Title: Quantum Computational Advantage: Recent Progress and Next Steps

Speakers: Xun Gao

Series: Machine Learning Initiative

Date: February 10, 2023 - 11:00 AM

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Abstract:

This talk is motivated by the question: why do we put so much effort and investment into quantum computing? A short answer is that we expect quantum advantages for practical problems. To achieve this goal, it is essential to reexamine existing experiments and propose new protocols for future quantum advantage experiments. In 2019, Google published a paper in Nature claiming to have achieved quantum computational advantage, also known as quantum supremacy. In this talk, I will explain how they arrived at their claim and its implications. I will also discuss recent theoretical and numerical developments that challenge this claim and reveal fundamental limitations in their approach. Due to these new developments, it is imperative to design the next generation of experiments. I will briefly mention three potential approaches: efficient verifiable quantum advantage, hardware-efficient fault-tolerance, and quantum algorithms on analog devices, including machine learning and combinatorial optimization.

Zoom Link: https://pitp.zoom.us/j/96945612624?pwd=ckRKMFJqZ0Q0dGtFOU91c1hnMzIzZz09

Quantum Computational Advantage — Recent Progress and Next Steps

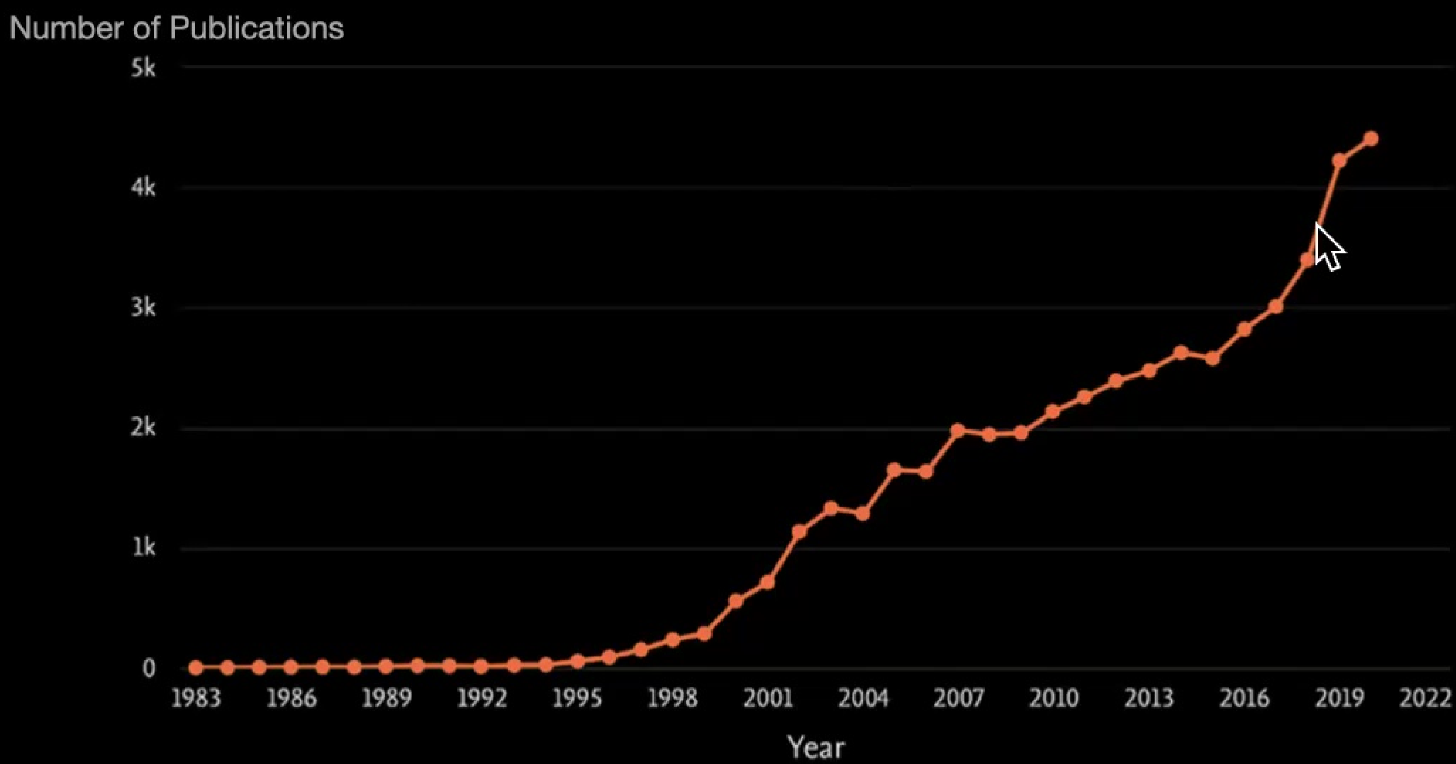
Xun Gao
Department of Physics, Harvard University

PIQuIL Group Meeting

Outline

- What is quantum computational advantage (a.k.a. quantum supremacy)? Why is it important?
- Has quantum computational advantage been achieved? Perhaps not!
- Next generation of quantum advantage protocols:
  - Efficient classical verification
  - Solving combinatorial optimization problems
  - Quantum machine learning (quantum ChatGPT?)

Quantum computing effort



Resouces: Quantum computing research trends report. Elsevier.

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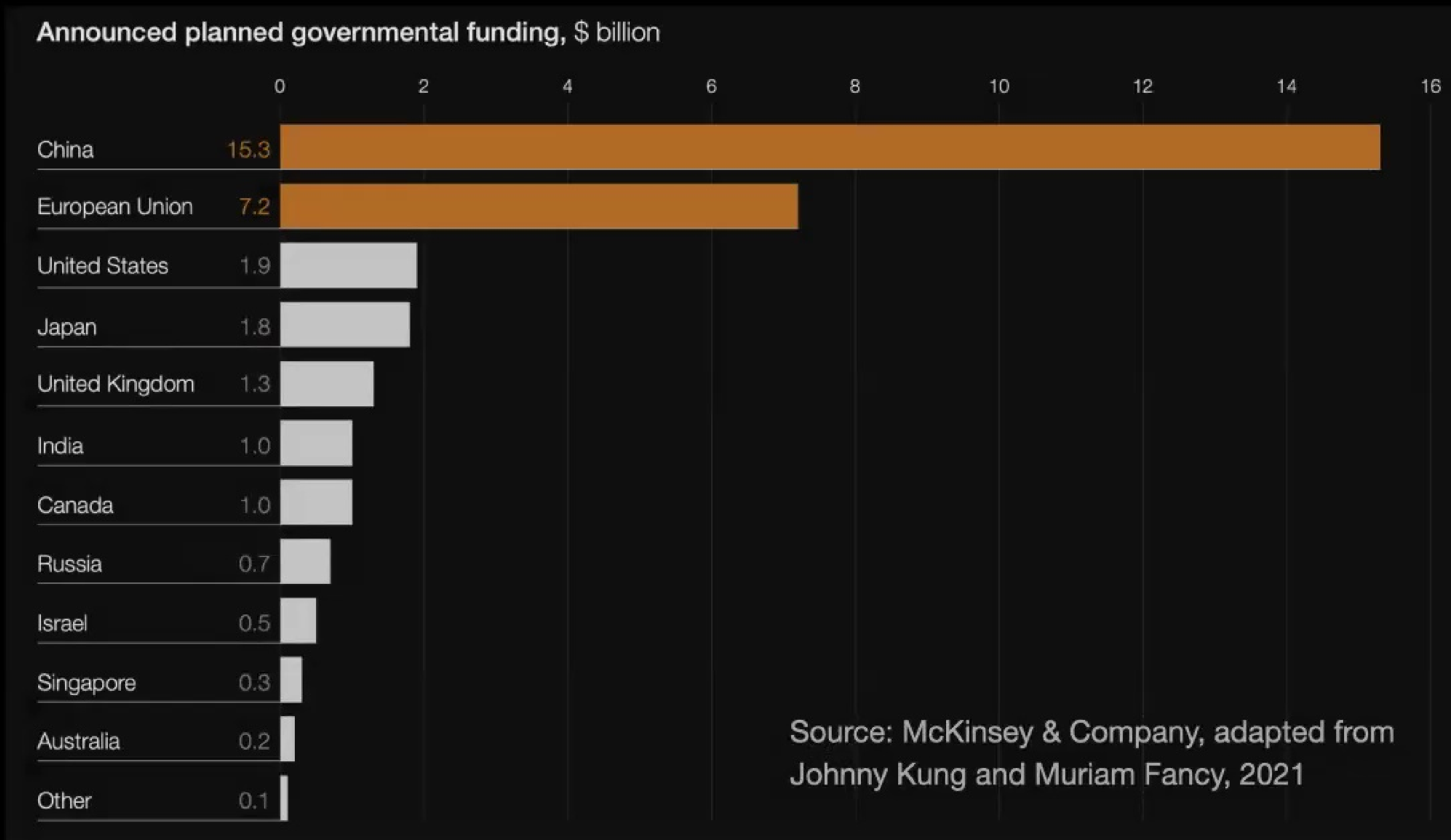
Quantum computing effort



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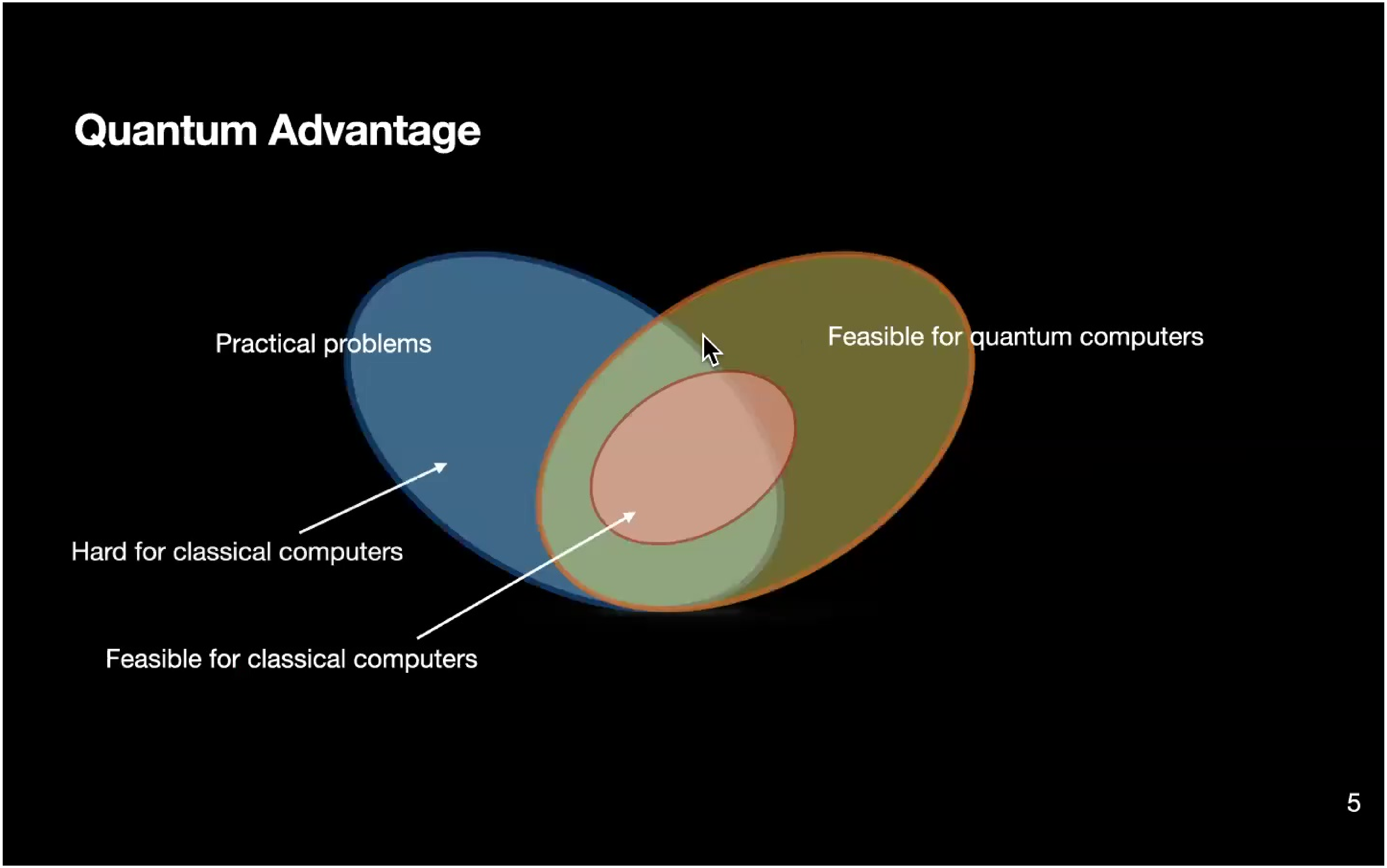
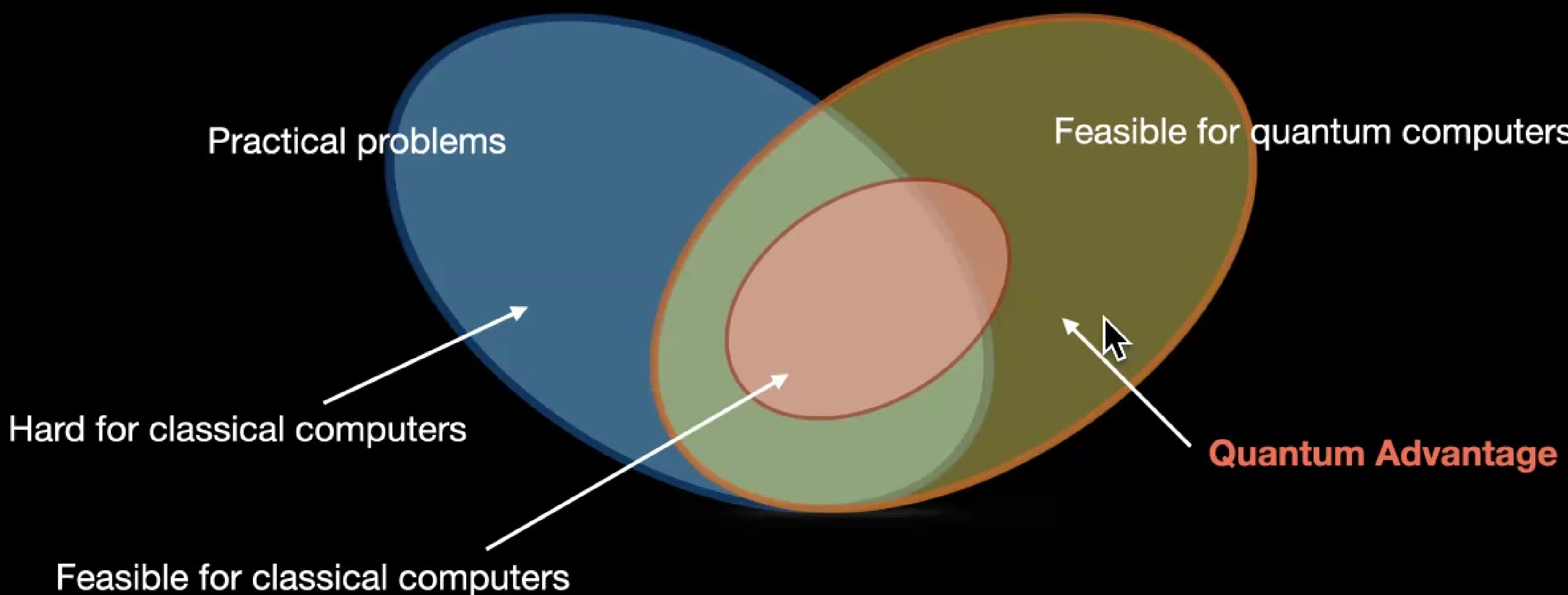

Quantum Advantage
Practical problems
Feasible for quantum computers
Hard for classical computers
Feasible for classical computers
5

Quantum Advantage



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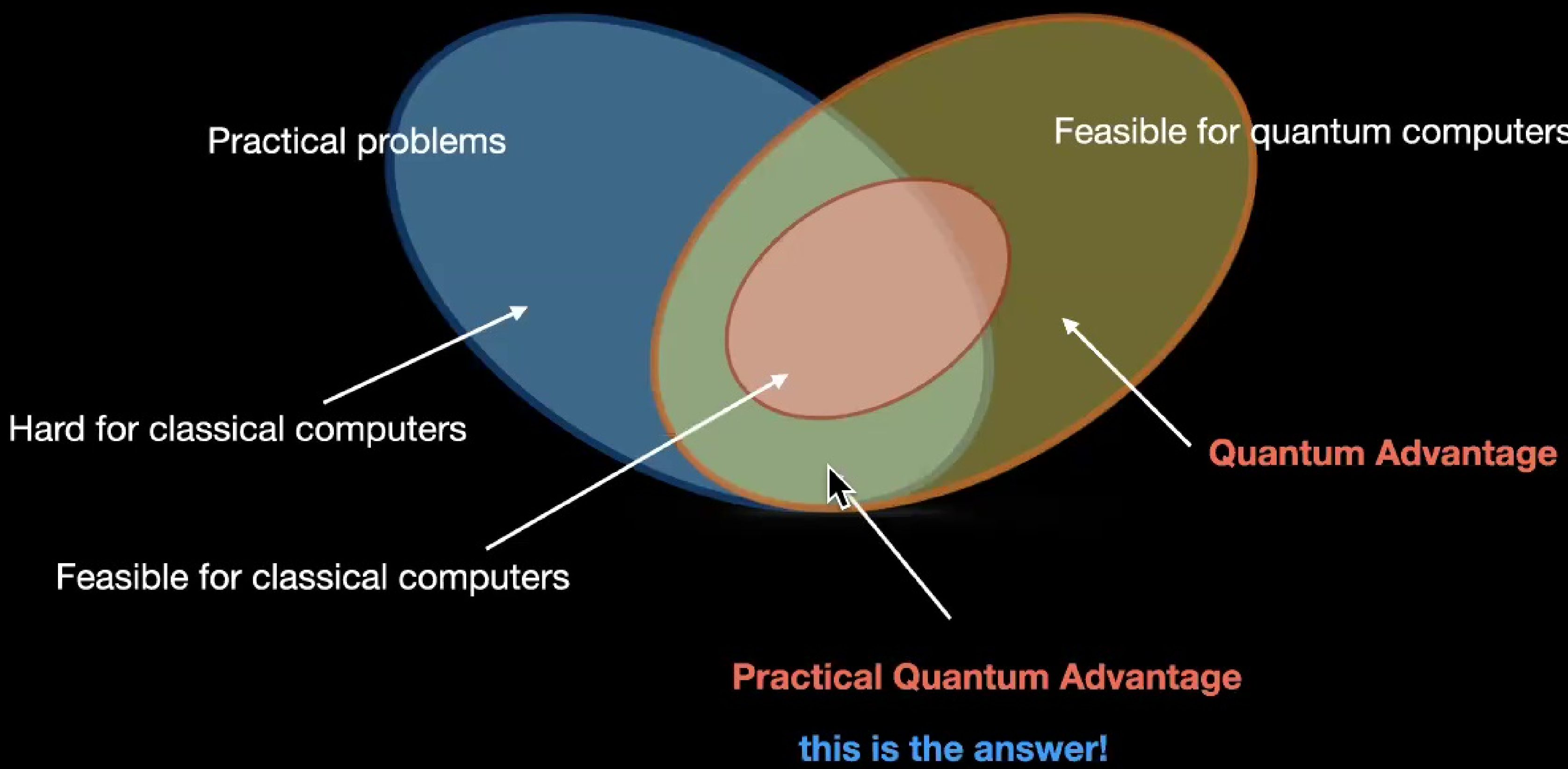
Quantum Advantage



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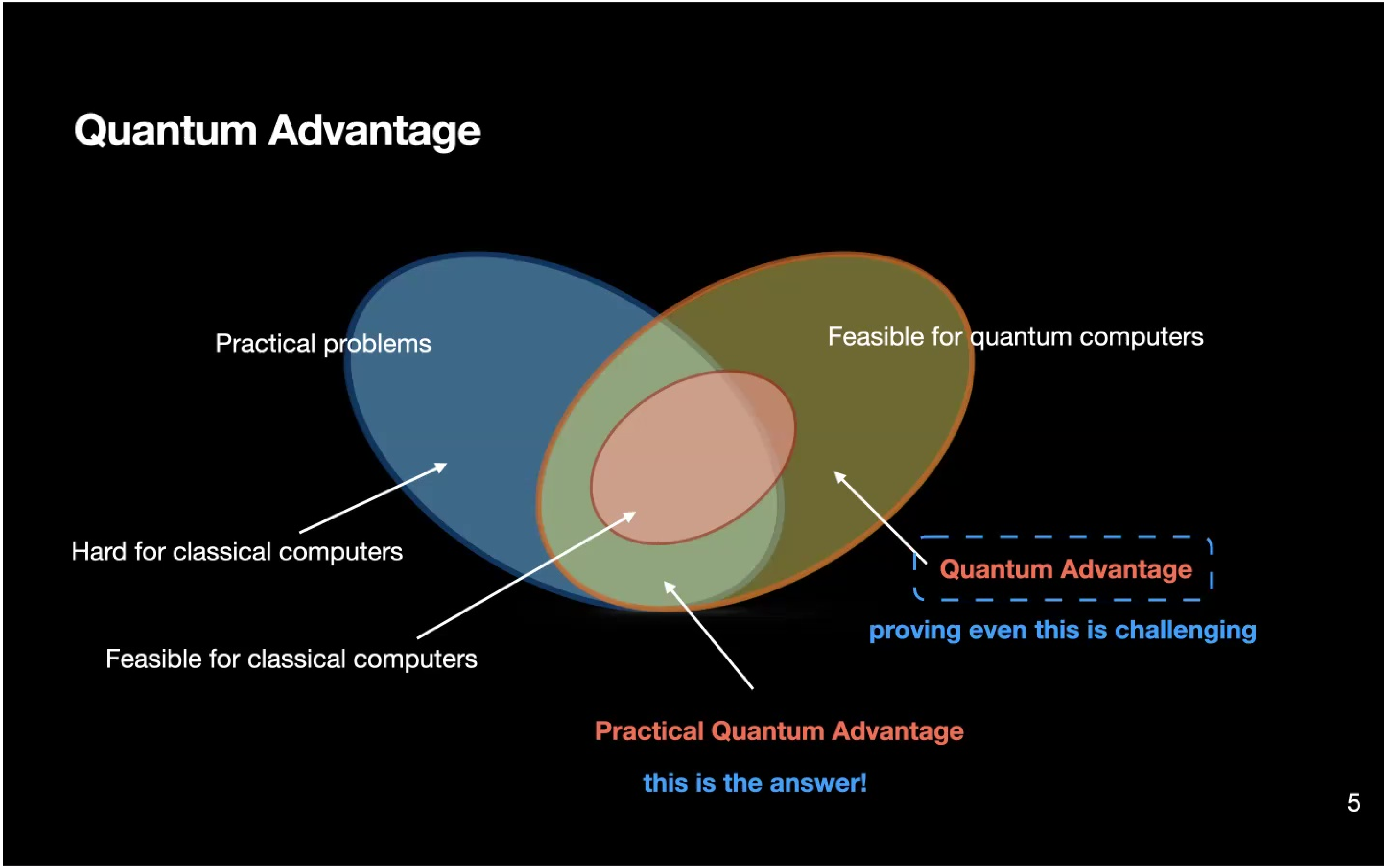
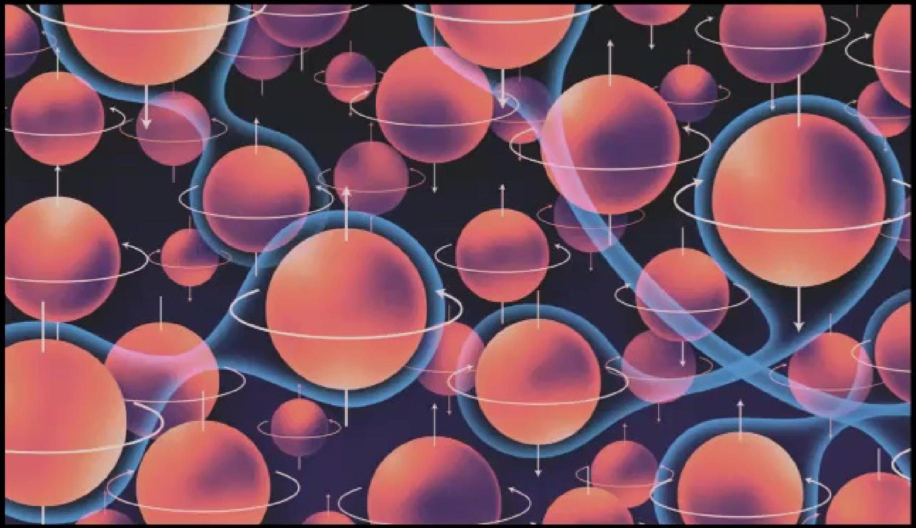

Quantum Advantage
Practical problems
Feasible for quantum computers
Hard for classical computers
Quantum Advantage
proving even this is challenging
Feasible for classical computers
Practical Quantum Advantage
this is the answer!
5

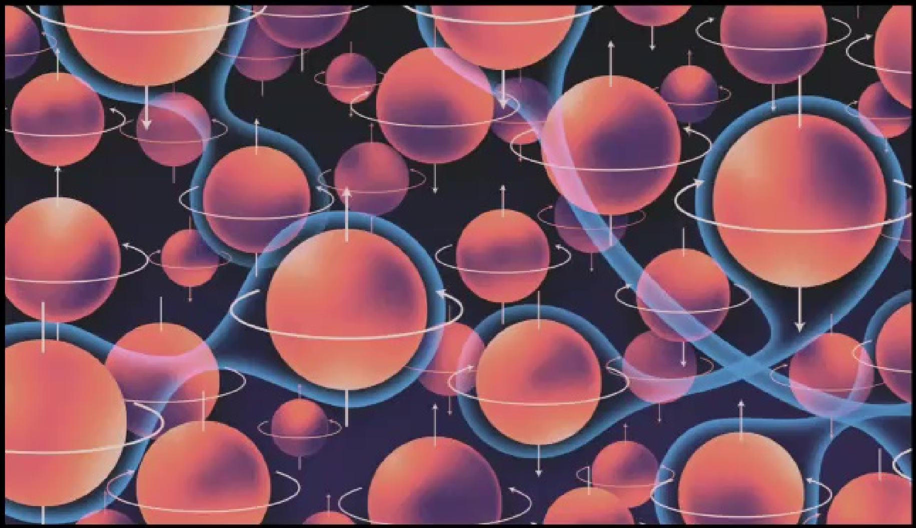
Intuition why we expect Quantum Advantage



a large amount of entanglement
hard to simulate classically

6

Intuition why we expect Quantum Advantage



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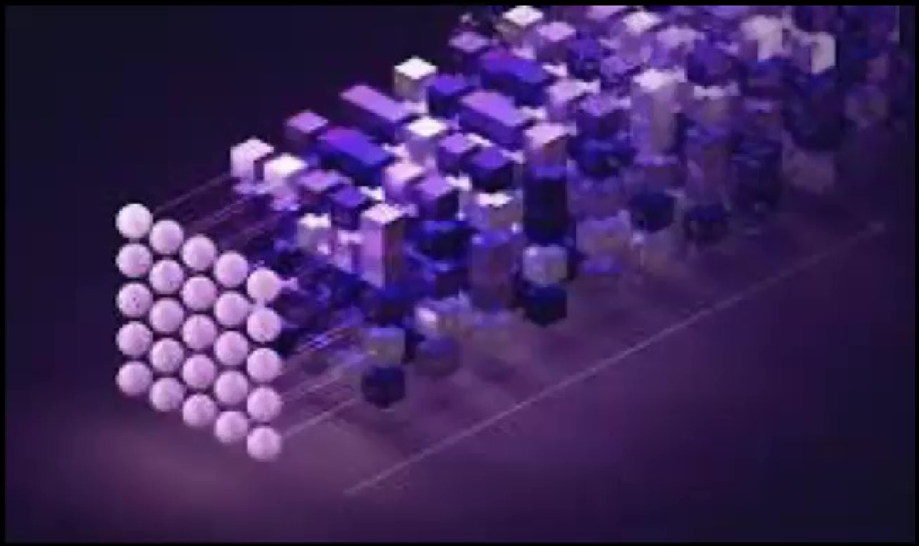
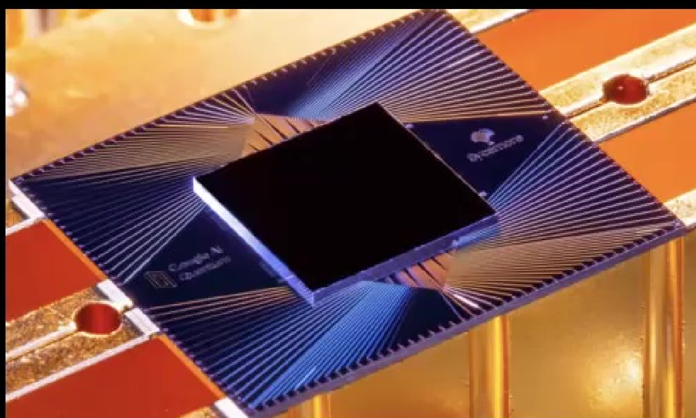
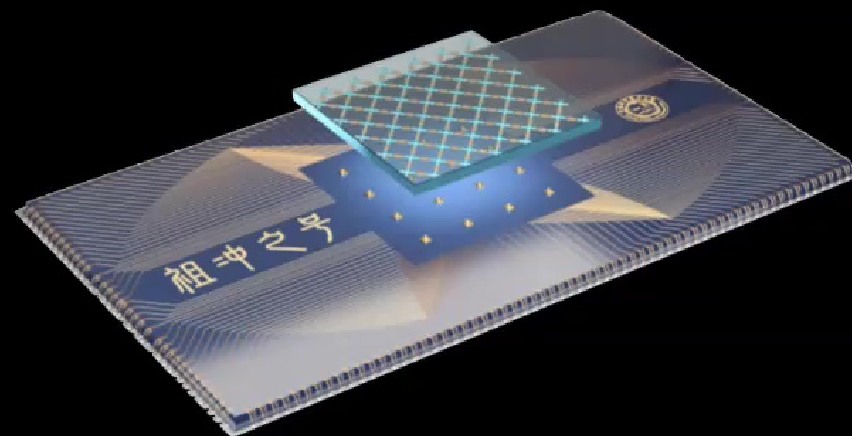


random quantum circuit can generate
a large amount of entanglement

Quantum Computational Advantage (quantum supremacy)





Sycamore (53 qubits)
by Google, Oct. 2019

Zuchongzhi (56 qubits)
by USTC, Jun. 2021

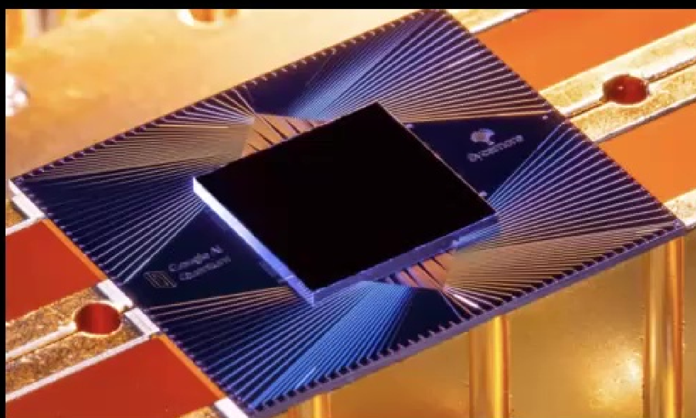
Zuchongzhi-2 (60 qubits)
by USTC, Sep. 2021

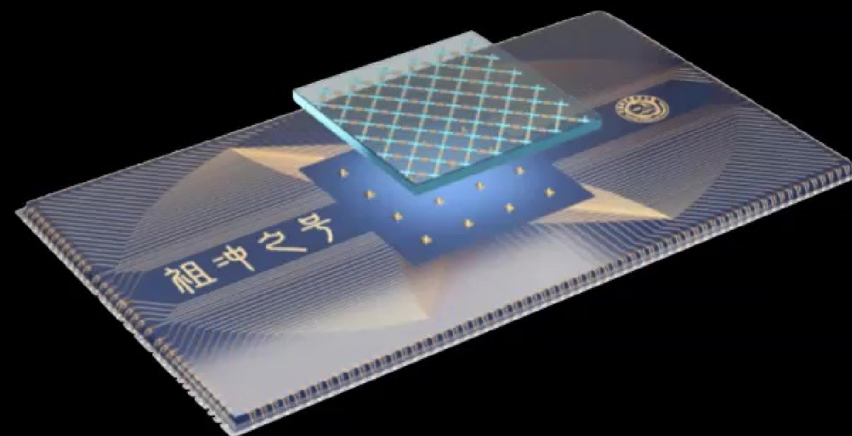
original claim:10,000 yr by classical computers

48,000yr

7

Quantum Computational Advantage (quantum supremacy)





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- constitute a milestone of quantum technology (on the way to a practical quantum computer)
- **test quantum physics with high complexity: violation of extended Church-Turing thesis**

  Any physical realistic computers are equivalent to Turing machine (up to only poly overhead)

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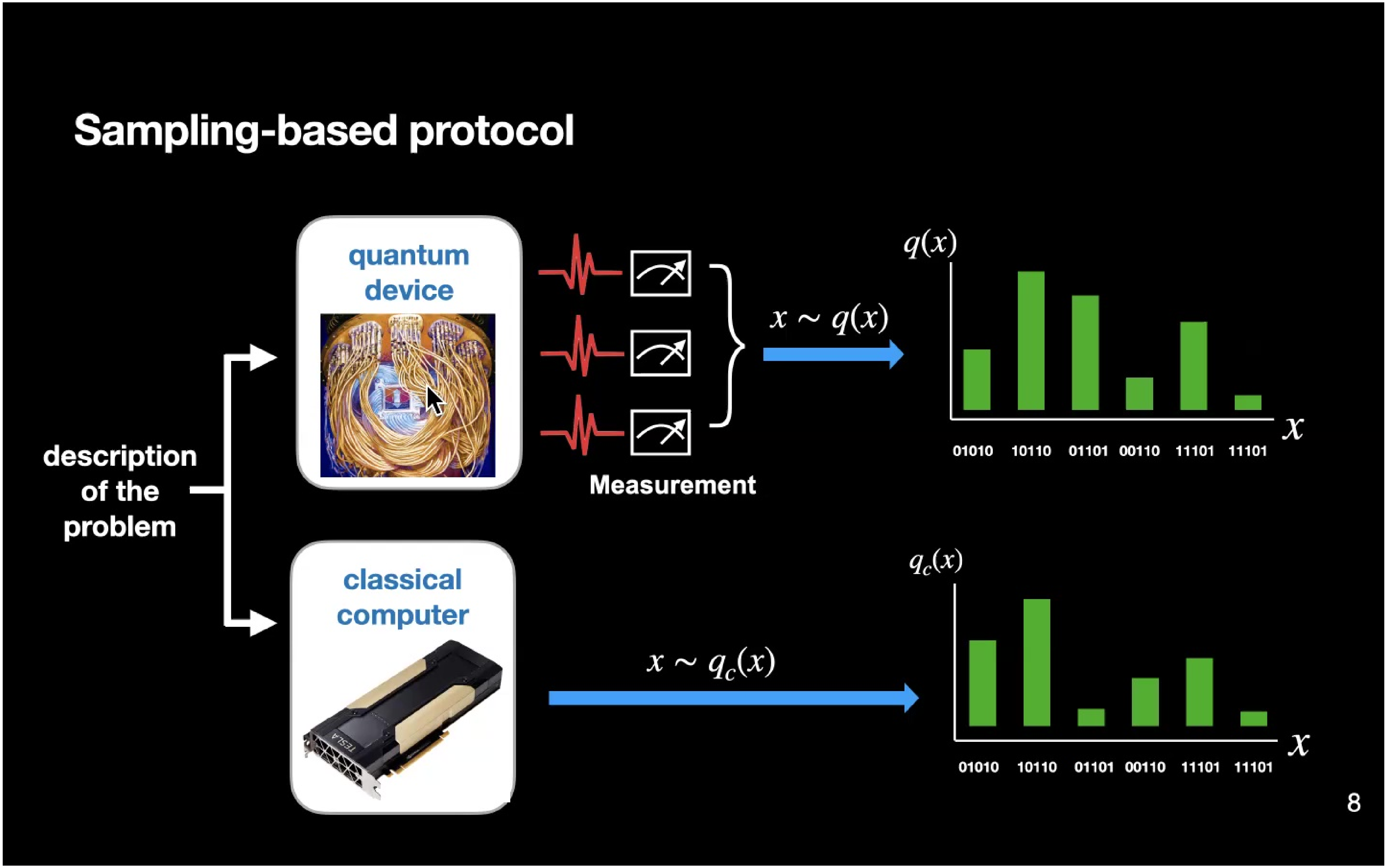
Sampling-based protocol
description of the problem
quantum device
Measurement
$x \sim q(x)$
$q(x)$
x
01010
10110
01101
00110
11101
11101
classical computer
TESLA
$x \sim q_c(x)$
$q_c(x)$
x
01010
10110
01101
00110
11101
11101
8

Analogy to Bell test

	Bell test	Quantum Computational Advantage
The scale of proof of quantumness	few bodies	many-body / high complexity
Precursors	not excluding hidden variable theory spectrum of hydrogen atom EPR paradox	lack of complexity-theoretic foundation strongly correlated condensed matter (FQHE, high Tc SC) quantum simulations Shor's algorithm perfect but not near-term

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Analogy to Bell test

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The scale of proof of quantumness	few bodies	many-body / high complexity
Precursors	spectrum of hydrogen atom EPR paradox	strongly correlated condensed matter (FQHE, high Tc SC) quantum simulations Shor's algorithm
Restrictions	2 players space-like separation	polynomial time algorithm to produce samples
Landmark	violation of Bell's inequality	**violation of Extended Church-Turing thesis** the foundation of computational complexity theory: define efficiency in a device-independent way

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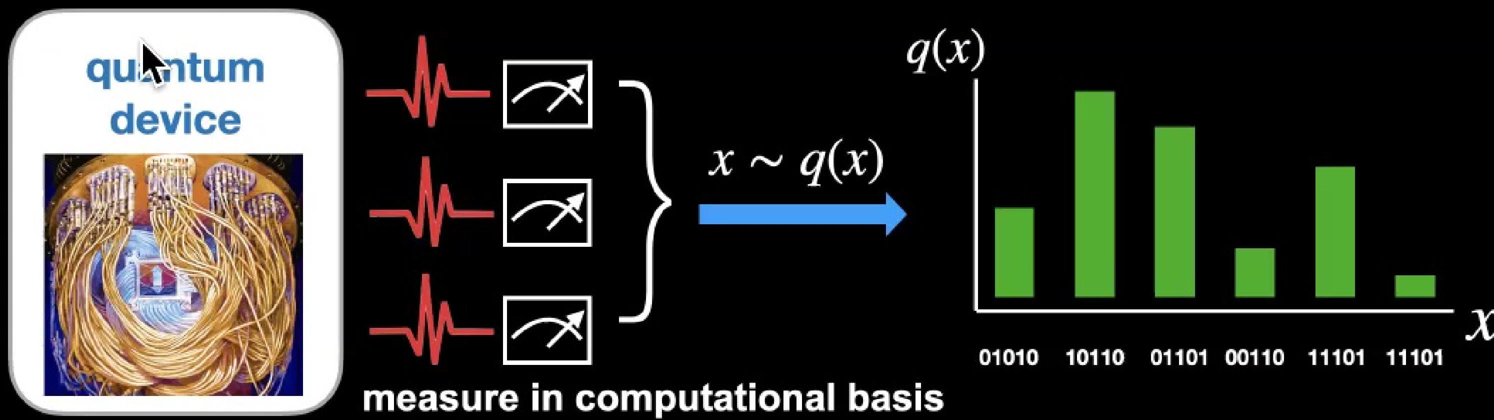
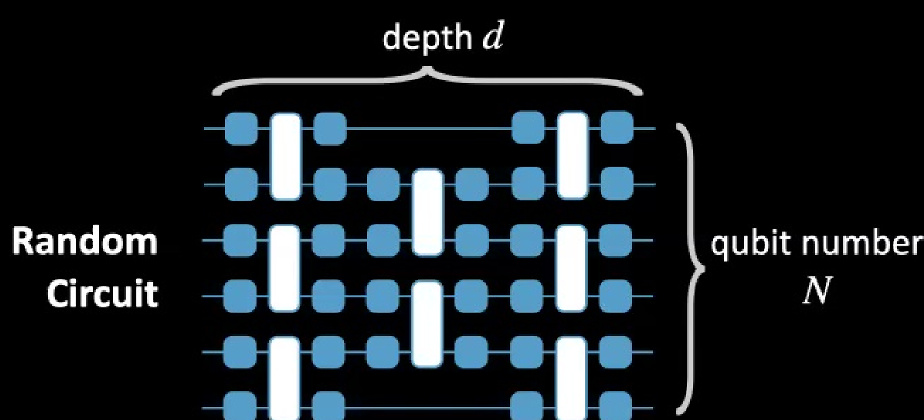
Random circuit sampling



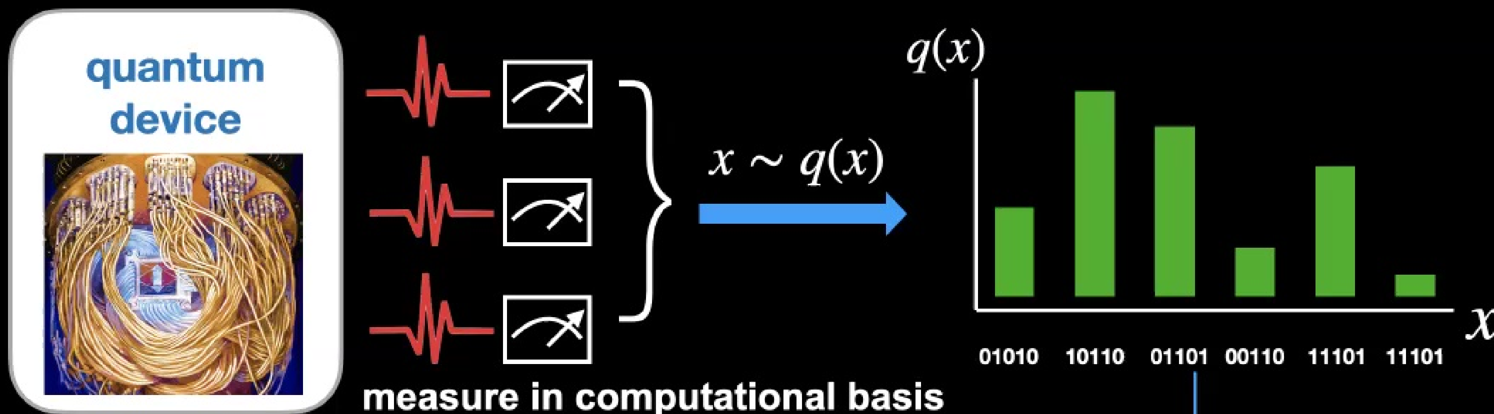


Google: $N = 53,\ d = 20$

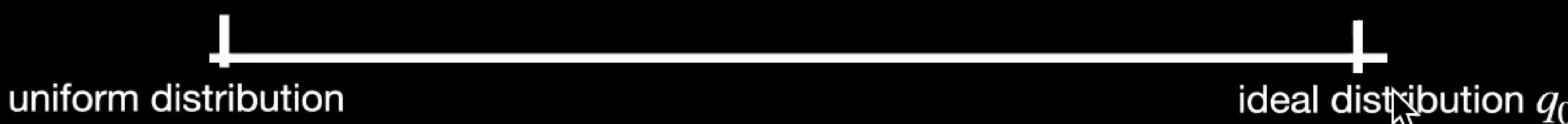
USTC-1: $N = 56,\ d = 20$

USTC-2: $N = 60,\ d = 24$

Random circuit sampling



It was believed that the statistics are hard to produce classically



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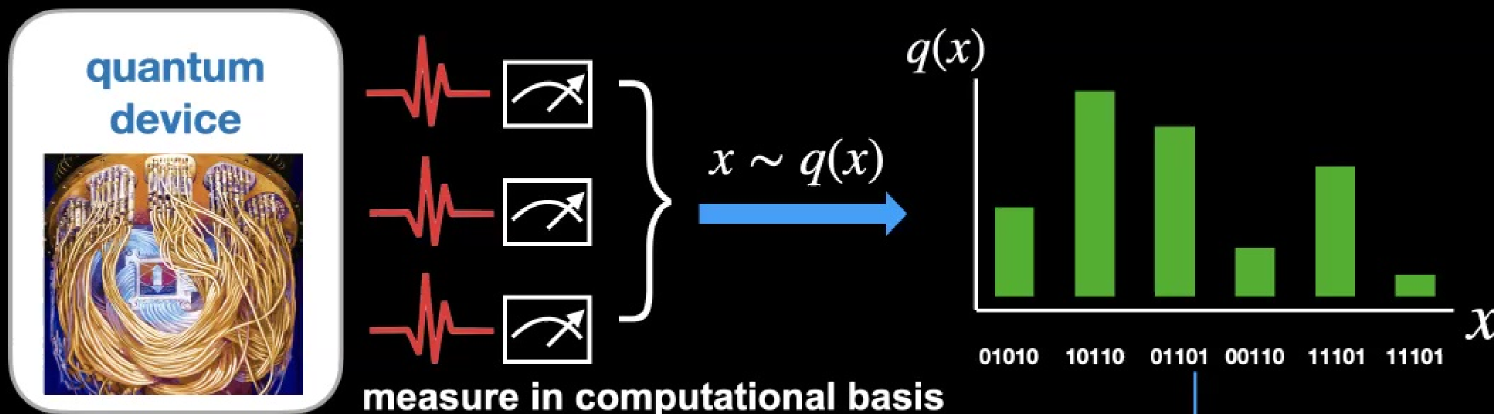

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Adam Bouland, et al., Nature Physics (2019)
Ramis Movassagh, ArXiv (2019)
Adam Bouland, et al., FOCS (2022)
Yasuhiro Kondo, et al., FOCS (2022)
Hari Krovi, et al., ArXiv (2022)

Linear Cross-Entropy Benchmark (XEB)

ideal circuit — ideal circuit (e.g., by supercomputer)

$$\mathrm{XEB} = 2^N \sum_x q_0(x) q(x) - 1 = \mathbb{E}_{x\sim q}[2^N q_0(x) - 1]$$

experiment — experiment

- $q \approx q_0$, XEB ≈ 1 (experiment is perfect)
- q and q_0 uncorrelated, XEB $= 0$ (experiment is too noisy)
- non-vanishing XEB —> non-trivial correlation q and q_0

conjecture (by Google): XEB≈fidelity of the quantum state

XEB as a complexity version of Bell's inequality?

Google. Nature (2019)
Google. Nature Physics (2018)

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XEB values of current experiments:
Google: 2×10^{-3} USTC-1: 6×10^{-4} USTC-2: 3×10^{-4}

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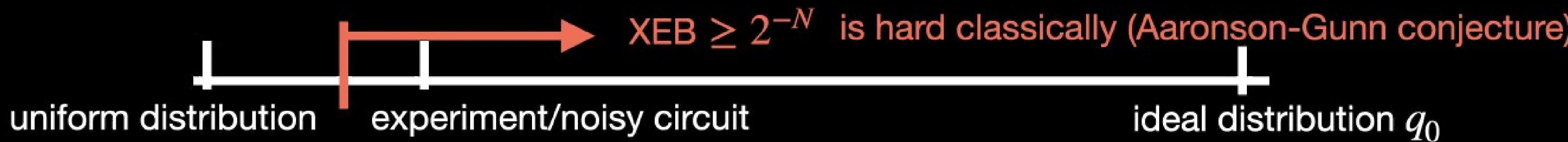
Complexity-theoretic foundation



Scott Aaronson and Lijie Chen. CCC (2017), Scott Aaronson and Sam Gunn. Theory of Computation (2020)

- **certification of quantum advantage:**
  even a tiny **non-trivial** XEB is hard to achieve classically => based on direct complexity-theoretic evidence (Aaronson-Gunn conjecture)

Key idea (path integral argument):

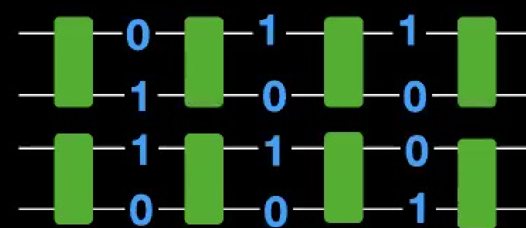
$$\langle 0^n | U | 0^n \rangle = \sum_{x_1,\cdots,x_{d-1}} \langle 0^n | U_1 | x_1 \rangle \langle x_1 | U_2 | x_2 \rangle \cdots \langle x_{d-1} | U_d | 0^n \rangle$$

in total 2^{Nd} paths

each path has roughly equal contribution $\sim 2^{-Nd}$

poly classical algorithms $\sim$ poly$(N) 2^{-Nd} \ll 2^{-N}$

$$I = |0\rangle\langle 0| + |1\rangle\langle 1|$$



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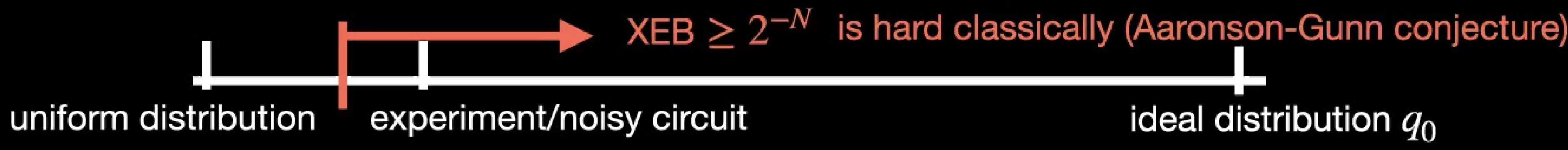
Spoofing algorithms (asymptotic)



Our result: a linear time algorithm (respect to Nd)

achieving XEB $= 2^N \sum_x q_0(x) q_c(x) - 1$

$$= 2^{-O(d)} \gg 2^{-N}$$

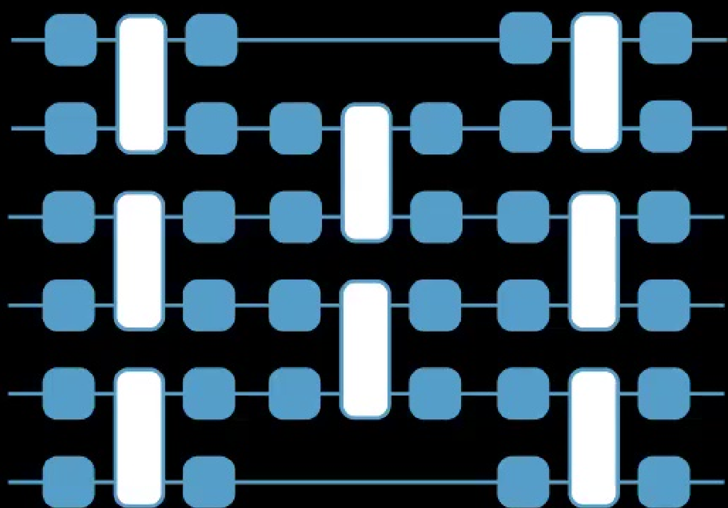
refuting Aaronson-Gunn conjecture

XEB≈fidelity is also refuted (in general)

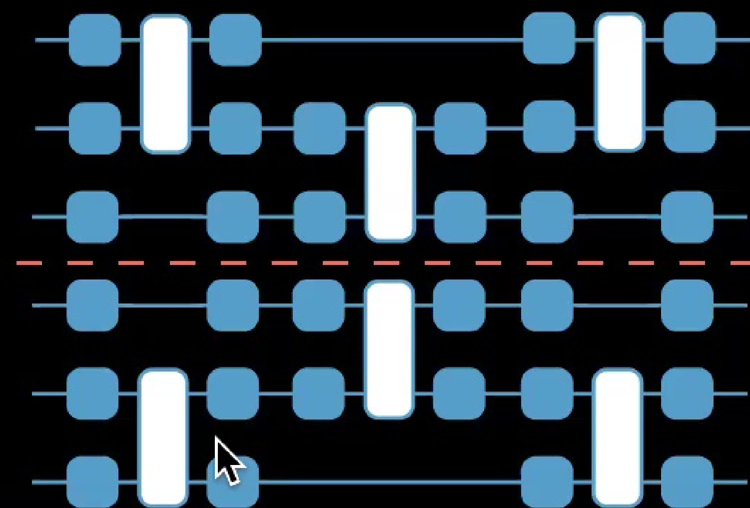
XG, Marcin Kalinowski, Chi-Ning Chou, Mikhail Lukin, Boaz Barak, Soonwon Choi (ArXiv:2112.01657)

Our algorithm: intuition

small XEB in experiments:
noises effectively truncate correlations



in our algorithm:
introduce the truncation explicitly



subsystems decouple —> much easier to simulate
if subsystem sizes are constant —> linear time alg
with other algorithmic ingredients

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Spoofing algorithms (finite-size)

Oct., 2019

Google

53 qubits, depth 20

XEB>2×10^{-3}

Their original claim: 10,000 yr even for supercomputer

Not true!

Skeptics:

IBM (2019), ……
finally: Feng Pan, Keyang Chen, Pan Zhang (Nov, 2021):
15 hours, 512 GPUs

- Tensor network contraction (sum over many paths in fewer steps, but still exponential complexity)
- not scalable! (no complexity implication)

Spoofing algorithms (finite-size)

Oct., 2019

Google
53 qubits, depth 20
XEB>2×10^{-3}

Jun., 2021

USTC
56 qubits, depth 20
XEB>6×10^{-4}

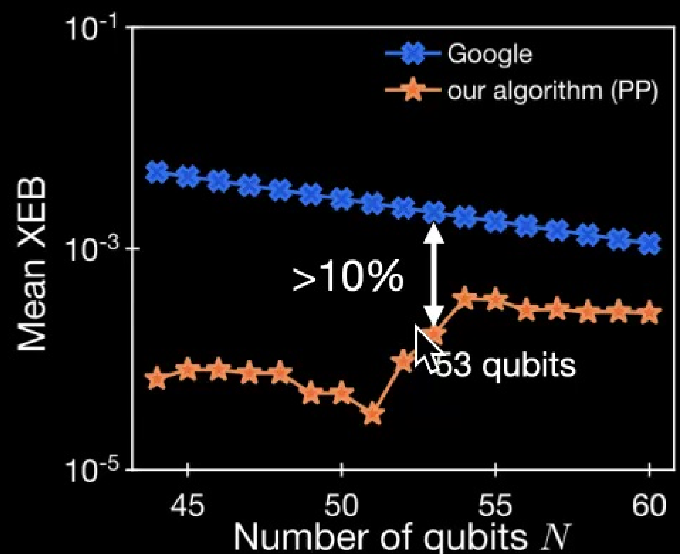
Sep., 2021

USTC-2
60 qubits, depth 24
XEB>3×10^{-4}

48,000yr

to avoid these attacks => larger system sizes

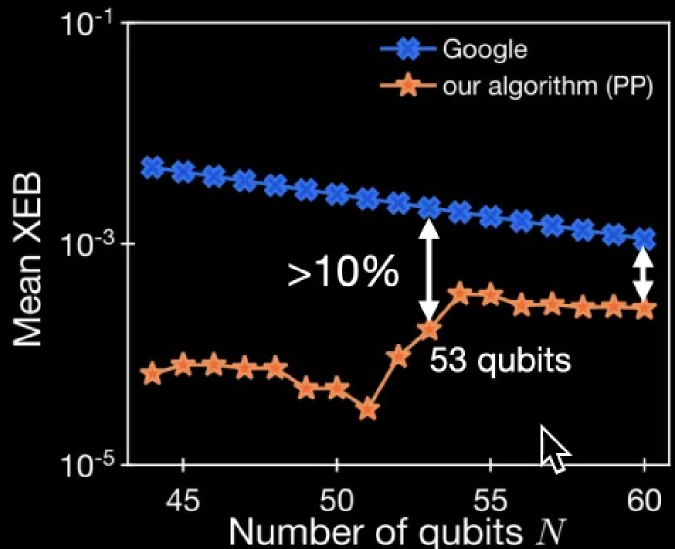
Our spoofing algorithm



1 GPU & 0.6 seconds
comparable XEB values by very limited computational resources

XG, Marcin Kalinowski, Chi-Ning Chou, Mikhail Lukin, Boaz Barak, Soonwon Choi (ArXiv:2112.01657)

Our spoofing algorithm



outperform Google at 70 qubits?
(keeping fidelity & depth)

1 GPU & 0.6 seconds
comparable XEB values by
very limited computational resources

questions quantum supremacy
at a much more fundamental level

1. Based on the understanding of noisy circuits
2. Very efficient (polynomial time) algorithm
3. Better performance when scaling up

5%-15% for USTC's circuits

XG, Marcin Kalinowski, Chi-Ning Chou, Mikhail Lukin, Boaz Barak, Soonwon Choi (ArXiv:2112.01657)

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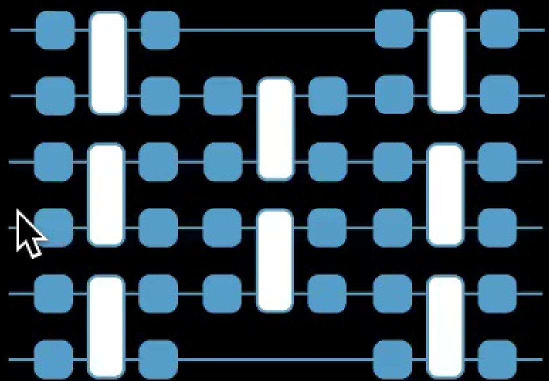
How to analyze the performance of our algorithm?

$$\text{XEB} = 2^N \sum q_0(x) q_c(x) - 1$$

Random Circuit



analyzing individual quantum circuit/dynamics is extremely difficult (q_0 is hard to compute)

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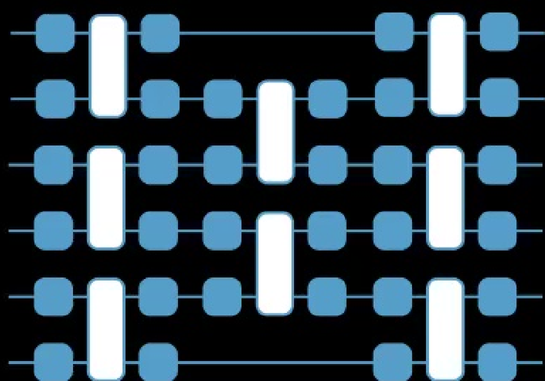
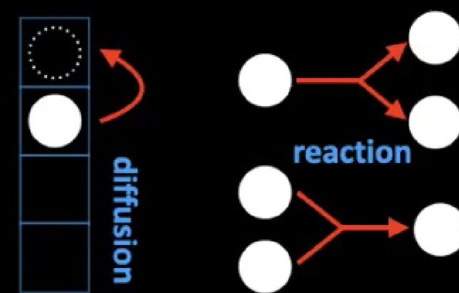
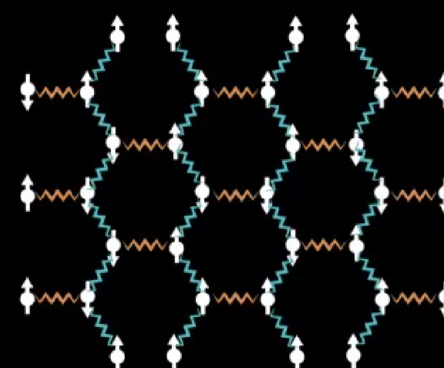


averaging over unitaries

Diffusion-reaction model



Ising spin model



analyzing individual quantum circuit/dynamics is extremely difficult (q_0 is hard to compute)

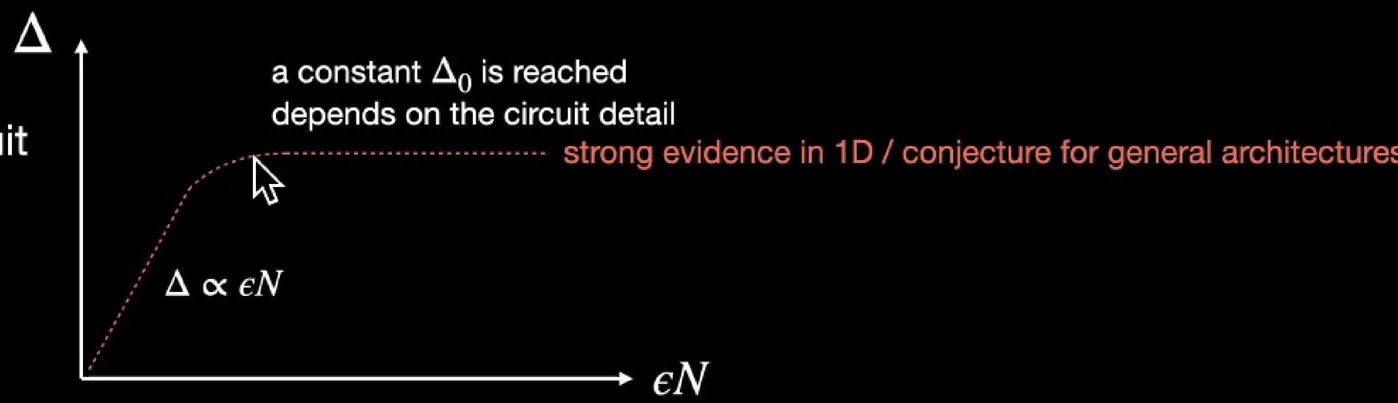
the average behavior via emergent statistical mechanics models

1. **more intuitive physical pictures**
2. **efficient semi-analytical tools**

Out spoofing algorithm: potential limitations

for noisy random circuit
XEB $\approx e^{-\Delta d}$

ϵ: strength of noise



Noisy circuit $e^{-\Delta_0 d}$ vs. our algorithm $e^{-\Delta_c d}$
$\Delta_0 \geq \Delta_c$ at least for 1D (mapping to spontaneous magnetization of Ising model)

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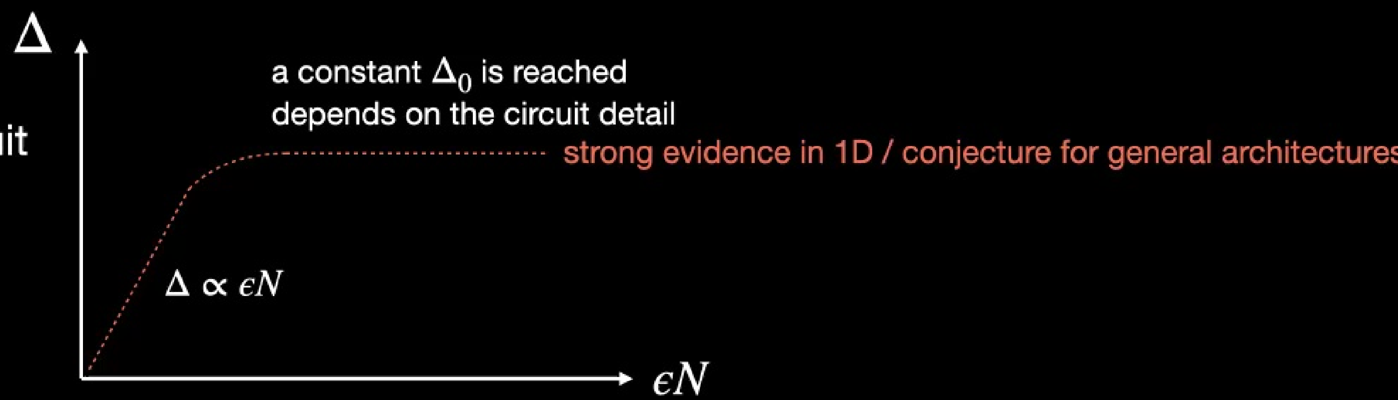

Noisy circuit $e^{-\Delta_0 d}$ vs. our algorithm $e^{-\Delta_c d}$

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- modified Aaronson-Gunn conjecture? (1D is excluded)
- other benchmarks beyond XEB (e.g., log XEB? …unlikely)
- geometrically local —> high connectivity architecture

Spoofing algorithm for any benchmarks and architectures

Our **new result** (informal)*:
no statistical test can distinguish between

- M samples from noisy circuits under constant noise per gate
- M samples from a classical algorithm in poly(N, M) time

Dorit Aharanov, **XG**, Zeph Landau, Yunchao Liu, Umesh Vazirani (ArXiv:2211.03999), STOC 2023, QIP 2023 long plenary talk
See also Quanta Magazine "closing the window of quantum supremacy"

* Note: not as practical as the previous algorithm, a large polynomial overhead

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No violation of extended Church-Turing thesis in noisy random circuit model

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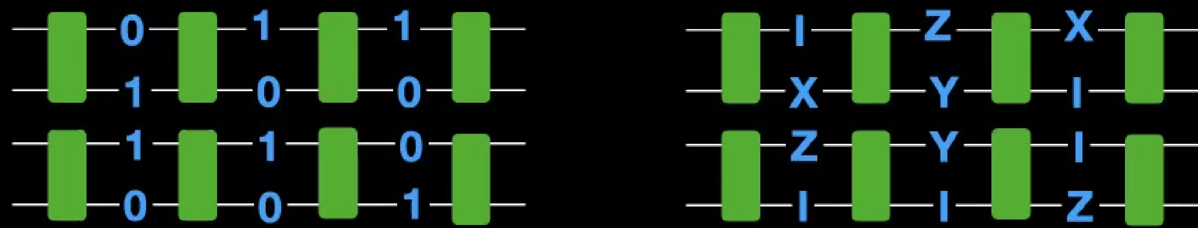
High level idea of our algorithm



Computational basis —> Pauli basis path integral
(density matrix)

- exponential decay with # non-I Pauli
- noises make the decay faster
- sparsity: many paths are 0

XG, Luming Duan (ArXiv:1810.03176)

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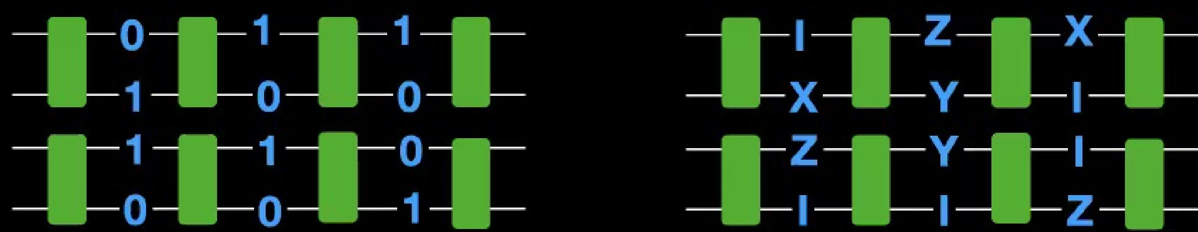

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an equivalent point of view: **Fourier expansion**

output of quantum circuits: $f_\epsilon(\theta) = \sum_{k \in \text{Paulis}} e^{-|k|\epsilon} \cdot \hat{f}(k) e^{i\theta \cdot k}$

θ: the parameters of quantum circuits

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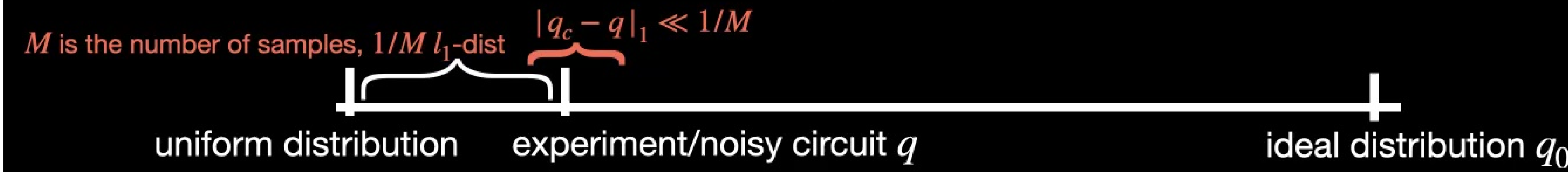
More formal statement of our result



Our algorithm:

achieving small l_1-distance by time $(NM)^{1/\epsilon}$

assumptions: 1. constant depolarizing noise per gate: $\epsilon = \text{const}$

2. anti-concentration condition (a technical condition, also used to show quantum supremacy)

3. enough randomness in the gate set

Dorit Aharanov, **XG**, Zeph Landau, Yunchao Liu, Umesh Vazirani (ArXiv:2211.03999)

Has quantum computational advantage really been achieved?

- analogy to Bell test
  - severe loopholes
  - fundamental problems
- "finite-size quantum advantage"
  - finite-size, no complexity guarantee: quantum simulation a "useful" candidate?
  - improving the spoofing algorithm (e.g. combined with tensor network methods)?

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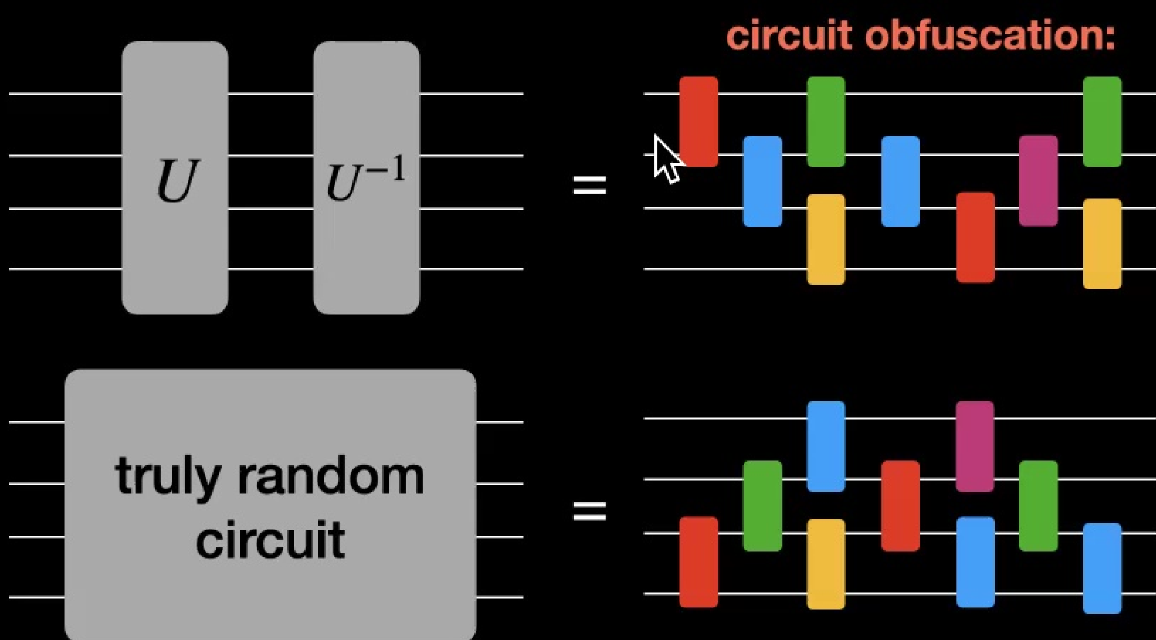
imperative to seek the next generation of quantum advantage experiments with a more solid theoretical foundation

Efficient verifiable quantum advantage

Scott Aaronson's proposal (from his talk, see also from Xiaodi Wu's github)



a simple circuit looks like a random circuit

- hard to distinguish by classical computers
- easy to verify by quantum computers

1. theoretical foundation has not been established
2. the effect of noise is unclear

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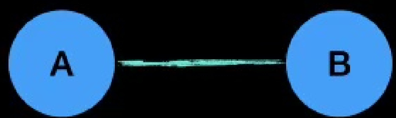
Efficient verifiable quantum advantage



Alice measure
X or Z basis

Bob's state: one of
$|0\rangle, |1\rangle, |+\rangle, |-\rangle$
but not know which one

then measure
X+Y or X-Y basis

"computational Bell test":

Kahanamoku-Meyer, Gregory D., Soonwon Choi, Umesh V. Vazirani, and Norman Y. Yao. *Nature Physics* (2022)

quantum computer

no Alice
Bob prepares one of the four states
not know which one

classical crypto technique:
solving $x^2 \equiv b \mod N$

no communication —>
computational hardness

similar to Shor's alg & QKD
but the circuit size is smaller

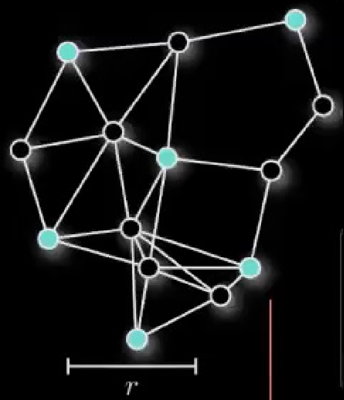
trade-off between
problem size vs. fidelity

Current status: high many-body fidelity (fidelity of the whole circuit) required
more careful analysis of the effect of noise is required

Outline

- What is quantum computational advantage (a.k.a. quantum supremacy)? Why is it important?
- Has quantum computational advantage been achieved? severe loops & fundamental problems
- Next generation of quantum advantage protocols:
  - Efficient classical verification
  - Solving combinatorial optimization problems
  - Quantum Machine Learning (quantum ChatGPT?)

Classical Combinatoric Optimization Problems



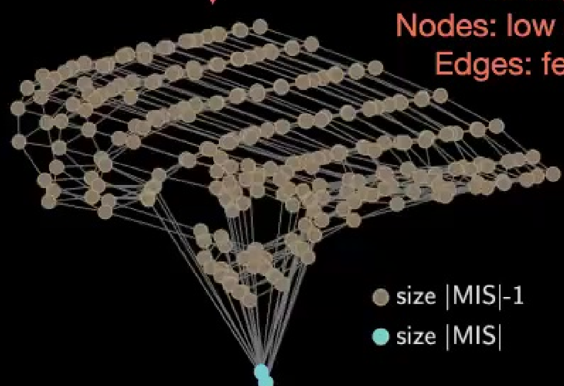
Rydberg blockade naturally encodes
MIS (maximum independent set) problems

Ebadi, Sepehr, ... **XG**, et al. "Quantum optimization of maximum independent set using Rydberg atom arrays." Science (2022)

shows evidence of super-linear quantum speedup for MIS instances

MIS: minimization of classical local Hamiltonian on a graph

Configuration graph
Nodes: low energy configurations
Edges: few-body transitions



quantum optimization algorithm can be viewed as **Quantum Walk** in the configuration graph

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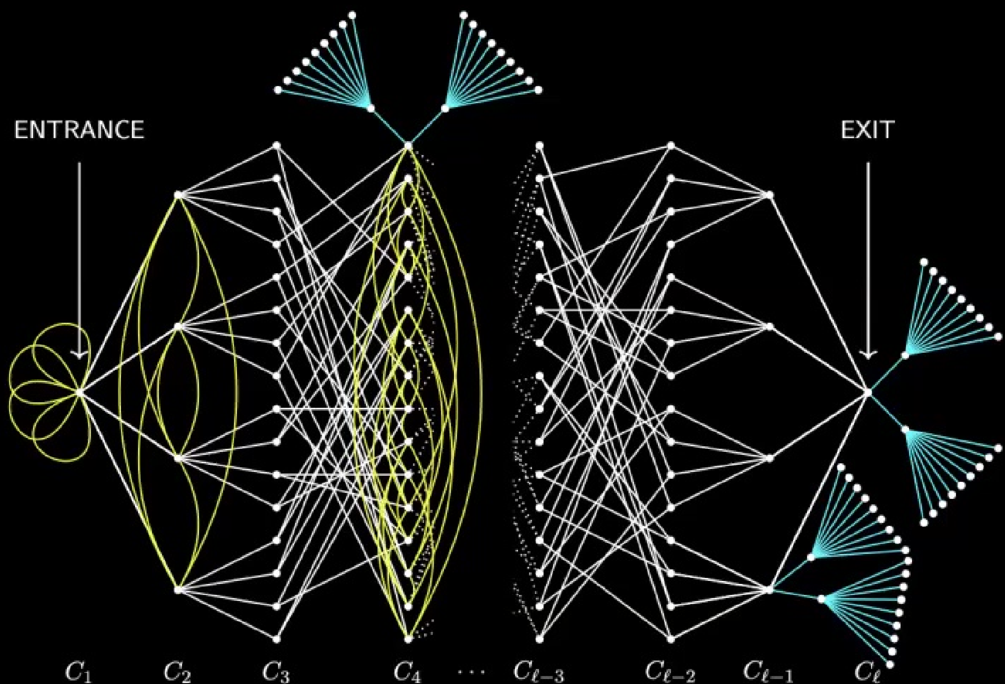
Potential for proving exponential speedup

Which kind of MIS instances can show **exponential** quantum speedup?



Gilyén, András, Matthew B. Hastings, and Umesh Vazirani. "(Sub) Exponential advantage of adiabatic Quantum computation with no sign problem." STOC 2021.

- configuration graph with exponential quantum speedup
- adiabatic algorithm can achieve this speedup

How to encode it by an MIS or other combinatorial problems?

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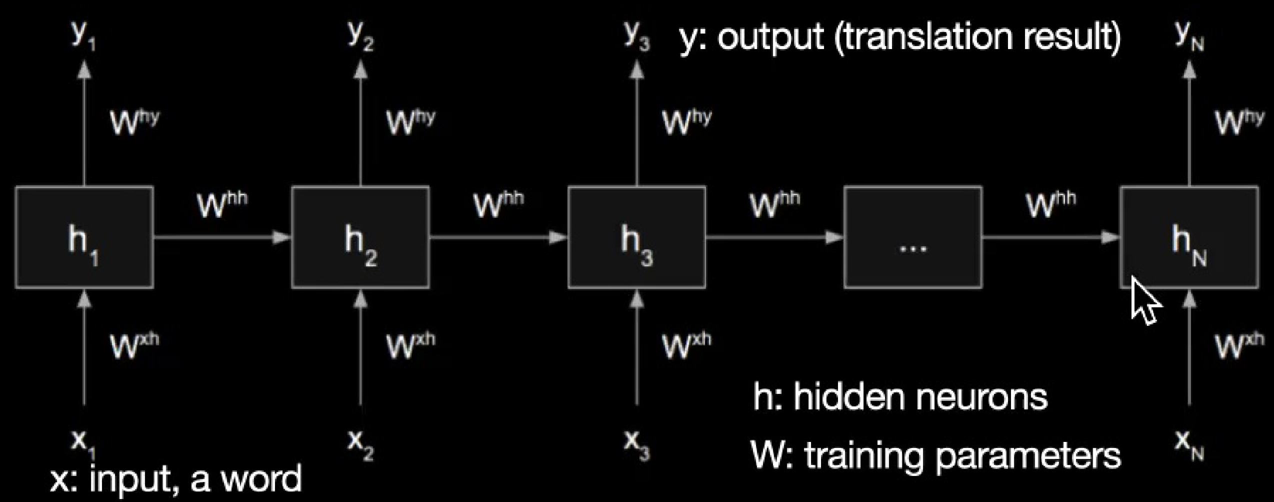
Quantum Advantage in Neural Sequence Modeling

Neural translation model



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Quantum Advantage in Neural Sequence Modeling

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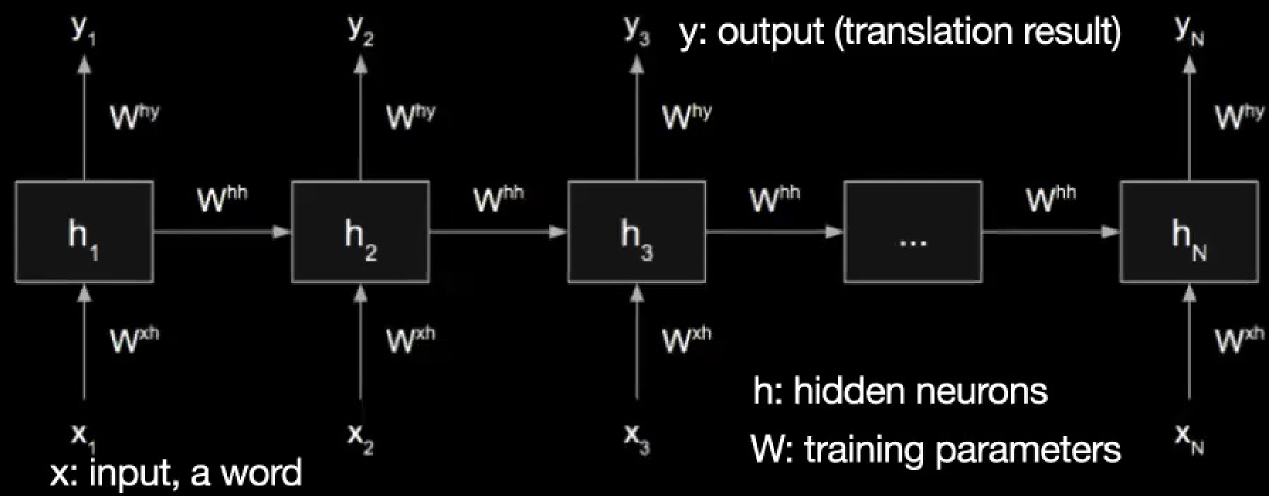

Our quantum model: Eric Anschuetz, Hongye Hu, Jinlong Huang, **XG** (ArXiv: 2209.14353)
x: description of measurement basis; y: measurement result; W: Gaussian unitary
$h \rightarrow |h\rangle$ quantum states to carry information along time direction

Concretely, a slight modification of Gaussian bosonic systems **(measure $e^{i(a\hat{x}+b\hat{p})}$)**

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Rigorous proof based on quantum contextuality

Rigorous proof: there exist formal language translation problems, such that
1. quantum model: n hidden neurons (bosonic modes);
2. any classical models: at least $n^2/2$ hidden neurons **due to contextuality**.

See also similar results for discrete models (Hidden Markov Models vs. Matrix Product State):
XG, Anschuetz, E. R., Wang, S. T., Cirac, J. I., & Lukin, M. D. PRX (2022).

+1	+1	+1
+1	−1	−1
−1	+1	?

- Product of each row = +1
- Product of each column = -1

no solution

$Z\otimes I$	$I\otimes Z$	$Z\otimes Z$
$I\otimes X$	$X\otimes I$	$X\otimes X$
$Z\otimes X$	$X\otimes Z$	$Y\otimes Y$

- Product of each row = +I
- Product of each column = -I
- Each row/column commute (called contexrt)

solution by operators

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Quantum contextuality vs. linguistic contextuality

quantum contextuality	to predict measurement results need to memorize the "context"	constraints in a context
linguistic contextuality	the meaning of a word depends on the context	grammar, fixed phrases, etc.

Numerical results: Spanish-to-English translation

Input	"Debemos limpiar la cocina."
Truth	"We must clean up the kitchen."
CRNN	"We must clean the kitchen."
GRU	"We have to turn the right address."

Input	"Admití que estaba equivocada."
Truth	"I admitted that I was wrong."
CRNN	"I was wrong to say that."
GRU	"They had a thing to be true."

CRNN: Contextual Recurrent Neural Network which is the quantum model
GRU (gated-recurrent-unit): (almost) state-of-the-art classical deep learning model (variation of LSTM)
here we restrict both models with **just 26 neurons** s.t. we can simulate CRNN

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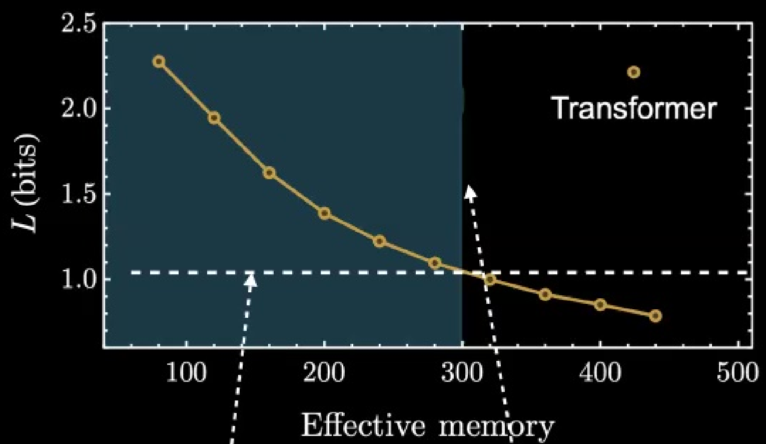
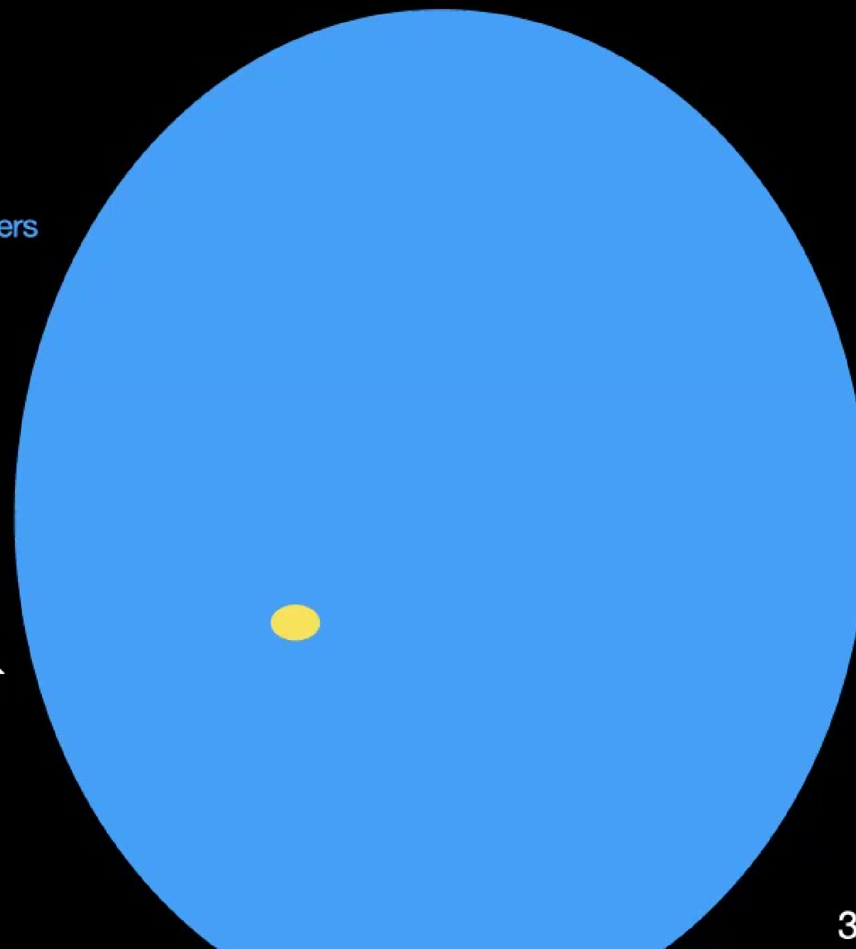
Towards quantum ChatGPT

Numerical results: Spanish-to-English translation



size of ChatGPT:
17×10^{10} parameters

assume $n^2/2$ scaling
our model: 6×10^5



our model at $n = 26$

Transformer has a
similar performance at $\sim n^2/2$
the same as our proof for RNN

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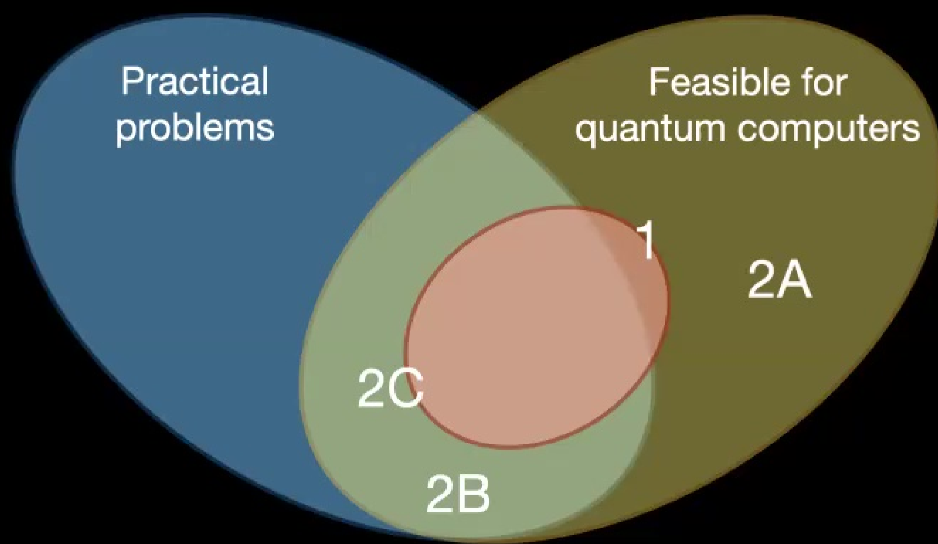
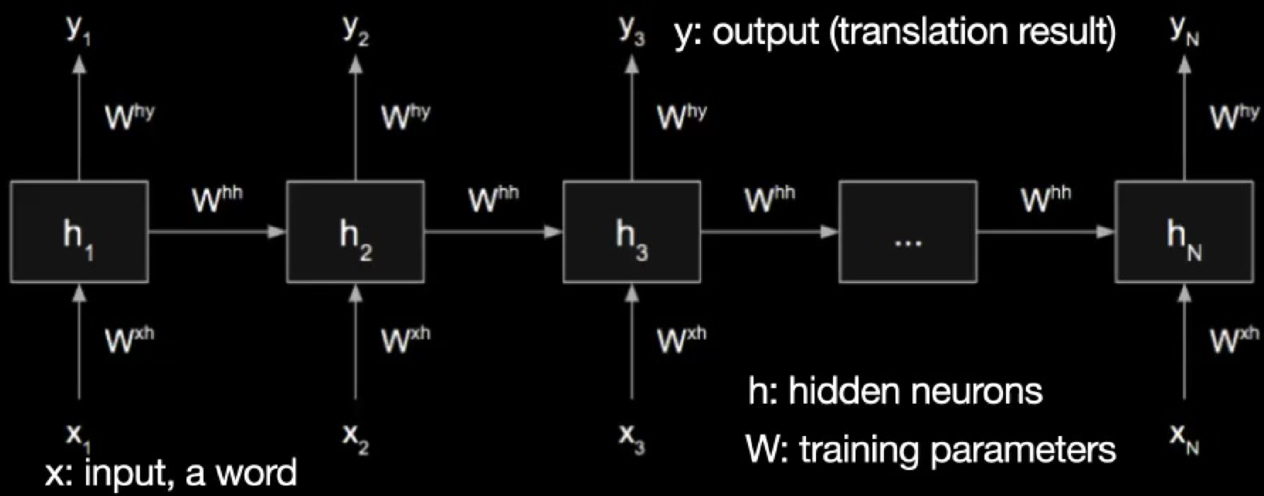
Summary

1. In current quantum advantage experiments: severe loopholes & fundamental problems
2. The next generation quantum advantage:
   A. efficient verifications
   B. optimization
   C. machine learning



Quantum Advantage in Neural Sequence Modeling

Neural translation model



Quantum Advantage in Neural Sequence Modeling

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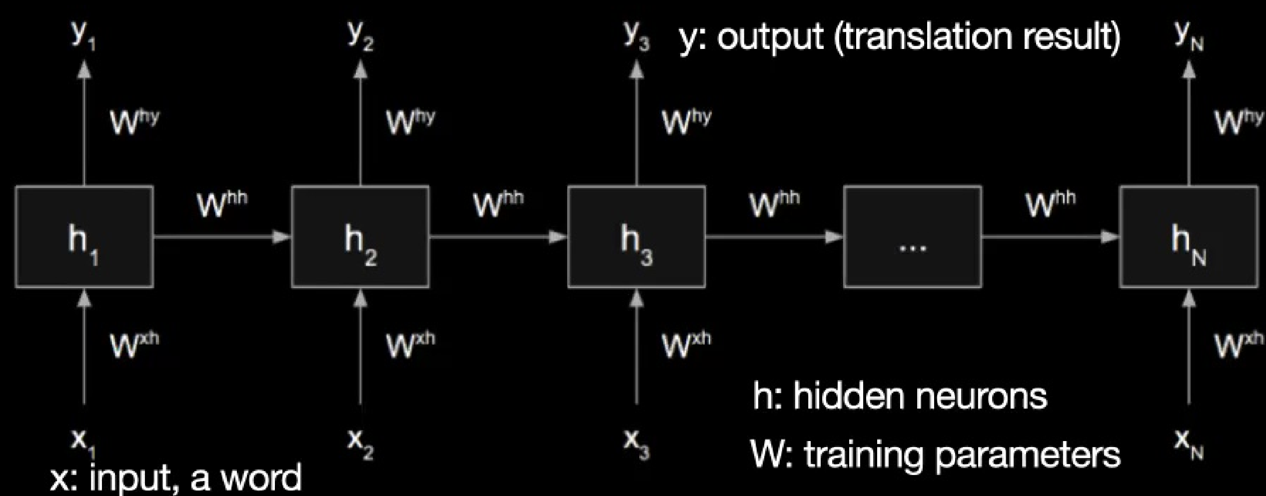

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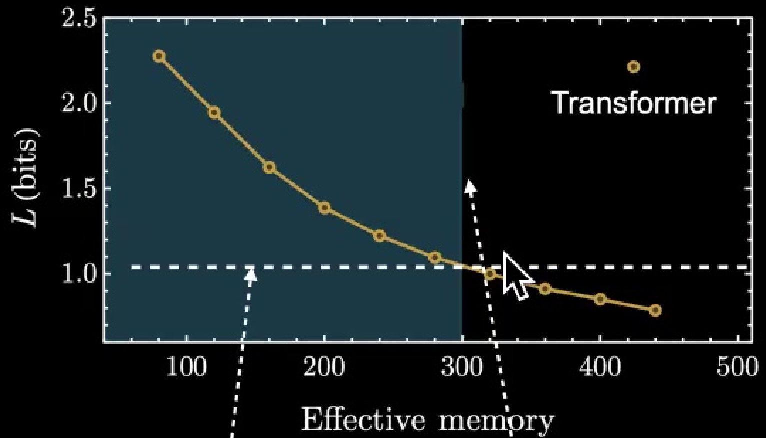

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