Title: Deriving the Simplest Gauge-String Duality

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Series: Quantum Fields and Strings

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Abstract: The simplest large N gauge theory is, arguably, the Gaussian matrix (or more generally, one hermitian matrix) integral. We will explicitly show that arbitrary correlators of single trace operators in this theory (without any double scaling limit) are identical to corresponding physical correlators in a dual topological string description. We will present both a novel A-model dual and also a mirror B-model Landau-Ginzburg description. The equality of correlators arises via open-closed-open string triality and a surprising relation to the c=1 string theory. The goal will be, however, to go beyond demonstrating equality but rather to make the duality manifest. For the B-model description this involves Eynard's recasting of topological recursion relations in terms of intersection numbers on moduli space. For the A-model this goes through the relation of Gaussian correlators to the special Belyi covering maps or equivalently, discrete volumes of moduli space. Finally, we also briefly mention the significance of these results for the gauge-string duality of N=4 Super Yang-Mills theory.

Zoom Link: https://pitp.zoom.us/j/93961992131?pwd=MkxlekxWU1ZGbzNZeWpzeU1TMzMrdz09

DERIVING THE SIMPLEST GAUGE-STRING DUALITY

Rajesh Gopakumar

Perimeter Institute, 10th Feb. 2023

Based on work w/ Edward Mazenc:

DSD I: Open-Closed-Open Triality (hep-th/2212.05999)

DSD II: B-Model (to appear)

DSD III: A-Model (to appear)

THE SIMPLEST LARGE N THEORY

- $\langle \mathrm{Tr}M^{l_1} \ldots \mathrm{Tr}M^{l_n} \rangle_g = \int [DM]_{N\times N}\, e^{-\frac{N}{2t}\mathrm{Tr}M^2}\, \mathrm{Tr}M^{l_1} \ldots \mathrm{Tr}M^{l_n}\Big|_g \equiv \mathcal{N}_{g,n}(l_1, \ldots, l_n)$

- Feynman diags. (Wick contractions) encode the combinatorics of the free large N 'Hooft expansion.

- Generalises to an interacting matrix model $\mathrm{Tr}M^2 \to \mathrm{Tr}V(M)$. [BIPZ'80][Cf. Gross-Migdal; Douglas-Shenker; Brezin-Kazakov]

- Ought to have a dual string description - without a double scaling limit [Cf. Dijkgraaf-Vafa]. A stripped-down-to-essentials version of gauge string duality - both `tensionless' (free) and away from Gaussian.

- Can we derive such a duality? Make equality of correlators manifest? (Cf. AdS_3/AdS_5 in tensionless limit) [Eberhardt-Gaberdiel-R.G.'18-19; Gaberdiel-R.G.-Knighton-Maity '20;Gaberdiel-R.G.'21].

ROADMAP

- Proposing the Simplest Gauge-String Duality
- Verifying the Simplest Gauge-String Duality
- Deriving the Simplest Gauge String Duality
- The Big(ger) Picture

BARE BONES GAUGE-STRING DUALITY

Gaussian Matrix Model:
$\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1}\ldots\frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g$

Open-Closed
Duality

A-Model Kazama-Suzuki on $\frac{SL(2,\mathbb{R})_1}{U(1)}$
(with mom. background)
$\langle V^{l_1}\ldots V^{l_n}\rangle_g$

Mirror
Symmetry

B-Model: Top. L-G theory $W(Z)=\frac{1}{Z}+tZ$
$\langle \mathcal{T}_{-l_1}(Z)\ldots\mathcal{T}_{-l_n}(Z)\rangle_g$

THE SIMPLEST DUALITY CHAIN COMPLEX

Gaussian Matrix Model: $\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1}\ldots\frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g$

Open-Closed Duality [R.G.-Mazenc]

A-Model: $\frac{SL(2,\mathbb{R})_1}{U(1)}$ (With Mom. Deformation) $\langle V^{l_1}\ldots V^{l_n}\rangle_g$

Mirror Symmetry

B-Model: Top.L-G theory $W(Z) = \frac{1}{Z} + tZ$ $\langle \mathscr{T}_{-l_1}(Z)\ldots\mathscr{T}_{-l_n}(Z)\rangle_g$

Open-Closed-Open Triality [R.G.-Mazenc]

[Mukhi-Vafa] [Nakamura-Niarchos] [Ashok-Murthy-Troost]

[Ghoshal-Mukhi; Hanany-Oz-Plesser]

Imbimbo-Mukhi Matrix Model (with deformation)

[Cf. Kontsevich-Penner Model Chekhov et.al.; Bonora-Xiong]

[W_∞ identities]

$c = 1$ String theory at self dual radius (with T_2 background)

[Gross-Klebanov] [Plesser-Moore-Ramgoolam]

[Ghoshal-Vafa] [Ooguri-Vafa; Aganagic et.al]

B-Model on deformed conifold [Dijkgraaf-Vafa]

VERIFYING THE SIMPLEST DUALITY

Gaussian Matrix Model:
$\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1} \ldots \frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g$

Open-Closed-Open Triality

Exact equality of n-point correlators for all genus g.

Imbimbo-Mukhi Matrix Model (with deformation)

$$Z(t,\bar{t}) = \frac{1}{Z_N}\int [DK][DM]_{N\times N}\, e^{\frac{1}{g}\mathrm{Tr_N}[V(K)-K(M-Y)]} \prod_{a=1}^{Q} det_N(x_a - M)$$

$$= \frac{1}{Z_Q}\int [DA][DB]_{Q\times Q}\, e^{-\frac{1}{g}\mathrm{Tr_Q}[V(A)+A(B-X)]} \prod_{i=1}^{N} det_Q(y_i - B)$$

Using Hubbard-Stratonovich trick with fermions. [cf. Maldacena-Moore-Seiberg-Shih; Aganagic-Dijkgraaf-Klemm-Marino-Vafa; Goel-H. Verlinde]

For the Gaussian case: [Altland-Sonner]

(`Compact Branes' - ZZ Branes)

$$Z_G(t,\bar{t}) = \frac{1}{Z_N}\int [DM]_N\, e^{-N\mathrm{Tr_N}[\frac{1}{t_2}M^2+\sum_k \bar{t}_k M^k]} \qquad [N\bar{t}_k = \frac{1}{k}Tr_Q X^{-k}]$$

(`Noncompact Branes' - FZZT Branes)

$$= \frac{Z_{penner}}{Z_Q}\int [DA]_{Q\times Q}\, e^{-\frac{1}{g}\mathrm{Tr_Q}[A^2-AX]-(N+Q)\mathrm{Tr}\ln A} = Z_{IM}(t_2 \neq 0, \bar{t}_k)$$

Exchanges graphs with dual graphs - "V-type" ↔ "F-type" duality [R. G.-Jo'burg workshop'11; Jiang-Komatsu-Vescovi].

VERIFYING THE SIMPLEST DUALITY

Gaussian Matrix Model:
$\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1} \ldots \frac{1}{l_n}\mathrm{Tr}M^{l_n} \rangle_g$

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$\mathrm{Tr}K^2$

$$Z(t,\bar{t}) = \frac{1}{Z_N}\int [DK][DM]_{N\times N}\, e^{\frac{1}{g}\mathrm{Tr_N}[V(K)-K(M-\cancel{Y})]} \prod_{a=1}^{Q} det_N(x_a - M)$$

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$$Z_G(t,\bar{t}) = \frac{1}{Z_N}\int [DM]_N\, e^{-N\mathrm{Tr_N}[\frac{1}{t_2}M^2 + \sum_k \bar{t}_k M^k]} \qquad [N\bar{t}_k = \frac{1}{k}Tr_Q X^{-k}]$$

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Gaussian Matrix Model:
$\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1} \dots \frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g$

Open-Closed-Open Triality

Exact equality of n-point correlators for all genus g.

Imbimbo-Mukhi Matrix Model (with deformation)

$[W_\infty$ identities$]$

$c = 1$ String theory at self dual radius (with T_2 background and $\mu = -iN$)

$$Z_G(t,\bar{t}) = \frac{1}{Z_N}\int [DM]_N \, e^{-N\mathrm{Tr_N}[\frac{1}{t_2}M^2 + \sum_k \bar{t}_k M^k]} \qquad [N\bar{t}_k = \frac{1}{k}Tr_Q X^{-k}]$$

$$[\mu = -iN]$$

$$\propto \frac{Z_{penner}}{Z_Q}\int [DA]_{Q\times Q}\, e^{i\mu \mathrm{Tr_Q} AX - (i\mu + Q)\mathrm{Tr}\ln A - i\mu t_2 \mathrm{Tr_Q} A^2} = Z_{IM}(t_2 \neq 0, \bar{t}_k)$$

Captures the S-Matrix of c=1 string theory of -ve momentum Tachyons in the presence of T_2 background

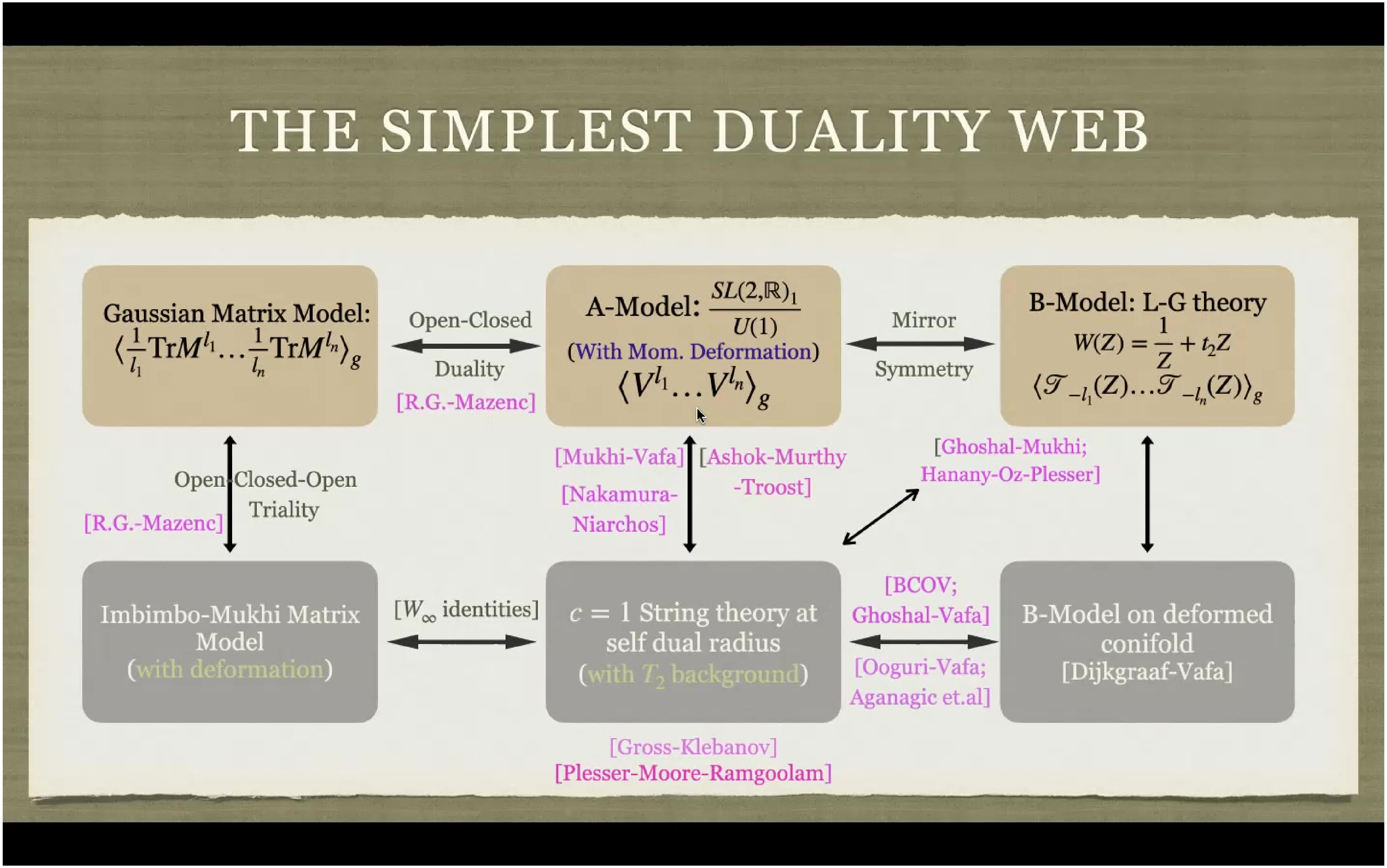
[Moore-Plesser-Ramgoolam]

Exact expression for all genus in an expansion in $\mu^{2-2g} = (-1)^g N^{2-2g}$

Sanity checks: Gaussian 1-pt. function $\langle \mathrm{Tr}_N M^{2l}\rangle = \langle\langle T_{-2l}\rangle\rangle \propto \langle (T_2)^l T_{-2l}\rangle$ for all genus

[R.G.-Mukhi ('95) - unpublished]

3 and 4 pt. functions at genus zero.


THE SIMPLEST DUALITY WEB
Gaussian Matrix Model: $\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1}\ldots\frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g$
Open-Closed
Duality
[R.G.-Mazenc]
A-Model: $\frac{SL(2,\mathbb{R})_1}{U(1)}$
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[R.G.-Mazenc]
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[Dijkgraaf-Vafa]
[Gross-Klebanov]
[Plesser-Moore-Ramgoolam]

DERIVING THE SIMPLEST DUALITY

Topological Recursion Relations (TRR)

Gaussian Matrix Model:

$N_{g,n} = \langle : \frac{1}{l_1}\mathrm{Tr}M^{l_1} : \ldots : \frac{1}{l_n}\mathrm{Tr}M^{l_n} : \rangle_g$

Counting Belyi Maps; Discrete Volumes of Moduli Space

B-Model: L-G theory

$W(Z) = \frac{1}{Z} + t_2 Z$

$\langle \mathcal{T}_{-l_1}(Z) \ldots \mathcal{T}_{-l_n}(Z) \rangle_g$

[Eynard-Orantin; Eynard]

[Bauer-Itzykson; R.G.; R.G-Pius; de Mello Koch-Ramgoolam; Razamat]

A-Model: $\frac{SL(2,\mathbb{R})_1}{U(1)}$

(With Mom. Deformation)

$\langle V^{l_1} \ldots V^{l_n} \rangle_g$

- Spectral curve $\mathcal{S}$ of MM: $X(z) = \frac{1}{z} + t_2 z;\ Y(z) = z$.
- Two branchpoints/critical points: $dX(z) = 0$.
- Machinery of TRR (with Bergman kernel $B(z, z')$) for

$W^{(\mathcal{S})}_{g,n}(z_1, \ldots z_n) = \langle \prod_{i=1}^{n} \mathrm{Tr}\left(\frac{dX(z_i)}{X(z_i) - M}\right) \rangle$ generates the $N_{g,n}$.

- Expressed in terms of integrals over moduli space.

- Correlators $N_{g,n}$ combinatorially account for special holomorphic Belyi maps.
- Covering maps $\Sigma_{g,n} \to \mathbb{P}^1$ of degree $\ell = \sum_i l_i$ with exactly three branch points $(0,1,\infty)$
- Branching profile $[l_i], \ldots, [l_n]$ at ∞; $[2]^{\frac{l}{2}}$ branching at 1.
- Via integer length Strebel differentials counts lattice points on moduli spaces [Mulase-Penkava; Norbury-Scott].

DERIVING THE SIMPLEST DUALITY

Topological Recursion Relations (TRR)

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DERIVING THE B-MODEL

Gaussian Matrix Model:
$N_{g,n} = \langle : \frac{1}{l_1}\mathrm{Tr}M^{l_1} : \ldots : \frac{1}{l_n}\mathrm{Tr}M^{l_n} : \rangle_g$

$\xrightarrow[\text{Intersection Numbers}]{\text{Topological Recursion Relns.}}$

B-Model: L-G theory
$W(Z) = \frac{1}{Z} + t_2 Z$
$\langle \mathcal{T}_{-l_1}(Z) \ldots \mathcal{T}_{-l_n}(Z) \rangle_g$

- $N_{g,n}$ (or $W_{g,n}$) are integrals over moduli space generalising Kontsevich's intersection numbers [Eynard]:

Mumford Kappa class

$$N_{g,n} = \langle \Lambda(\mathcal{S}) \prod_{i=1}^{n} \mathcal{O}_{l_i} \rangle_{\mathcal{M}^{(2)}_{g,n}} \qquad [\mathrm{Tr}M^l \leftrightarrow \mathcal{O}_l = \sum_{d=0}^{\infty} C_{k,d}\psi^d] \qquad [\Lambda(\mathcal{S}) = e^{\sum_{k=0} \tilde{s}_k \kappa_k} e^{\sum_\delta \sum_{k,l} l_{\delta *} \hat{B}_{k,l} \psi_k \psi_l}]$$

- $\mathcal{M}^{(2)}_{g,n}$: two copies of moduli space associated to the two branch points $dX = 0$ of spectral curve.
- View as moduli space of constant maps to the two critical points of the LG superpotential $dW(z) = 0$.
- 2d Top. Gravity w/ LG matter after integrating out the matter fields. Alternatively view solutions of TRR as LG matter integrals (after integrating out 2d gravity) [Cf. Losev]
- Also connection to Kodaira-Spencer theory on spectral curve: TRR as Schwinger-Dyson equations [Dijkgraaf-Vafa; Post-v.d-Heijden-E. Verlinde; Altland-Post-Sonner-v.d.Heijden-E. Verlinde].

DERIVING THE A-MODEL

Gaussian Matrix Model: $N_{g,n} = \langle : \frac{1}{l_1}\mathrm{Tr}M^{l_1} : \ldots : \frac{1}{l_n}\mathrm{Tr}M^{l_n} : \rangle_g$

$\xrightarrow[\text{Localisation as in } AdS_3]{\text{Belyi Branched Coverings}}$

A-Model: $\frac{SL(2,\mathbb{R})_1}{U(1)}$ (With Mom. Deformation) $\langle V^{l_1} \ldots V^{l_n} \rangle_g$

- Localisation to special points on moduli space also seen in tensionless limit of AdS_3 strings [Eberhardt-Gaberdiel-R.G.]. [cf. Komatsu-Maity]
- Followed from Ward Identities of $k = 1$, $sl(2,\mathbb{R})$ worldsheet theory for spectrally flowed vertex operators.
- Contributions only from points on moduli space which admit holomorphic covering maps to $\mathbb{P}^1$ with specified branching.
- Here also SUSY $sl(2,\mathbb{R})_1$ theory. Physical vertex operators V_l are in the $D^{(l)}_{j=\frac{1}{2}}$ repn [Ashok-Murthy-Troost].
- Hence same WI apply but no spacetime position dependence. Thus branching $[l_1]\ldots[l_n]$ at ∞: compactified cigar end.
- $T_2 = e^{i\sqrt{2}X}$ background → `clean Belyi maps' - with simple $[2]^{\frac{l}{2}}$-branching at 1 and Liouville wall interactions at 0.

THE BMN LIMIT

- In both B-Model and A-model pictures one recovers the observables of pure 2d gravity in a BMN-like limit. Zoom into the edge of the eigenvalue distribution - double scaling limit.

  [Cf. Okounkov;Okounkov-Pandharipande]

- B-Model: $\lim_{l_i\to\infty}\langle \frac{1}{l_1}\mathrm{Tr}M^{l_1}\ldots\frac{1}{l_n}\mathrm{Tr}M^{l_n}\rangle_g \propto \sum_{d_1+\ldots d_n=d_{g,n}}\langle\prod_{i=1}^{n}\psi_i^{d_i}\rangle_{\mathcal{M}_{g,n}}\prod_i v_i^{d_i+\frac{1}{2}}$. $\quad [l_i = \ell v_i]$

- A-Model: The discrete volumes $N_{g,n}$ of lattice points on $\mathcal{M}_{g,n}$ goes over in the large l_i limit to the continuum Kontsevich volumes. [Norbury]

- Similar continuum approach as in large twist limit of symmetric product CFTs dual to tensionless limit of AdS_3. [Gaberdiel-R.G.-Knighton-Maity]

THE BIGGER PICTURE

Gaussian Matrix Model ($\frac{1}{2}$-BPS sector) $\subset$ Chiral Algebra Sector $\subset$ $\mathcal{N} = 4$ Super Yang-Mills

Simplest Gauge-String Duality

Twisted Holography [Costello-Gaiotto; Budzik-Gaiotto; Costello-Paquette]

AdS/CFT Holography

B-Model with $w(Z) = \frac{1}{Z} + tZ \leftrightarrow$ Deformed Conifold $\subset$ B-Model on $SL(2,\mathbb{C}) \approx AdS_3 \times S^3$ $\subset$ IIB Strings on $AdS_5 \times S^5$

[Make the relation to the deformed conifold precise?]

[What is the relation to tensionless AdS_3/CFT_2?]

[Embed the matrix duality in tensionless AdS_5/CFT_4 ?]

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