Title: Non-Invertible Theta Symmetries

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Abstract: The modern point of view is that the global symmetries of a quantum field theory are described by topological defects/operators of the theory. In general such symmetries are non-invertible, i.e. the associated topological defects do not admit an inverse under fusion. I will describe a general construction of such non-invertible topological defects by coupling lower-dimensional topological quantum field theories (TQFTs) to discrete gauge fields living in a higher-dimensional bulk. The associated symmetries would be referred to as theta symmetries, as this construction can be understood as a generalization of the notion of theta angle. Mathematically, this construction is connected to interesting fusion higher-categories like those formed by higher-representations of groups and higher-groups. I will briefly explain this mathematical connection. I will also describe how the study of theta symmetries includes within it, as a special case, the study of topological phases of matter pursued in condensed matter physics. Towards the end of the talk, I will discuss some works in progress regarding possible physical applications of non-invertible symmetries. Based on ArXiv: 2212.06159, 2208.05973.

Zoom Link: https://pitp.zoom.us/j/92668739313?pwd=ZmdteFQybU9SbTlPNVQxV3l5dE5FQT09

Non-Invertible Theta Symmetries

Universal Symmetries and Beyond

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ArXiv: 2204.06564, 2208.05973, 2212.06159, 2212.06842

with Lea Bottini, Sakura Schafer-Nameki, Apoorv Tiwari and Jingxiang Wu

Invertible Global Symmetries: Physical Applications

(1) **IR Applications**: SSB ⟸ 't Hooft Anomaly

Degenerate Vacua or Moduli Space (Standard)

(Higher-)Gauge Theory, Deconfinement (Higher-form)

[Gaiotto, Kapustin, Seiberg, Willett 2014]

Moduli Stacks (Higher-Group)

[in progress LB, Bullimore, Ferrari, Schafer-Nameki]

Invertible Global Symmetries: Physical Applications

① **IR Applications**: SSB $\Longleftarrow$ 't Hooft Anomaly

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(Higher-)Gauge Theory, Deconfinement (Higher-form)
[Gaiotto, Kapustin, Seiberg, Willett 2014]

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② **"UV" Applications**: Operator Spectrum (especially CFTs)

(Standard)

1. $G = SU(2), \quad q(\mathcal{O}_1) \neq 0$ under $\mathbb{Z}_2 \implies \exists\, \mathcal{O}_2$
2. $G = S_3, \quad q(\mathcal{O}_1) \neq 0$ under $\mathbb{Z}_3 \implies \exists\, \mathcal{O}_2 \in E$

$$S_3 \text{ irreps} = \{1,\ 1_-,\ E\}$$

Invertible Global Symmetries: Physical Applications

② **"UV" Applications**: (Higher-Form)

$$q(D_p) \neq 0 \implies D_p \text{ ———————• } D_{p-1} = 0$$

i.e. D_p is not "screened"

Invertible Global Symmetries: Physical Applications

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Also, twisted sector operators: e.g. in $2d$

$$b \longrightarrow \bullet\ \mathcal{O}_1^{\pm}$$

$$\Downarrow$$

$$\exists \quad ab \longrightarrow \bullet\ \mathcal{O}_2^{\pm} \quad \text{and} \quad a^2 b \longrightarrow \bullet\ \mathcal{O}_3^{\pm}$$

In higher d, involves higher-representation theory of $H \subseteq G$

[in progress LB, Schafer-Nameki]

Invertible Global Symmetries: Physical Applications

(2) **"UV" Applications**: (Higher-Form)

$$q(D_p) \neq 0 \implies \quad D_p \longrightarrow\!\!\bullet \; D_{p-1} = 0$$

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(Higher-Group = Bockstein 2-Group) [LB 2021]

$$G^{(0)} = SO(3), \quad G^{(1)} = \mathbb{Z}_2, \quad \text{Postnikov Class} = w_3 = \text{Bock}(w_2)$$

$SO(3)$ $\quad L \quad L^2 \quad SU(2) \quad L^3$

$$q(\mathcal{O}) \neq 0 \text{ under } \mathbb{Z}_2 \implies \mathcal{O} \in \{\text{non-genuine local operators}\}$$

Can be used to infer existence of Bockstein 2-group symmetries.

[Apruzzi, LB, Oh, Schafer-Nameki 2021], [Apruzzi, LB, Gould, Schafer-Nameki 2021]
[LB 2021], [LB, Bullimore, Ferrari, Schafer-Nameki 2022]

More generally, operators form higher-representations of higher-group

[in progress LB, Schafer-Nameki]

Non-Invertible Symmetries: Physical Applications

[in progress LB, Schafer-Nameki]

① **IR Applications**:

$$\mathcal{S} \circlearrowright \text{QFT} \xrightarrow{\text{if gapped}} \mathcal{S} \circlearrowright \text{TQFT}$$

e.g. a gapped 2d QFT with Ising symmetry must have 3n vacua.

$$P^2 = 1, \quad PS = SP = 1, \quad S^2 = 1 + P$$

i.e. SSB (2 vacua) and non-SSB (1 vacuum) phases of $\langle P \rangle = \mathbb{Z}_2 \subset$ Ising come combined together.

(Side remark: There are also relative Euler terms between the vacua)

Non-Invertible Symmetries: Physical Applications

[in progress LB, Schafer-Nameki]

② **<u>"UV" Applications</u>**:

e.g. consider a $2d$ QFT with Ising symmetry.

$q(\mathcal{O}_\sigma) \neq 0$ under $\mathbb{Z}_2$ $\qquad$ $q(\mathcal{O}_\mu) = 0$ under $\mathbb{Z}_2$

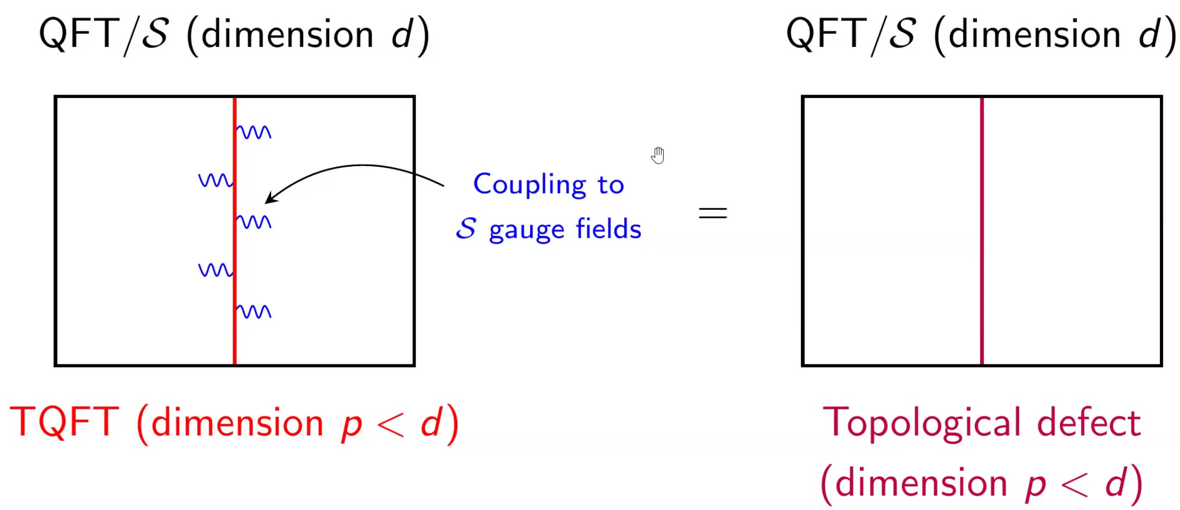
$\because$ $(\mathcal{O}_\sigma, \mathcal{O}_\mu)$ form an irreducible representation of Ising symmetry.

In Ising CFT:
1. $\mathcal{O}_\sigma$ can be taken as spin/order operator.
2. $\mathcal{O}_\mu$ is then the disorder operator.

Theta Symmetries: Construction

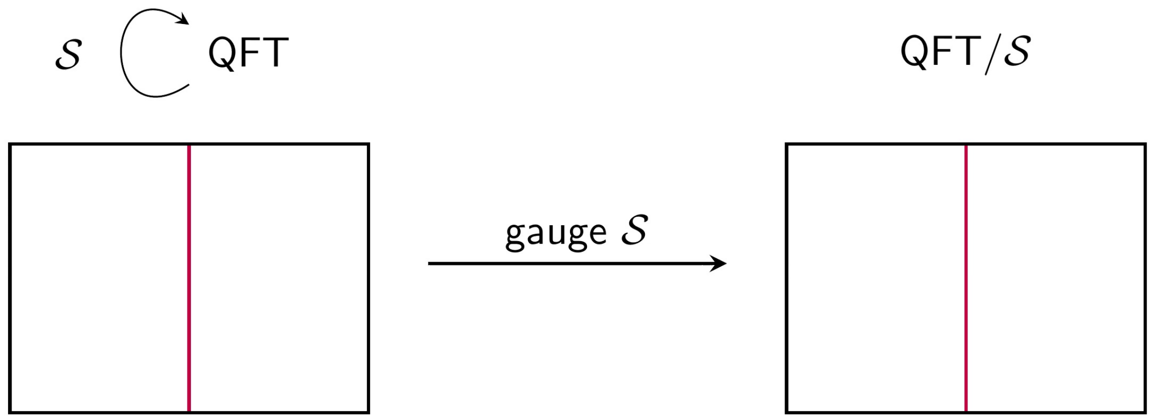
$$\mathcal{S} \circlearrowright \text{QFT} \xrightarrow{\text{gauge } \mathcal{S}} \text{QFT}/\mathcal{S}$$

$$\text{QFT}/\mathcal{S} \supseteq \mathcal{S} \text{ gauge fields}$$



Theta Symmetries: Construction (Equivalent)

[LB, Schafer-Nameki, Wu 2022]



$\mathcal{S}$-symmetric TQFT (dimension $p < d$)

Topological defect (dimension $p < d$)

- Analogous to Theta Angle ($p = d$, TQFT = Trivial).
- **Universal**. Carried by all QFT$/\mathcal{S}$.

Theta Symmetries: Structure for $p = 1$

- Take $\mathcal{S} = G$ standard 0-form symmetry (finite group).
- A 1d TQFT (TQM) is specified by a finite-dimensional vector space V (Hilbert space).
- A G-symmetric 1d TQFT requires additional specification of action of G on V, converting V into a representation R of G.
- Theta symmetries for $p = 1$ are specified by objects of $\text{Rep}(G)$.
- Fusion is non-invertible for non-abelian G.
- Can be identified with Wilson lines obtained after gauging.
- $\{\text{Theta Symmetries at } p = 1\} = \{\text{Dual/Quantum Symmetries}\}$.

Theta Symmetries: Structure for $p = 2$

- G-symmetric 2d TQFTs are classified (upto global Euler number counterterm) by:
  - $H \subseteq G$: spontaneously unbroken symmetry.
    Number of vacua $=$ Number of H-cosets.
  - $\sigma \in H^2(H, U(1))$: 2d SPT phase for unbroken symmetry H.
- A mixture of SPT and SSB phases.
- Theta symmetries for $p = 2$ are specified by objects of 2-Rep(G).
- Fusion is non-invertible even for abelian G!
- Generalization of dual/quantum symmetries.

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Example: $G = \mathbb{Z}_2$. Two irreducible 2-representations:

- Trivial 2-rep with dimension 1 (No SSB). Leads to identity surface.
- Non-trivial 2-rep with dimension 2 ($\mathbb{Z}_2$ SSB). Leads to non-identity topological surface $D_2^{(\mathbb{Z}_2)}$.
- Non-invertibility: $D_2^{(\mathbb{Z}_2)} \otimes D_2^{(\mathbb{Z}_2)} = 2D_2^{(\mathbb{Z}_2)}$.

Aside: Action of G on Lines

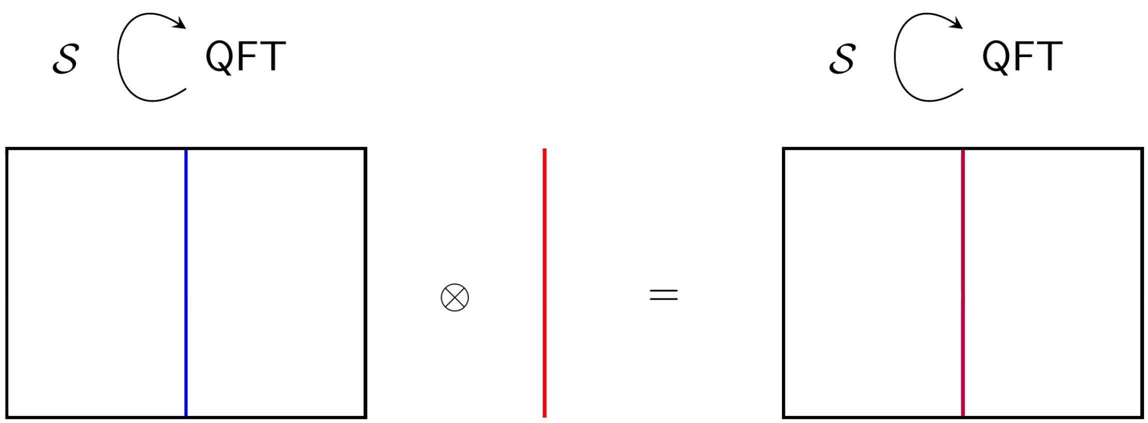
- Line operators in a d-dimensional QFT with standard 0-form symmetry group G form 2-representations of G.
- $H \subseteq G$ describes the subgroup that does not permute the line.
- $\sigma \in H^2(H, U(1))$ describes projective representation of H arising on the line.
  Known as symmetry fractionalization. [Delmastro, Gomis, Hsin, Komargodski 2022; Daniel Brennan, Cordova, Dumitrescu 2022]
- Local operators between lines are intertwiners (1-morphisms) between the two 2-representations.
- Very interesting structure for a 2-group $\mathfrak{G}$ and for higher-dimensional operators.

Theta Symmetries: Structure for $p \geq 3$

- Subtle. An infinite number of irreducible/simple 3d G-symmetric TQFTs.
- Due to the existence of topological order. Can have SET phases.
- Can be obtained by condensation/Karoubi completion of the trivial 3d TQFT. [Gaiotto, Johnson-Freyd 2019]
- At $p = 3$ should be classified by G-graded spherical fusion categories.

Twisted Theta Symmetries

[LB, Schafer-Nameki, Tiwari 2022]



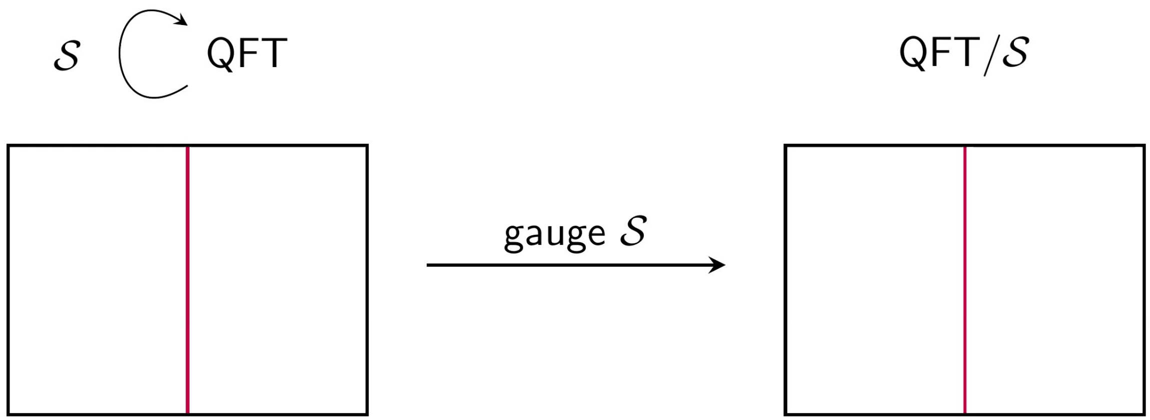
Topological Defect (dimension $p < d$)
Coupling to background $\mathcal{S}$ gauge fields not possible (anomaly, symm frac)

TQFT

Topological defect (dimension $p < d$)
Coupling to background $\mathcal{S}$ gauge fields possible

Twisted Theta Symmetries

[LB, Schafer-Nameki, Tiwari 2022]



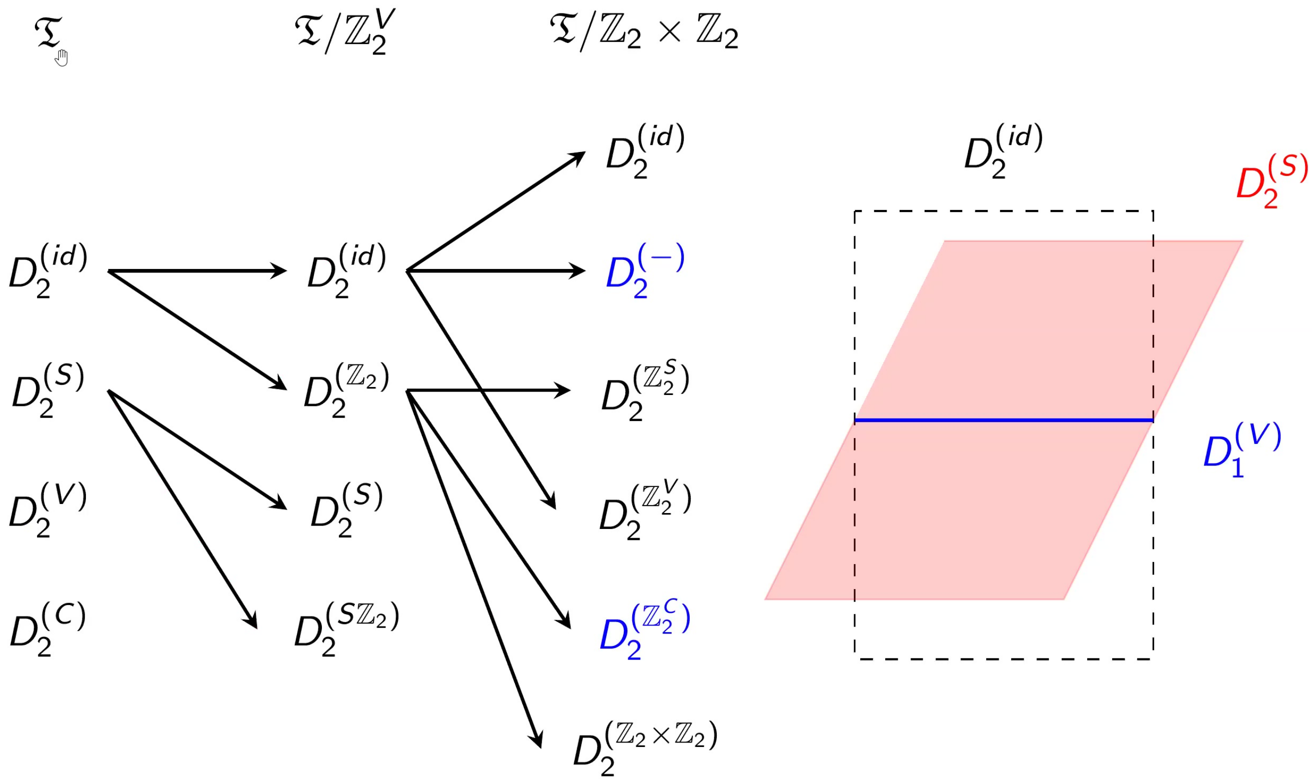
Topological Defect
(dimension $p < d$)
$+$ Choice of Coupling

Topological defect
(dimension $p < d$)

- Non-universal.
- Reproduces many known constructions of non-invertible symmetries.

[Kaidi, Ohmori, Zheng 2021; Choi, Cordova, Hsin, Lam, Shao 2021]

Example 1: Gauging $\mathbb{Z}_2 \times \mathbb{Z}_2$ by Gauging Two $\mathbb{Z}_2$



Summary

1. **Unification of Non-Invs**: (Twisted) Theta symmetries systematize/unify various constructions of non-invertible symmetries.
2. **Dual/Quantum Symmetries:** Theta symmetries are the most general dual/quantum symmetries that emerge after gauging a (higher-)group symmetry.
3. **Mathematical Connections**: These symmetries have deep connections with higher-representations and higher-projective representations of (higher-)groups. Physically, higher-reps can either be (dual/quantum) symmetries, or describe charged objects.
4. **Connections with Cond-Mat**: The study of theta symmetries includes within it the study of SPT, SSB, SET phases.
5. **Physical Applications:** Plethora of applications in constraining IR physics and the spectrum of local and extended operators.

Gauging $\mathbb{Z}_4$ by Gauging Two $\mathbb{Z}_2$ (Symmetry Frac.)