Title: Horizon entropy and the Einstein equation - Lecture 20230223

Speakers: Ted Jacobson

Collection: Horizon entropy and the Einstein equation

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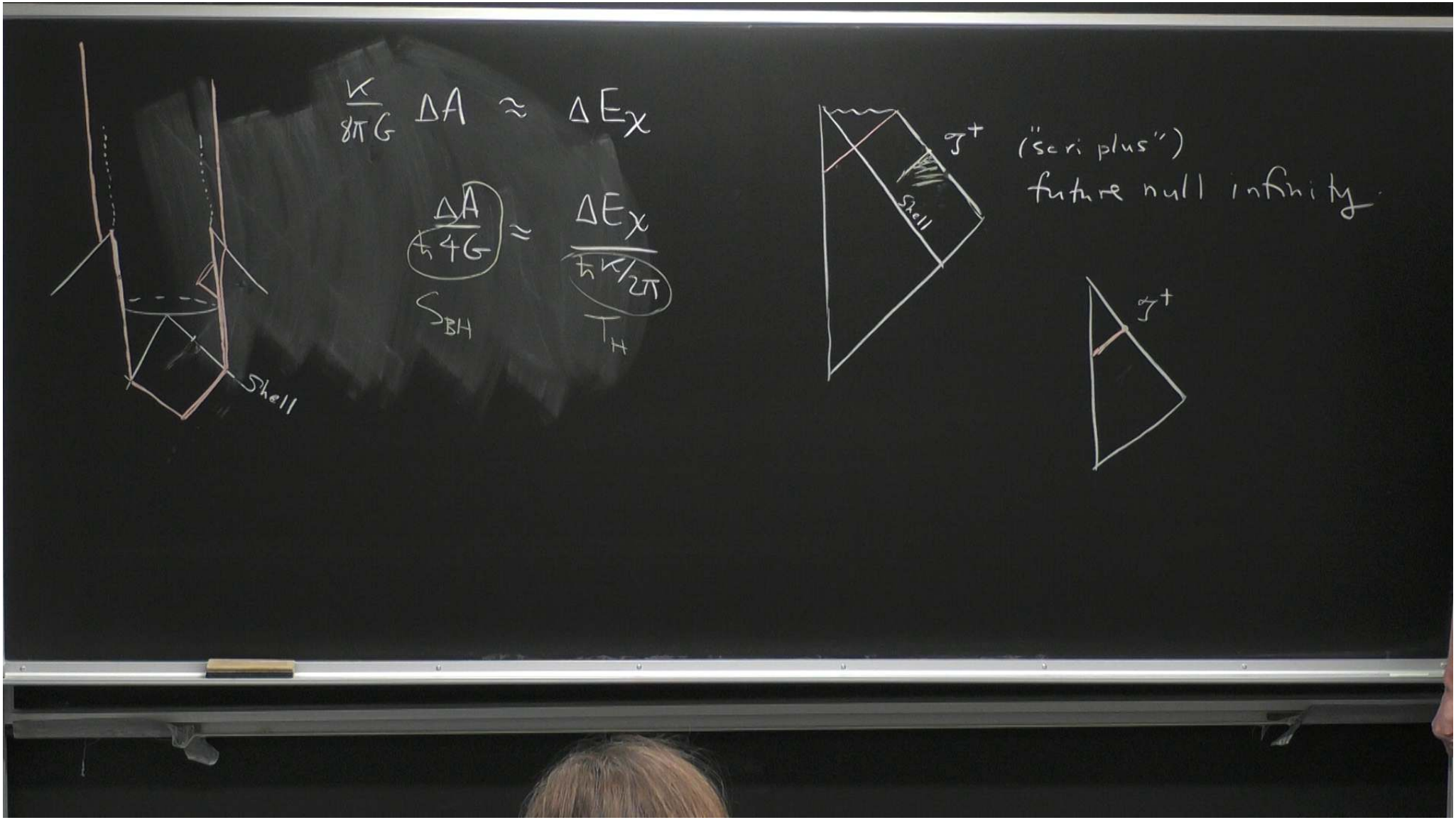
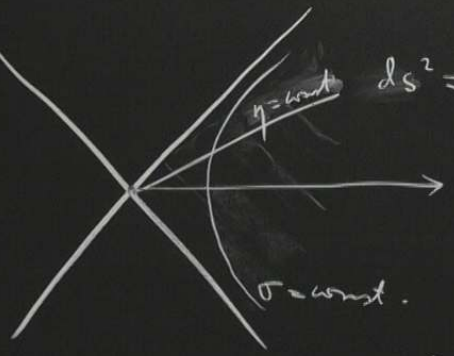
URL: https://pirsa.org/23020043

Abstract: Zoom link: https://pitp.zoom.us/j/96212372067?pwd=dWVaUFFFc3c5NTlVTDFHOGhCV2pXdz09


$\frac{\kappa}{8\pi G}\Delta A \approx \Delta E_X$
$\frac{\Delta A}{\hbar 4G} \approx \frac{\Delta E_X}{\hbar \kappa/2\pi}$
S_{BH}
T_H
Shell
$\mathscr{I}^+$
("scri plus")
future null infinity.
Shell
$\mathscr{I}^+$

Rindler horizon (in Minkowski space).



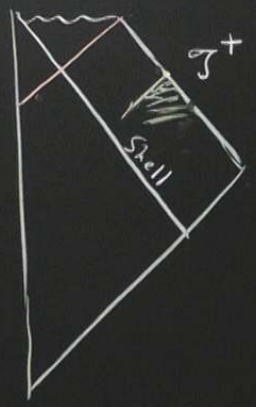
$ds^2 = \sigma^2 d\eta^2 - d\sigma^2$

see Horizon Entropy, (2003)
TJ & R. Parentani.

$\Delta A \approx \Delta E_\chi$

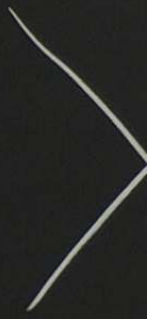
$\frac{\Delta A}{\hbar 4G} \approx \frac{\Delta E_\chi}{\hbar \kappa / 2\pi}$

S_{BH} T_H



("scri plus")
future null infinity.

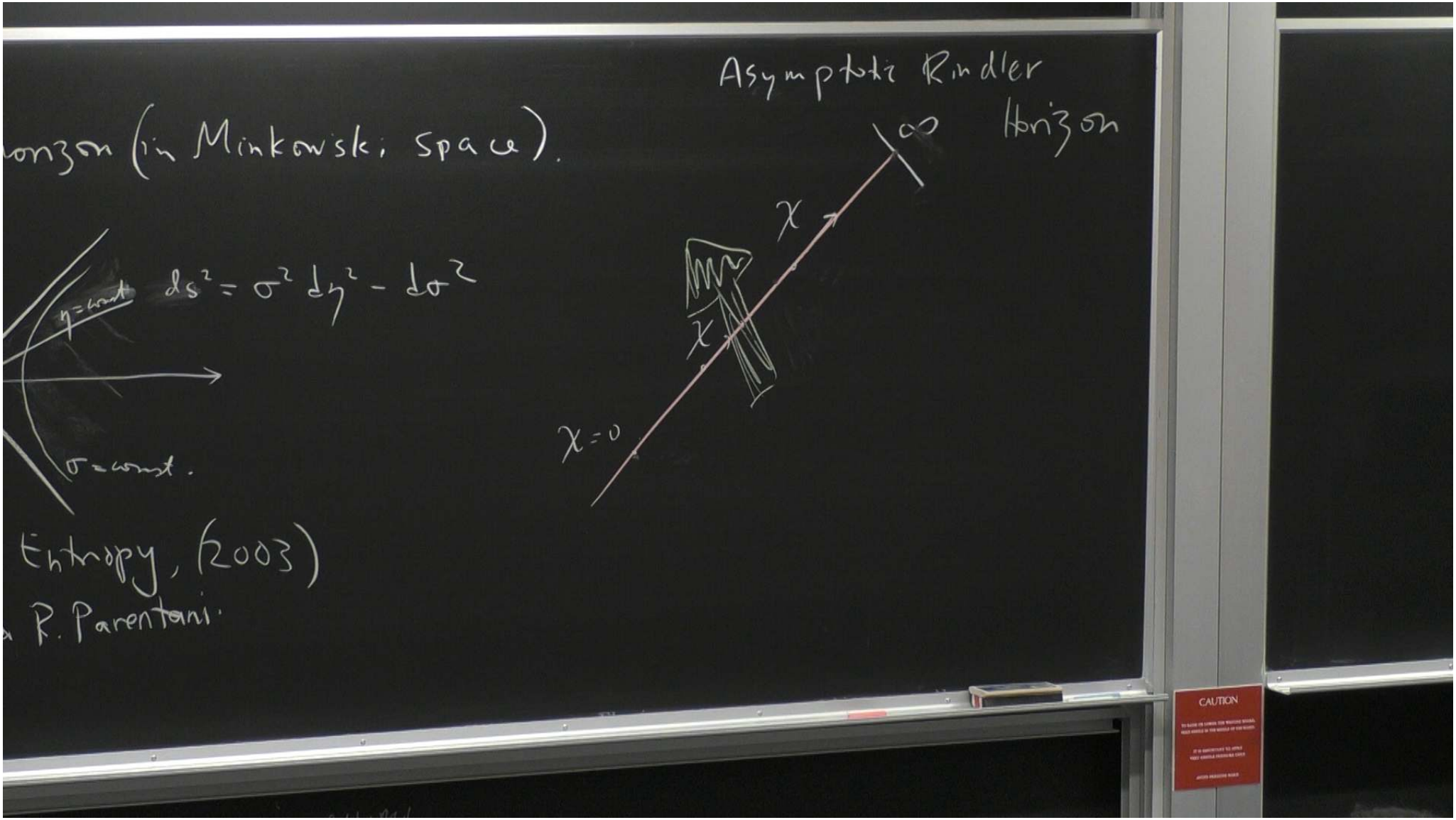
Rindler



See Horiz

TJ

CAUTION


Asymptotic Rindler
horizon (in Minkowski space).
$ds^2 = \sigma^2 d\eta^2 - d\sigma^2$
η = const
σ = const.
∞
x
x
$x = 0$
Entropy, (2003)
P. Parentani.
CAUTION

Asymptotic Rindler

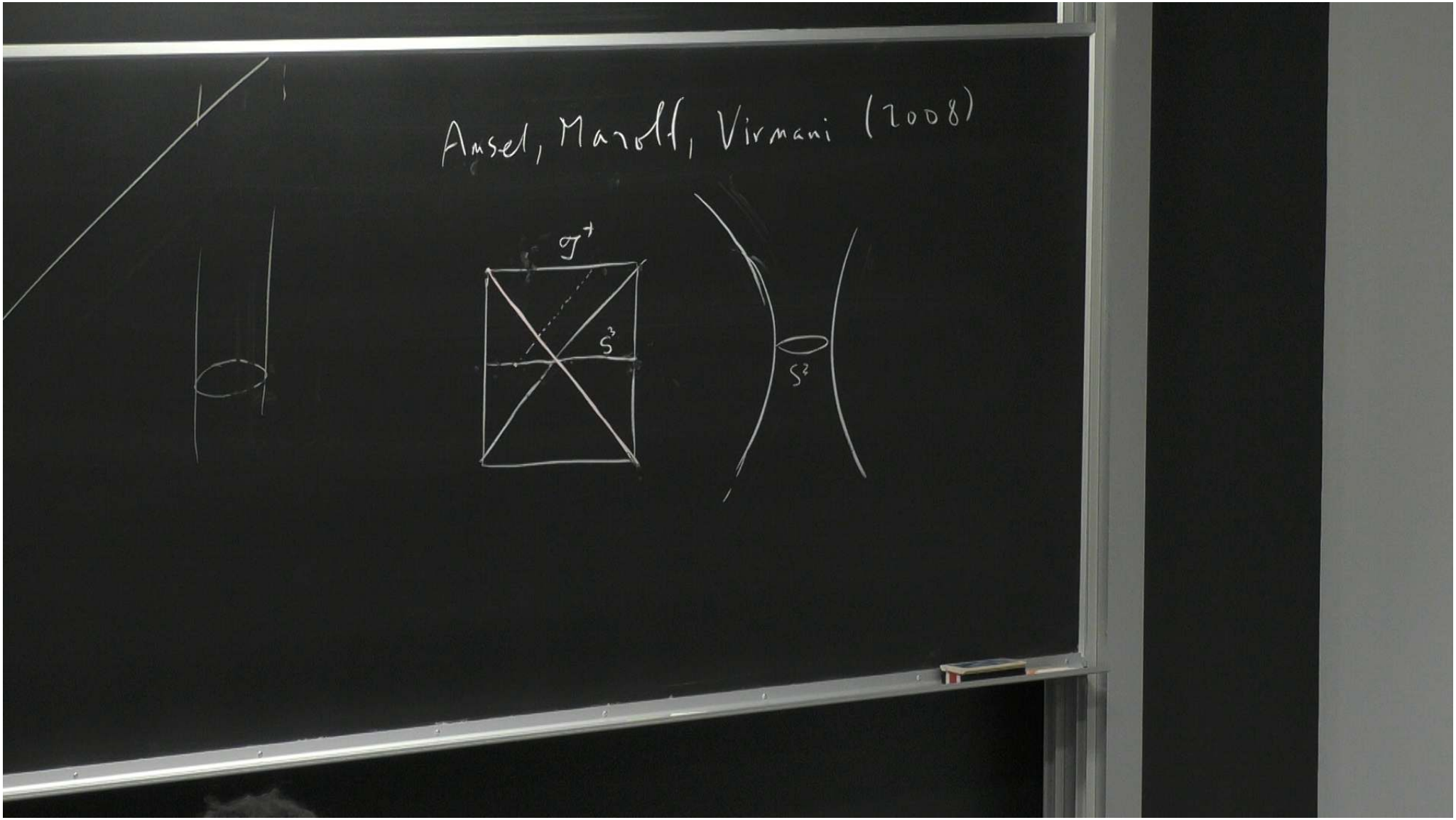
Space).

∞ Horizon

$x \to$

x

$x = 0$

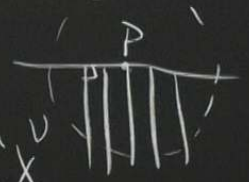

Ansel, Marolf, Virmani (2008)
$\mathcal{I}^+$
S^2
S^2

Localize this: local Rindler horizon (LRH)

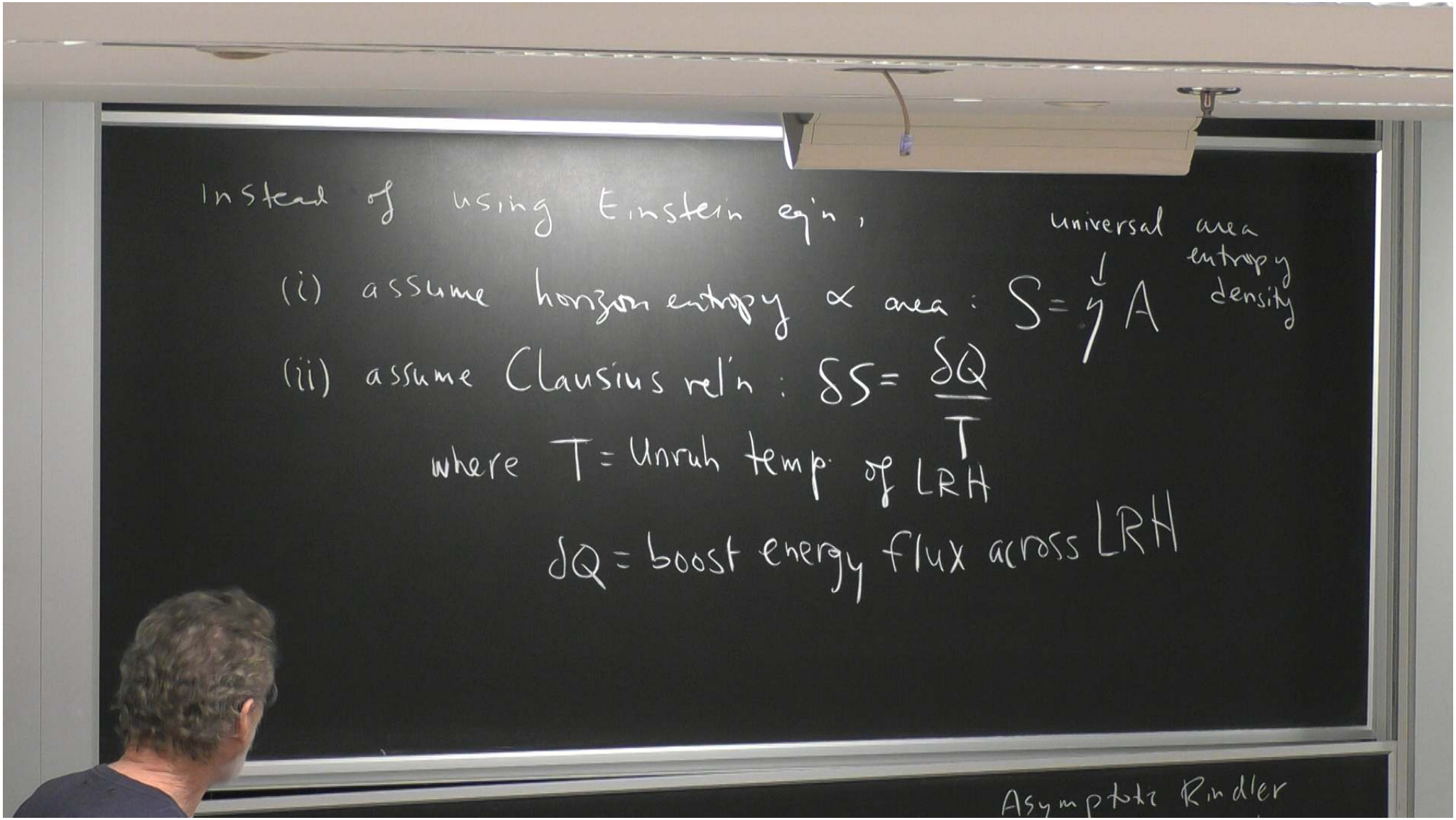
(at any point p in any spacetime)

Riemann normal coordinates:

(RNC) $g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^\mu x^\nu + O(x^3)$, $x^\mu|_p = 0$



$\theta = 0 = \sigma_{ab}$ at p


Instead of using Einstein eqn,
(i) assume horizon entropy ∝ area : $S = \eta A$
universal area entropy density
(ii) assume Clausius rel'n : $\delta S = \frac{\delta Q}{T}$
where T = Unruh temp. of LRH
δQ = boost energy flux across LRH
Asymptotic Rindler

(ii) assume Clausius rel'n: $\delta S = \frac{\delta Q}{T}$

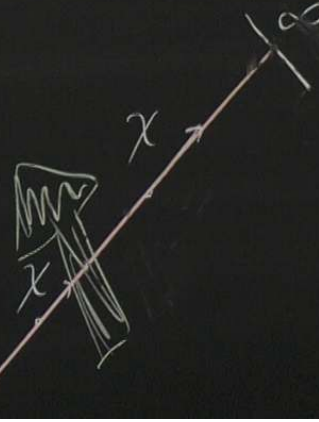
where T = Unruh temp. of LRH

δQ = boost energy flux across LRH

(iii) show this Implies Einstein Eq'n

Rindler horizon (in Minkowski space).

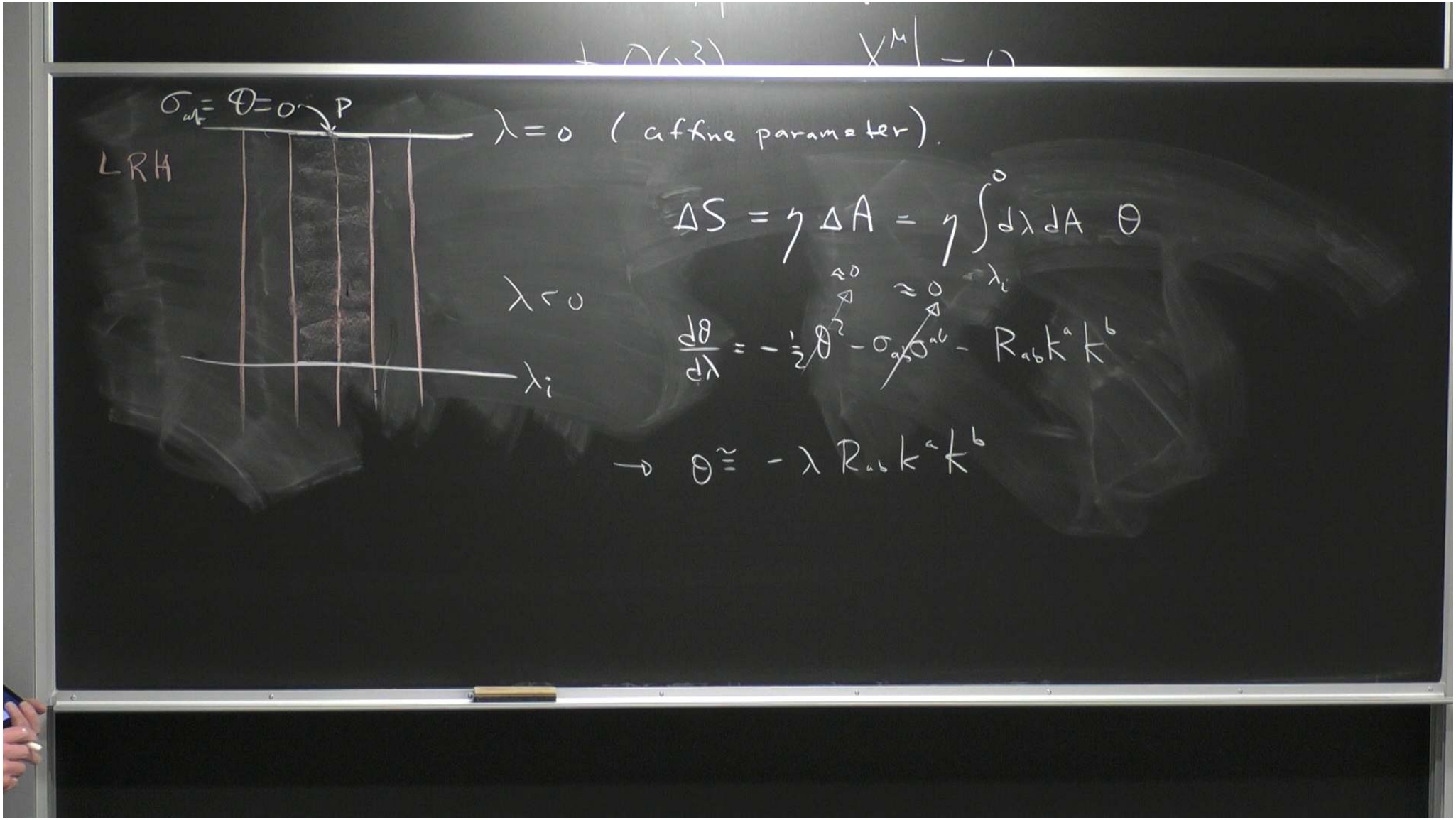
Asymptotic Rindler horizon

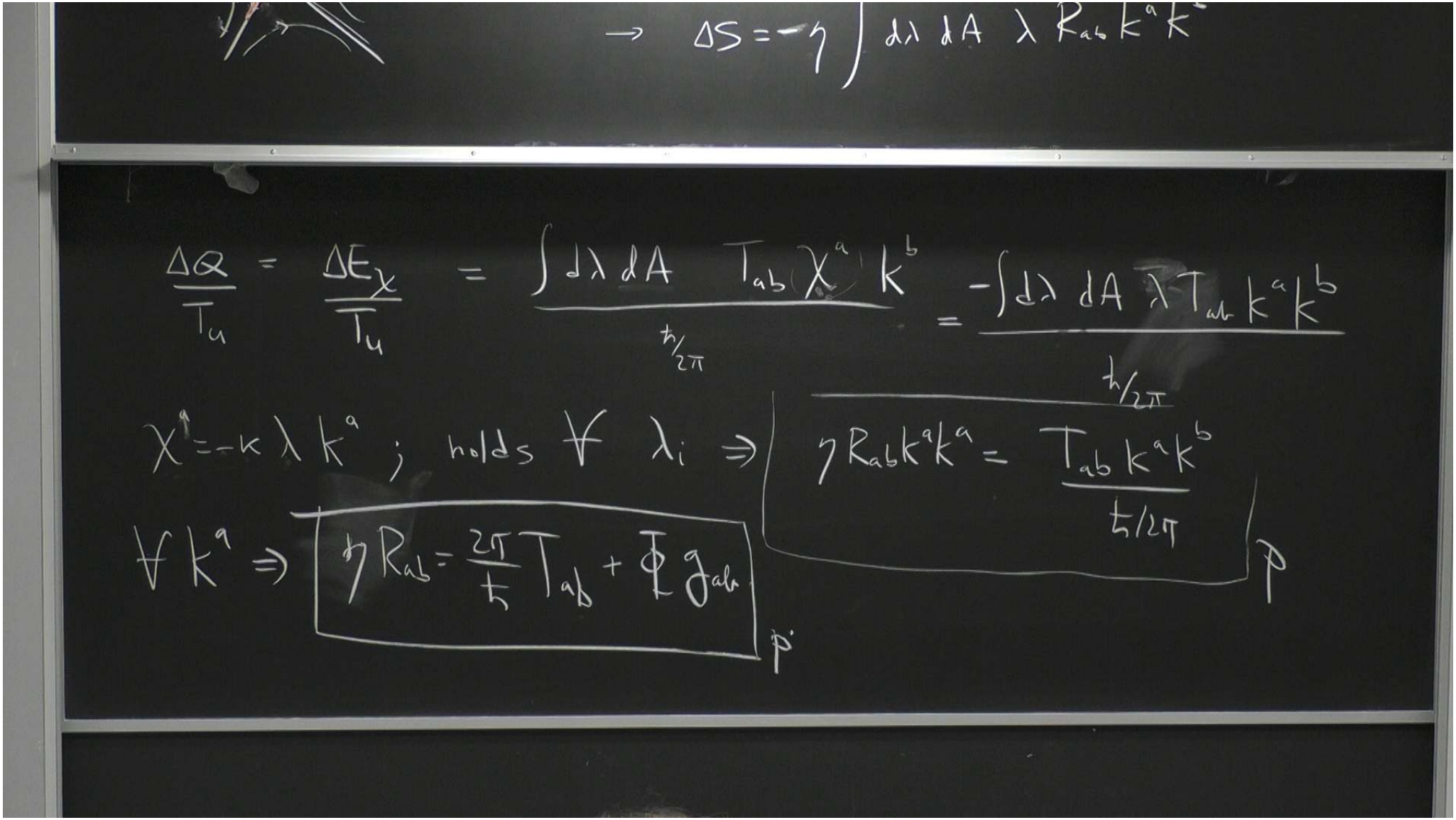


in vacuum of Poincaré inv. QFT, state restricted to any Rindler wedge is thermal w.r.t. the boost hamiltonian

H_B : boost generator

$$\mathrm{Tr}_L\,|0\rangle\langle 0| = \frac{e^{-H_B/T_u}}{\mathrm{Tr}\, e^{-H_B/T_u}}, \qquad T_u = \frac{\hbar\kappa}{2\pi}$$


$\sigma_{ab} = \theta = 0$
P
LRH
$\lambda = 0$ (affine parameter).
$\lambda < 0$
λ_i
$\Delta S = \eta \Delta A = \eta \int_{\lambda_i}^{0} d\lambda\, dA\, \theta$
$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$
$\theta \cong -\lambda R_{ab} k^a k^b$


$\rightarrow \Delta S = -\eta \int d\lambda\, dA\, \lambda R_{ab} k^a k^b$
$\frac{\Delta Q}{T_u} = \frac{\Delta E_\chi}{T_u} = \frac{\int d\lambda\, dA\, T_{ab} \chi^a k^b}{\hbar/2\pi} = \frac{-\int d\lambda\, dA\, \lambda T_{ab} k^a k^b}{\hbar/2\pi}$
$\chi^a = -\kappa \lambda k^a$; holds $\forall\, \lambda_i \Rightarrow$
$\eta R_{ab} k^a k^a = \frac{T_{ab} k^a k^b}{\hbar/2\pi}$
$\forall k^a \Rightarrow$
$\eta R_{ab} = \frac{2\pi}{\hbar} T_{ab} + \Phi g_{ab}$

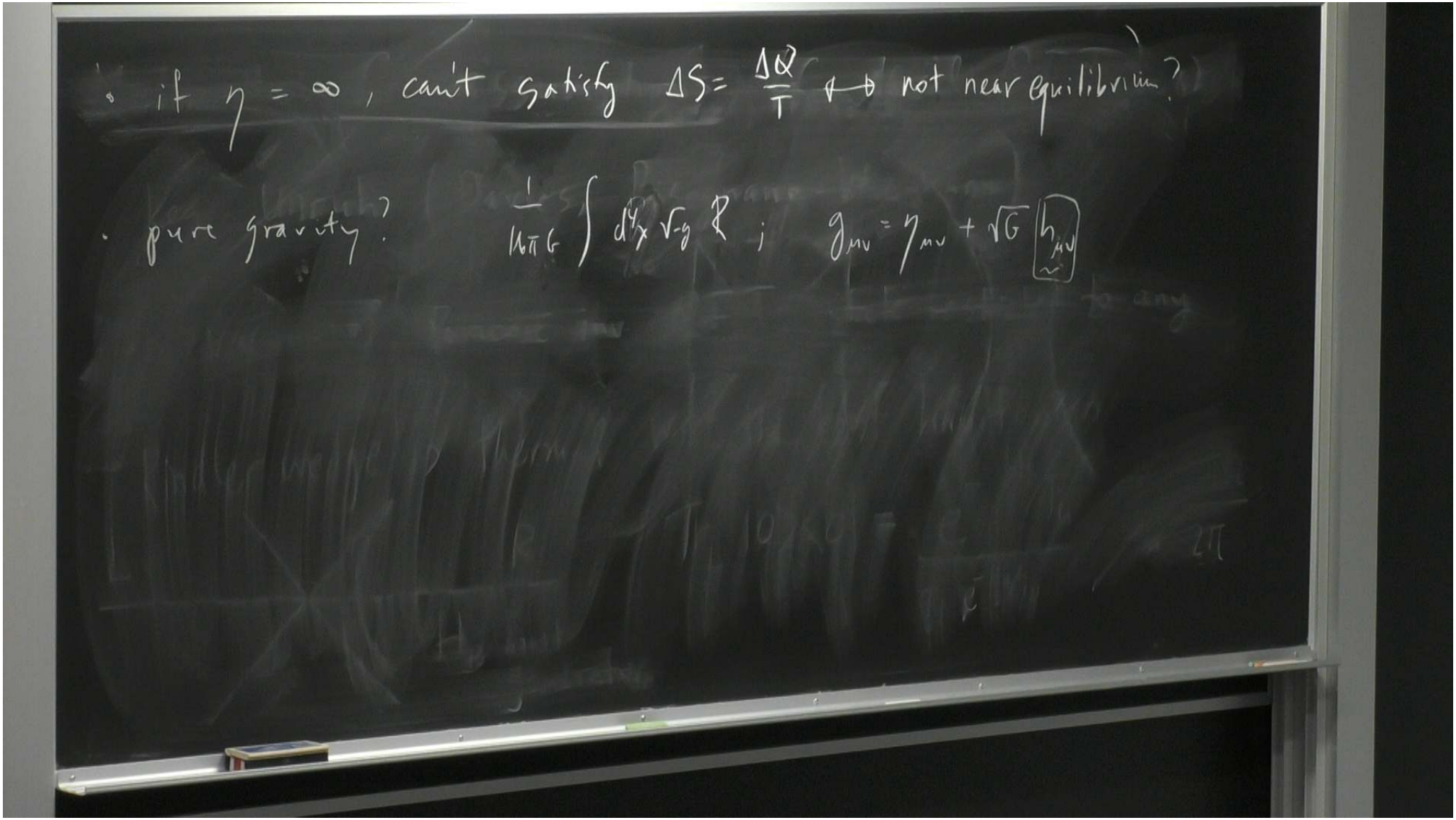
$$R_{ab} = \frac{2\pi}{t_{Pl}} T_{ab} + \frac{1}{\gamma} \Phi\, g_{ab}$$

$$\Leftrightarrow \quad G_{ab} = \frac{2\pi}{t_{Pl}} T_{ab} + \Phi'\, g_{ab} \quad \Rightarrow \quad \boxed{G_{ab} = \frac{2\pi}{t_{Pl}} T_{ab} + \Lambda\, g_{ab}}$$

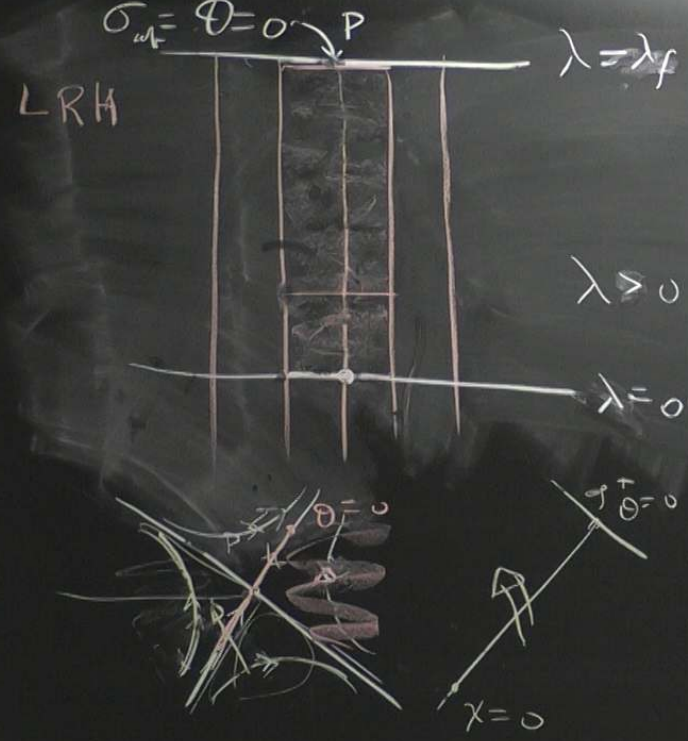
$\forall p, \quad \nabla^a G_{ab} \equiv 0$ (Bianchi id.)

$\Rightarrow \quad \nabla^a T_{ab} \propto \nabla_b \Phi' \Rightarrow \Phi' = \Lambda = \text{const}$ (some const Λ)

$\nabla^a T_{ab} = 0$


if $\eta = \infty$, can't satisfy $\Delta S = \frac{\Delta Q}{T}$ ↛ not near equilibrium?
pure gravity? $\frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R$; $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$

$+ O(x^3)$, $X^\mu|_p = 0$



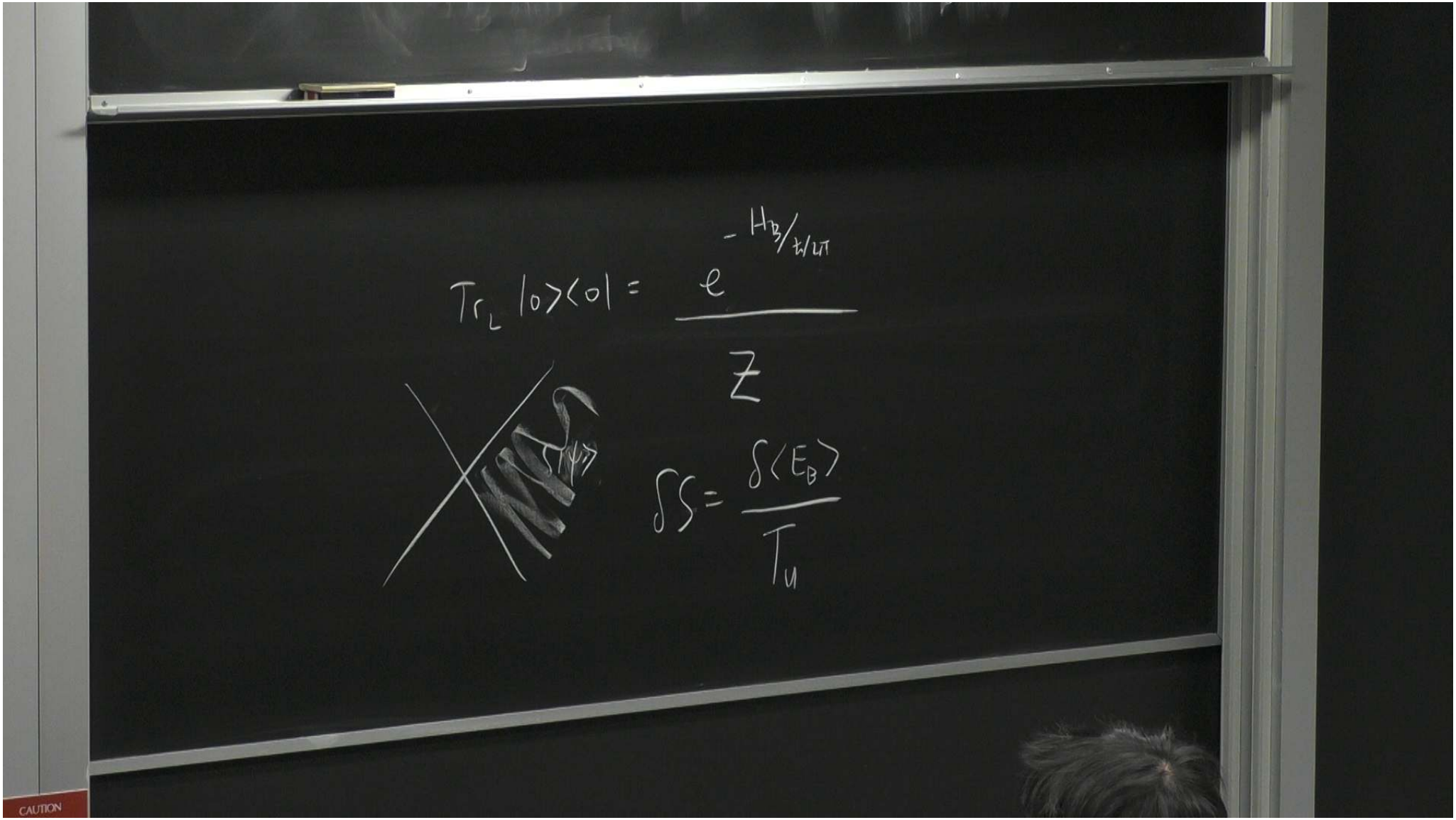
$$\Delta S = \eta \Delta A = \eta \int_0^{\lambda_f} d\lambda\, dA\, \theta$$

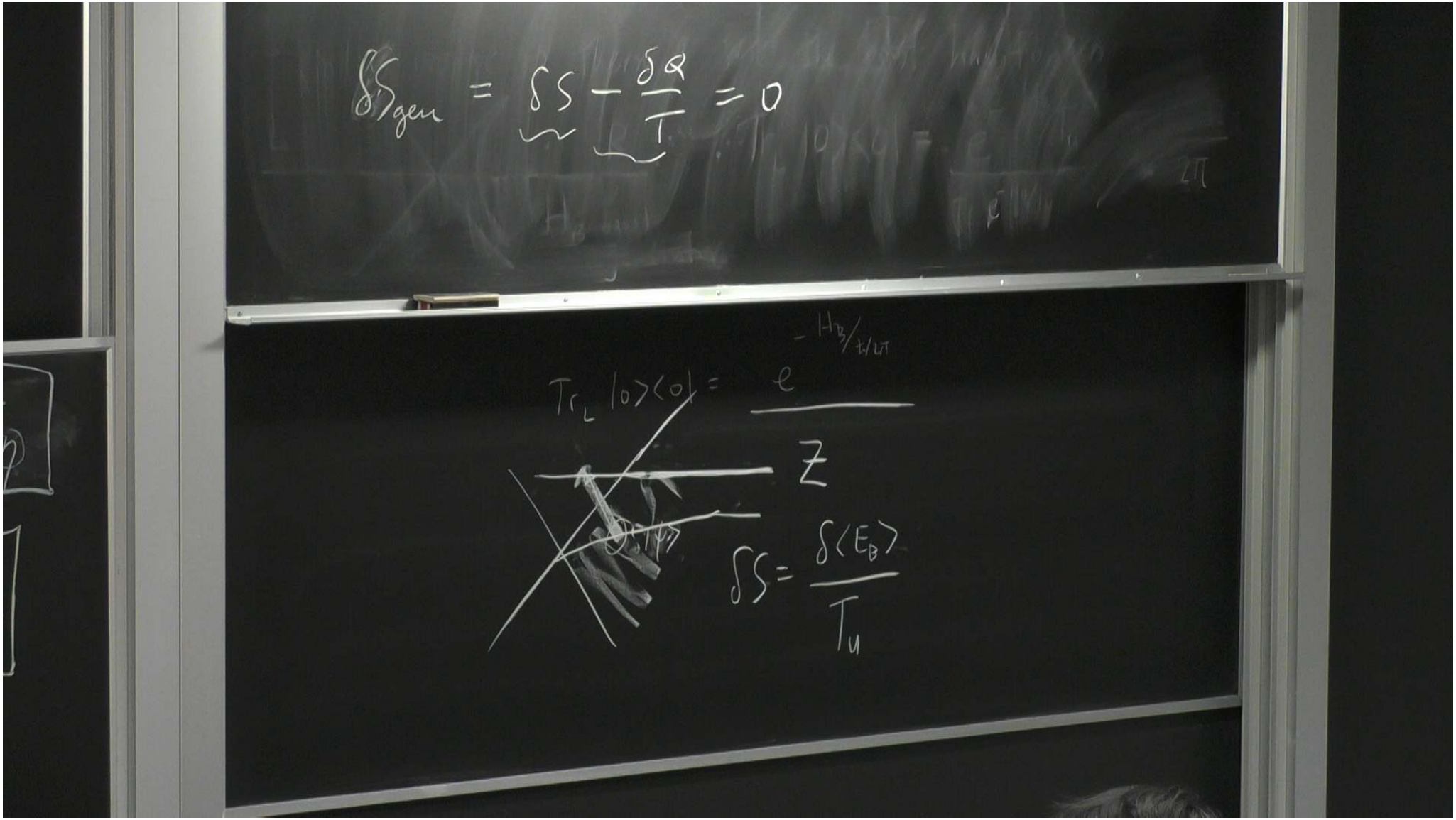
$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$$

$\theta \approx 0$ $\sigma \approx 0$ λ_i

$$\rightarrow \theta \cong -\lambda R_{ab} k^a k^b$$

$$\rightarrow \Delta S = -\eta \int d\lambda\, dA\, \lambda R_{ab} k^a k^b$$


$Tr_L |0\rangle\langle 0| = \frac{e^{-H_B/k_B T_U}}{Z}$
$\delta S = \frac{\delta \langle E_B \rangle}{T_U}$
CAUTION


$\delta S_{gen} = \delta S - \frac{\delta Q}{T} = 0$
$Tr_L |0\rangle\langle 0| = \frac{e^{-H_B/T_H/T}}{Z}$
$\delta S = \frac{\delta \langle E_B \rangle}{T_H}$