Title: Horizon entropy and the Einstein equation - Lecture 20230228

Speakers: Ted Jacobson

Collection: Horizon entropy and the Einstein equation

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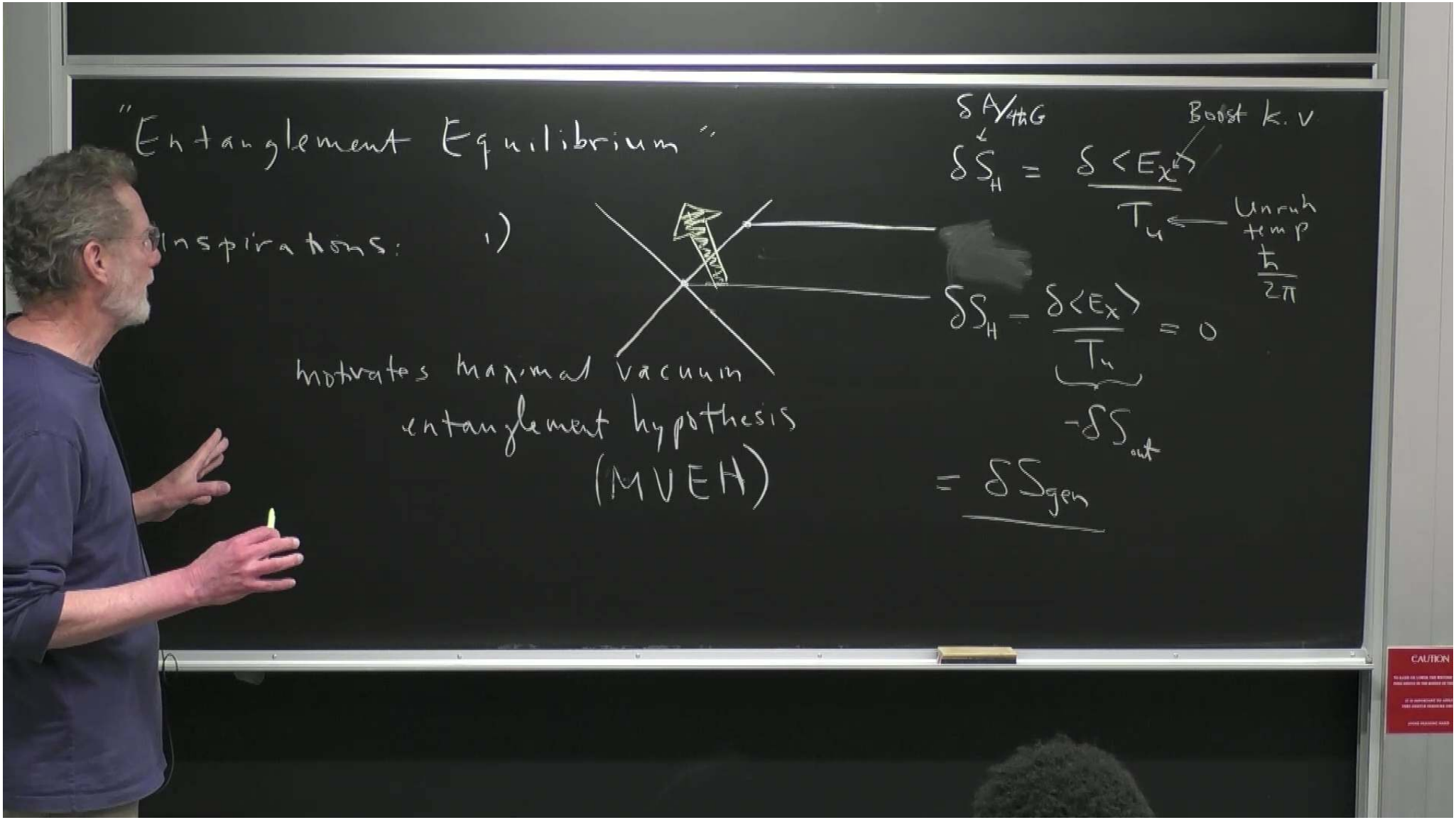
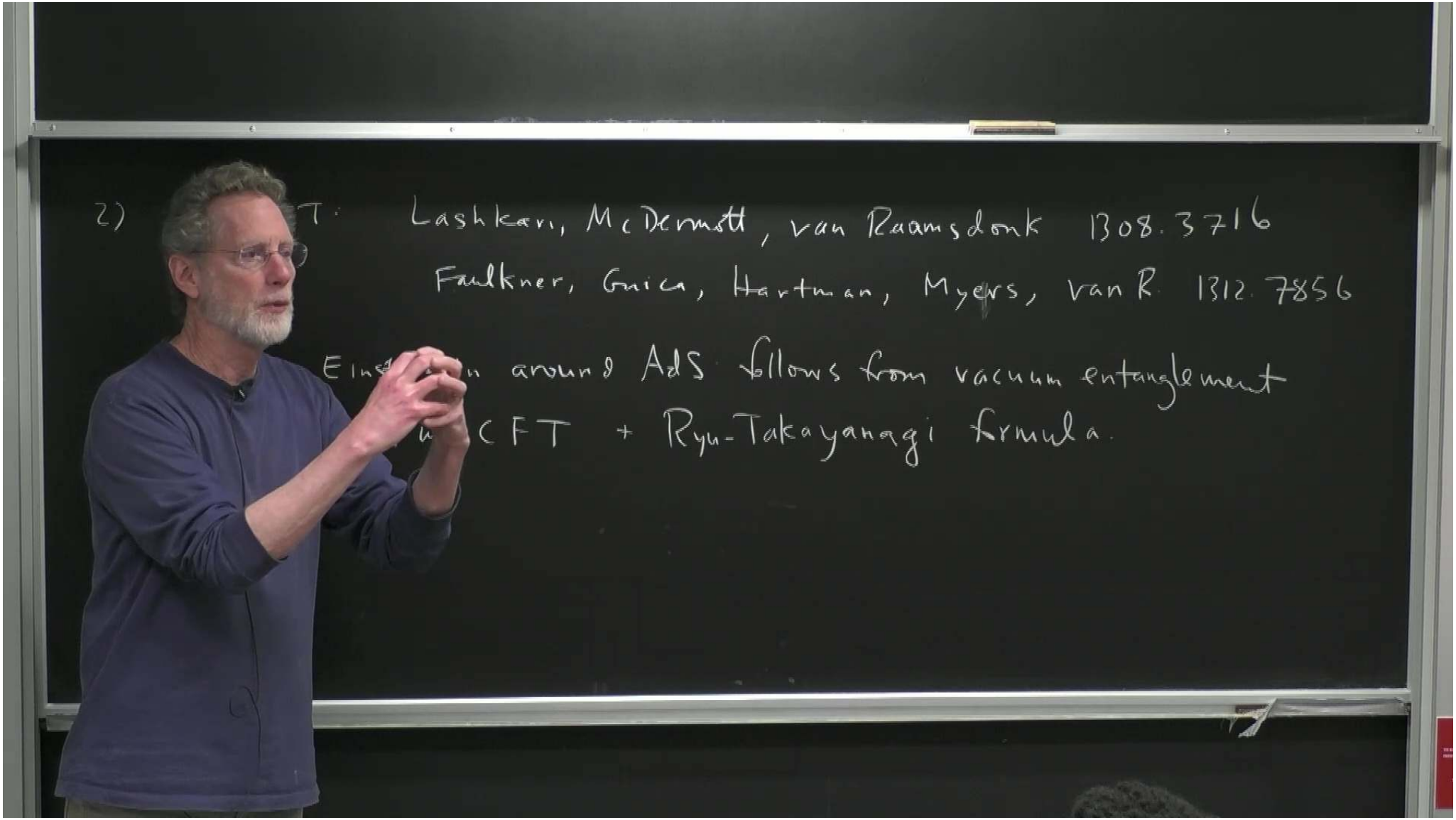
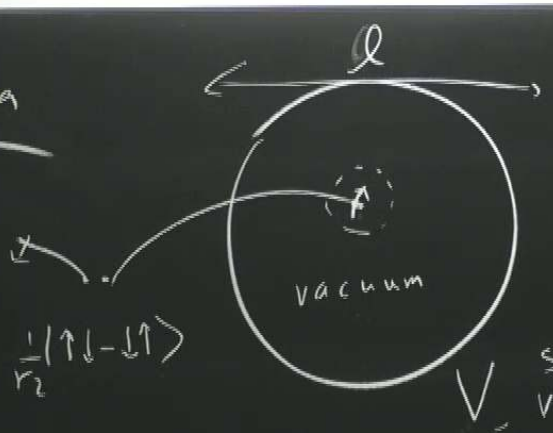
URL: https://pirsa.org/23020044

Abstract: Zoom link: https://pitp.zoom.us/j/96212372067?pwd=dWVaUFFFc3c5NTlVTDFHOGhCV2pXdz09


"Entanglement Equilibrium"
Inspirations: 1)
motivates maximal vacuum entanglement hypothesis (MVEH)
$\delta S_H = \frac{\delta \langle E_K \rangle}{T_U}$
$\delta A/4\hbar G$
Boost K.V
Unruh temp $\frac{\hbar}{2\pi}$
$\delta S_H - \frac{\delta \langle E_K \rangle}{T_U} = 0$
$-\delta S_{out}$
$= \delta S_{gen}$


2)
T:
Lashkari, McDermott, van Raamsdonk 1308.3716
Faulkner, Guica, Hartman, Myers, van R. 1312.7856
around AdS follows from vacuum entanglement
CFT + Ryu-Takayanagi formula.

Basic idea




hypothesis: entropy maximal in vacuum at fixed $V$.

to put $N$ qubits, must have placed $E \gtrsim N\frac{\hbar}{\ell}$.

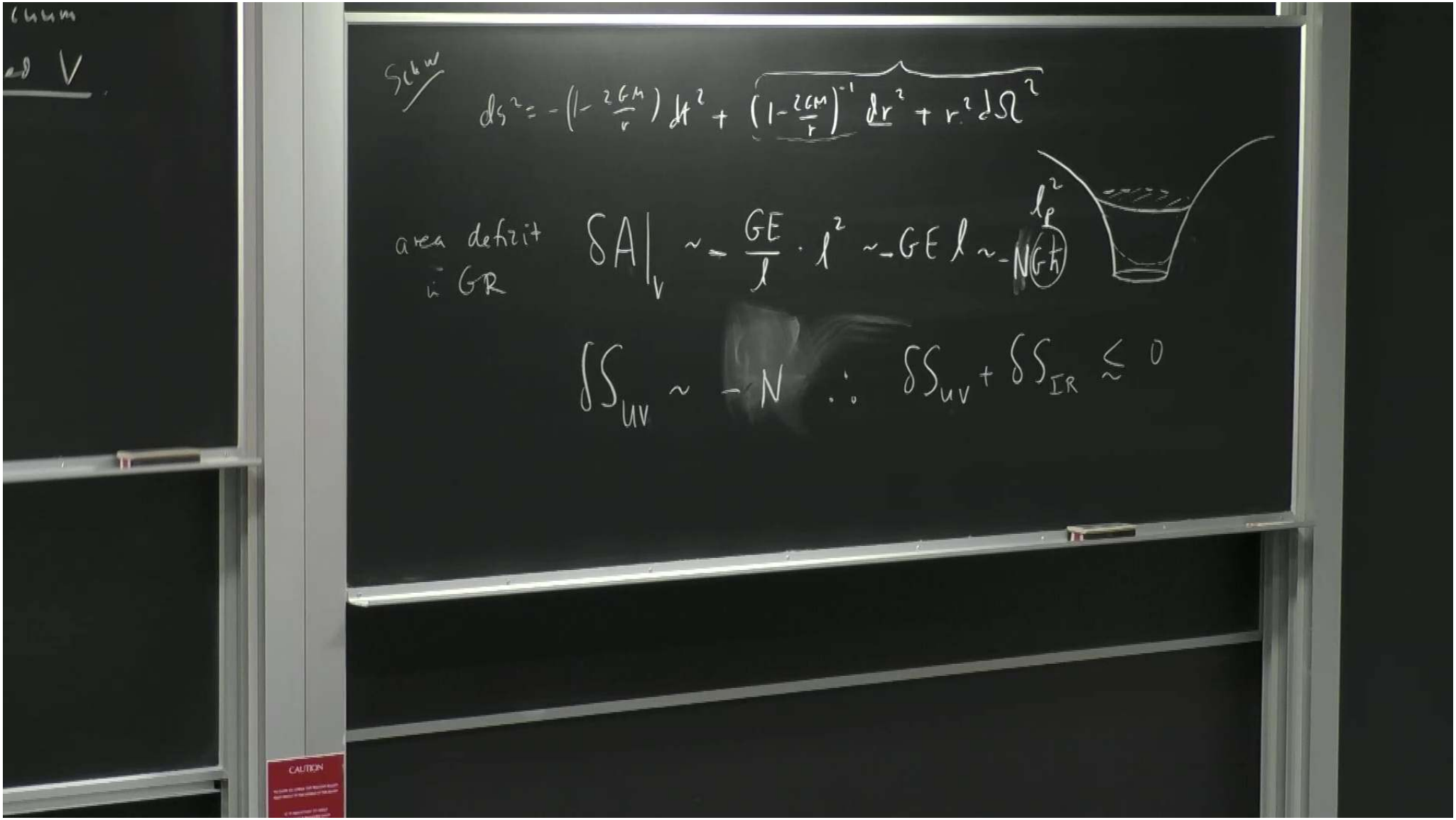
Schw

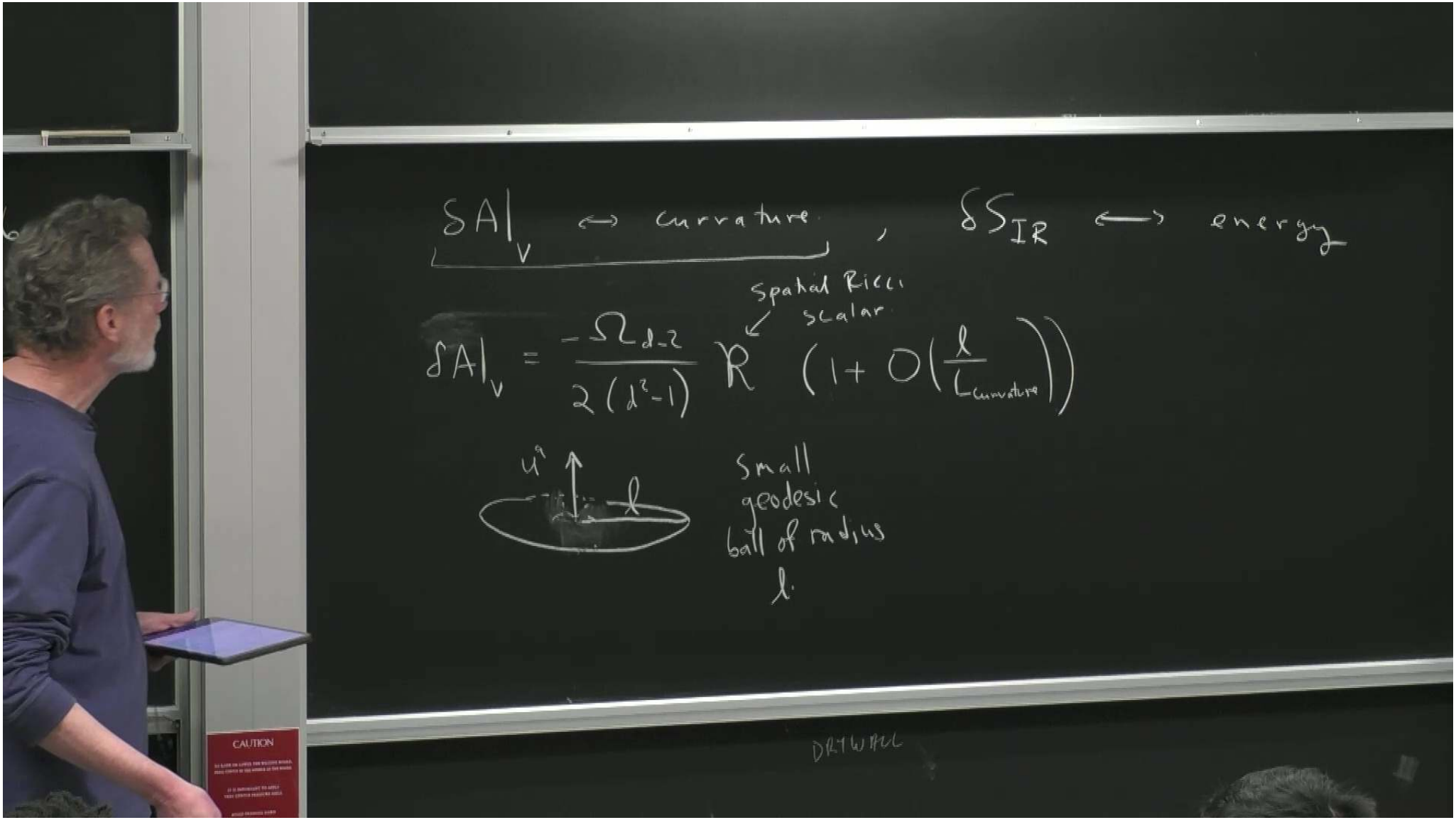
$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

area defrit in GR

$$\delta A\big|_V \sim -\frac{GE}{l} \cdot l^2 \sim -GEl \sim -N G\hbar$$

$$\delta S_{UV} \sim \frac{GEl}{}$$


Schw
$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$
area defizit in GR
$\delta A|_V \sim -\frac{GE}{l}\cdot l^2 \sim -GEl \sim -N\hbar$
$\delta S_{UV} \sim -N \quad \therefore \quad \delta S_{UV} + \delta S_{IR} \lesssim 0$


$\delta A|_V \longleftrightarrow$ curvature , $\delta S_{IR} \longleftrightarrow$ energy
Spatial Ricci scalar
$\delta A|_V = \frac{-\Omega_{d-2}}{2(d^2-1)} R^{(1+O(\frac{l}{L_{curvature}}))}$
$u^q$
$l$
Small geodesic ball of radius $l$.
CAUTION

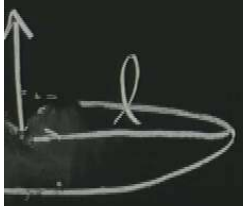
$\longleftrightarrow$ curvature , $\delta S_{IR} \longleftrightarrow$ energy

spatial Ricci scalar

$$\frac{-\Omega_{d-2}\, l^d}{2(d^2-1)} R \left(1 + O\left(\frac{l}{L_{curvature}}\right)\right)$$

how is $R$ related to spacetime Riemann tensor?

$$R = {}^{(s)}R_{ik} \quad {}^{ik\,(st)}R = {}^{ik}R_{ik}$$




small geodesic ball of radius $l$.

CAUTION

$\delta A|_V \longleftrightarrow$ curvature, $\quad \delta S_{IR} \longleftrightarrow$ en

$$\delta A|_V = -\frac{\Omega_{d-2}\, l^d}{2(d^2-1)}\, \mathcal{R}\, \left(1 + O\left(\frac{l}{L_{\text{curvature}}}\right)\right)$$

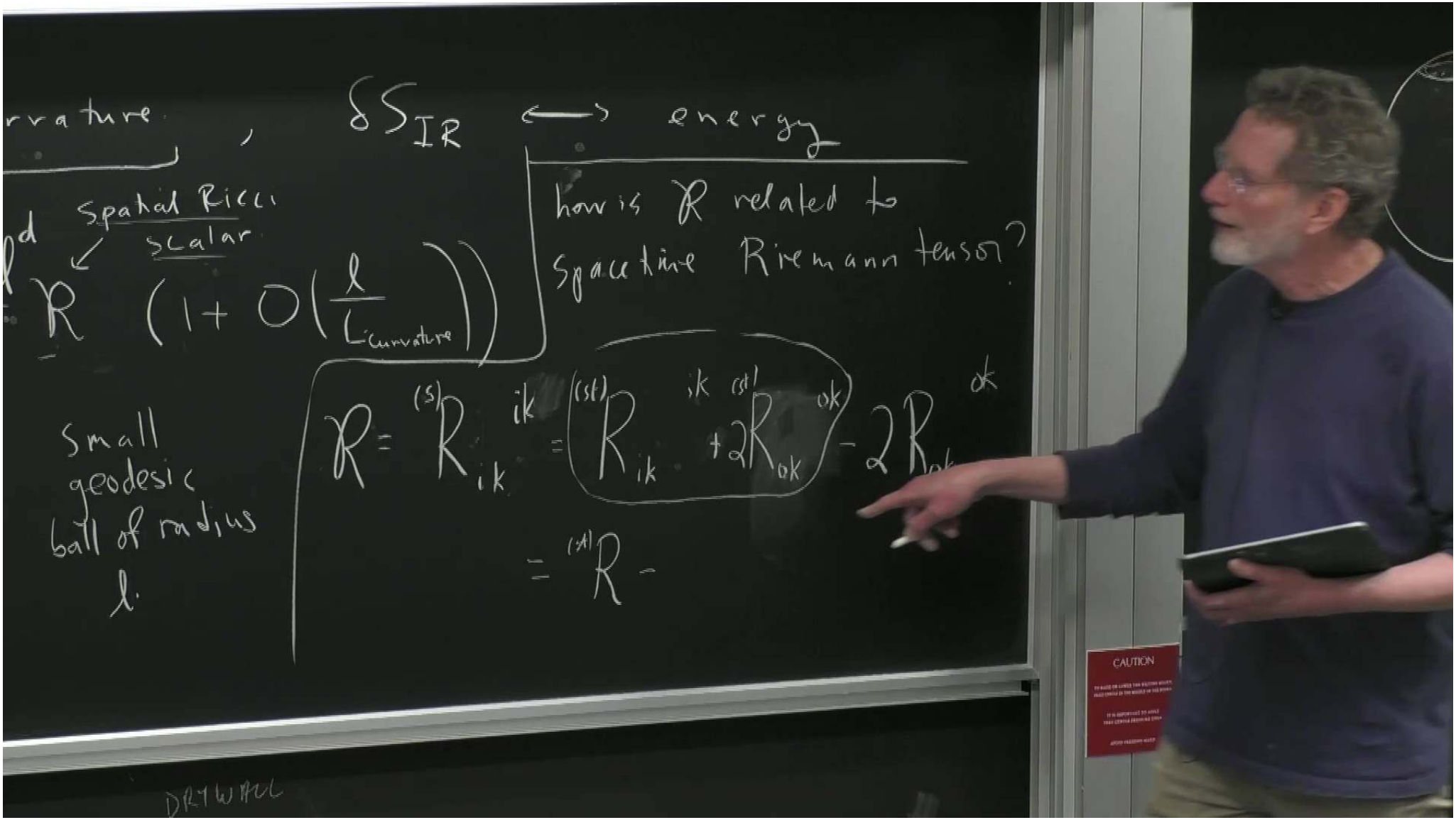
$\mathcal{R}$ = Spatial Ricci scalar

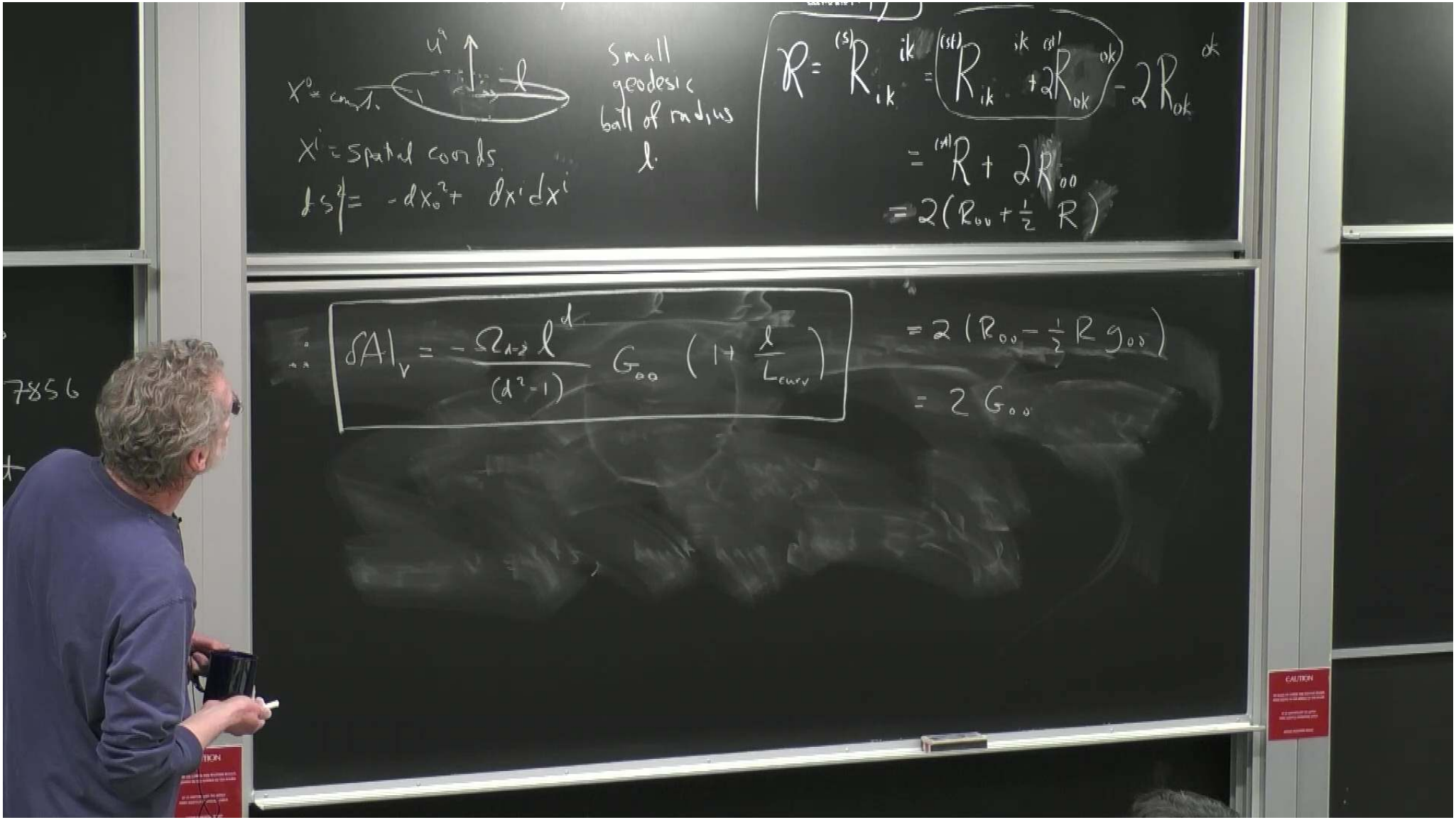
how is $\mathcal{R}$ Spacetime

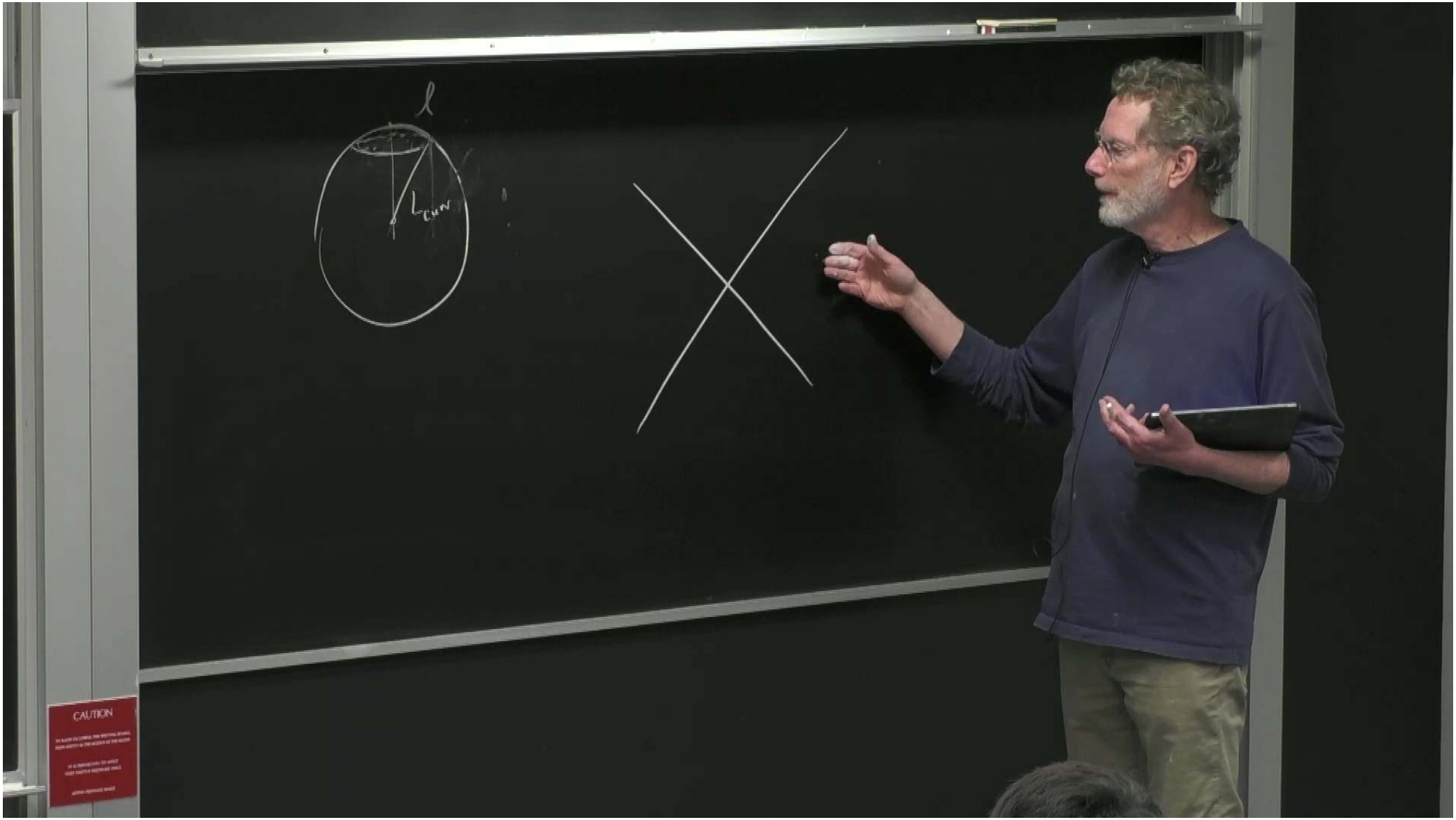
$$\mathcal{R} = {}^{(s)}R_{ik}{}^{ik} = {}^{(st)}R_{ik}{}^{ik}$$

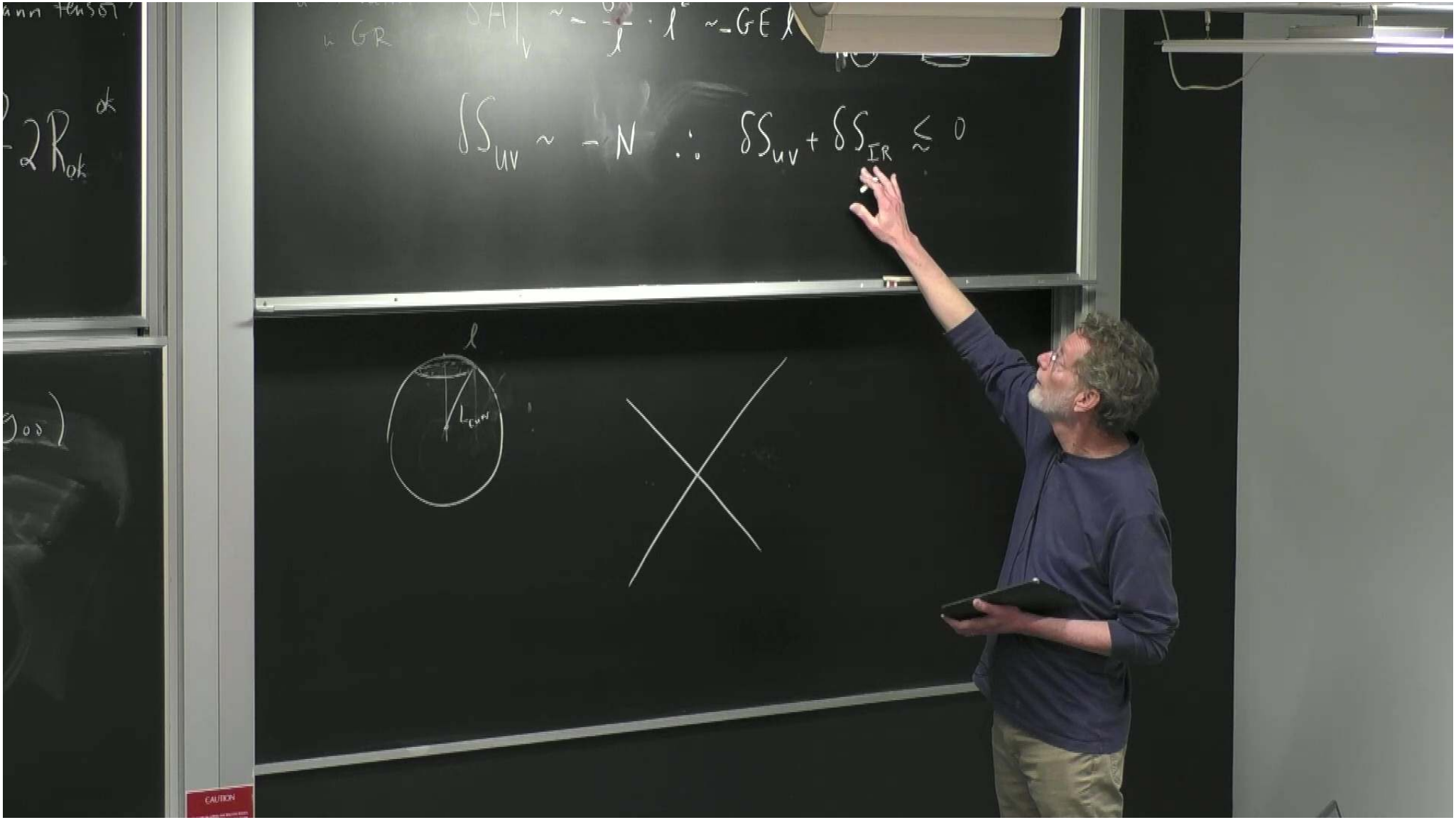
$x^i$ = Spatial coords.

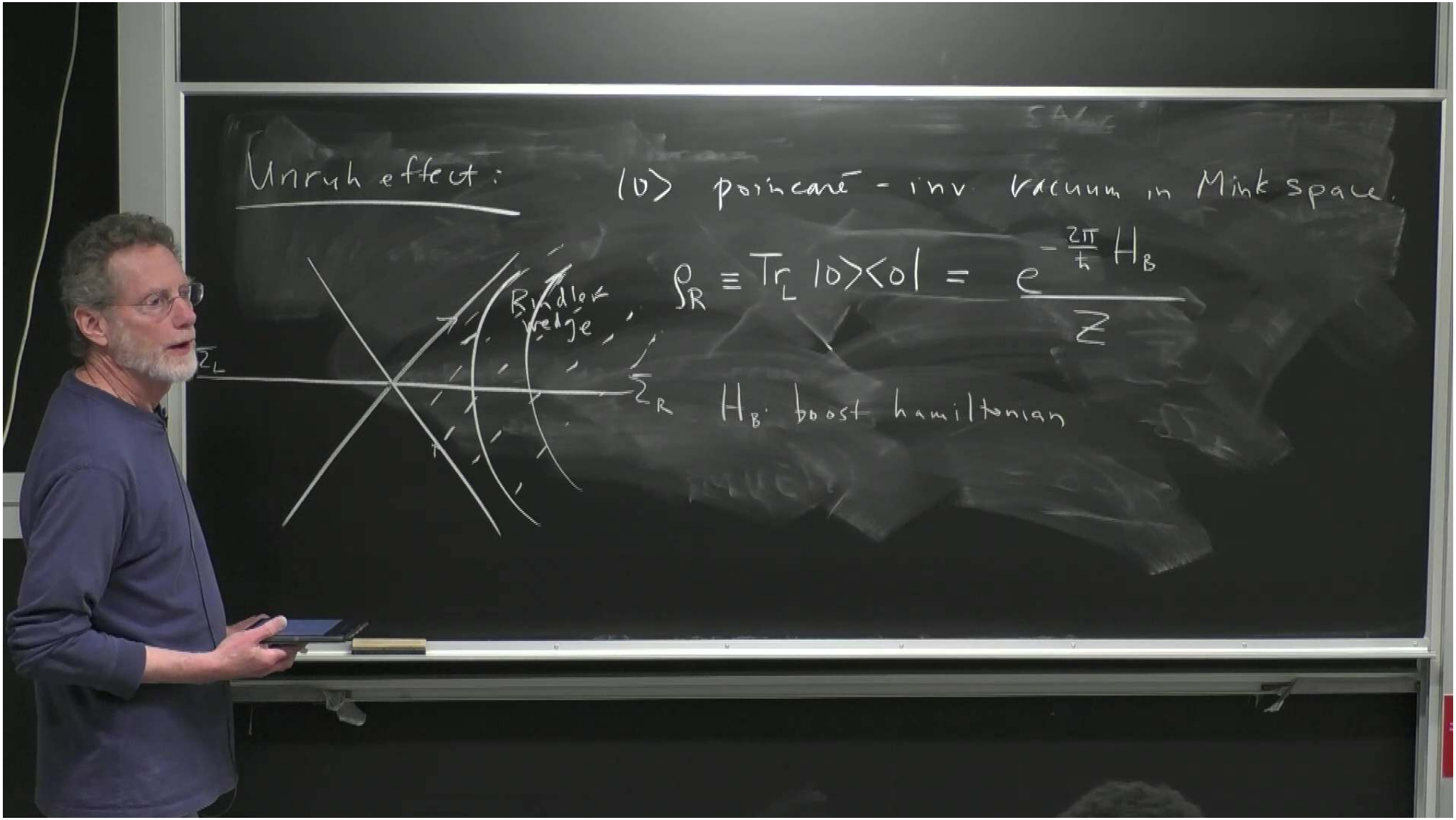
Small geodesic ball of radius $l$.

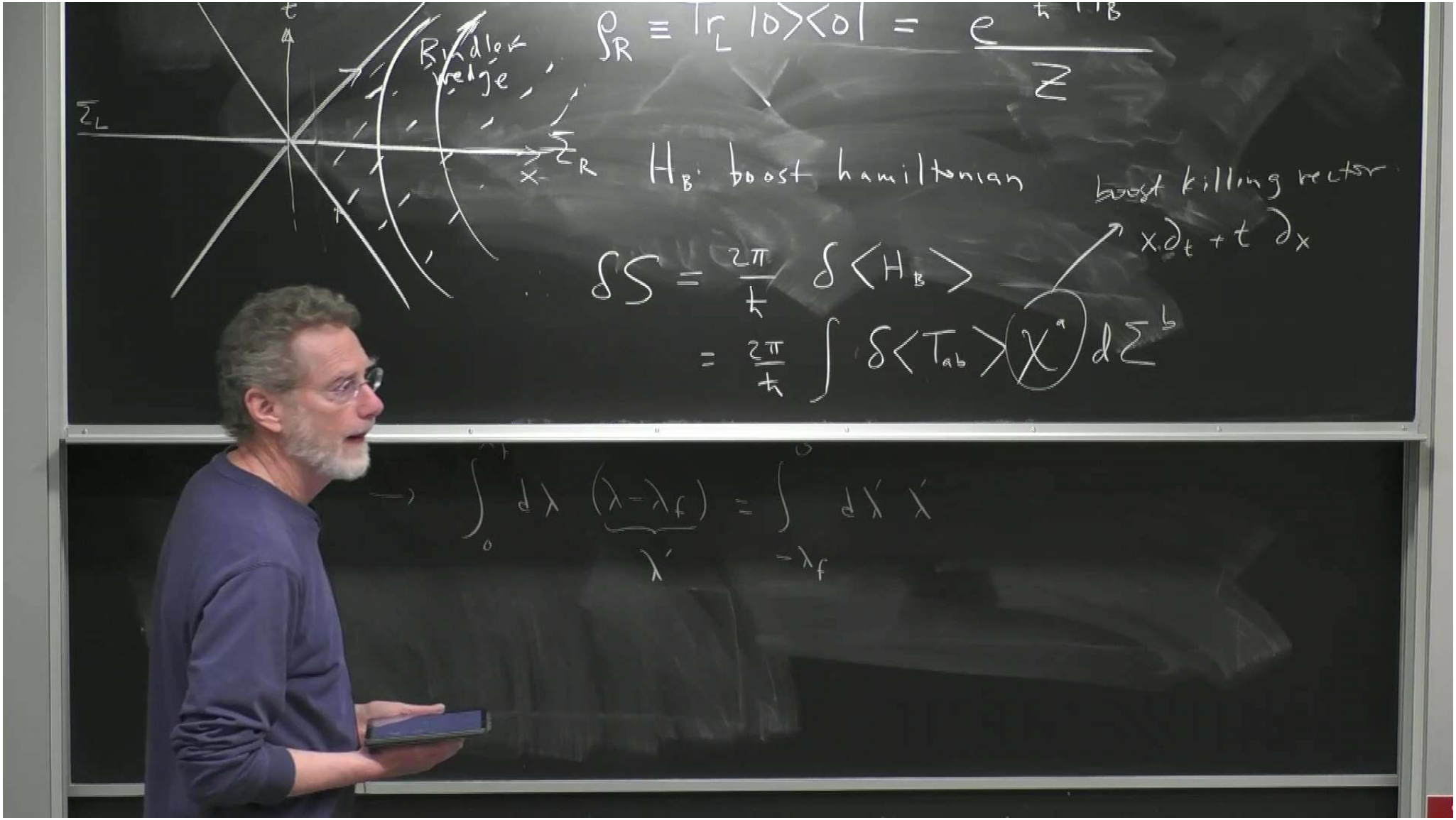

rvature
δS_IR ⟷ energy
Spatial Ricci scalar
How is ℛ related to Spacetime Riemann tensor?
Small geodesic ball of radius l.
CAUTION

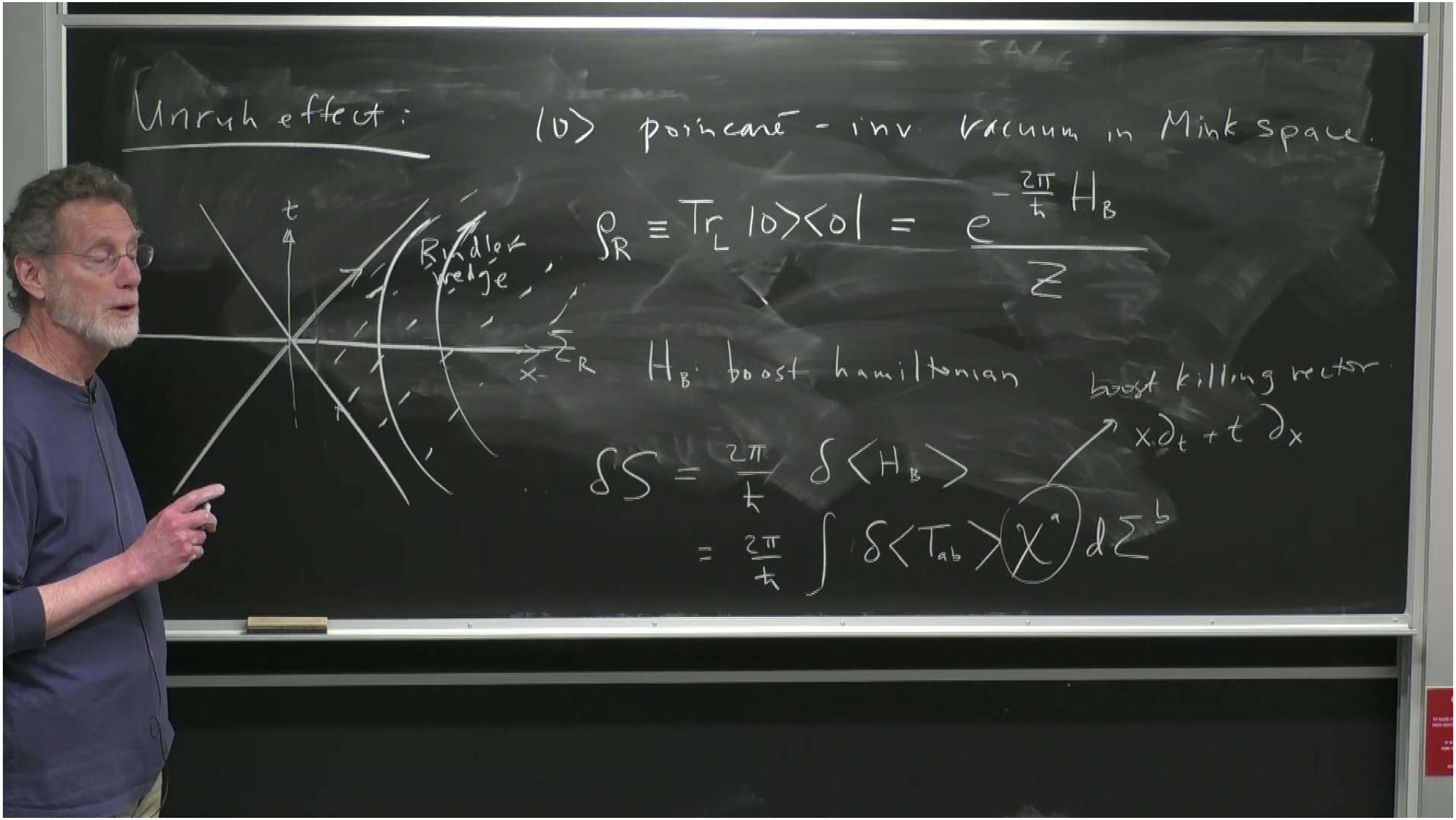

$x^0$ = const.
$x^i$
$\ell$
Small geodesic ball of radius $\ell$.
$x^i$ = spatial coords
$ds^2 = -dx_0^2 + \delta_{ij} dx^i dx^j$
$R = {}^{(3)}R_{ik}{}^{ik} = \left({}^{(3)}R_{ik}{}^{ik} + 2R_{ok}{}^{ok}\right) - 2R_{ok}{}^{ok}$
$= {}^{(3)}R + 2R_{00}$
$= 2(R_{00} + \frac{1}{2}R)$
$= 2(R_{00} - \frac{1}{2}R g_{00})$
$= 2G_{00}$
$\therefore \left[ \delta A|_V = -\frac{\Omega_{d-2}\,\ell^d}{(d^2-1)} G_{00} \left(1 + \frac{\ell}{L_{curv}}\right) \right]$


l
$L_{CURV}$
CAUTION


$\delta S_{UV} \sim -N \therefore \delta S_{UV} + \delta S_{IR} \lesssim 0$
$l$
$L_{CUV}$

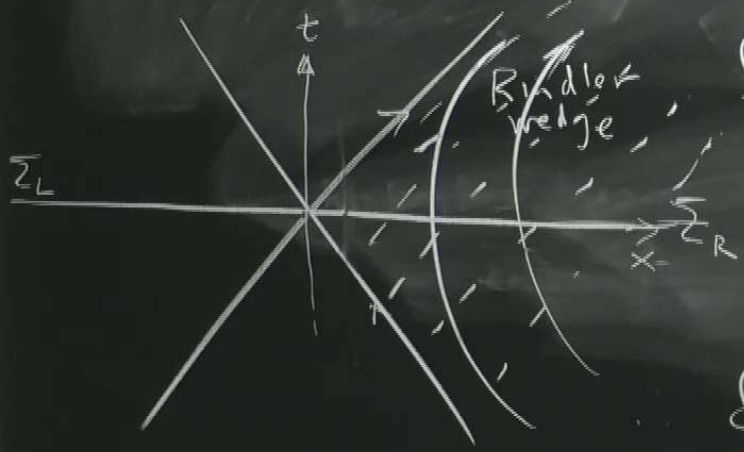

Unruh effect:
$|0\rangle$ poincaré - inv vacuum in Mink space.
Rindler wedge
$\rho_R \equiv Tr_L |0\rangle\langle 0| = \frac{e^{-\frac{2\pi}{\hbar} H_B}}{Z}$
$z_L$
$z_R$
$H_B$ boost hamiltonian


t
Rindler wedge
$\Sigma_L$
$\Sigma_R$
x
$\rho_R \equiv |r_L|0\rangle\langle 0| = \frac{e^{-\frac{1}{\hbar} T_B}}{Z}$
$H_B$ boost hamiltonian
boost killing vector
$x\partial_t + t\,\partial_x$
$\delta S = \frac{2\pi}{\hbar}\,\delta\langle H_B\rangle$
$= \frac{2\pi}{\hbar}\int \delta\langle T_{ab}\rangle \chi^a d\Sigma^b$
$\int_0^{\lambda_f} d\lambda\,(\lambda - \lambda_f) = \int_{-\lambda_f}^{0} d\lambda'\,\lambda'$
$\lambda'$


Unruh effect:
|0> poincaré - inv vacuum in Mink space.
t
Rindler wedge
x
$\Sigma_R$
$\rho_R \equiv Tr_L |0\rangle\langle 0| = \frac{e^{-\frac{2\pi}{\hbar} H_B}}{Z}$
$H_B$: boost hamiltonian
boost killing vector
$x\partial_t + t\partial_x$
$\delta S = \frac{2\pi}{\hbar} \delta \langle H_B \rangle$
$= \frac{2\pi}{\hbar} \int \delta \langle T_{ab} \rangle \chi^a d\Sigma^b$

Unruh effect:

$|0\rangle$ poincaré - inv vacuum in Mink space.




$$\rho_R \equiv \mathrm{Tr}_L\, |0\rangle\langle 0| = \frac{e^{-\frac{2\pi}{\hbar} H_B}}{Z}$$

(see 1212.6821 for path integral derivation.)

$H_B$: boost hamiltonian

boost killing vector: $x\partial_t + t\partial_x$

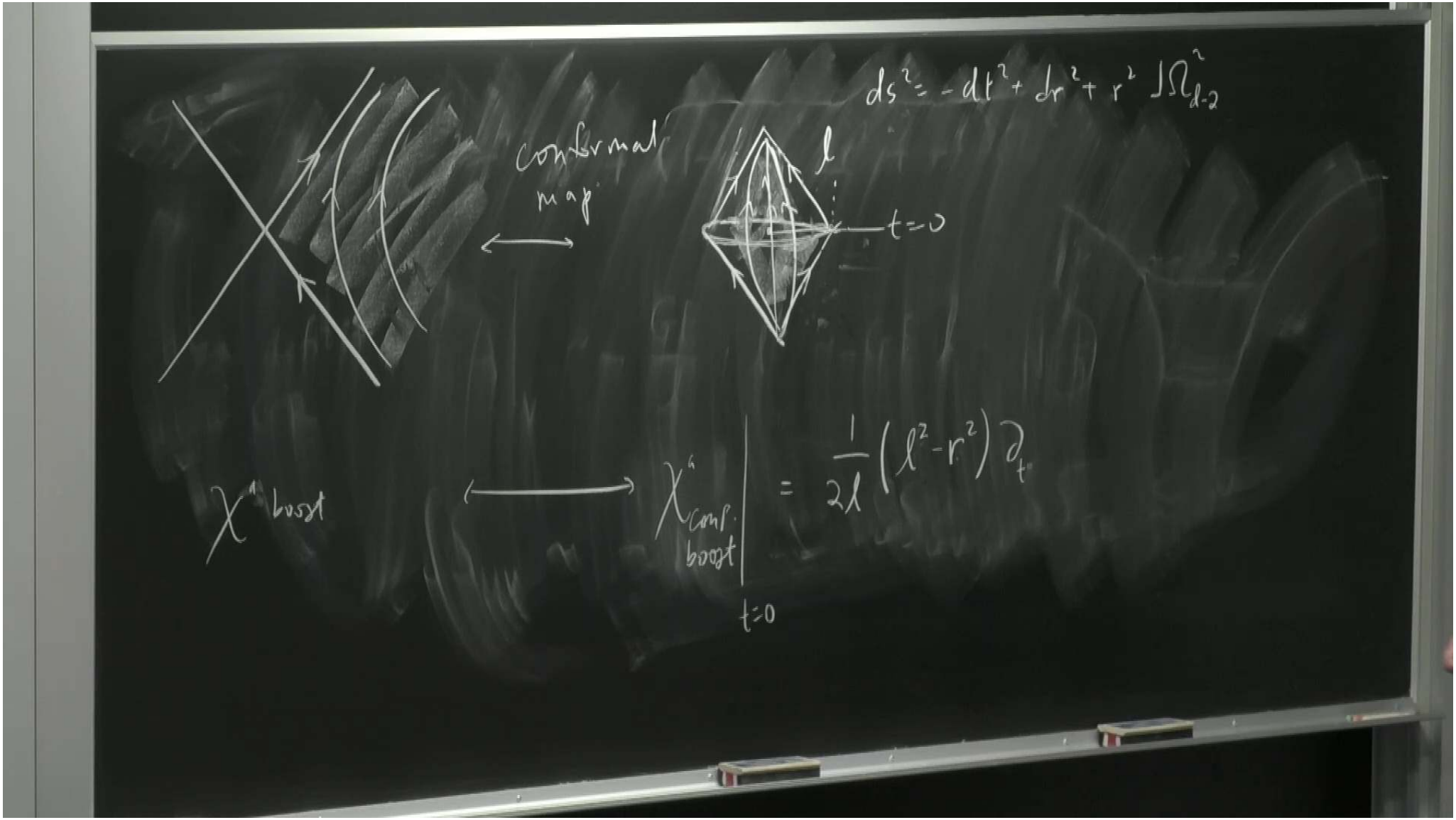
$$\delta S = \frac{2\pi}{\hbar}\,\delta\langle H_B\rangle = \frac{2\pi}{\hbar}\int \delta\langle T_{ab}\rangle \chi^a\, d\Sigma^b$$

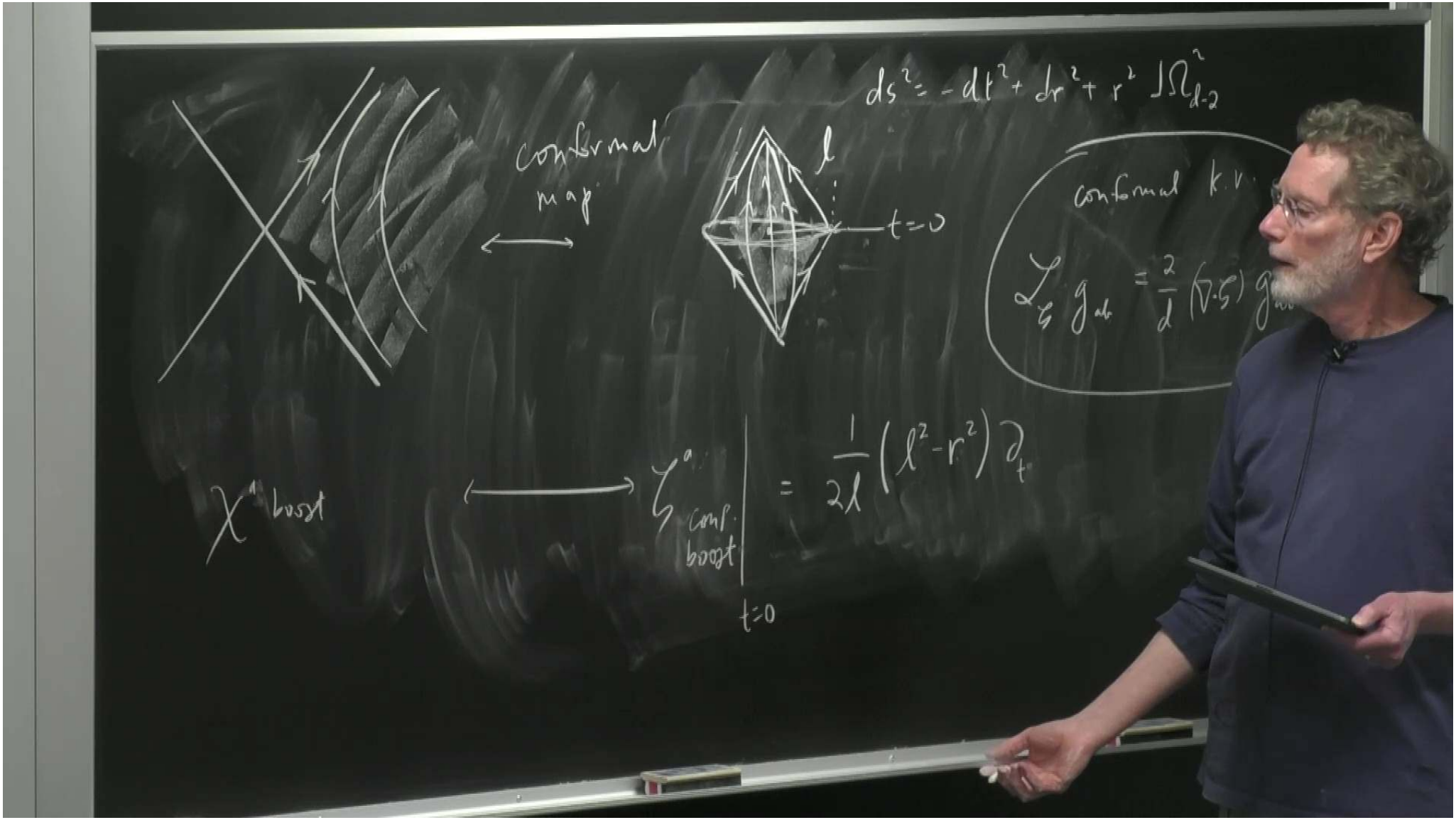
$$= \frac{2\pi}{\hbar} \int \delta \langle T_{ab} \rangle \chi^a \, d\Sigma^b$$

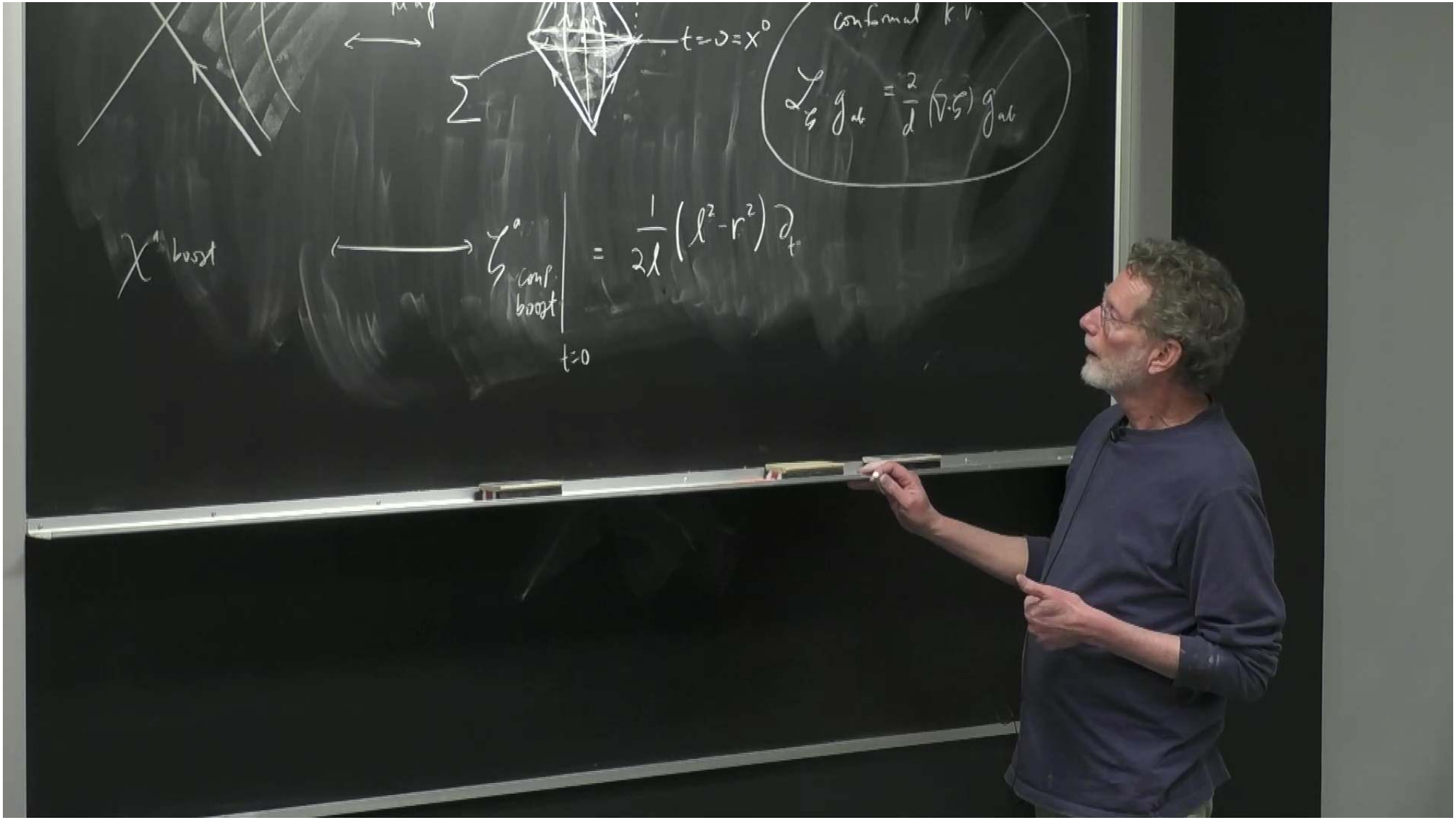
Great, but we need a formula relating $\delta S$ & $\delta \langle T_{ab} \rangle$

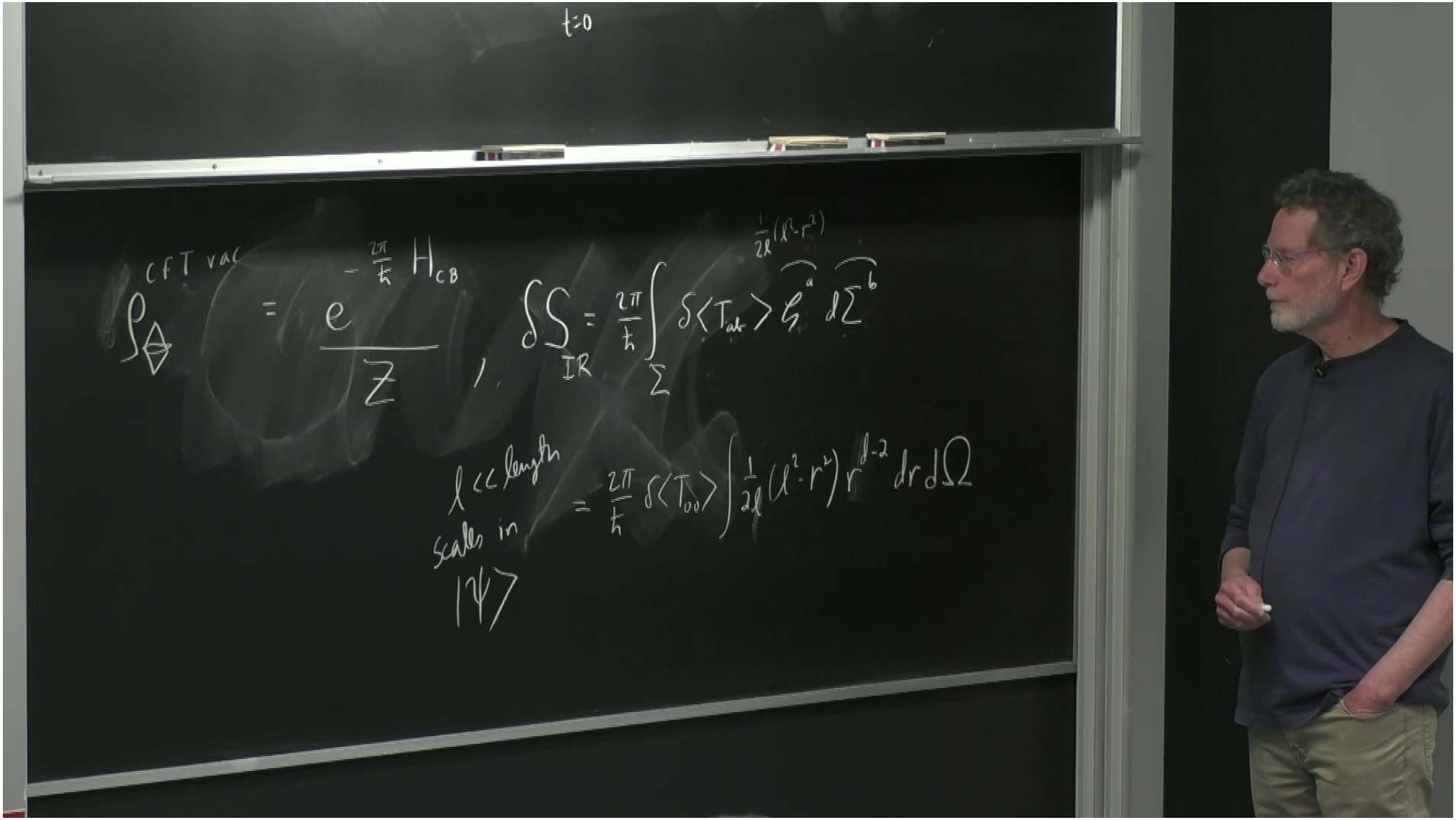
In a small ball

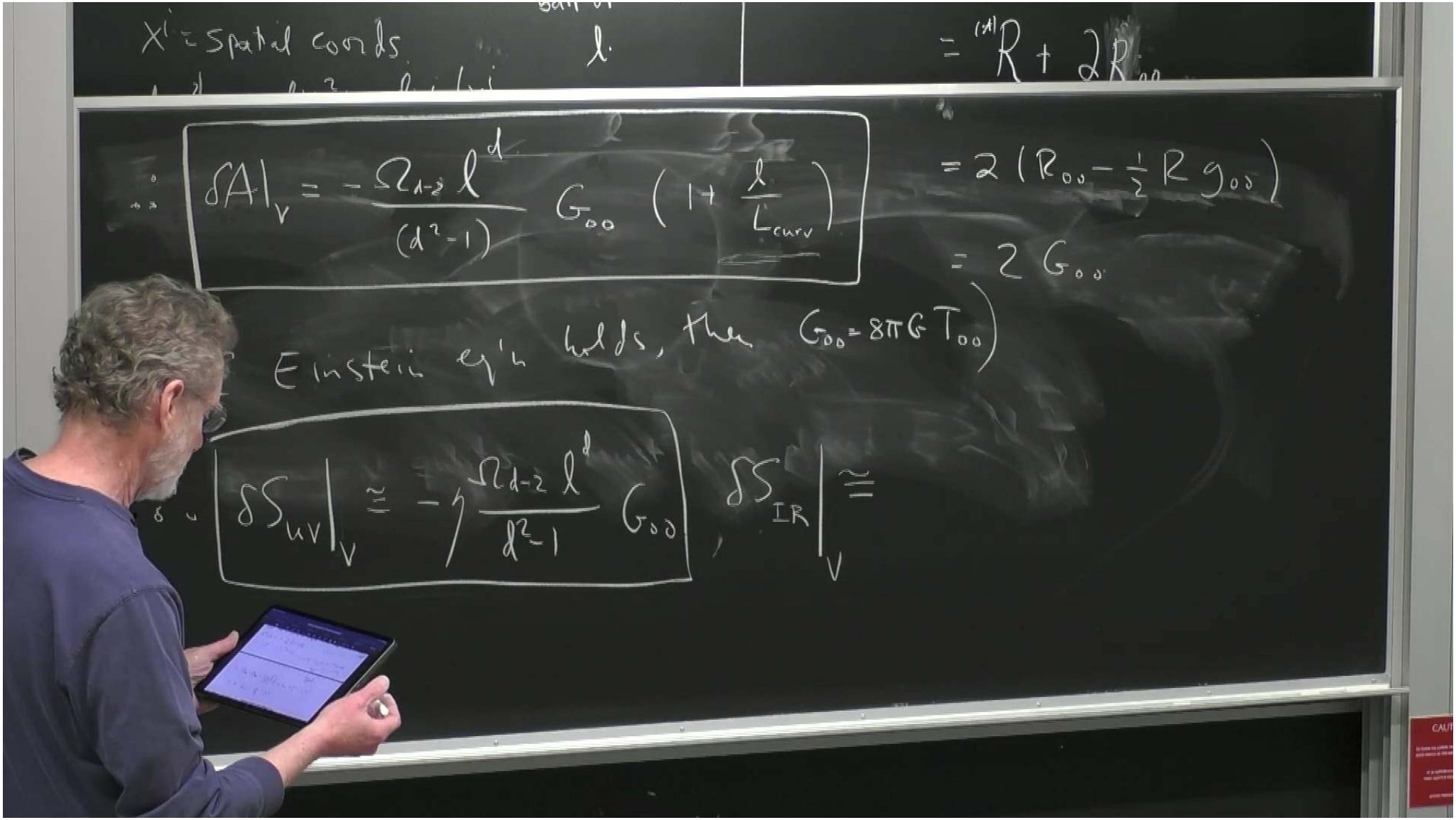
First cheat, consider only CFT (conformal field theory).

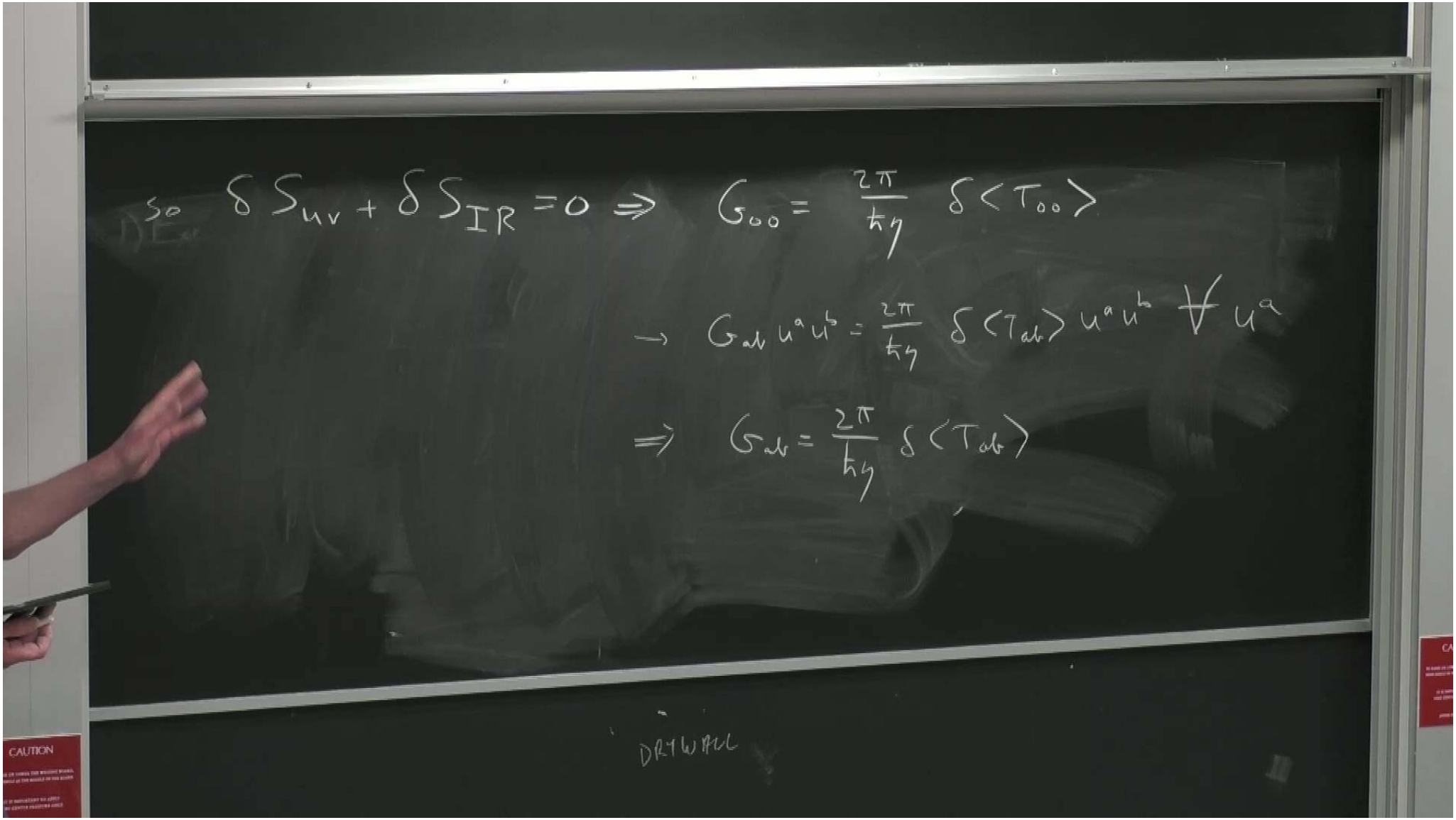

conformal map
$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2$
$t=0$
$\chi^a$ boost
$\chi^a_{\text{comp. boost}}\Big|_{t=0} = \frac{1}{2l}(l^2 - r^2)\partial_t$

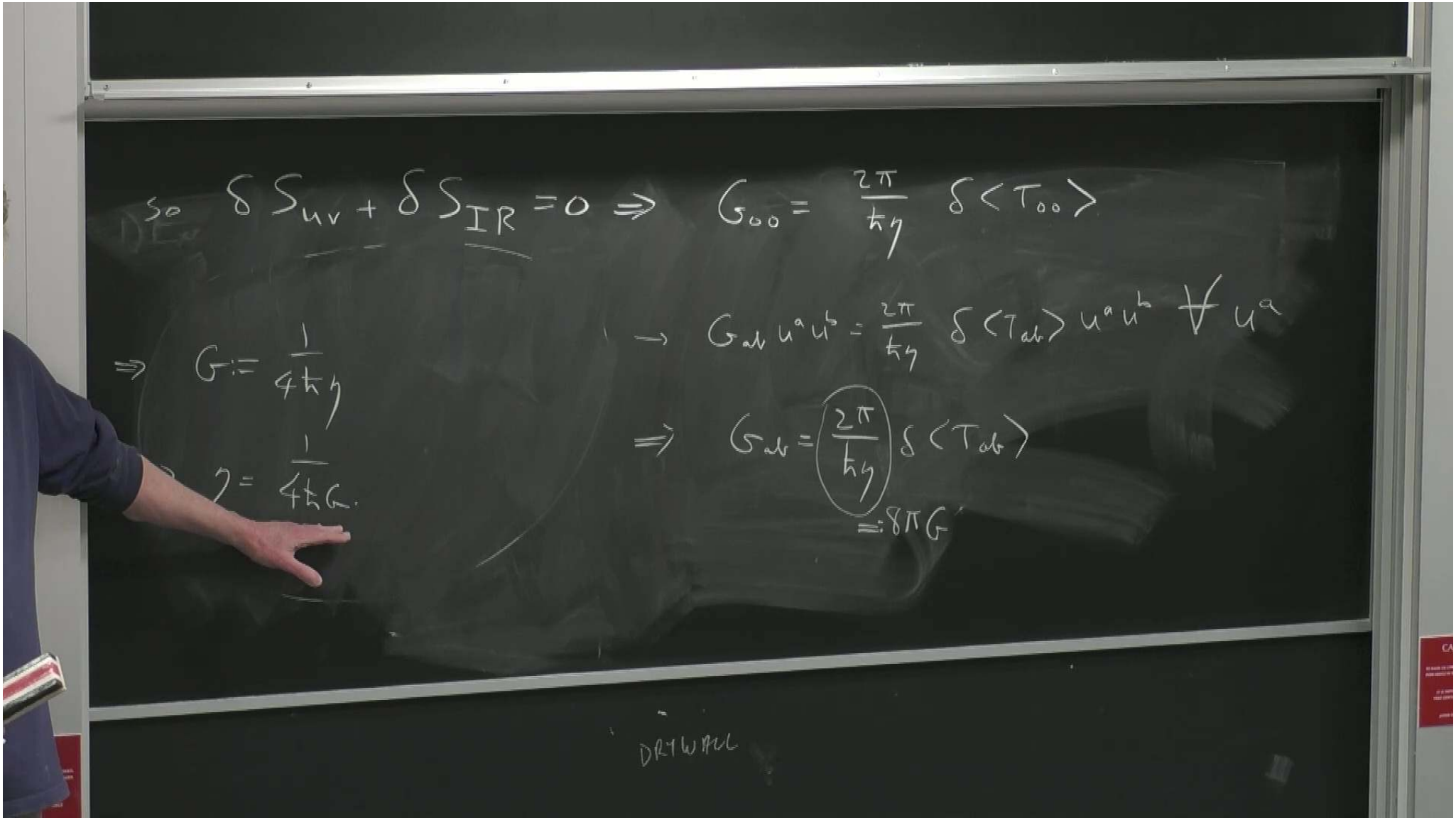

conformal map
$ds^2 = -dt^2 + dr^2 + r^2 d\Omega_{d-2}^2$
$t=0$
conformal k.v.
$\mathcal{L}_\xi g_{ab} = \frac{2}{d}(\nabla\cdot\xi) g_{ab}$
$X^a$ boost
$\xi^a$ conf. boost
$= \frac{1}{2l}(l^2 - r^2)\partial_t$
$t=0$

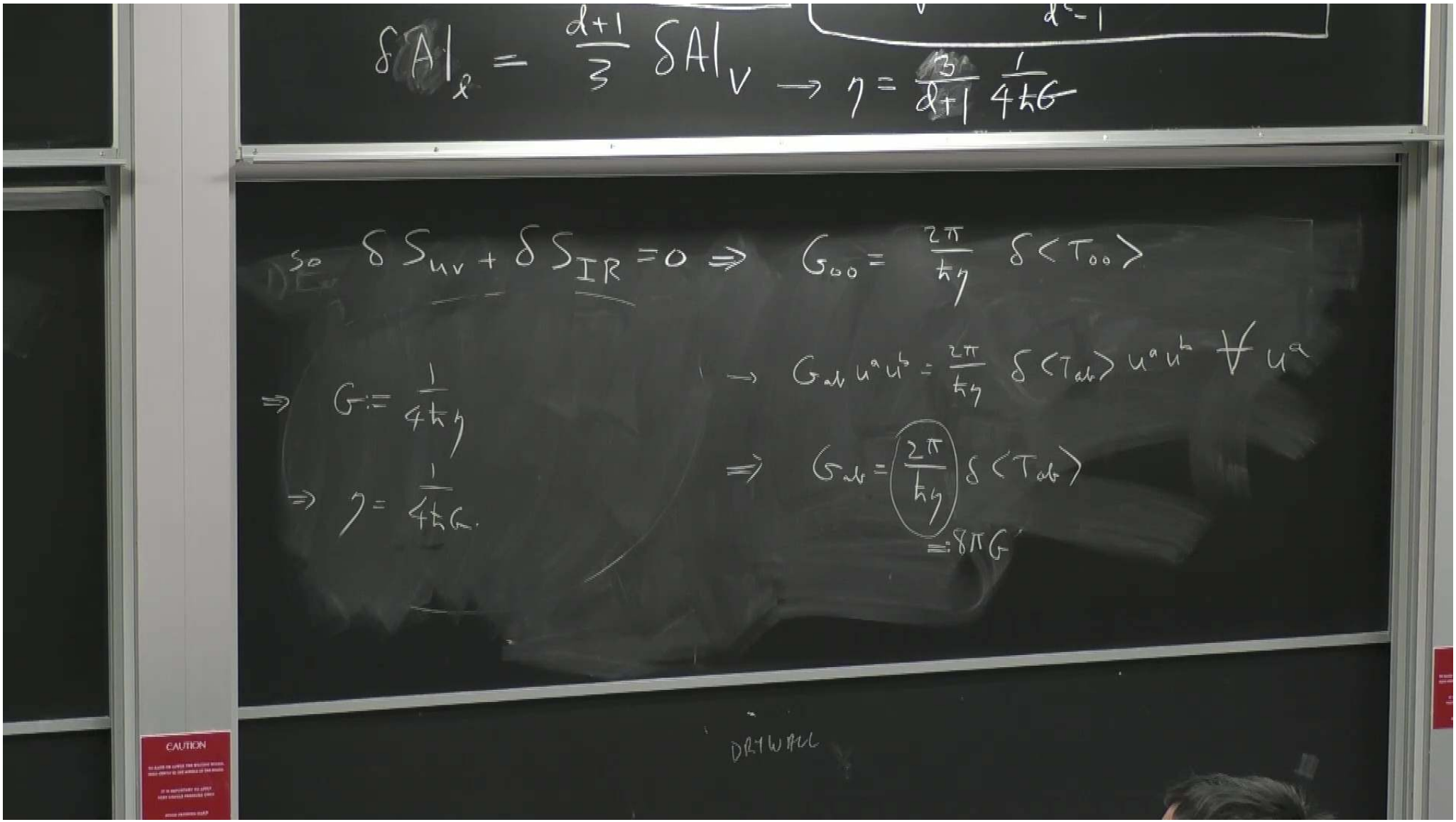

$t=0=x^0$
conformal
$\mathcal{L}_\xi g_{ab} = \frac{2}{d}(\nabla\cdot\xi) g_{ab}$
$X^a$ boost
$\xi^a_{\text{comp. boost}} = \frac{1}{2l}(l^2-r^2)\partial_t$
$t=0$


t=0
CFT vac
$\rho_{\diamond} = \frac{e^{-\frac{2\pi}{\hbar} H_{CB}}}{Z}$
$\delta S_{IR} = \frac{2\pi}{\hbar} \int_{\Sigma} \delta \langle T_{ab} \rangle \xi^a d\Sigma^b$
$\frac{1}{2R}(R^2 - r^2)$
$\ell \ll$ length scales in $|\Psi\rangle$
$= \frac{2\pi}{\hbar} \delta \langle T_{00} \rangle \int \frac{1}{2R}(R^2 - r^2) r^{d-2} dr d\Omega$


$X^i$ = spatial coords
$l$.
$= {}^{(d)}R + 2R_{00}$
$\delta A|_V = -\frac{\Omega_{d-2}\, l^d}{(d^2-1)}\, G_{00}\left(1 + \frac{l}{L_{curv}}\right)$
$= 2\left(R_{00} - \frac{1}{2} R\, g_{00}\right)$
$= 2\, G_{00}$
Einstein eq'n holds, then $G_{00} = 8\pi G\, T_{00}$
$\delta S_{UV}\big|_V \cong -\gamma \frac{\Omega_{d-2}\, l^d}{d^2-1}\, G_{00}$, $\delta S_{IR}\big|_V \cong$


So $\delta S_{UV} + \delta S_{IR} = 0 \Rightarrow G_{00} = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{00}\rangle$
$\rightarrow G_{ab}u^a u^b = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{ab}\rangle u^a u^b \;\forall\, u^a$
$\Rightarrow G_{ab} = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{ab}\rangle$
CAUTION


So $\delta S_{UV} + \delta S_{IR} = 0 \Rightarrow G_{00} = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{00}\rangle$
$\Rightarrow G_{ab}u^a u^b = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{ab}\rangle u^a u^b \;\forall\, u^a$
$\Rightarrow G := \frac{1}{4\hbar\eta}$
$\eta = \frac{1}{4\hbar G}$
$\Rightarrow G_{ab} = \frac{2\pi}{\hbar\eta}\,\delta\langle T_{ab}\rangle$
$= 8\pi G$
DRYWALL


$\delta A|_R = \frac{d+1}{3}\delta A|_V \rightarrow \eta = \frac{3}{d+1}\frac{1}{4\hbar G}$
$d=1$
So $\delta S_{UV} + \delta S_{IR} = 0 \Rightarrow G_{00} = \frac{2\pi}{\hbar\eta}\delta\langle T_{00}\rangle$
$\Rightarrow G := \frac{1}{4\hbar\eta}$
$\Rightarrow \eta = \frac{1}{4\hbar G}$
$\rightarrow G_{ab}u^a u^b = \frac{2\pi}{\hbar\eta}\delta\langle T_{ab}\rangle u^a u^b \quad \forall u^a$
$\Rightarrow G_{ab} = \frac{2\pi}{\hbar\eta}\delta\langle T_{ab}\rangle$
$= 8\pi G$
CAUTION
DRYWALL