Title: Horizon entropy and the Einstein equation - Lecture 20230228

Speakers: Ted Jacobson

Collection: Horizon entropy and the Einstein equation

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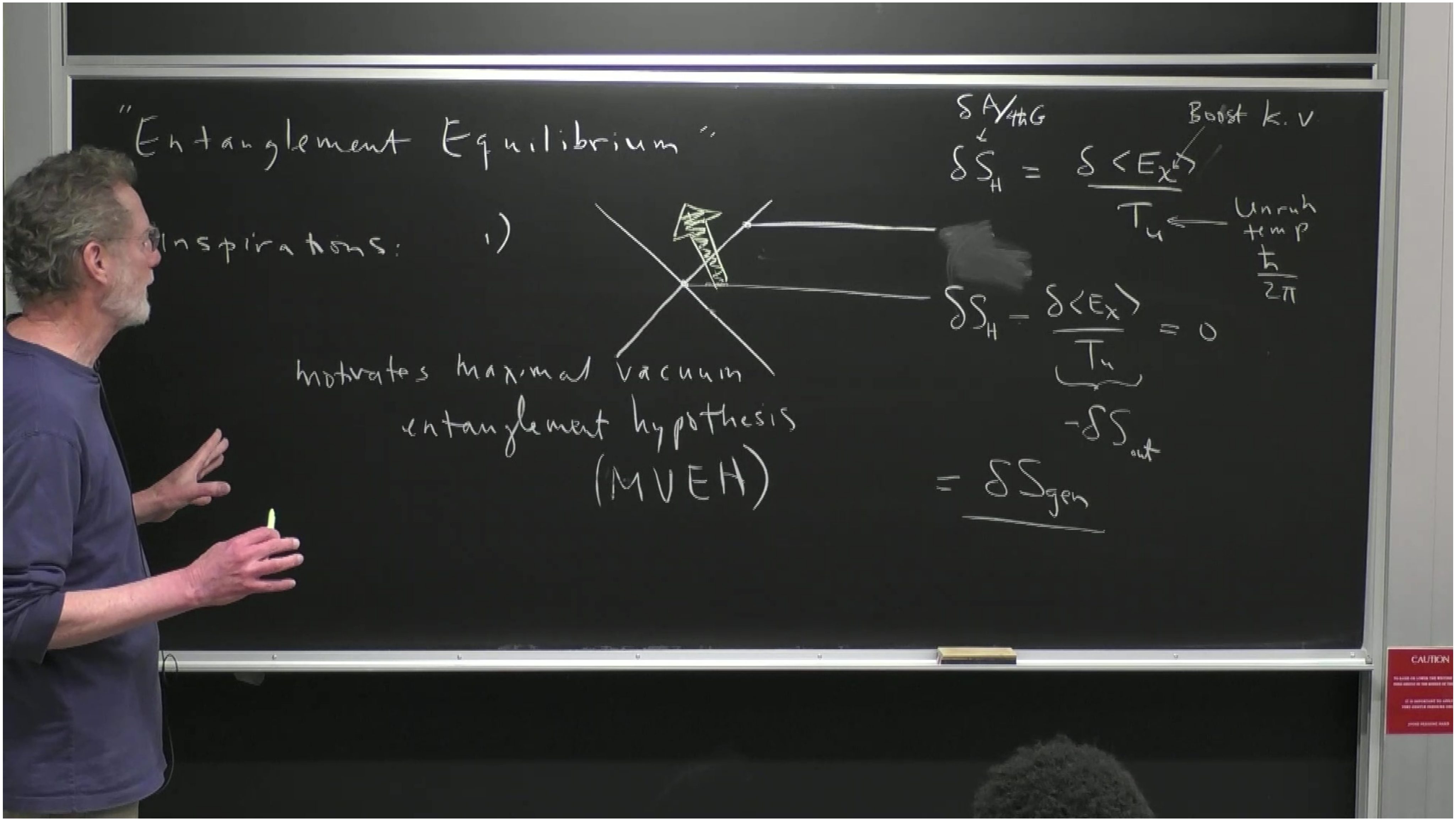
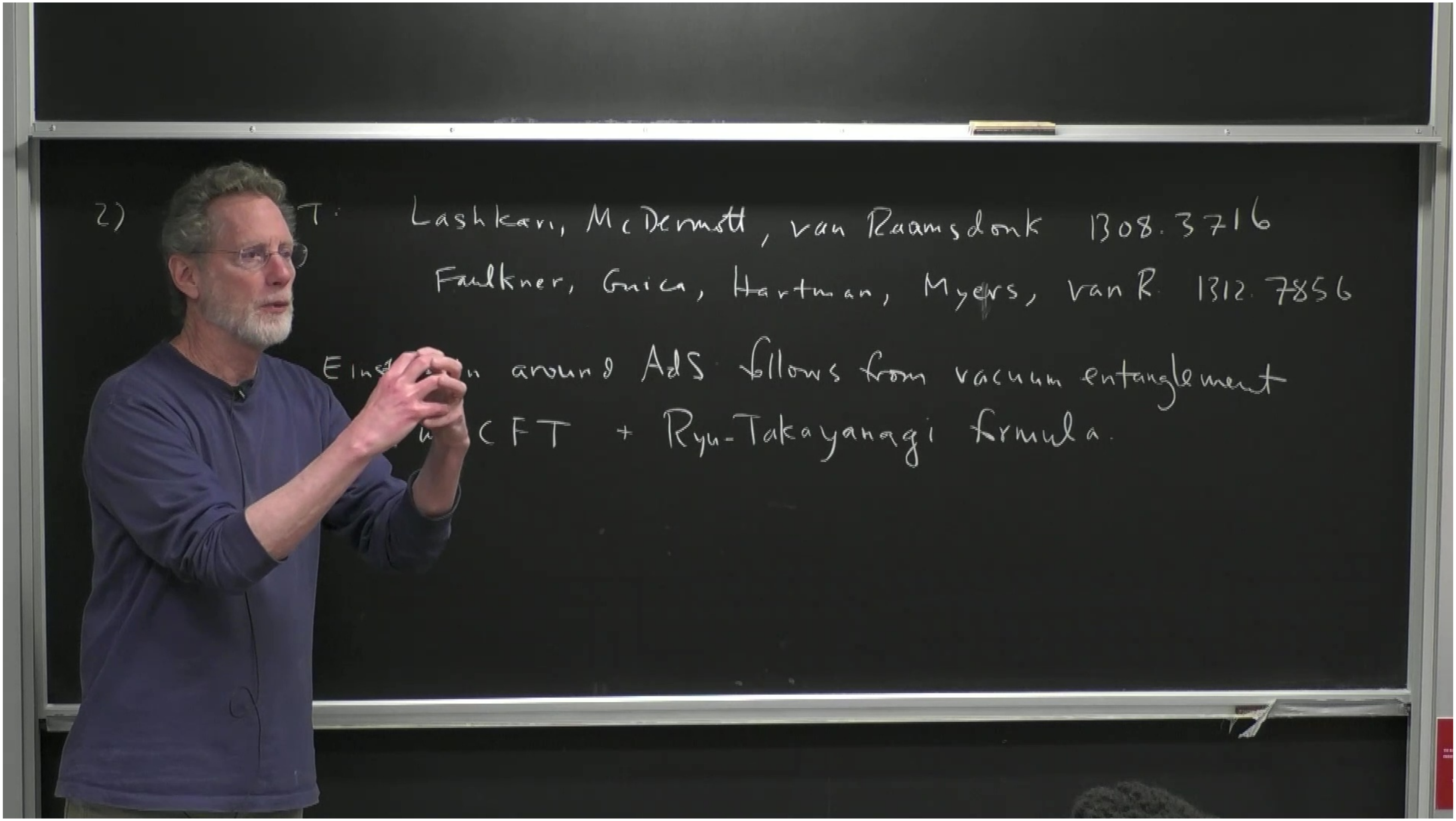
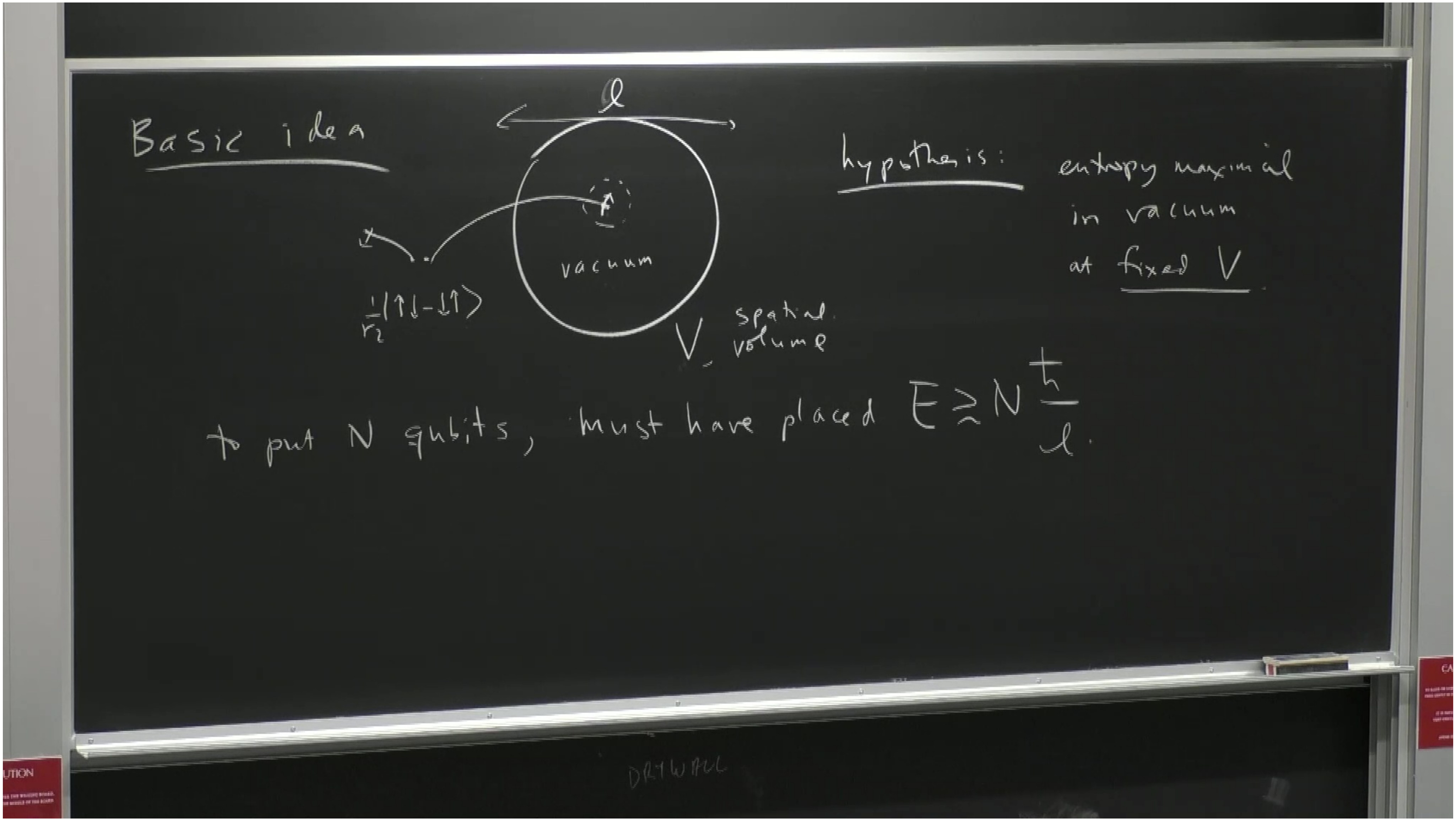
URL: https://pirsa.org/23020044

Abstract: Zoom link: https://pitp.zoom.us/j/96212372067?pwd=dWVaUFFFc3c5NTlVTDFHOGhCV2pXdz09


"Entanglement Equilibrium"
Inspirations: 1)
motivates maximal vacuum entanglement hypothesis
(MVEH)
δA/4ħG
δS_H = δ<E_x>/T_u
Boost k.v
Unruh temp
ħ/2π
δS_H − δ<E_x>/T_u = 0
−δS_out
= δS_gen


2)
Lashkari, McDermott, van Raamsdonk 1308.3716
Faulkner, Guica, Hartman, Myers, van R 1312.7856
around AdS follows from vacuum entanglement
CFT + Ryu-Takayanagi formula.


Basic idea
ℓ
vacuum
1/√2 |↑↓ − ↓↑⟩
V spatial volume
hypothesis:
entropy maximal in vacuum at fixed V
to put N qubits, must have placed E ≳ N ħ/ℓ.
DRYWALL

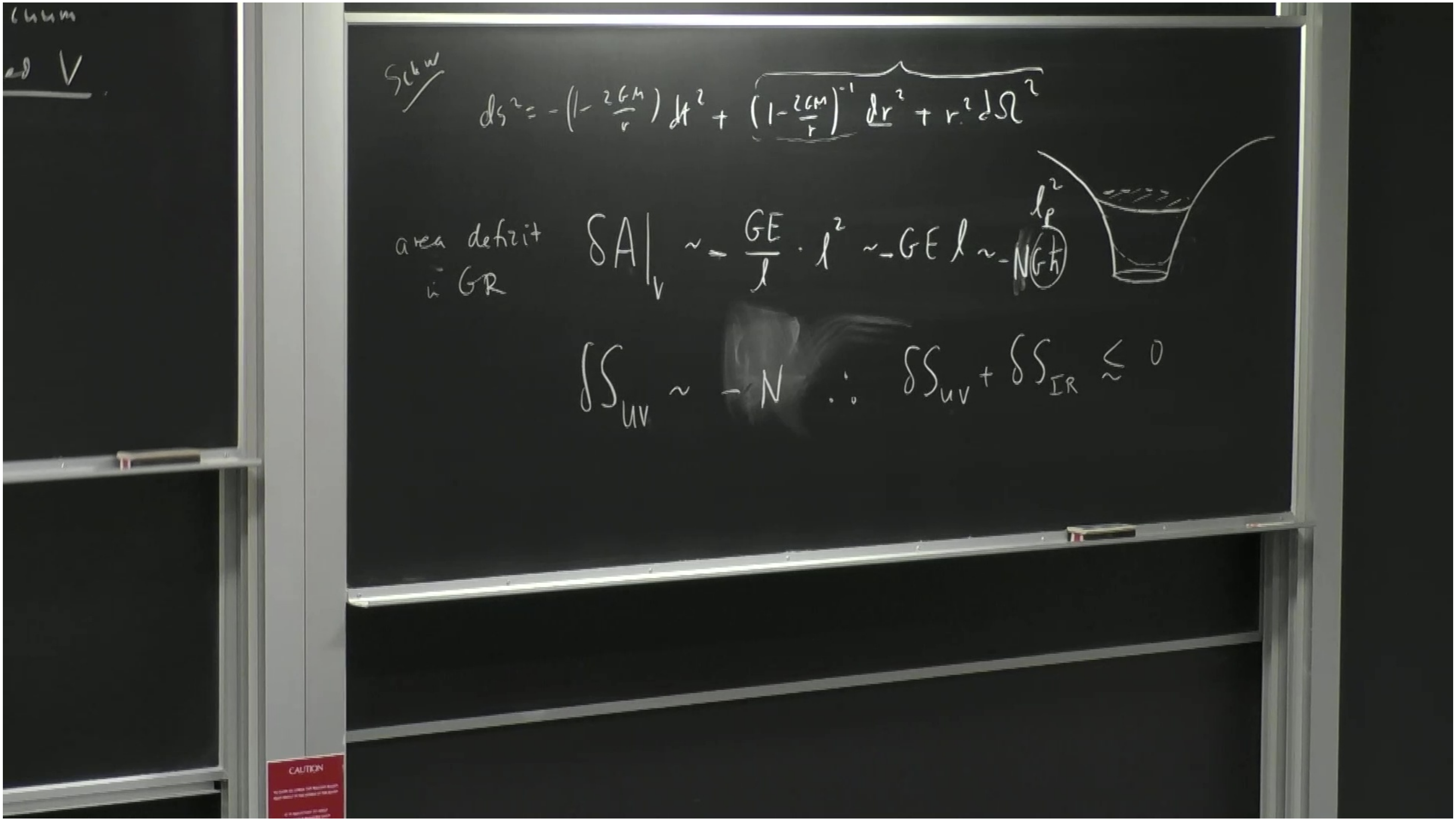
Schw

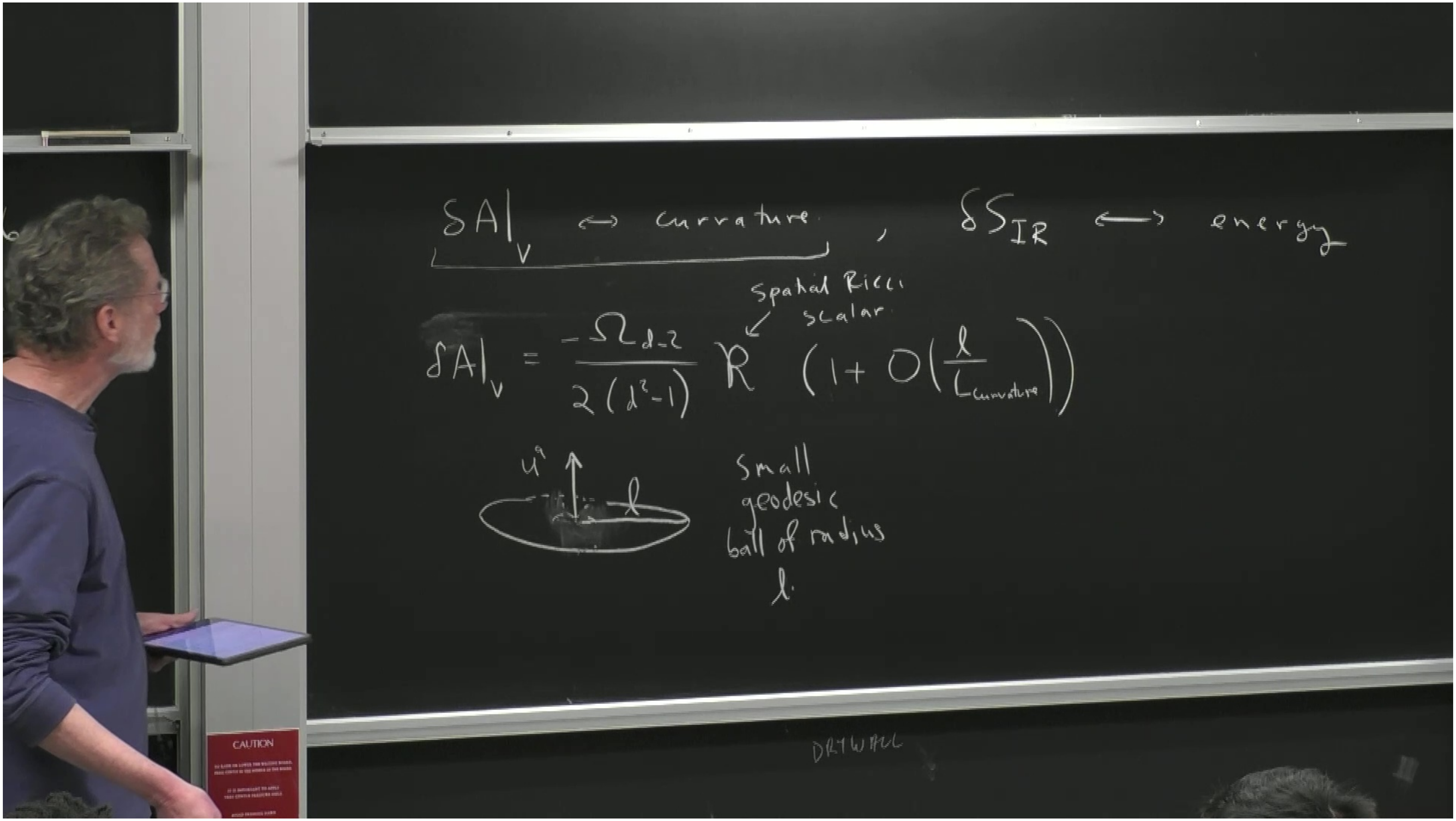
$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \overbrace{\left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2}$$

area deficit in GR

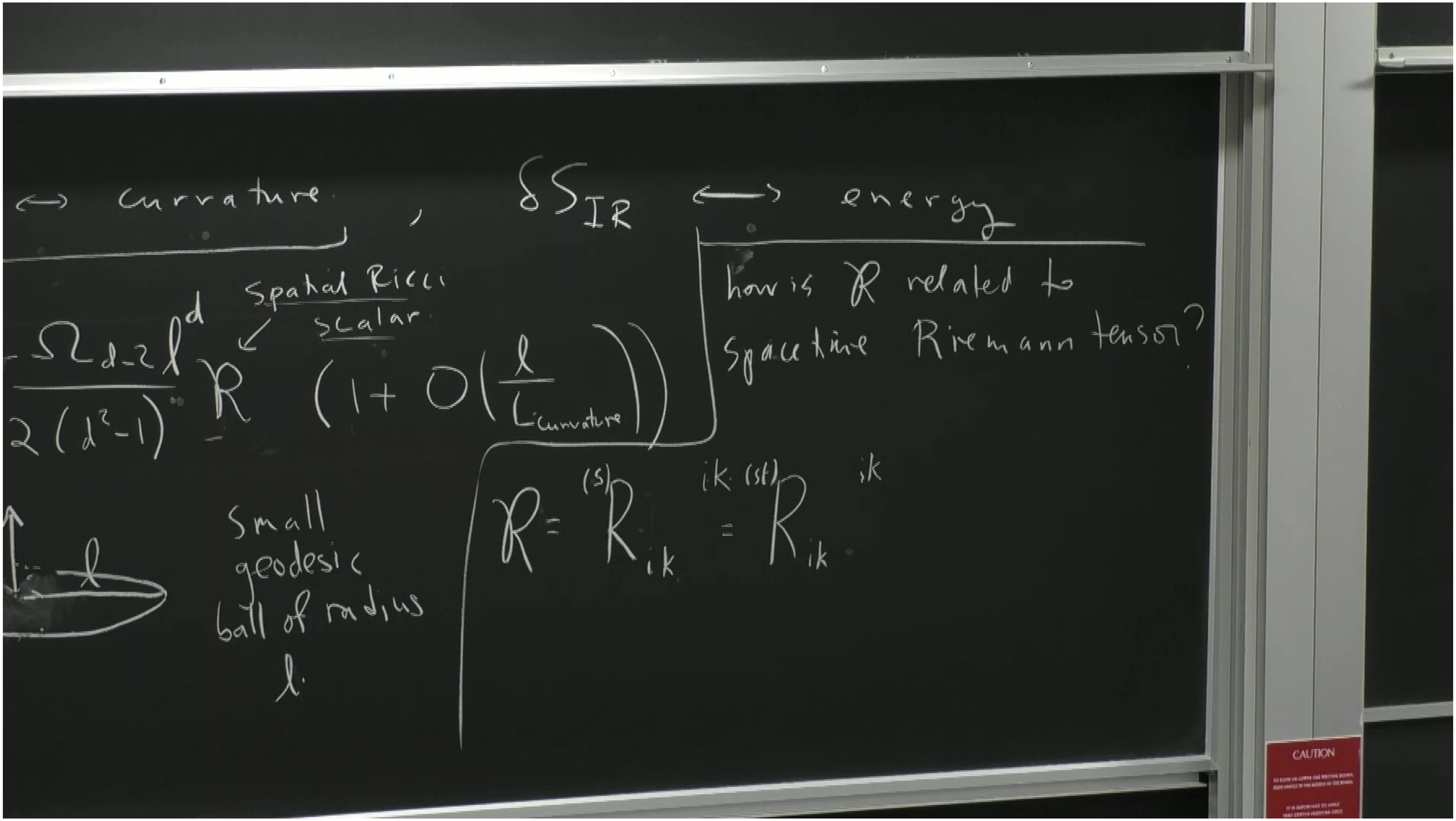
$$\delta A\Big|_V \sim -\frac{GE}{\ell}\cdot \ell^2 \sim -GE\ell \sim -NG\hbar$$

$$\delta S_{UV} \sim \frac{GE\ell}{}$$


Schw
$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$
area deficit in GR
$\delta A|_V \sim -\frac{GE}{l}\cdot l^2 \sim -GEl \sim -NG\hbar$
l_p^2
$\delta S_{UV} \sim -N \quad \therefore \quad \delta S_{UV} + \delta S_{IR} \lesssim 0$
CAUTION



$\delta A|_V \longleftrightarrow$ curvature , $\delta S_{IR} \longleftrightarrow$ energy
Spatial Ricci scalar
$\delta A|_V = \frac{-\Omega_{d-2}}{2(d^2-1)} R \left(1 + O\left(\frac{\ell}{L_{curvature}}\right)\right)$
u^a
ℓ
Small geodesic ball of radius ℓ.
DRYWALL
CAUTION


⟷ curvature
δS_{IR} ⟷ energy
Spatial Ricci scalar
$\frac{\Omega_{d-2}\, l^d}{2(d^2-1)} \mathcal{R} \left(1 + O\left(\frac{l}{L_{curvature}}\right)\right)$
how is $\mathcal{R}$ related to spacetime Riemann tensor?
$\mathcal{R} = {}^{(s)}R_{ik}{}^{ik} = {}^{(st)}R_{ik}{}^{ik}$
small geodesic ball of radius l.
l

$\delta A|_V \longleftrightarrow$ curvature , $\delta S_{IR} \longleftrightarrow$ en

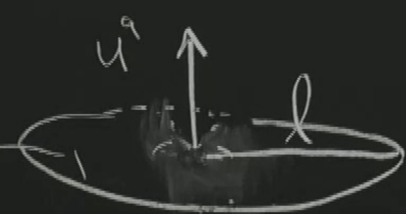
spatial Ricci scalar

$$\delta A|_V = -\frac{\Omega_{d-2}\,\ell^d}{2(d^2-1)}\,\mathcal{R}\left(1+O\left(\frac{\ell}{L_{curvature}}\right)\right)$$

how is $\mathcal{R}$

spacetime

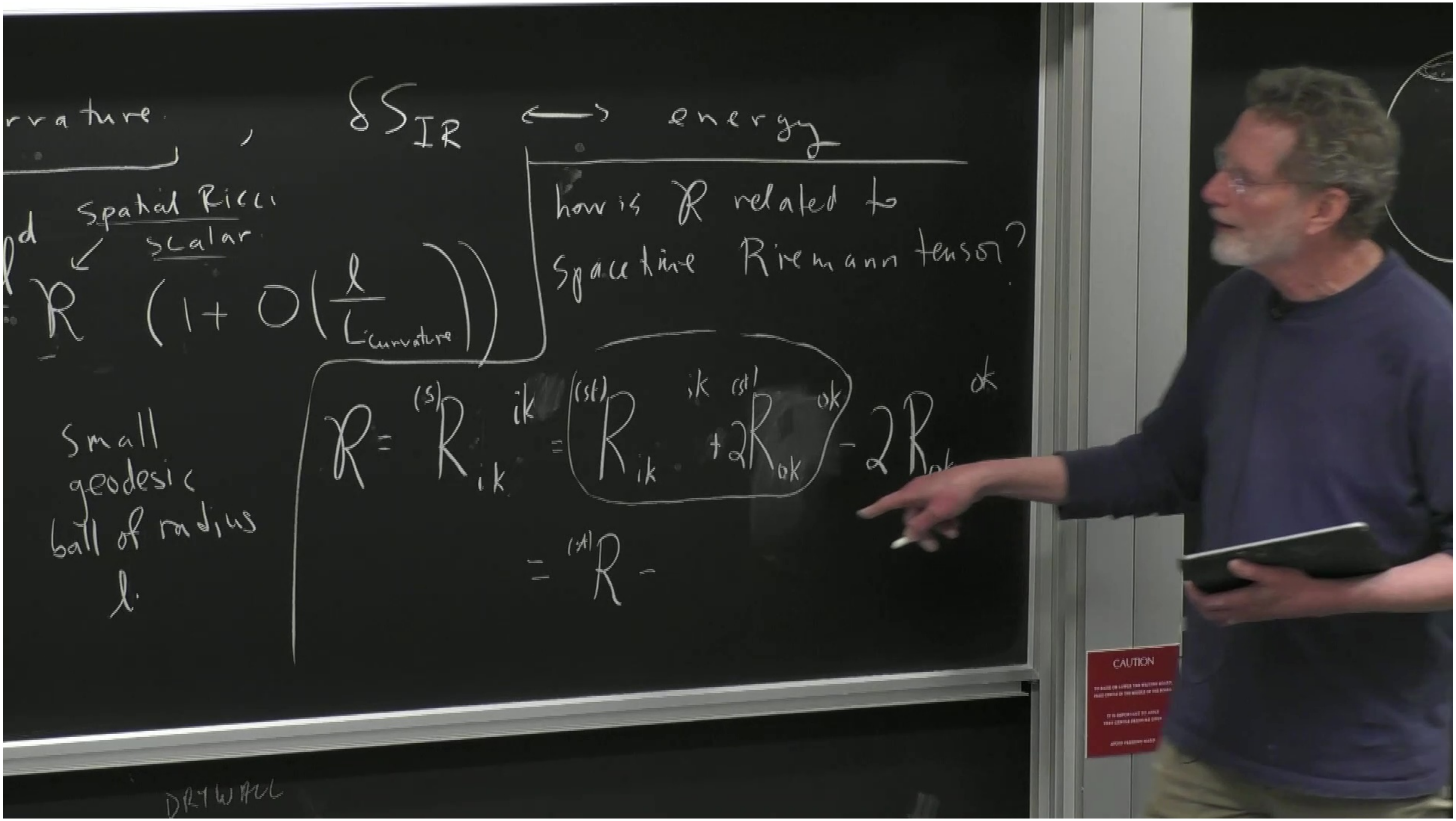
$$\mathcal{R} = {}^{(s)}R_{ik}{}^{ik} = {}^{(st)}R_{ik}{}^{ik}$$

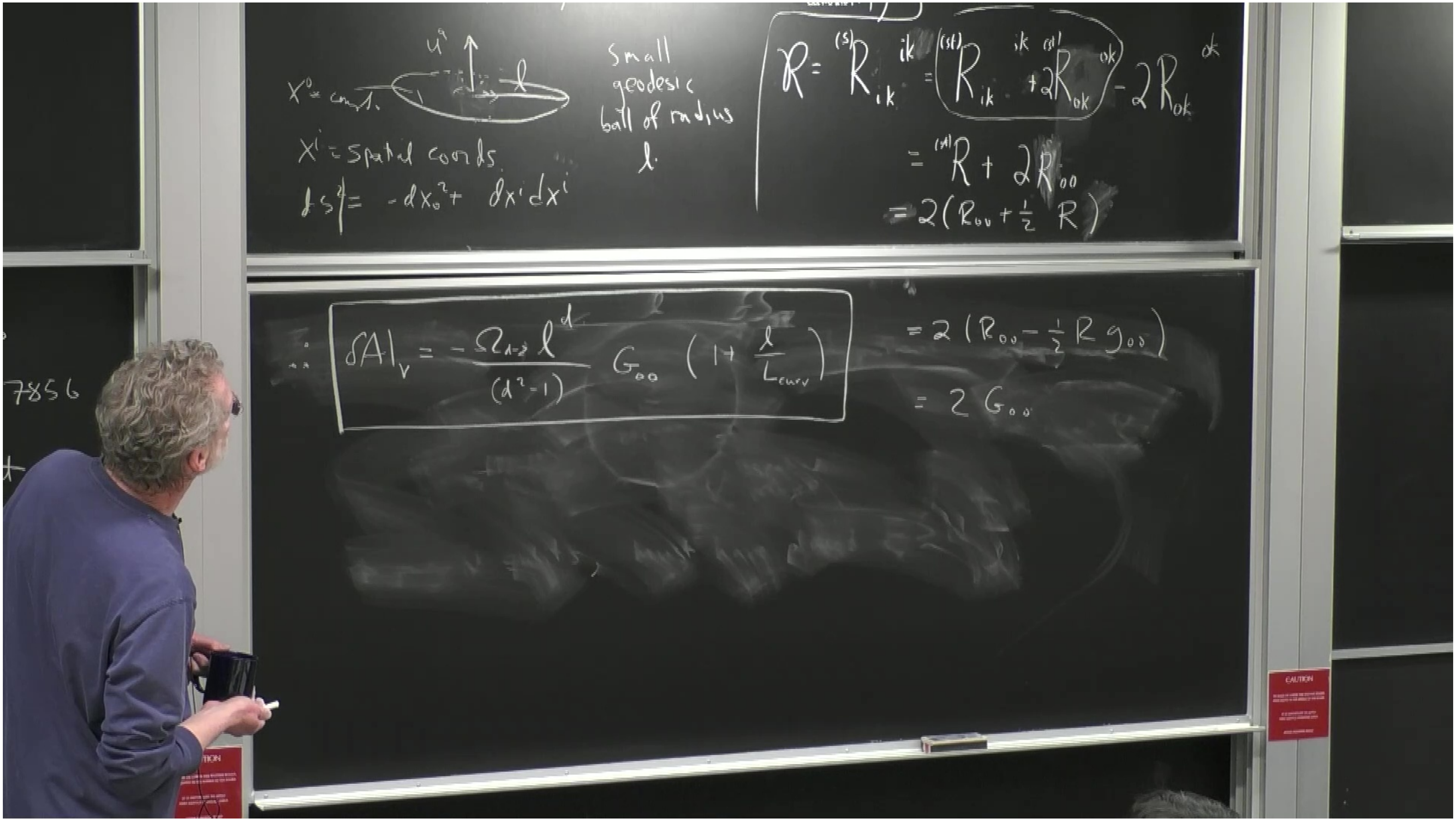


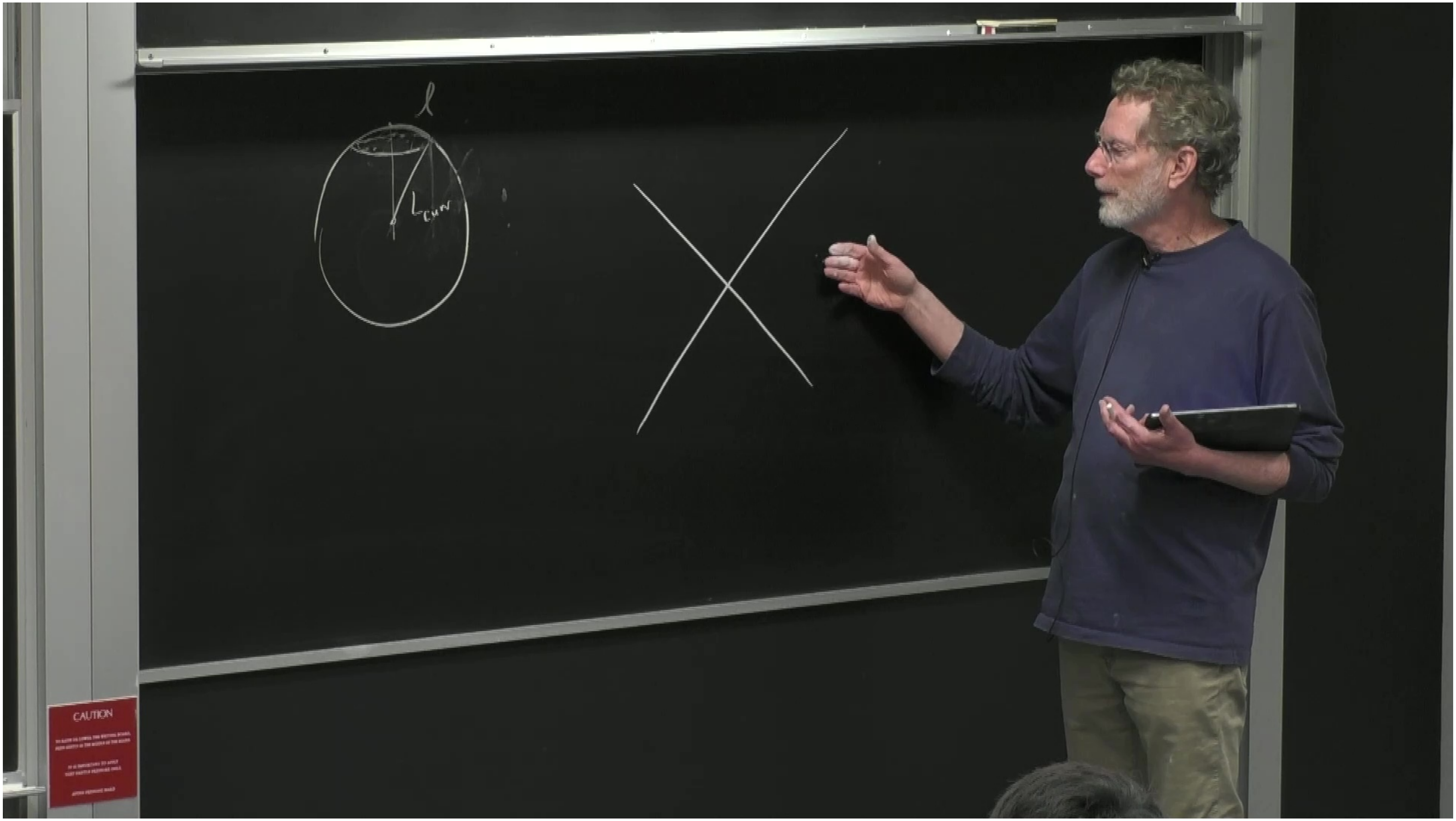
x^0 = const.

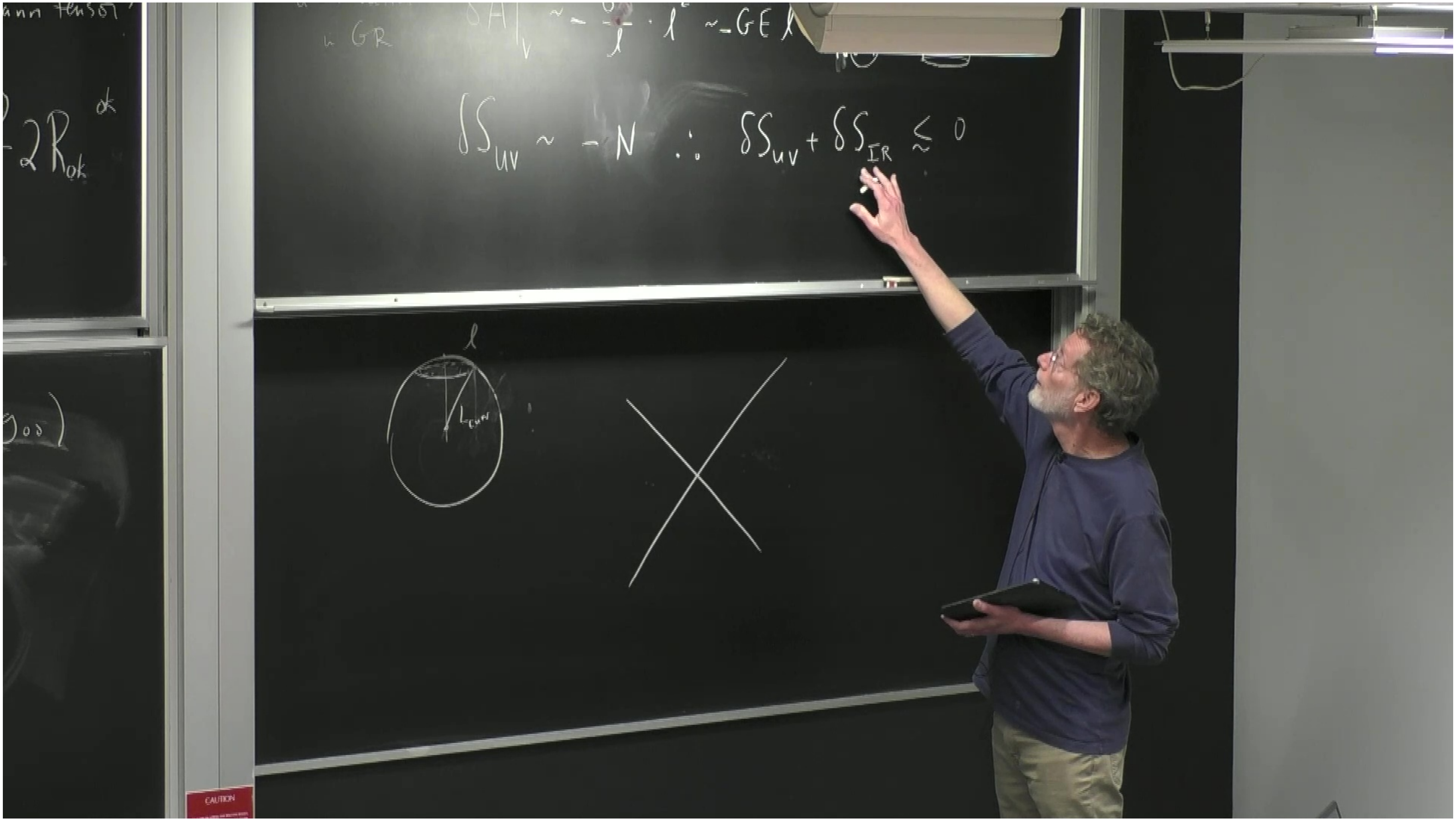
x^i = spatial coords.

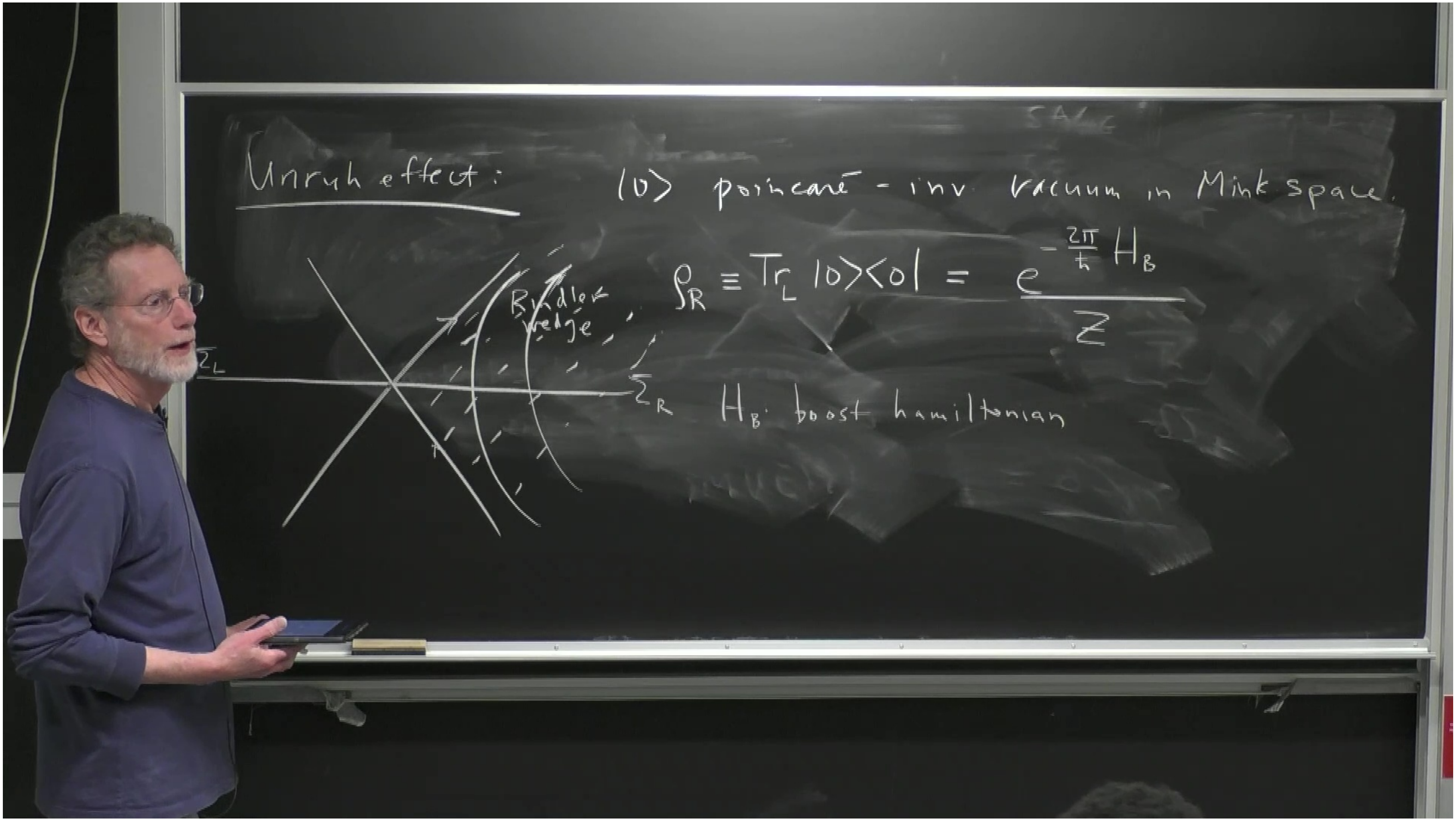
Small geodesic ball of radius ℓ.

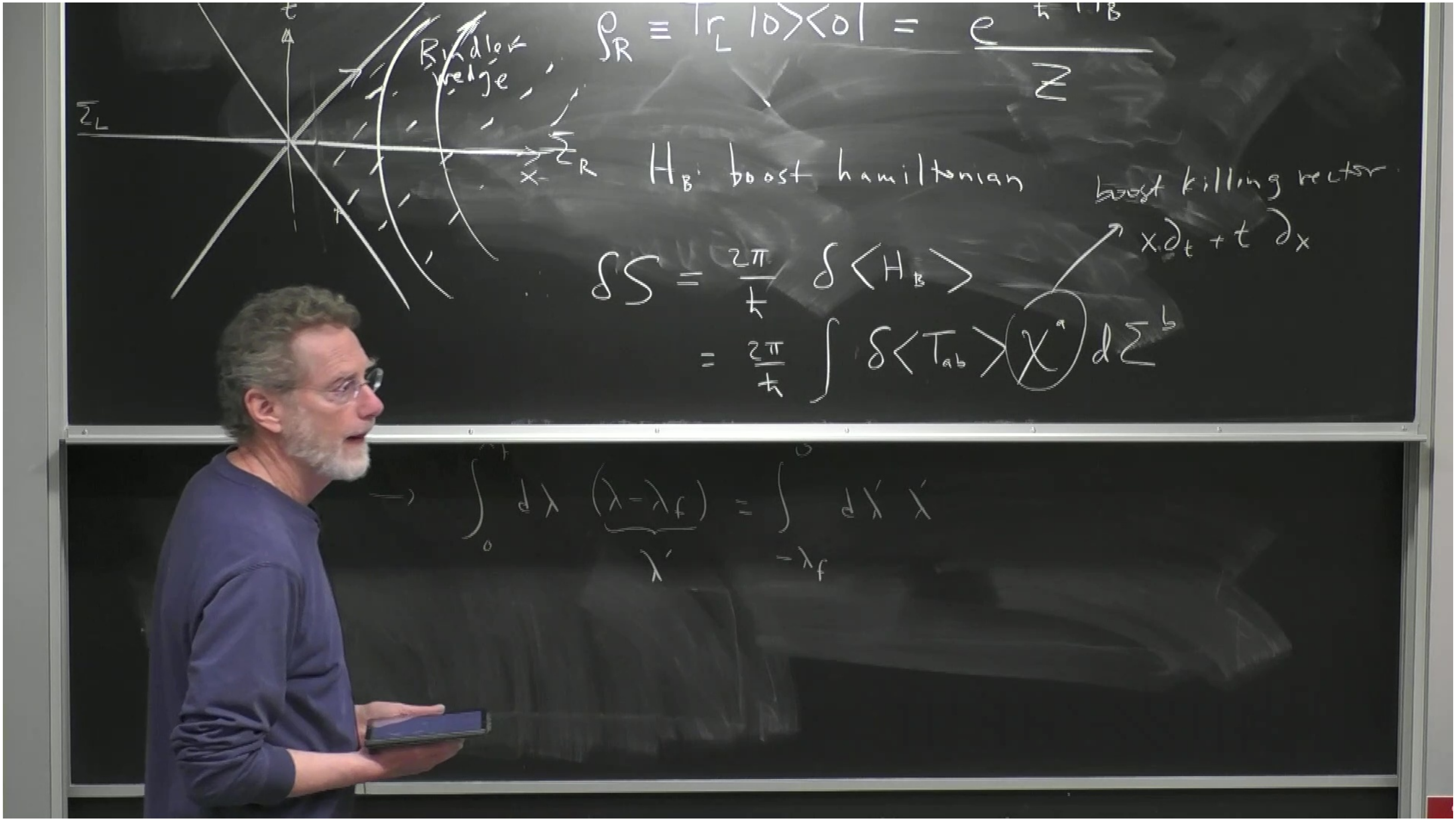

rvature , δS_IR ⟷ energy
Spatial Ricci scalar
R (1 + O(l / L_curvature))
how is R related to spacetime Riemann tensor?
Small geodesic ball of radius l.
R = (s)R_ik^ik = (st)R_ik^ik + 2 (st)R_ok^ok − 2 R_ok^ok
= (st)R −
DRYWALL
CAUTION

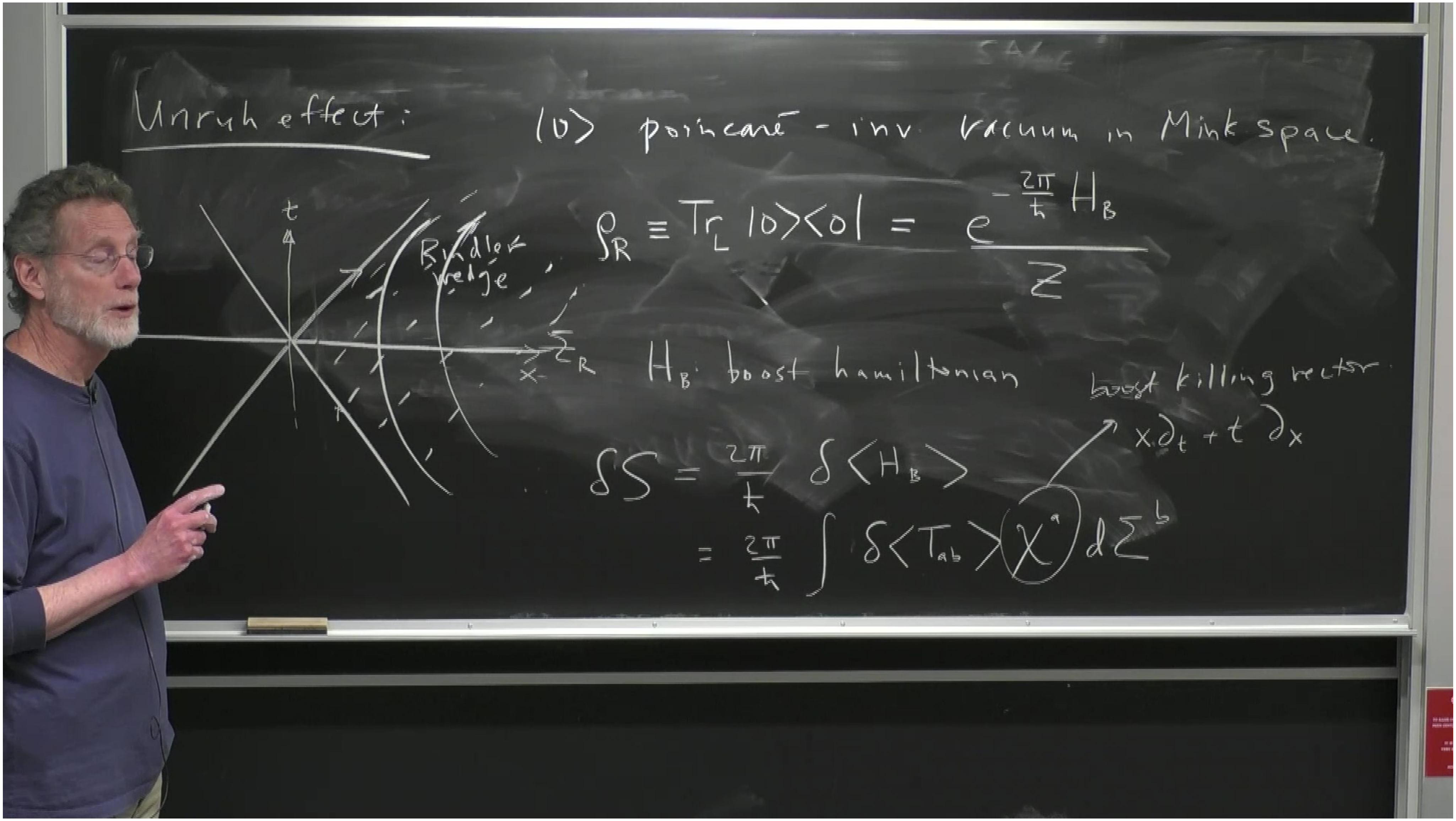

x^0 = const.
u^a
l
small geodesic ball of radius l.
x^i = spatial coords.
$ds^2 = -dx_0^2 + dx^i dx^i$
$R = {}^{(s)}R_{ik}{}^{ik} = \left({}^{(st)}R_{ik}{}^{ik} + 2\,{}^{(st)}R_{0k}{}^{0k}\right) - 2R_{0k}{}^{0k}$
$= {}^{(st)}R + 2R_{00}$
$= 2(R_{00} + \frac{1}{2} R)$
$= 2(R_{00} - \frac{1}{2} R g_{00})$
$= 2 G_{00}$
$\therefore \delta A|_V = -\frac{\Omega_{d-2}\, l^d}{(d^2-1)} G_{00} \left(1 + \frac{l}{L_{curv}}\right)$
7856


ℓ
L_CMW
CAUTION


δS_UV ~ −N ∴ δS_UV + δS_IR ≲ 0
ℓ
CAUTION

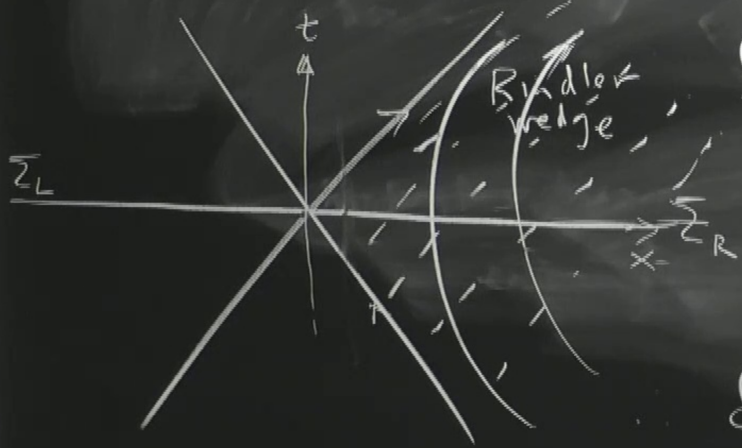

Unruh effect:
|0> poincaré - inv vacuum in Mink space.
$\rho_R \equiv Tr_L |0\rangle\langle 0| = \frac{e^{-\frac{2\pi}{\hbar} H_B}}{Z}$
Rindler wedge
Σ_L
Σ_R
H_B: boost hamiltonian


Rindler wedge
Σ_L
Σ_R
$\rho_R \equiv \mathrm{Tr}_L |0\rangle\langle 0| =$
Z
H_B: boost hamiltonian
boost killing vector
$x\,\partial_t + t\,\partial_x$
$\delta S = \frac{2\pi}{\hbar}\,\delta\langle H_B\rangle$
$= \frac{2\pi}{\hbar}\int \delta\langle T_{ab}\rangle\,\chi^a\,d\Sigma^b$
λ'
$-\lambda_f$


Unruh effect:
|0> poincaré - inv vacuum in Mink space.
t
Rindler wedge
Σ_R
x
$\rho_R \equiv \mathrm{Tr}_L |0\rangle\langle 0| = \frac{e^{-\frac{2\pi}{\hbar} H_B}}{Z}$
H_B: boost hamiltonian
~~boost~~ killing vector
$x\partial_t + t\partial_x$
$\delta S = \frac{2\pi}{\hbar} \delta\langle H_B\rangle$
$= \frac{2\pi}{\hbar} \int \delta\langle T_{ab}\rangle \chi^a d\Sigma^b$

Unruh effect:

$|0\rangle$ poincaré - inv vacuum in Mink space.



$$\rho_R \equiv \mathrm{Tr}_L |0\rangle\langle 0| = \frac{e^{-\frac{2\pi}{\hbar} H_B}}{Z}$$

(see 1212.6821 for path integral derivation)

H_B: boost hamiltonian

boost killing vector $x\partial_t + t\partial_x$

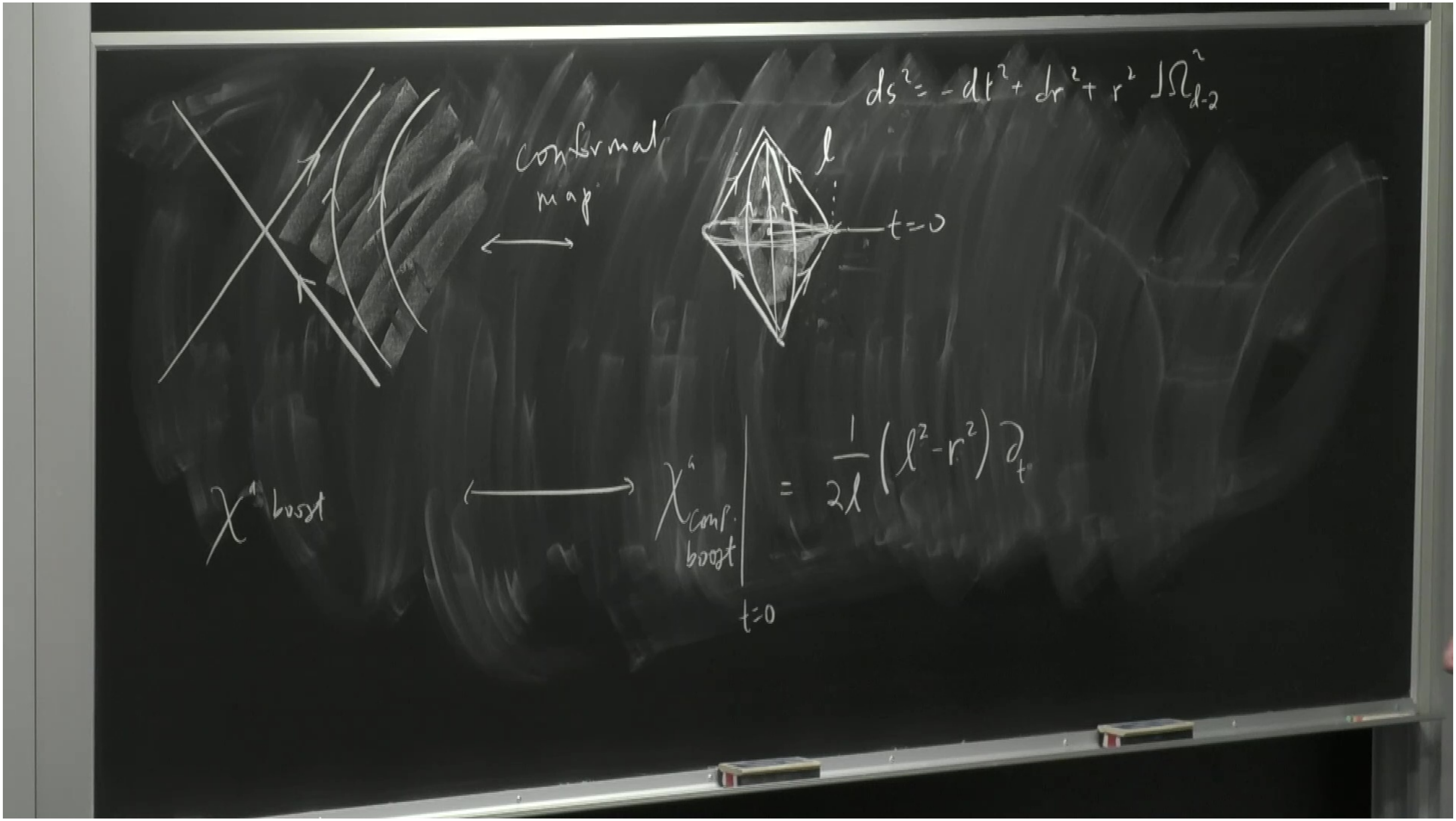
$$\delta S = \frac{2\pi}{\hbar}\,\delta\langle H_B\rangle$$

$$= \frac{2\pi}{\hbar}\int \delta\langle T_{ab}\rangle \chi^a\, d\Sigma^b$$

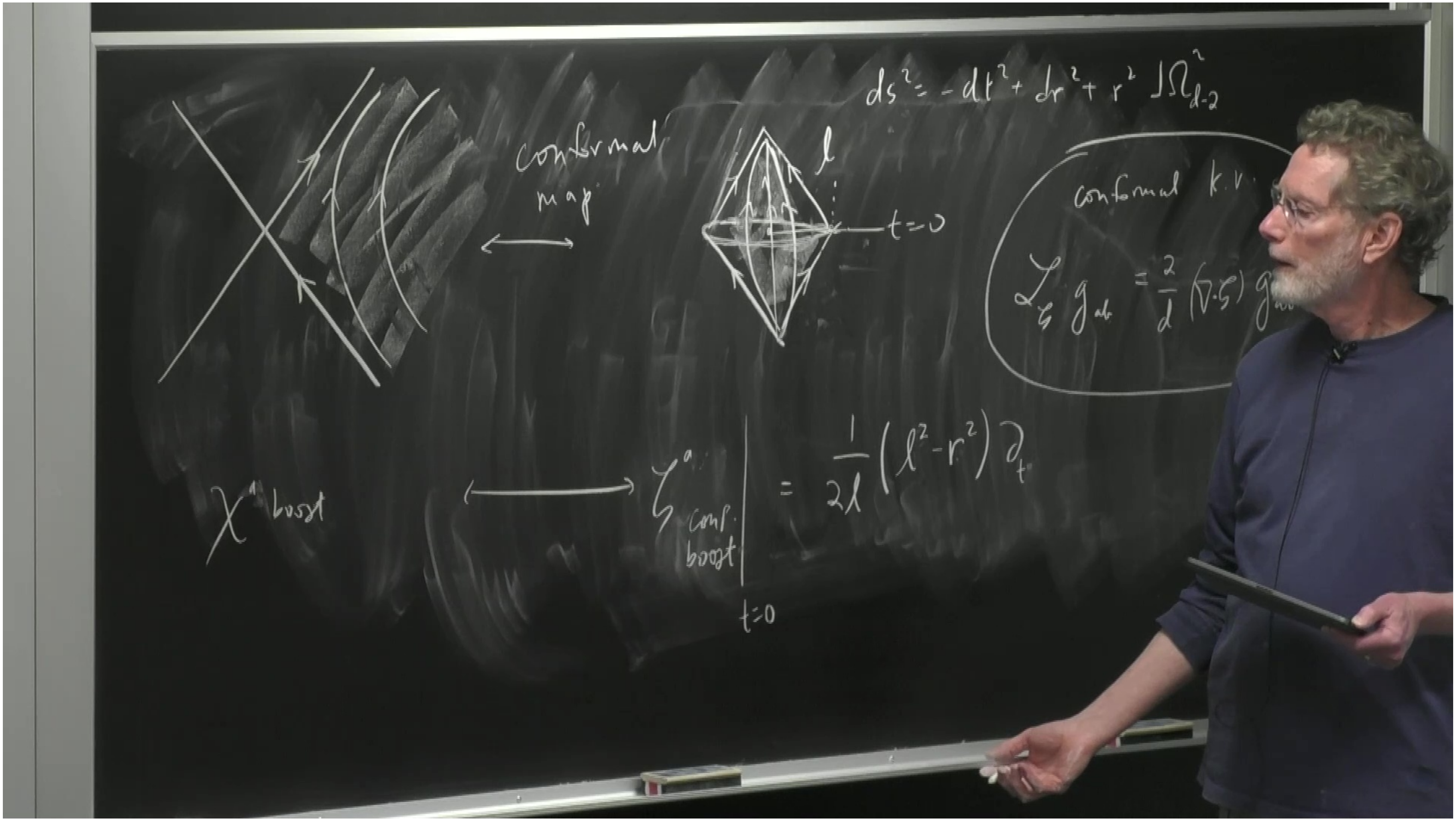
$$= \frac{2\pi}{\hbar} \int \delta \langle T_{ab} \rangle \chi^a \, d\Sigma^b$$

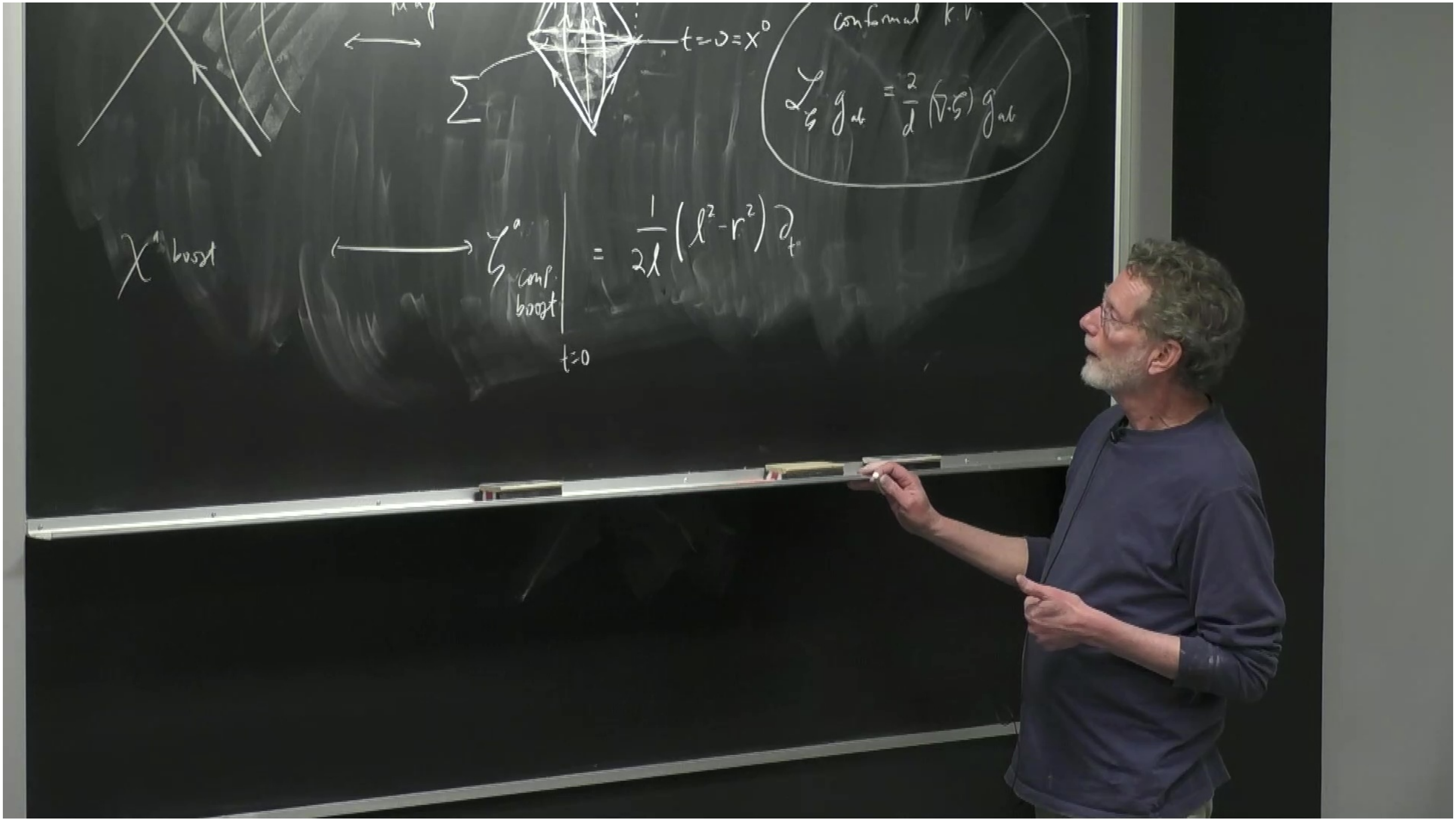
Great, but we need a formula relating δS & $\delta \langle T_{ab} \rangle$ in a <u>small</u> <u>ball</u>.

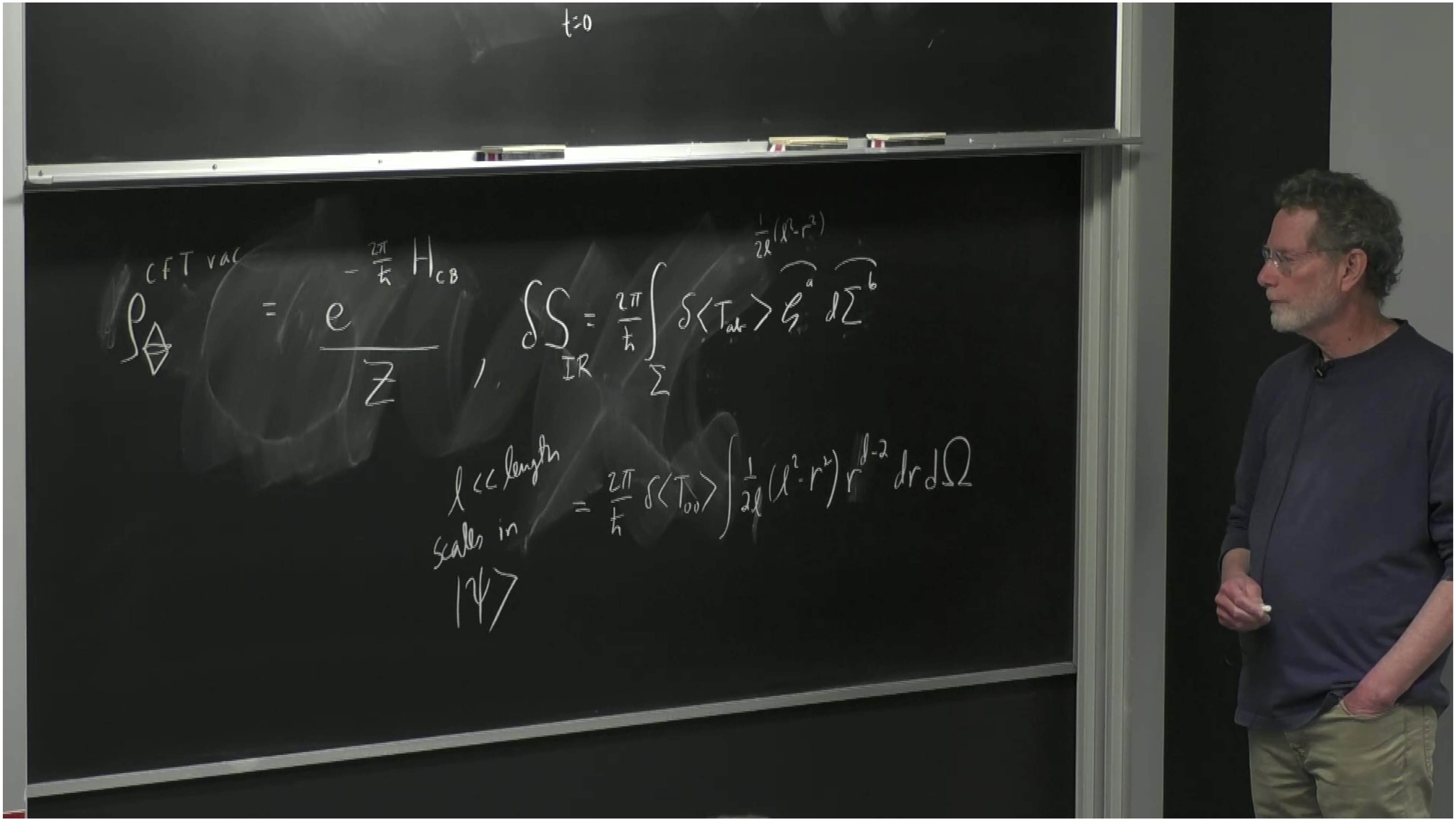
First: cheat, consider only CFT (conformal field theory).

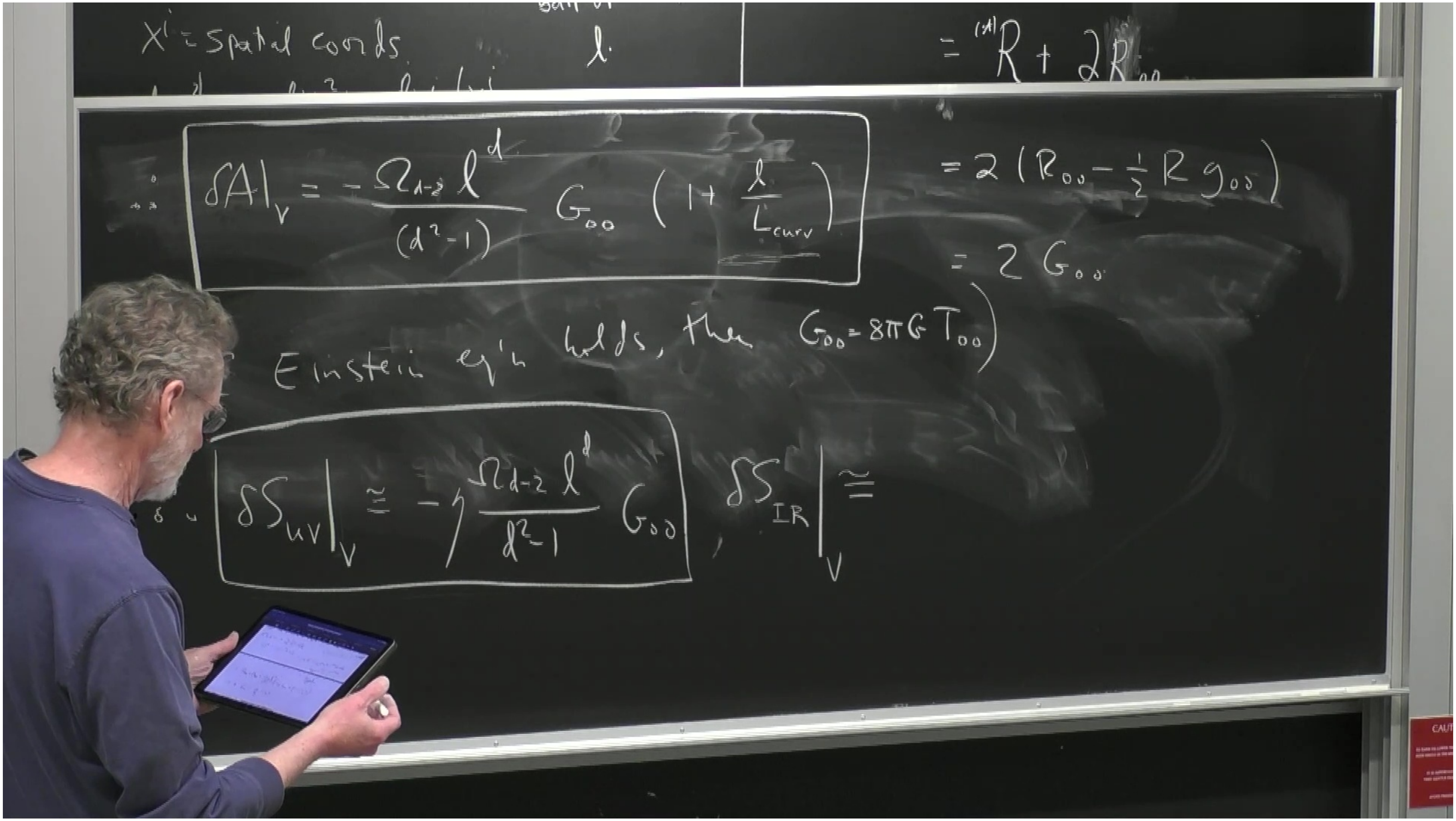


$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2$
conformal map
ℓ
$t=0$
χ^a boost
$\chi^a_{\text{comp. boost}}\Big|_{t=0} = \frac{1}{2\ell}(\ell^2 - r^2)\partial_t$


$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2$
conformal map
ℓ
$t=0$
conformal k.v.
$\mathcal{L}_\xi g_{ab} = \frac{2}{d}(\nabla\cdot\xi)\, g_{ab}$
χ^a boost
$\xi^a_{\text{conf. boost}}\Big|_{t=0} = \frac{1}{2\ell}(\ell^2 - r^2)\,\partial_t$


conformal k.v.
boost
t=0


$t=0$
$\rho^{\text{CFT vac}}_{\diamond} = \dfrac{e^{-\frac{2\pi}{\hbar} H_{CB}}}{Z}$, $\delta S_{IR} = \dfrac{2\pi}{\hbar} \int_{\Sigma} \delta\langle T_{ab} \rangle \overbrace{\xi^a}^{\frac{1}{2\ell}(\ell^2-r^2)} \widehat{d\Sigma^b}$
$\ell <<$ length scales in $|\psi\rangle$
$= \dfrac{2\pi}{\hbar} \delta\langle T_{00}\rangle \int \frac{1}{2\ell}(\ell^2 - r^2) r^{d-2} \, dr \, d\Omega$


x^i = spatial coords
l.
$= {}^{(d)}R + 2R_{00}$
$\delta A|_V = -\frac{\Omega_{d-2}\, l^d}{(d^2-1)} G_{00} \left(1 + \frac{l}{L_{curv}}\right)$
$= 2(R_{00} - \frac{1}{2} R g_{00})$
$= 2 G_{00}$
Einstein eq'n holds, then $G_{00} = 8\pi G T_{00}$
$\delta S_{UV}|_V \cong -\eta \frac{\Omega_{d-2}\, l^d}{d^2-1} G_{00}$
$\delta S_{IR}|_V \cong$

so $\delta S_{uv} + \delta S_{IR} = 0 \Rightarrow G_{00} = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{00}\rangle$

$\rightarrow G_{ab}u^a u^b = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{ab}\rangle u^a u^b \quad \forall u^a$

$\Rightarrow G_{ab} = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{ab}\rangle$

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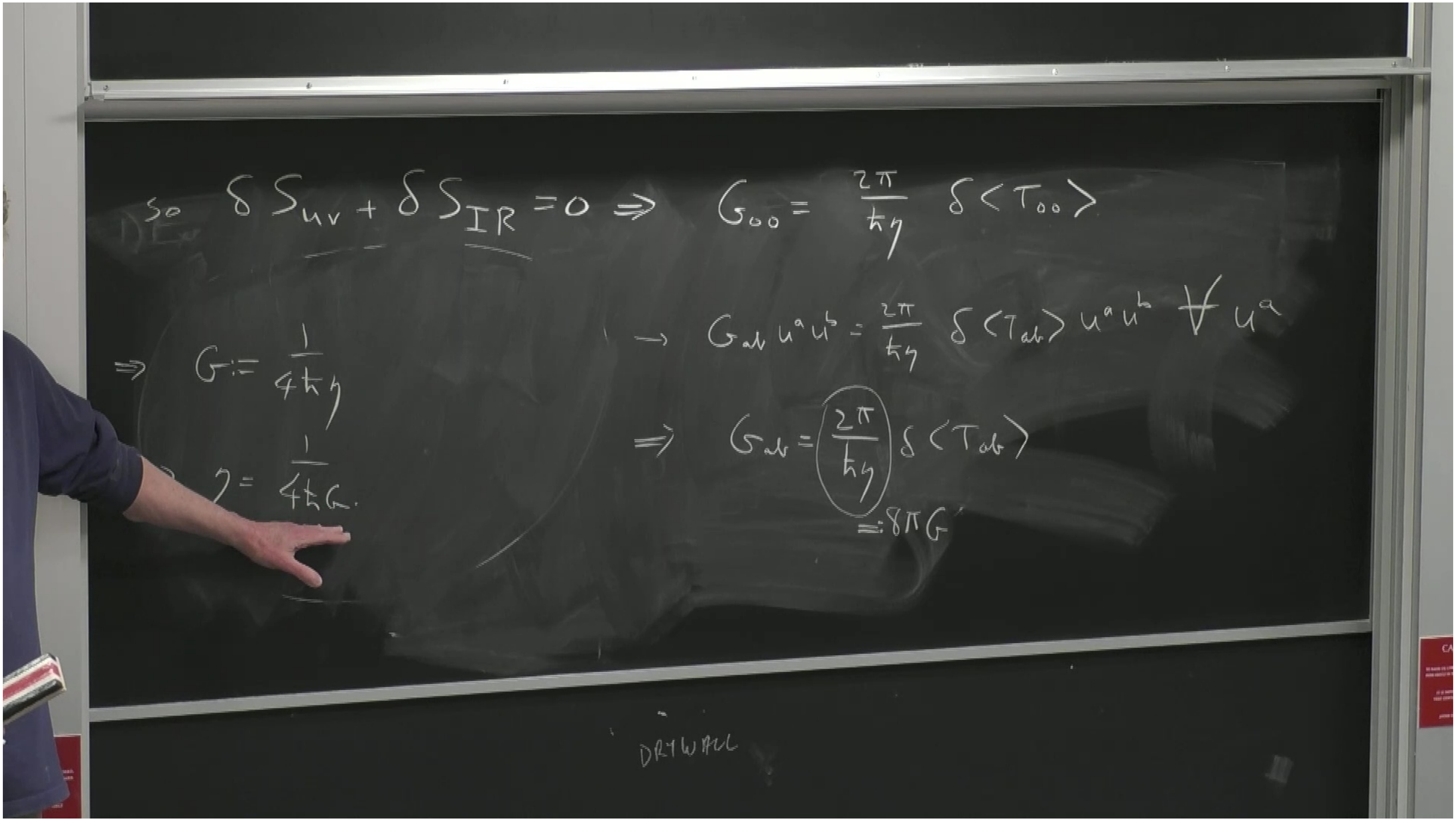
so $\delta S_{UV} + \delta S_{IR} = 0 \Rightarrow G_{00} = \frac{2\pi}{\hbar\eta}\, \delta\langle T_{00}\rangle$
$\rightarrow G_{ab} u^a u^b = \frac{2\pi}{\hbar\eta}\, \delta\langle T_{ab}\rangle u^a u^b \ \forall u^a$
$\Rightarrow G := \frac{1}{4\hbar\eta}$
$\Rightarrow G_{ab} = \frac{2\pi}{\hbar\eta}\, \delta\langle T_{ab}\rangle$
$=: 8\pi G$
$\eta = \frac{1}{4\hbar G}$
DRYWALL

$$\delta A|_\ell = \frac{d+1}{3}\,\delta A|_V \;\longrightarrow\; \eta = \frac{3}{d+1}\,\frac{1}{4\hbar G}$$

so $\delta S_{UV} + \delta S_{IR} = 0 \Rightarrow G_{00} = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{00}\rangle$

$$\longrightarrow G_{ab}u^a u^b = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{ab}\rangle u^a u^b \quad \forall u^a$$

$$\Rightarrow G := \frac{1}{4\hbar\eta}$$

$$\Rightarrow G_{ab} = \underbrace{\frac{2\pi}{\hbar\eta}}_{=:\,8\pi G}\,\delta\langle T_{ab}\rangle$$

$$\Rightarrow \eta = \frac{1}{4\hbar G}$$

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