\ 1+2\Delta_r,\ -2\Delta_w,\ K^{-1})$$

confluent hypergeometric

$$K = \frac{1}{16\sqrt{\lambda}} e^{t}$$

- Coincides with the result from Schwarzian
- expansion in $1/\sqrt{\lambda}$ reproduces 3-loop bootstrap result by [Ferrero, Meneghelli]

includes $Li_3(1/x)$ etc

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Reparametrization Talk

5. Conclusion & Future

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Summary

- Importance of reparametrization modes
- Conformal gauge + repara = static gauge
- Maximal chaos without Schwarzian
- Double scaled OTOC from reparametrization

Future

- Role of reparametrization for $\langle W \rangle$?

$$\lambda^{-3/4}\sqrt{\frac{2}{\pi}}\, e^{\sqrt{\lambda}}$$

- Flat space analogue? Fluxtube S-matrix in conformal gauge?

e^{iS}

$$A_{\text{Douglas}} = \int d\tau\, d\tau' \frac{\left(x(\alpha(\tau)) - x(\alpha(\tau'))\right)^2}{\left(2\sin\left(\frac{\tau-\tau'}{2}\right)\right)^2}$$

- Full reparametrization action from integrability?

[Kruczenski]

- How does the reparametrization mode arise from weak coupling? Can we use integrability?

Double scaling limit (result)

$$\langle V W(t)\, V W(t) \rangle$$

$$= \frac{1}{K^{2\Delta_V}}\, \underset{\sim}{U}\left(2\Delta_V,\ 1+2\Delta_V,\ -2\Delta_W,\ K^{-1}\right)$$

confluent hypergeometric

$$K = \frac{1}{16\sqrt{\lambda}}\, e^{t}$$

- Coincides with the result from Schwarzian
- expansion in $1/\sqrt{\lambda}$ reproduces 3-loop bootstrap result by [Ferrero, Meneghelli]

includes $Li_3(1/x)$ etc

Symmetries of Schwarzian vs. AdS string

- Schwarzian

$$\int d\theta \left\{\tan\frac{\tau}{2}, \theta\right\}$$

$$\sim \int d\theta \; \ddot{\epsilon}(\theta)^2 - \dot{\epsilon}(\theta)^2$$

local

- AdS_2 string

$$\int d\theta\, d\theta' \; \frac{(\dot{\epsilon}(\theta) - \dot{\epsilon}(\theta'))^2 - (\epsilon(\theta) - \epsilon(\theta'))^2}{\left(2\sin\frac{\theta-\theta'}{2}\right)^2}$$

non-local

- Left $SL(2,R)$: $\tau(\theta) \to f(\tau(\theta))$ $\quad f \in SL(2,R)$

⇝ Schwarzian gauge $SL(2,R)$ invariant

AdS string worldsheet $SL(2,R)$ invariant

- Right $SL(2,R)$: $\tau(\theta) \to \tau(f(\theta))$

⇝ Schwarzian physical $SL(2,R)$ broken

AdS string target-space $SL(2,R)$ invariant

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