Title: Horizon entropy and the Einstein equation - Lecture 20230223

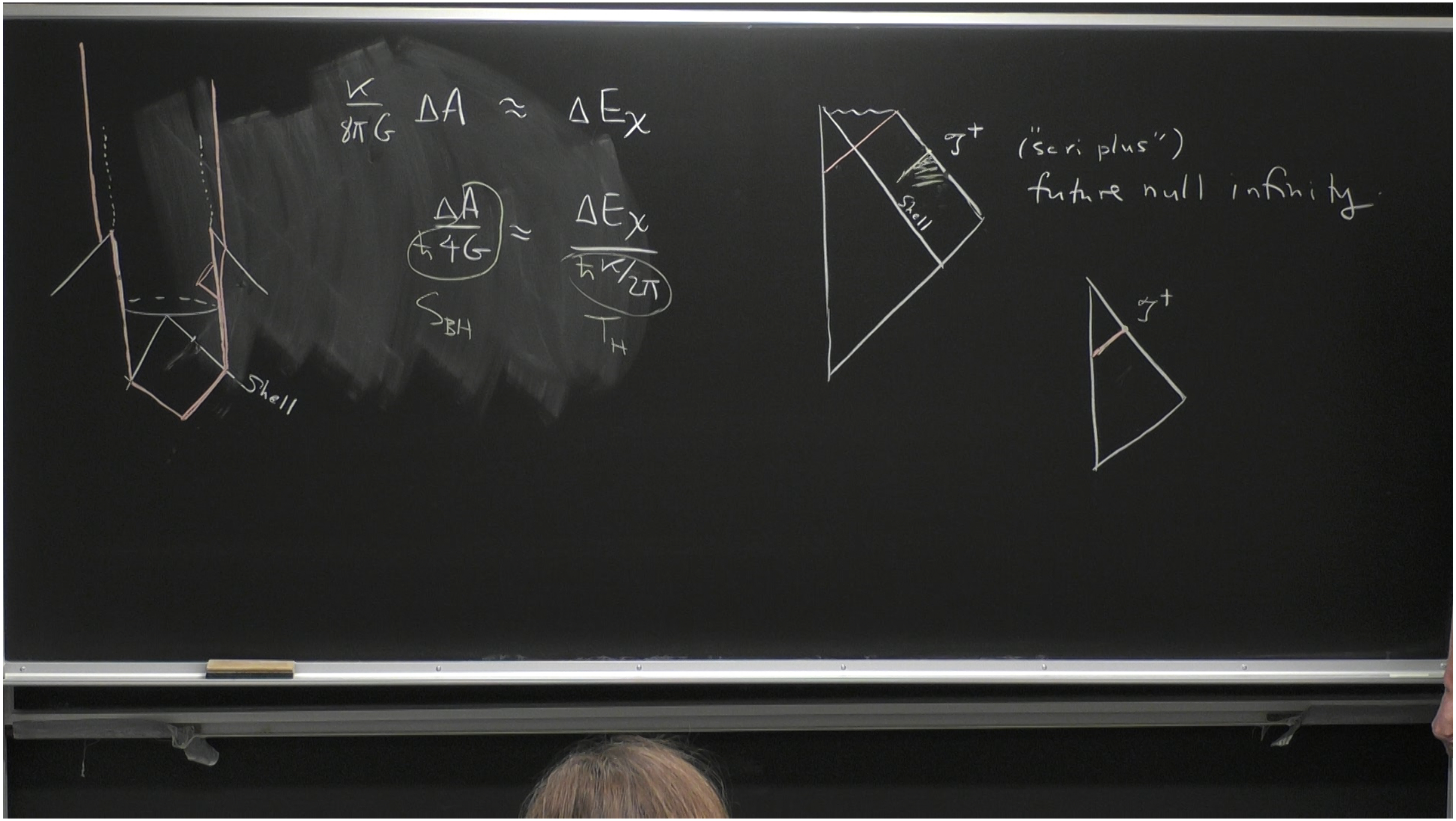
Speakers: Ted Jacobson

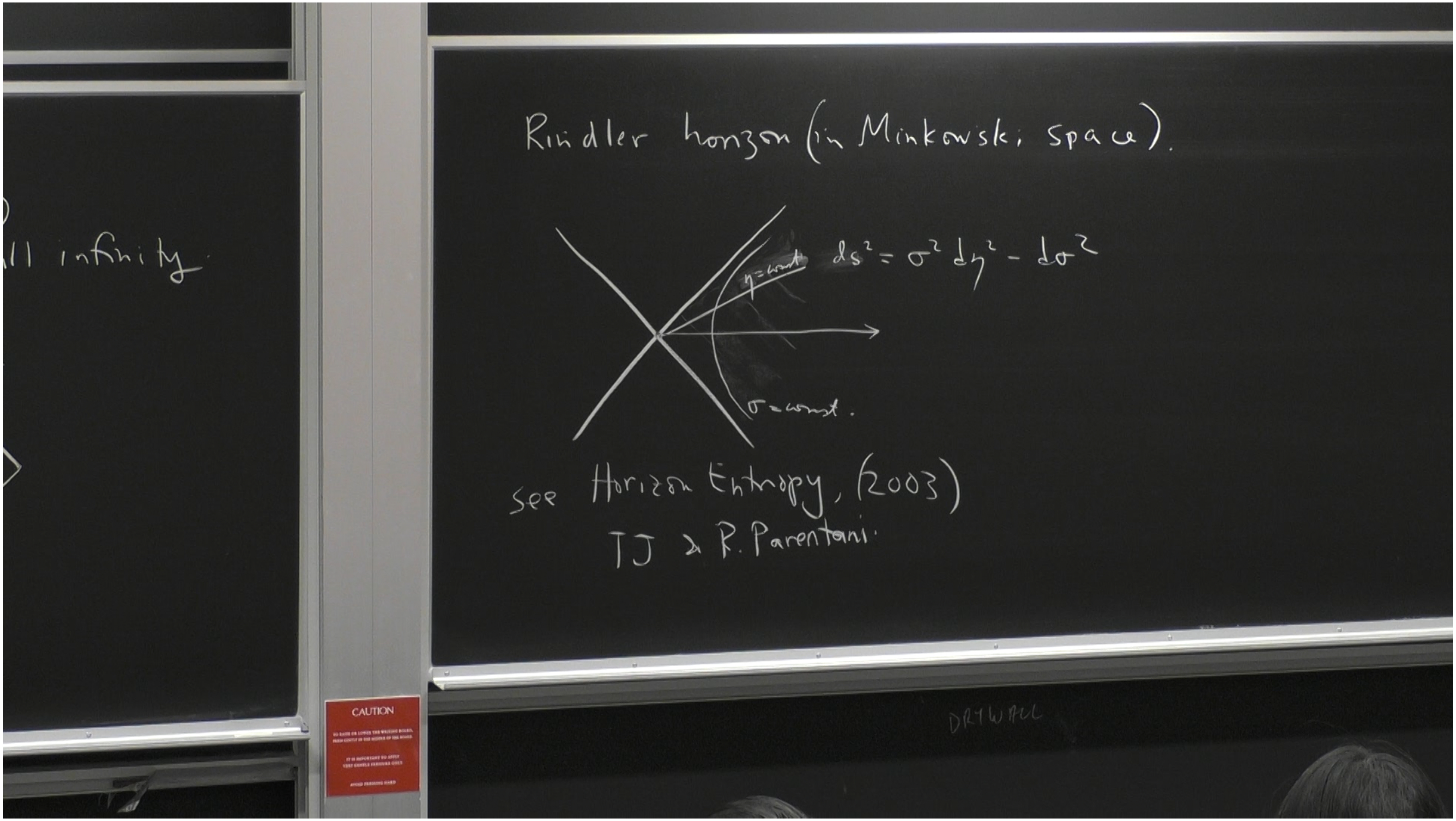
Collection: Horizon entropy and the Einstein equation

Date: February 23, 2023 - 10:00 AM

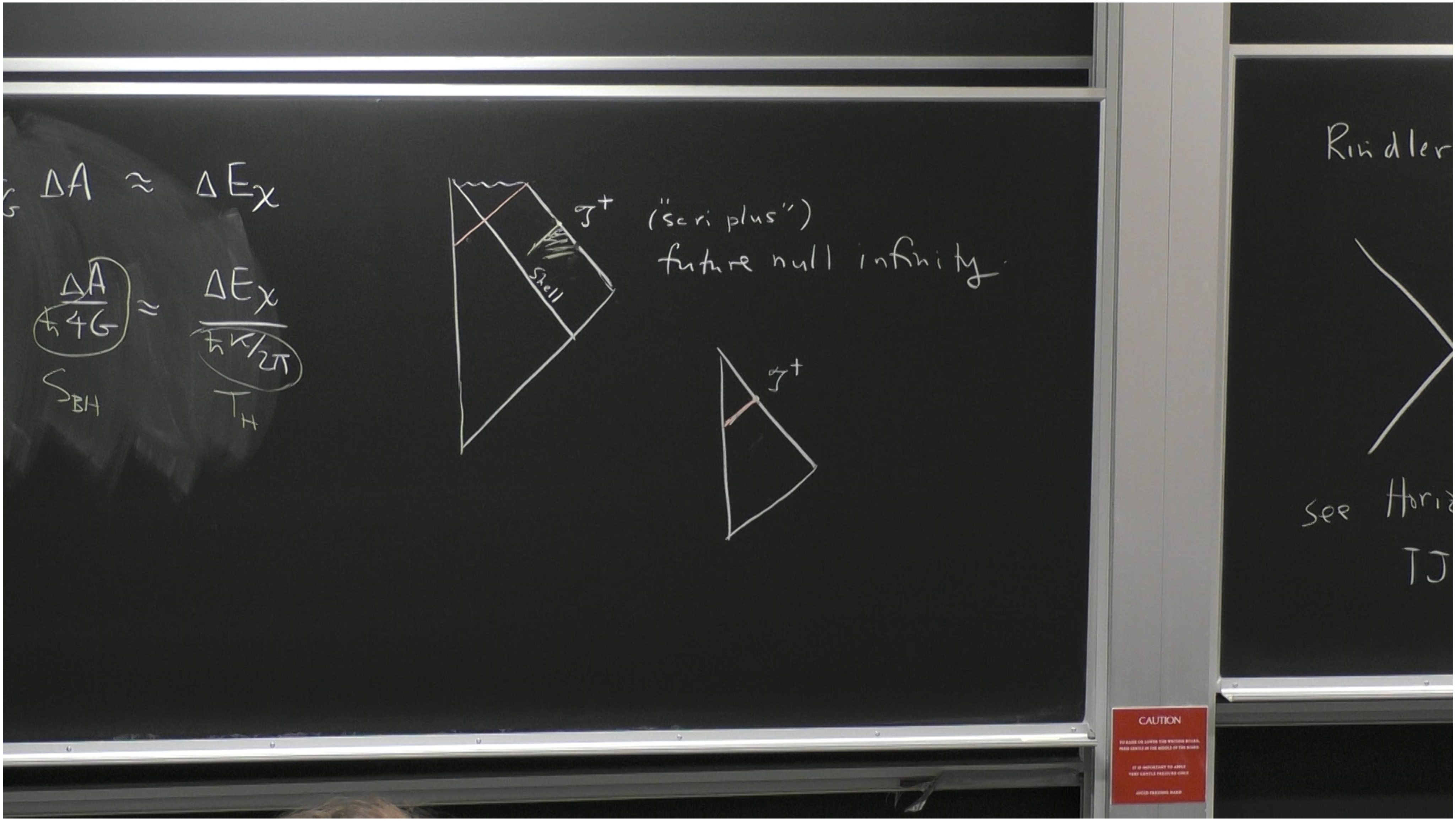
URL: https://pirsa.org/23020043

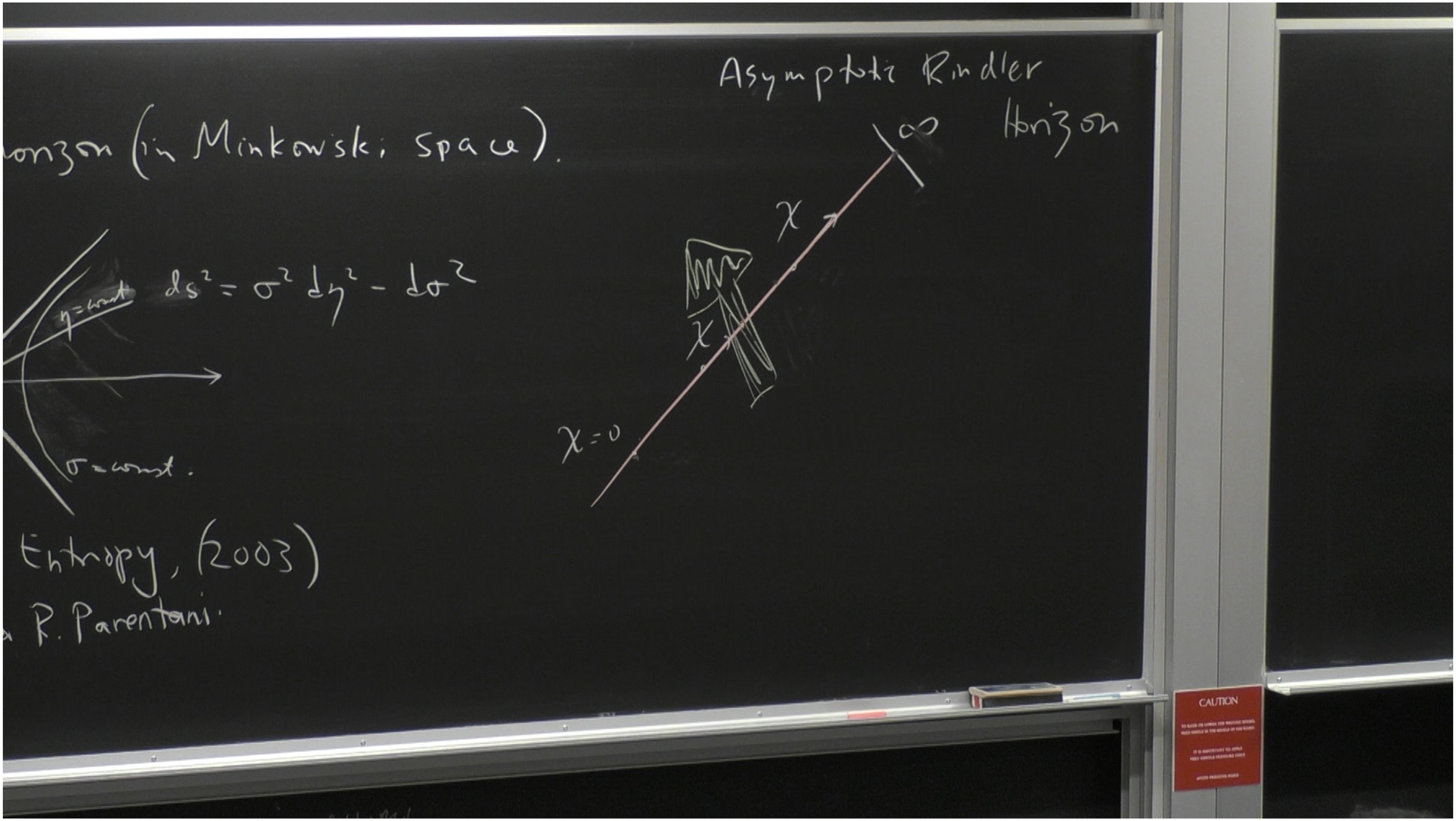
Abstract: Zoom link: https://pitp.zoom.us/j/96212372067?pwd=dWVaUFFFc3c5NTlVTDFHOGhCV2pXdz09

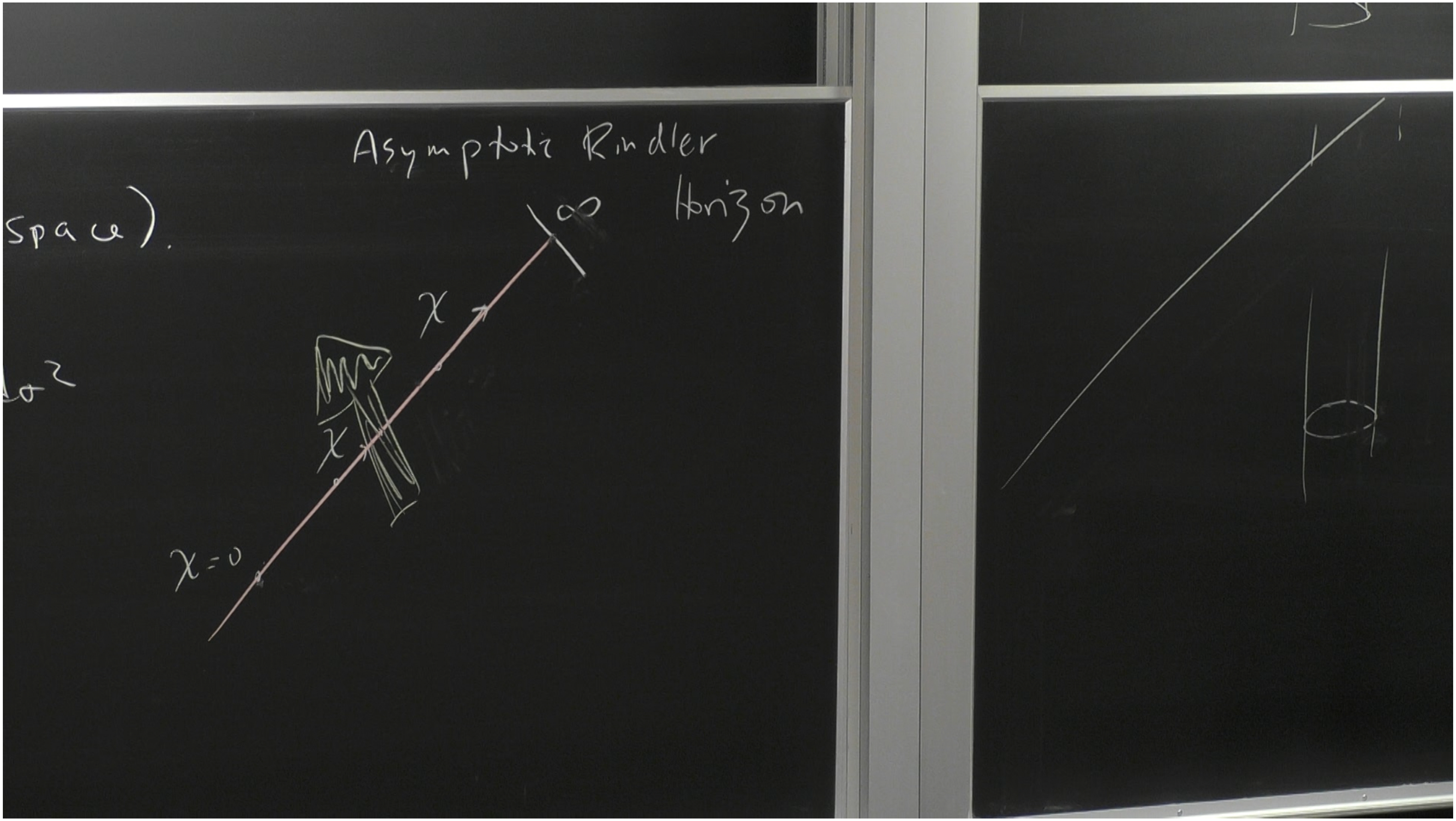

$\frac{\kappa}{8\pi G} \Delta A \approx \Delta E_\chi$
$\frac{\Delta A}{\hbar 4G} \approx \frac{\Delta E_\chi}{\hbar \kappa / 2\pi}$
S_{BH}
T_H
Shell
Shell
$\mathscr{I}^+$
("scri plus")
future null infinity
$\mathscr{I}^+$

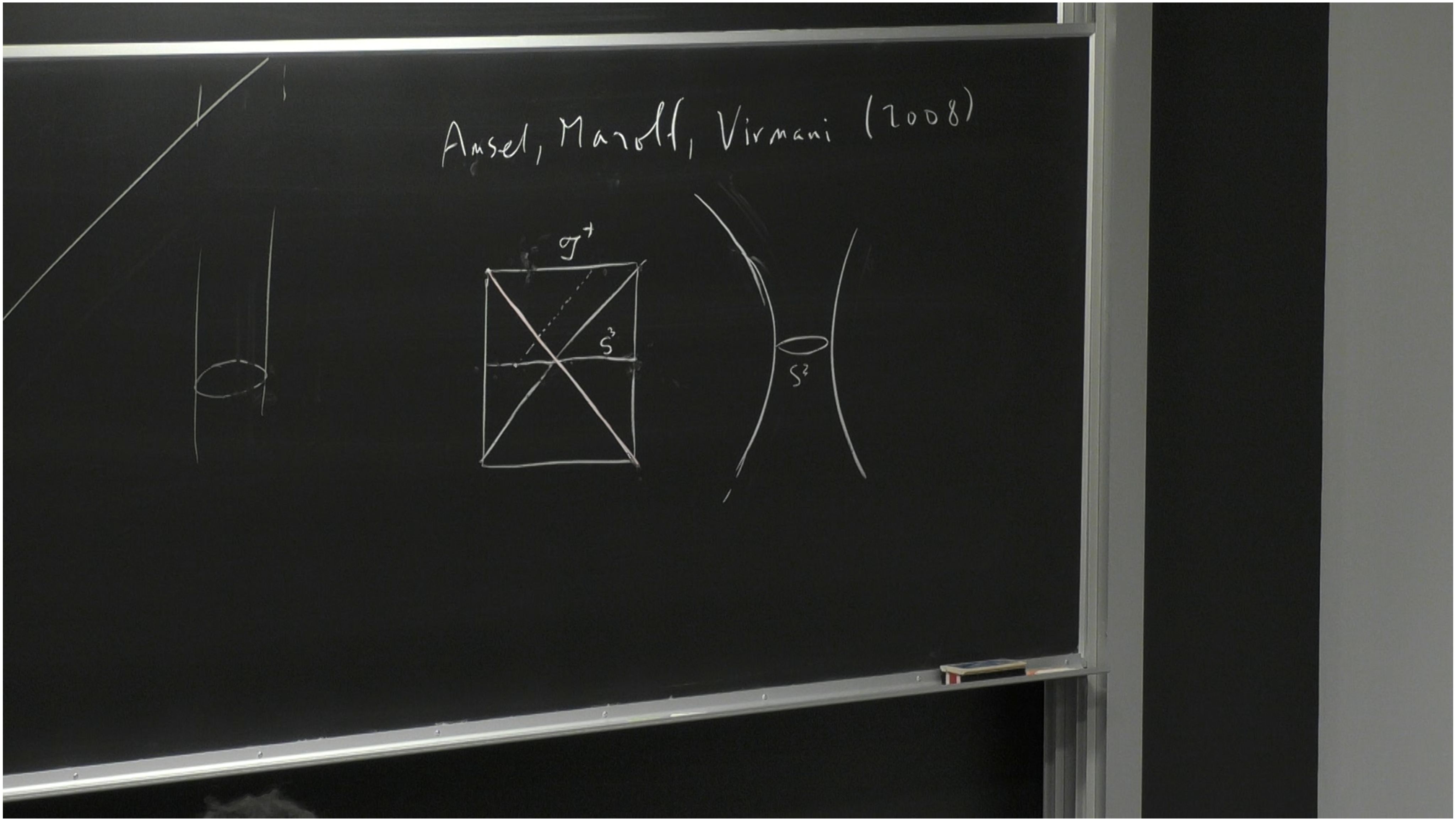


ll infinity.
Rindler horizon (in Minkowski space).
η=const
$ds^2 = \sigma^2 d\eta^2 - d\sigma^2$
σ=const.
see Horizon Entropy, (2003)
TJ & R. Parentani.
CAUTION
DRYWALL


ΔA ≈ ΔEx
ΔA / ħ4G ≈ ΔEx / ħκ/2π
S_BH
T_H
ℐ+ ("scri plus")
future null infinity
Shell
ℐ+
Rindler
see Horiz
TJ


Asymptotic Rindler
Horizon
onzon (in Minkowski space).
$ds^2 = \sigma^2 d\eta^2 - d\sigma^2$
η=const
σ=const.
∞
χ
χ
$\chi=0$
Entropy, (2003)
R. Parentani.


Asymptotic Rindler
Horizon
space).
dσ²
∞
χ
χ
χ=0



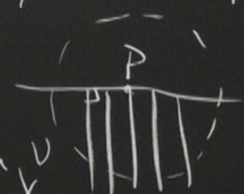
Ansel, Marolf, Virmani (2008)
$\mathcal{J}^+$
S^3
S^2

localize this: local Rindler horizon (LRH)

(at any point p in any spacetime).

Riemann normal coordinates:

(RNC)



$\theta = 0 = \sigma_{ab}$ at $\underline{p}$.

$$g_{\alpha\beta} = \eta_{\alpha\beta} - \frac{1}{3} R_{\alpha\mu\beta\nu} x^{\mu} x^{\nu} + O(x^3) \,, \quad x^{\mu}\big|_p = 0$$

Instead of using Einstein eq'n,

(i) assume horizon entropy $\propto$ area : $S = \underset{\substack{\uparrow \\ \text{universal area entropy density}}}{\eta} A$

(ii) assume Clausius rel'n : $\delta S = \dfrac{\delta Q}{T}$

where T = Unruh temp. of LRH

δQ = boost energy flux across LRH

Asymptotic Rindler

(ii) assume Clausius rel'n : $\delta S = \dfrac{\delta Q}{T}$

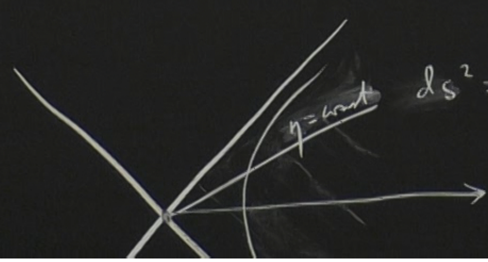
where T = Unruh temp. of LRH

δQ = boost energy flux across LRH

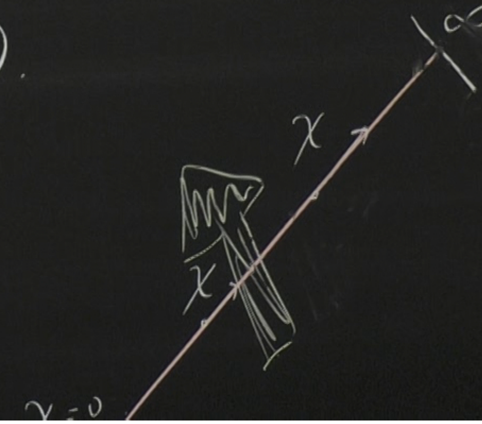
(iii) show this Implies Einstein Eq'n

Asymptotic Rindler Horizon

Rindler horizon (in Minkowski space).



$ds^2 = \sigma^2 d\eta^2 - d\sigma^2$



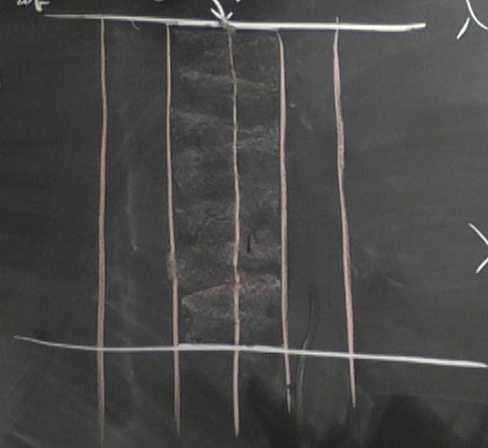
in vacuum of Poincaré inv. QFT, state restricted to any Rindler wedge is thermal w.r.t the boost hamitonian

H_B: boost generator

$$\mathrm{Tr}_L |0\rangle\langle 0| = \frac{e^{-H_B/T_u}}{\mathrm{Tr}\, e^{-H_B/T_u}}, \qquad T_u = \frac{\hbar\kappa}{2\pi}$$

$\sigma_{ab} = \theta = 0$ P

LRH



$\lambda = 0$ (affine parameter).

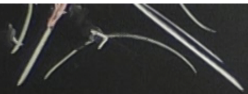
$\lambda < 0$

λ_i

$$\Delta S = \eta \, \Delta A = \eta \int_{\lambda_i}^{0} d\lambda \, dA \; \theta$$

$$\frac{d\theta}{d\lambda} = -\underbrace{\cancel{\tfrac{1}{2}\theta^2}}_{\approx 0} - \underbrace{\cancel{\sigma_{ab}\sigma^{ab}}}_{\approx 0} - R_{ab}k^a k^b$$

$$\rightarrow \theta \simeq -\lambda R_{ab} k^a k^b$$



$$\rightarrow \quad \Delta S = -\eta \int d\lambda \, dA \, \lambda \, R_{ab} k^a k^b$$

$$\frac{\Delta Q}{T_u} = \frac{\Delta E_\chi}{T_u} = \frac{\int d\lambda \, dA \; T_{ab} \chi^a k^b}{\hbar/2\pi} = \frac{-\int d\lambda \, dA \, \lambda \, T_{ab} k^a k^b}{\hbar/2\pi}$$

$$\chi^a = -\kappa \lambda k^a \; ; \quad \text{holds } \forall \; \lambda_i \Rightarrow \boxed{\eta R_{ab} k^a k^a = \frac{T_{ab} k^a k^b}{\hbar/2\pi}}_{\,p}$$

$$\forall k^a \Rightarrow \boxed{\eta R_{ab} = \frac{2\pi}{\hbar} T_{ab} + \Phi g_{ab}}_{\,p}$$

... this implies Einstein eq^n.

$$R_{ab} = \frac{2\pi}{\hbar\eta} T_{ab} + \frac{1}{\eta}\Phi\, g_{ab}$$

$$\Leftrightarrow \quad G_{ab} = \frac{2\pi}{\hbar\eta} T_{ab} + \Phi' g_{ab} \quad \Rightarrow \quad \boxed{G_{ab} = \frac{2\pi}{\hbar\eta} T_{ab} + \Lambda g_{ab}}$$

(some const. Λ)

$\forall p, \quad \nabla^a G_{ab} \equiv 0$ (Bianchi id.)

$$\Rightarrow \quad \underbrace{\nabla^a T_{ab}}_{=0} \propto \nabla_b \Phi' \quad \Rightarrow \quad \Phi' = \Lambda = \text{const}$$

• if $\eta = \infty$, can't satisfy $\Delta S = \frac{\Delta Q}{T}$ ↔ not near equilibrium?

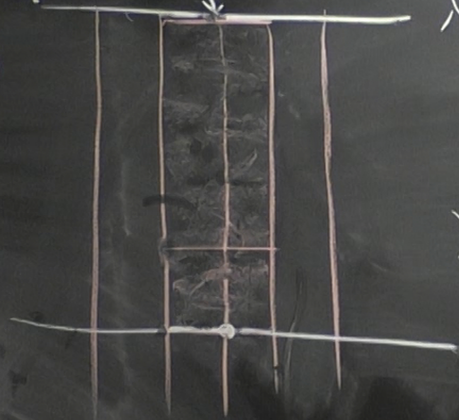
• pure gravity? $\frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R$; $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, \boxed{h_{\mu\nu}}$

$$+\,O(x^3)\,, \qquad X^\mu\big|_p = 0$$

LRH

$\sigma_{ab} = \theta = 0 \to p$

$\lambda = \lambda_f$ (affine parameter)

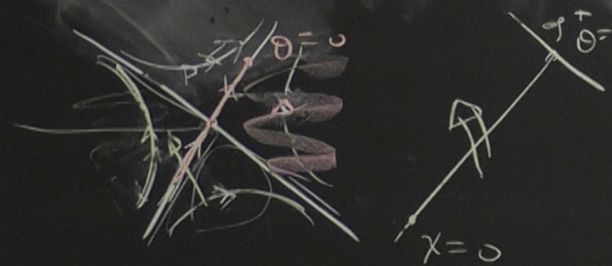


$$\Delta S = \eta\,\Delta A = \eta \int_{\lambda_i}^{0} d\lambda\, dA\ \theta$$

$\lambda > 0$

$$\frac{d\theta}{d\lambda} = -\underbrace{\cancel{\tfrac{1}{2}\theta^2}}_{\approx 0} - \underbrace{\cancel{\sigma_{ab}\sigma^{ab}}}_{\approx 0} - R_{ab}k^a k^b$$

$\lambda = 0$

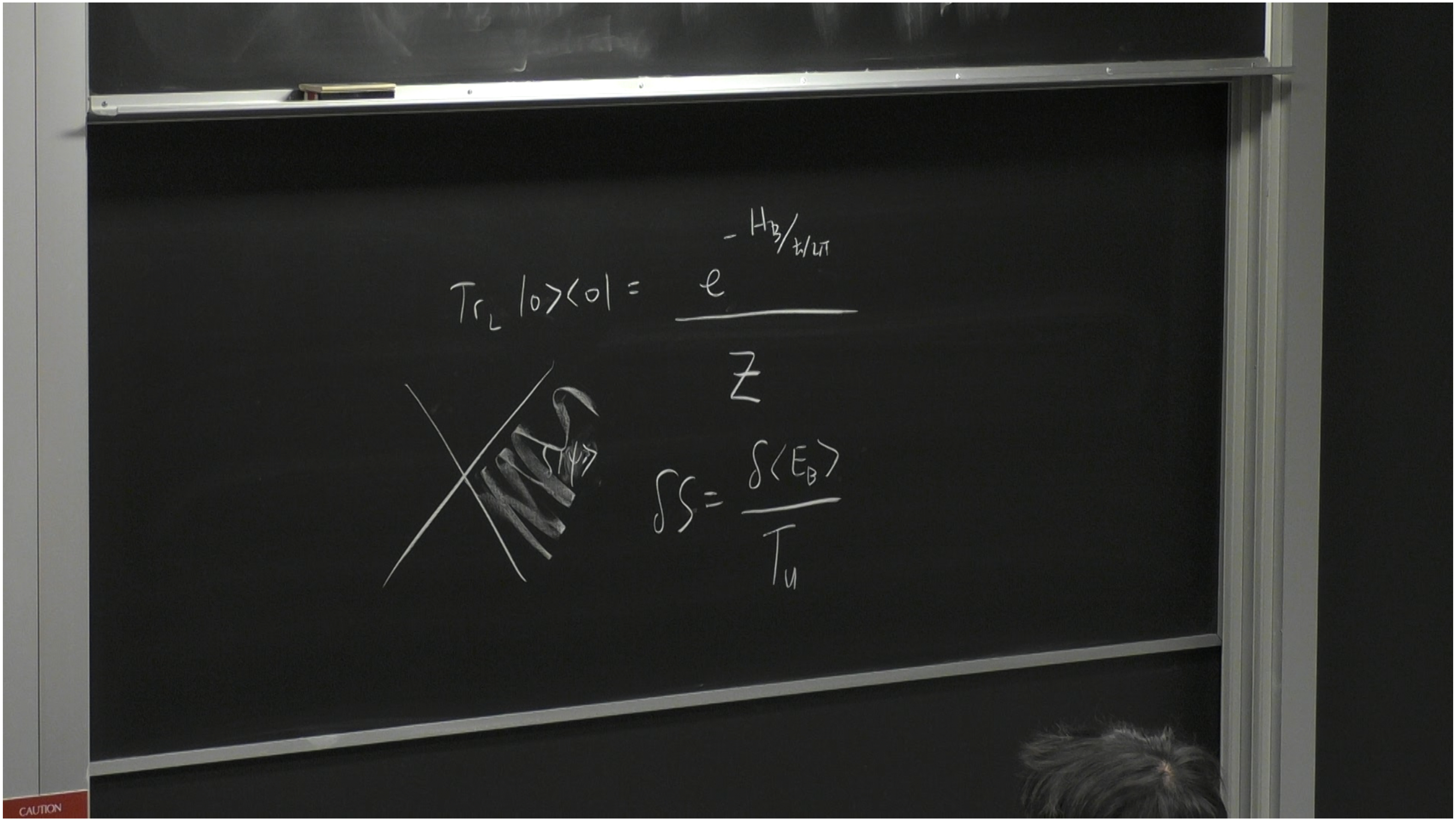


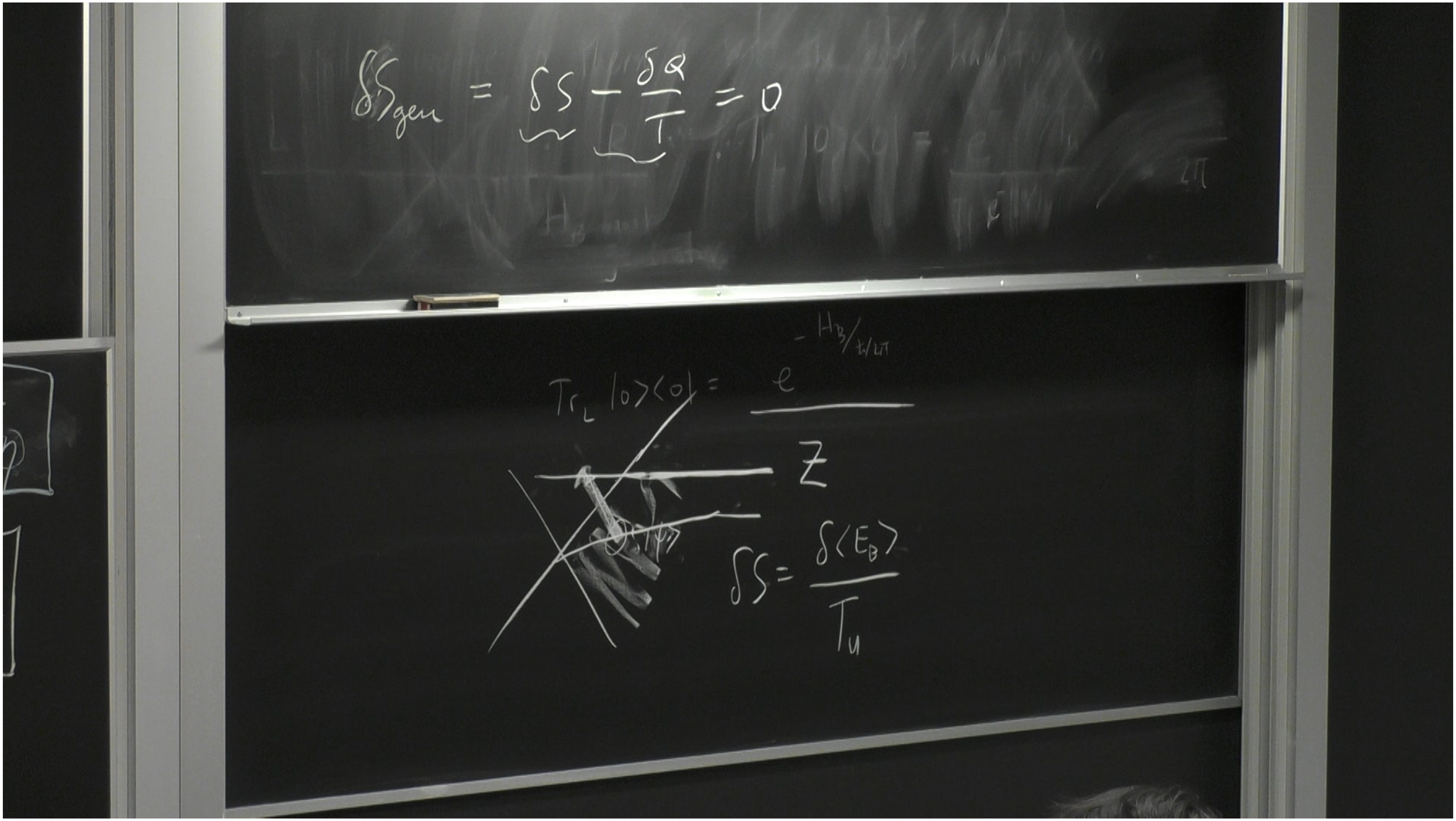
$\sigma, \theta = 0$

$x = 0$

$$\to \theta \cong -\lambda R_{ab} k^a k^b$$

$$\to \Delta S = -\eta \int d\lambda\, dA\ \lambda R_{ab} k^a k^b$$


$\mathrm{Tr}_L |0\rangle\langle 0| = \frac{e^{-H_B/\hbar/2\pi}}{Z}$
$\delta|\psi\rangle$
$\delta S = \frac{\delta\langle E_B\rangle}{T_U}$
CAUTION


$\delta S_{gen} = \delta S - \frac{\delta Q}{T} = 0$
$\mathrm{Tr}_L |0\rangle\langle 0| = \frac{e^{-H_B/\hbar/2\pi}}{Z}$
$\delta S = \frac{\delta\langle E_B\rangle}{T_U}$