Title: Beyond the linear tide: impact of the non-linear tidal response of neutron stars on gravitational waveforms from binary inspirals

Speakers: Hang Yu

Series: Strong Gravity

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Abstract: Tidal interactions in coalescing binary neutron stars modify the dynamics of the inspiral, and hence imprint a signature on their gravitational-wave (GW) signals in the form of an extra phase shift. We need accurate models for the tidal phase shift in order to constrain the supranuclear equation of state from observations. In previous studies, GW waveform models were typically constructed by treating the tide as a linear response to a perturbing tidal field. In this work, we incorporate non-linear corrections due to hydrodynamic three- and four-mode interactions and show how they can improve the accuracy and explanatory power of waveform models. We set up and numerically solve the coupled differential equations for the orbit and the modes, and analytically derive solutions of the system's equilibrium configuration. Our analytical solutions agree well with the numerical ones up to the merger and involve only algebraic relations, allowing for fast phase shift and waveform evaluations for different equations of state over a large parameter space. We find that, at Newtonian order, nonlinear fluid effects can enhance the tidal phase shift by >~ 1 radian at a GW frequency of 1000 Hz, corresponding to a 10-20% correction to the linear theory. The scale of the additional phase shift near the merger is consistent with the difference between numerical relativity and theoretical predictions that account only for the linear tide. Nonlinear fluid effects are thus important when interpreting the results of numerical relativity, and in the construction of waveform models for current and future GW detectors.

Zoom link: https://pitp.zoom.us/j/91730134841?pwd=U0JXNHhFbWdQYno3aUpDWHRxWEtkUT09

Impact of the nonlinear tidal response of neutron stars on gravitational waveforms from binary inspirals

Hang Yu

Collaborators: N. N. Weinberg, P. Arras,

J. Kwon, T. Venumadhav

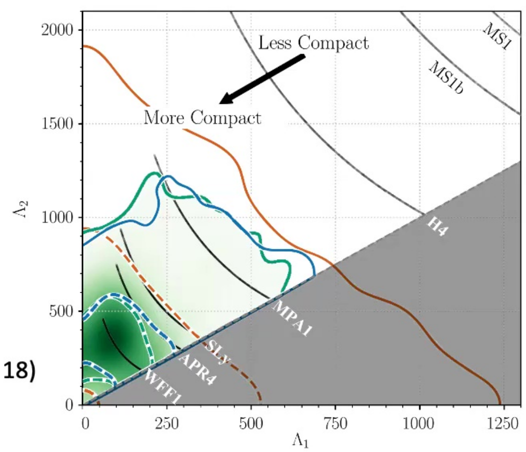
MNRAS 519, 4325 (2023)

Strong gravity seminar @ Perimeter Institute

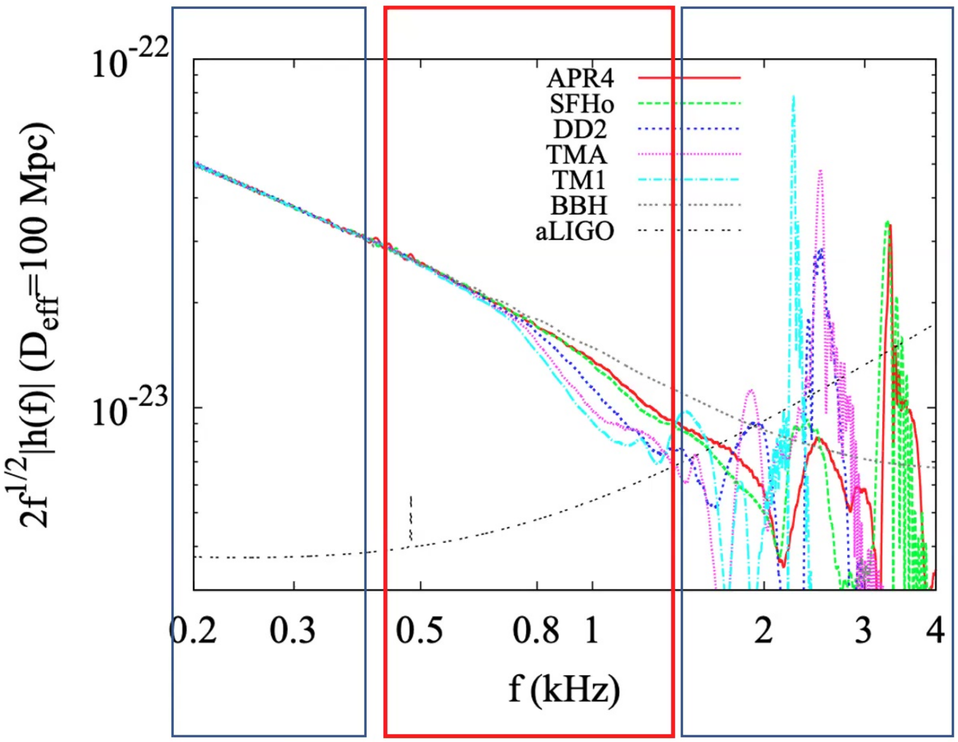
Feb 2, 2023

Introduction

- Binary neutron stars (BNSs):
  - sources for gravitational wave (GW) observations
- Tidal deformation: phase shift in the waveform
- Understanding extreme gravity
- NS equation of state
- E.g., GW170817



(LVC 18)



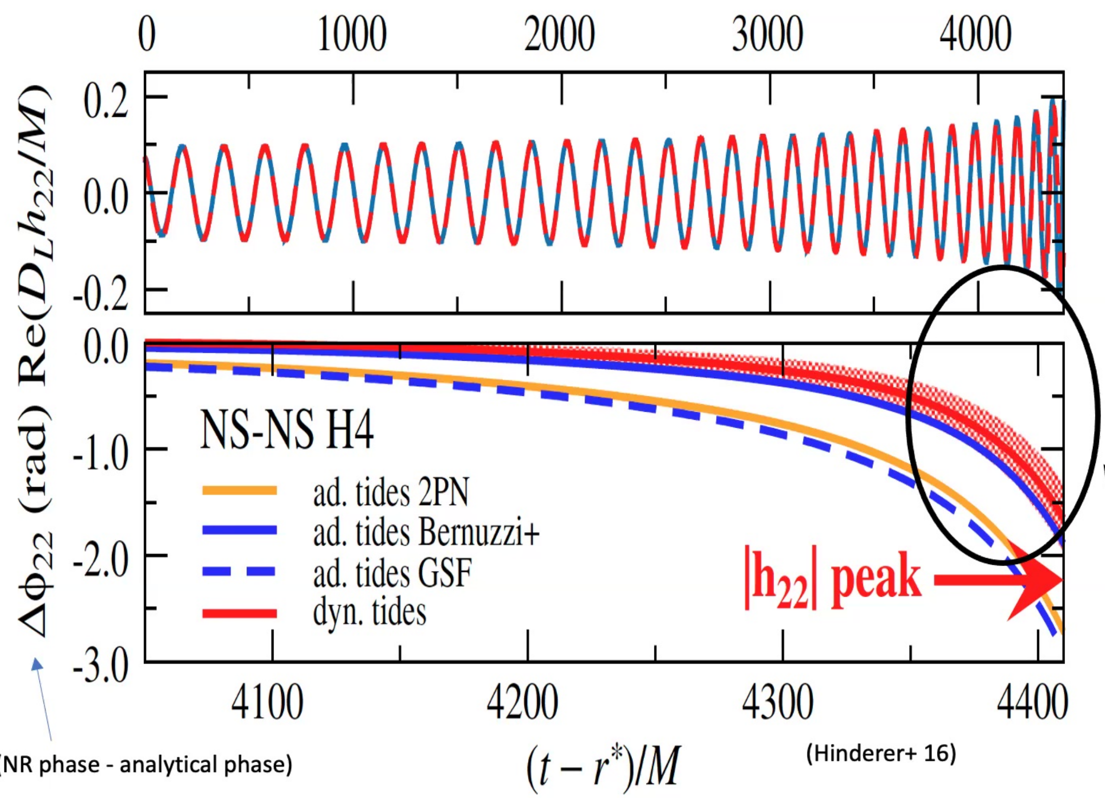
Point particles
<~ 300 Hz

Tidal deformation
400 - 1,500 Hz
(this talk!)

Remanent oscillation
>~ 2,000 Hz

(Hotokezaka+ 16)

Numerical relativity vs. linear tidal theory

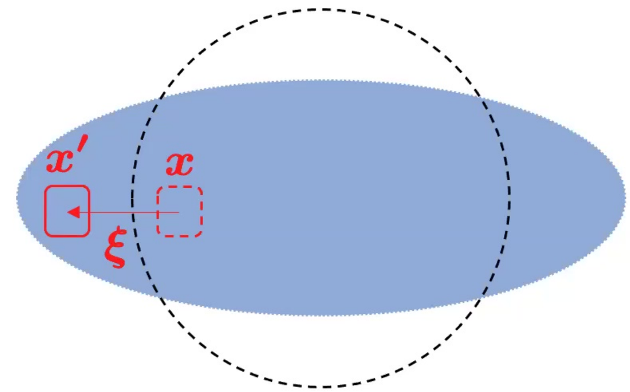


(NR phase - analytical phase)

(Hinderer+ 16)

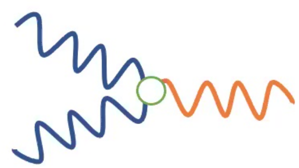
- NR: accurate but expensive
  (Hotokezaka+ 15; Foucart+ 19; …)
- Analytical waveforms:
  - Efficient generation
  - Insights to physics
  (Lai+93, 94a, 94b; Flanagan & Hinder 08; Hinderer+10; Bini & Damour 14; Bernuzzi 15; Hinderer+ 16; Nagar+ 18; Steinhoff+ 21; …)
- State-of-the-art waveforms include just the linear tide
- Y-axis: (NR phase - analytical phase)
- Linear theory misses~ 1 rad!

Nonlinear tide: the explanation?



- Lagranian displacement $\boldsymbol{\xi}$ = $\boldsymbol{x}'$ - $\mathbf{x}$
- Nonlinear in $\boldsymbol{\xi}$
- Conservative effect (not the pg-instability)

Three-mode interaction



$$H_{3\mathrm{md}} = -\frac{E_0}{3}\sum_{abc}\kappa_{abc}c_a c_b c_c$$

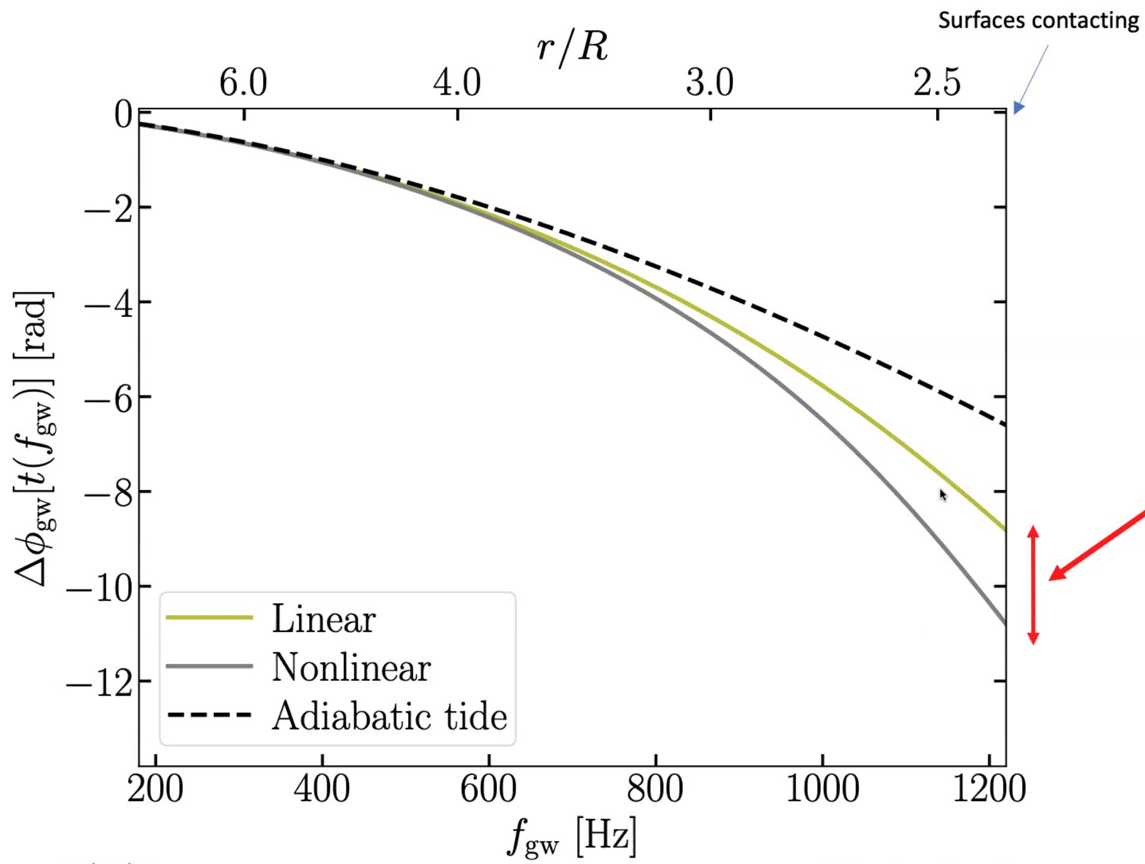
$$\rho\ddot{\boldsymbol{\xi}} = \boldsymbol{f}_1[\boldsymbol{\xi}] + \boldsymbol{f}_2[\boldsymbol{\xi},\boldsymbol{\xi}] + \rho\boldsymbol{a}_{\mathrm{tide}}.$$

O($\boldsymbol{\xi}^2$) contribution to the mass quadrupole

$$H_{\mathrm{int}} = \frac{1}{2}\mathcal{E}_{ij}Q^{ij}_{\mathrm{ns}}$$

$$\boldsymbol{a}_{\mathrm{tide}} = -\frac{1}{\rho}\frac{\delta H_{\mathrm{int}}}{\delta\boldsymbol{\xi}} = -\boldsymbol{\nabla}U - (\boldsymbol{\xi}\cdot\boldsymbol{\nabla})\boldsymbol{\nabla}U.$$

Nonlinear tide: the explanation?

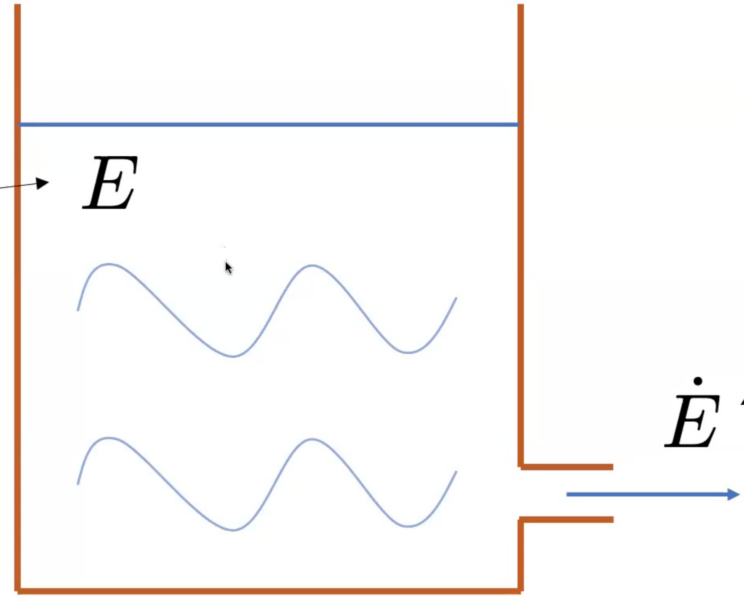


- The nonlinear tide enhances the phase shift by ~1 rad.
- Consistent with the discrepancy

- How does the tide affect the GW signal?

Point-particle case

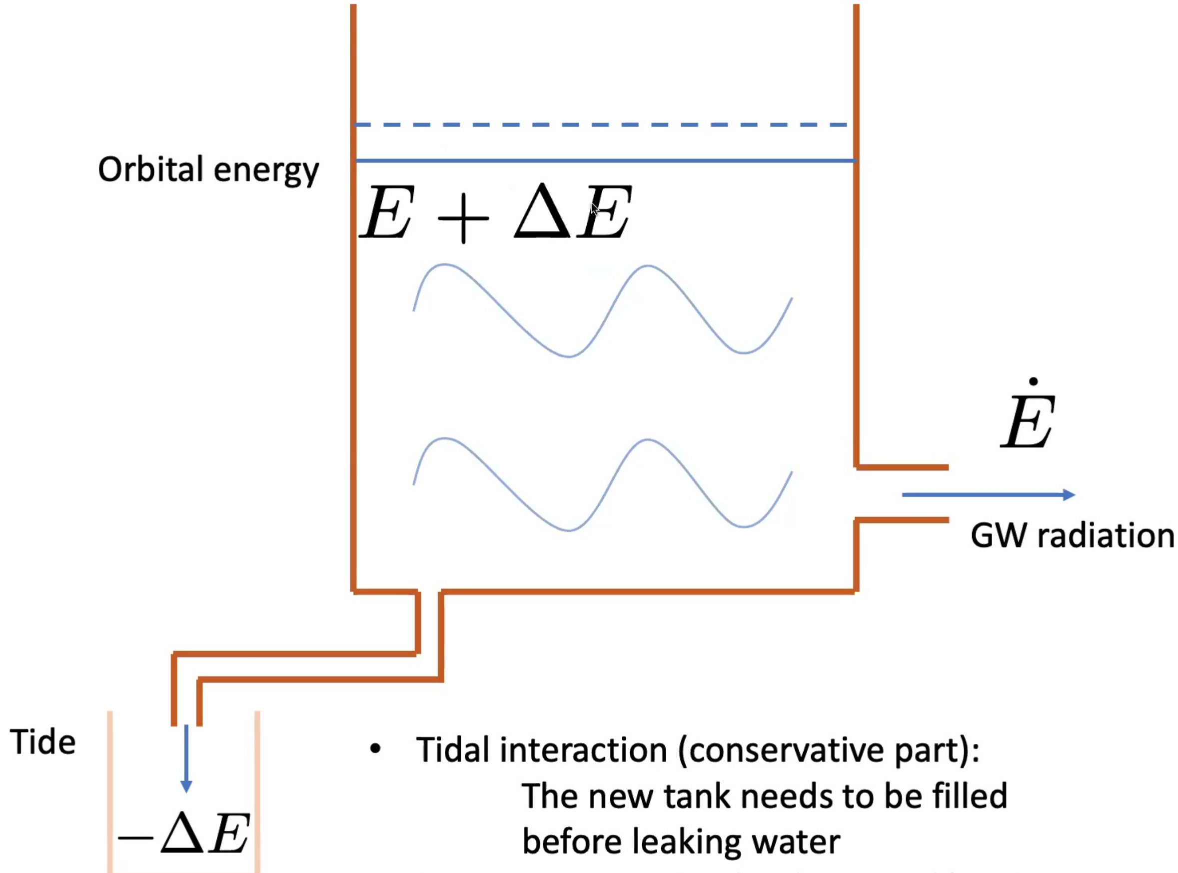
Orbital energy:
Tank of water



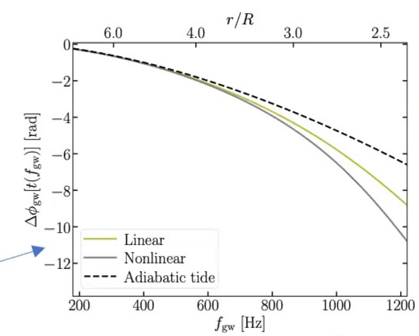
GW radiation:
Hole on the tank

What we measure from the GW signal:
Accumulated phase of the waveform
-> time to empty the tank

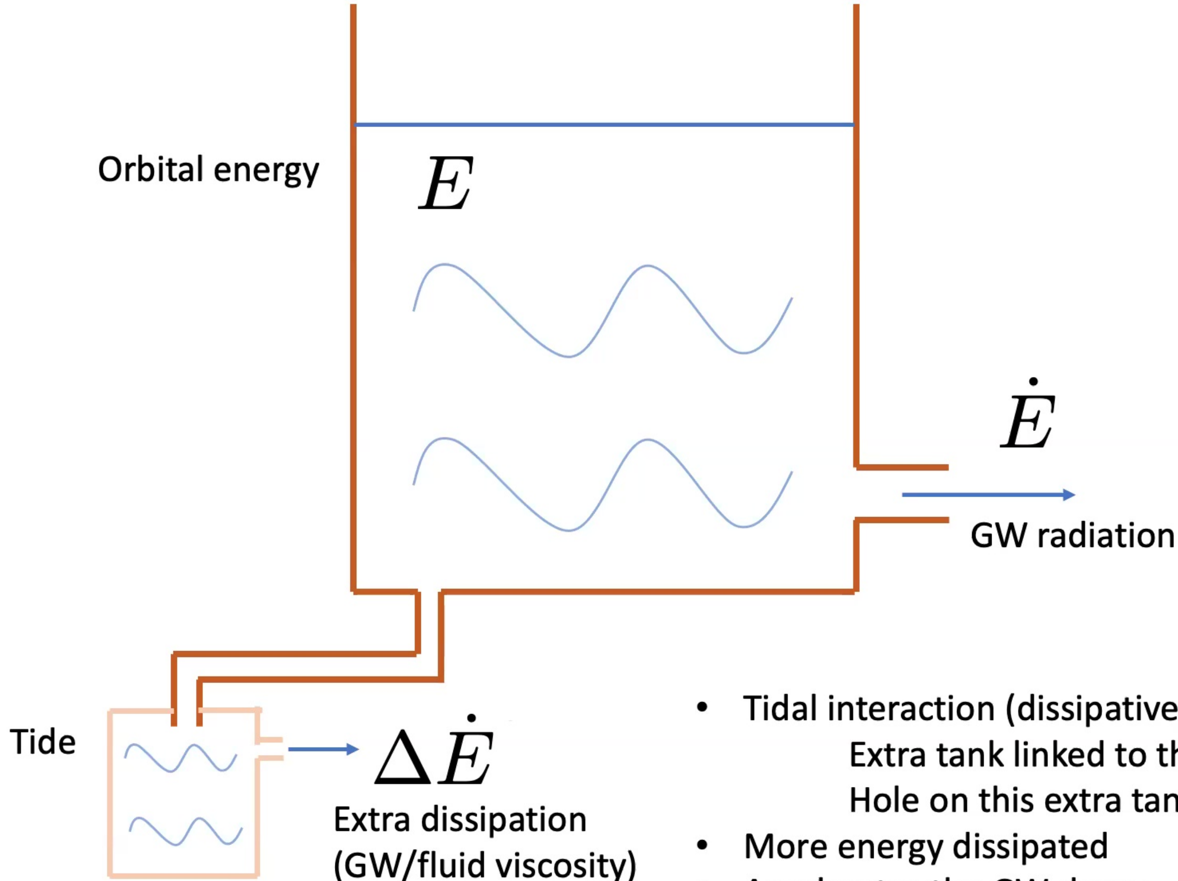
Conservative tidal interaction (this work!)



- Tidal interaction (conservative part):
  The new tank needs to be filled before leaking water
- Less energy need to be dissipated by GW
- Faster GW decay (negative phase shift)



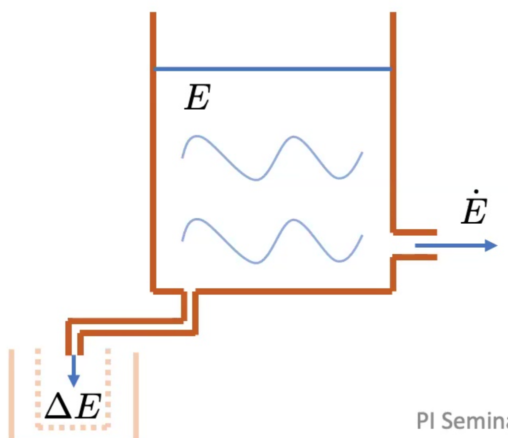
Dissipative tidal interaction



- Tidal interaction (dissipative part):
  - Extra tank linked to the main one
  - Hole on this extra tank
- More energy dissipated
- Accelerates the GW decay

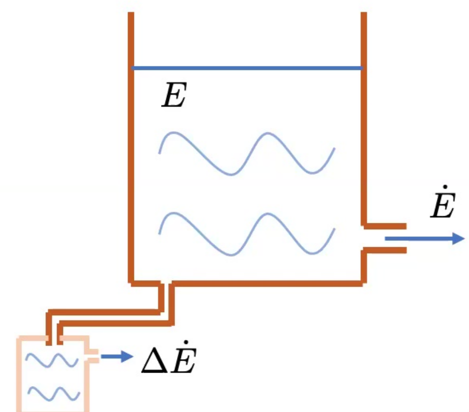
$$\frac{\mathrm{d}\Delta\phi_{\mathrm{gw}}}{\mathrm{d}f_{\mathrm{gw}}} = \frac{2\pi f_{\mathrm{gw}}}{\dot{E}} \left(\frac{\mathrm{d}\Delta E_{\mathrm{eq}}}{\mathrm{d}f_{\mathrm{gw}}} - \frac{\mathrm{d}E_{\mathrm{eq}}}{\mathrm{d}f_{\mathrm{gw}}} \frac{\Delta \dot{E}}{\dot{E}} \right).$$

- Conservative part
- Changing energy to be dissipated
- This work!



- Dissipative part
- Changing energy dissipation rate
- Nonlinear pg-instability

(Weinberg+ 13; Venumadhav+14; Weinberg 16; Essick+ 16; …
Work in progress by James Kwon, Teja Venumadhav, and HY)



$$\frac{\mathrm{d}\Delta\phi_{\mathrm{gw}}}{\mathrm{d}f_{\mathrm{gw}}} = \frac{2\pi f_{\mathrm{gw}}}{\dot{E}} \left(\underbrace{\frac{\mathrm{d}\Delta E_{\mathrm{eq}}}{\mathrm{d}f_{\mathrm{gw}}}} - \underbrace{\frac{\mathrm{d}E_{\mathrm{eq}}}{\mathrm{d}f_{\mathrm{gw}}} \frac{\Delta\dot{E}}{\dot{E}}} \right).$$

- Conservative part
- Changing energy to be dissipated
- This work!

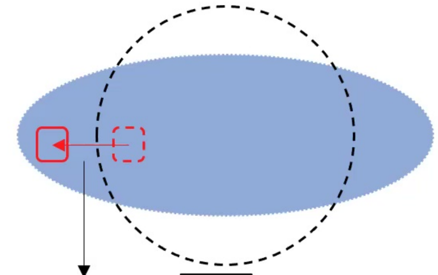
- Dissipative part
- Changing energy decay rate
- Nonlinear pg-instability

(Weinberg+ 13; Venumadhav+14; Weinberg 16; Essick+ 16; …)

This work	Nonlinear pg-instability
Conservative (up to the GW radiation from f-modes)	Dissipative
Important >~ 1000 Hz	Important ~ 50-100 Hz
Interaction among f-modes (long wavelength)	Coupling with high-order p/g-modes (very short wavelength)
Resolvable in numerical relativity	Cannot be resolved by NR

- Why is nonlinear tide important?

How much energy transferred to the tide?



$$\boldsymbol{\xi} = \sum_a c_a \boldsymbol{\xi}_a$$

Mode amplitude

$$\ddot{c}_a + \omega_a^2 c_a = \omega_a^2 U_a$$

Harmonic oscillator

$$\omega_a \simeq \sqrt{M/R^3} \simeq 2\pi \times 2000\,\mathrm{Hz}$$

Tidal forcing

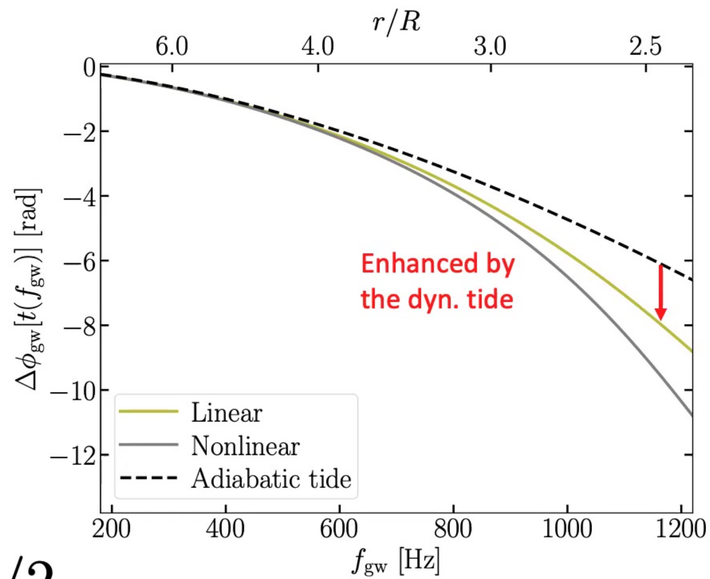
$$U_a \sim \exp(-i2\Omega t)$$

- Prior to resonance, $c_a \simeq \dfrac{\omega_a^2}{\omega_a^2 - (2\Omega)^2} U_a$
- Most part of the inspiral, equilibrium/adiabatic tide. (Lai+93, 94a, 94b; Flanagan & Hinderer 08; Hinderer+10; …)

How much energy transferred to the tide?

$$\ddot{c}_a + \omega_a^2 c_a = \omega_a^2 U_a$$

$$c_a \simeq \frac{\omega_a^2}{\omega_a^2 - (2\Omega)^2} U_a$$



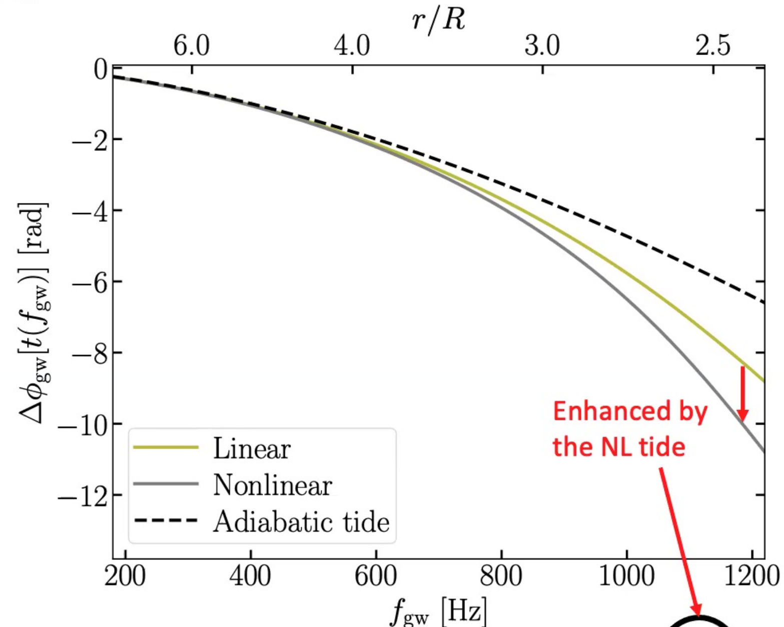
- Near merger $\Omega \simeq \sqrt{(M+M)/(2R)^3} \simeq \omega_a/2$
- Mode resonance (dynamical tide) becomes important (Hinderer+ 16)
- Sensitive to perturbations on ω_a!

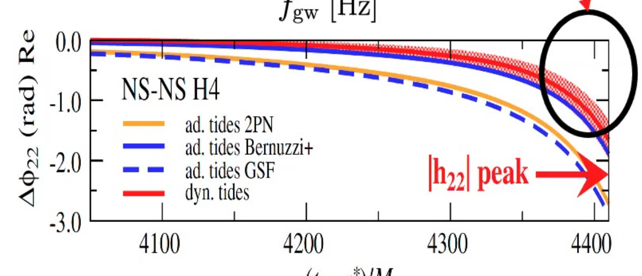
Frequency shift by the nonlinear tide

$$\ddot{c}_a + \omega_a^2 c_a = \omega_a^2 \left[U_a + 2\kappa c_b c_a + ... \right],$$

$$\ddot{c}_a + \omega_a^2 \underbrace{(1 - 2\kappa c_b)}_{1 + 2\frac{\Delta\omega_a}{\omega_a}} c_a = \omega_a^2 \left[U_a + ... \right]$$

- Nonlinear tide creates a frequency shift
- $\Delta\omega_a / \omega_a < 0$
- Mode resonance enhanced!





(Hinderer+ 16)

Frequency shift by the nonlinear tide

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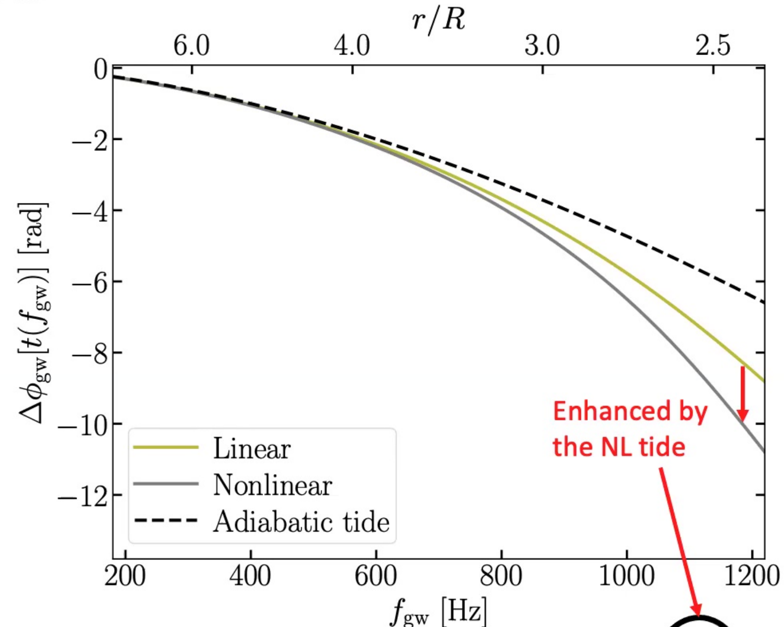
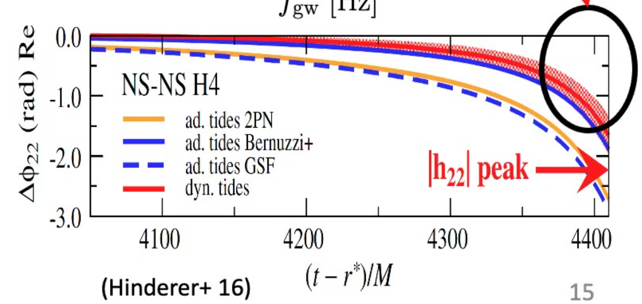
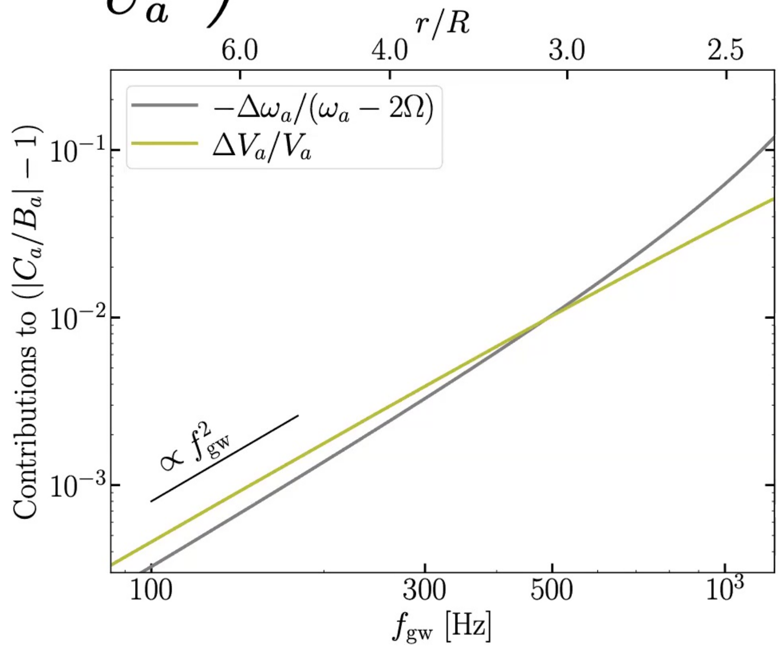




(Hinderer+ 16)

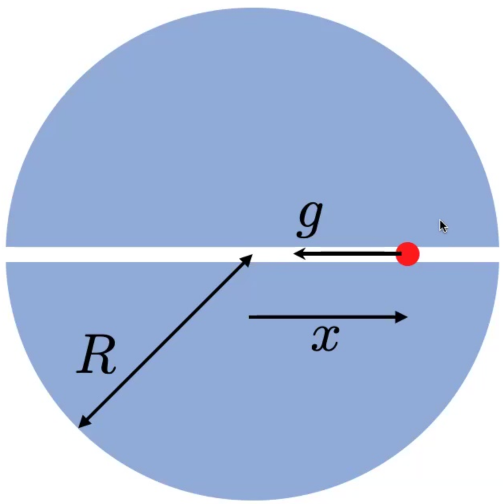
Analytical model

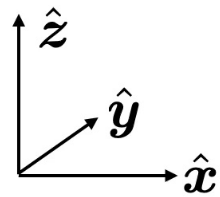
- $c_a = \frac{\omega_a^2}{\omega_a^2(1+2\Delta\omega_a/\omega_a)-(2\Omega)^2} U_a \left(1+\frac{\Delta U_a}{U_a}\right)$
- $\frac{|\Delta\omega_a|}{\omega_a} \sim \frac{\Delta U_a}{U_a} \sim \left(\frac{R}{r}\right)^3 \propto f^2$
- Fractional correction > (R/r)^3
- NL tide: formally 8 PN
- More than 8 PN due to mode resonance!



- Why is the frequency shift negative?

Toy model to demonstrate the frequency shift



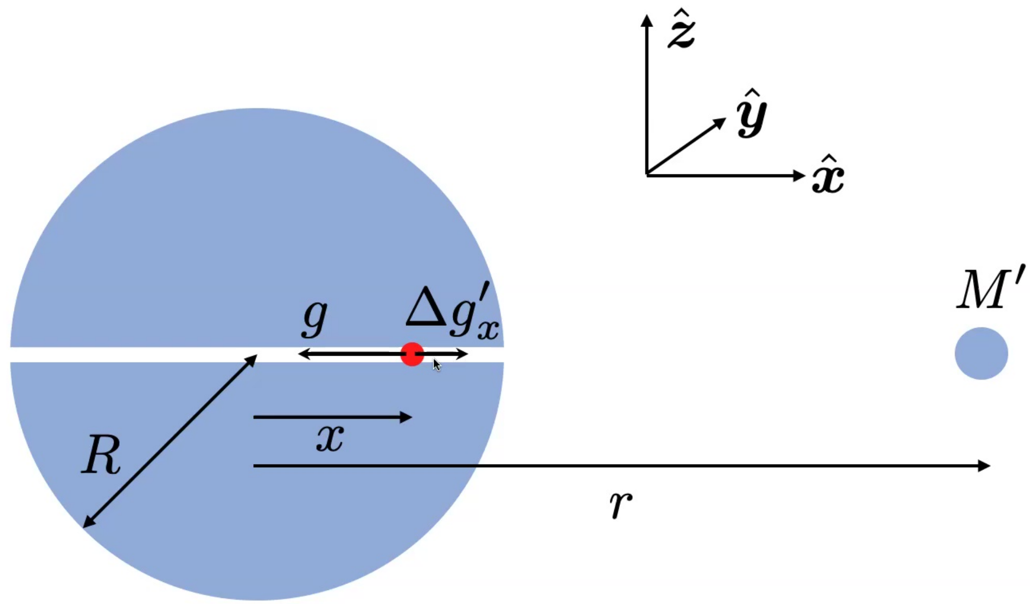


- Test particle in a small hole through the star:

$$\ddot{x} = -g \simeq -\rho x = -\omega^2 x$$

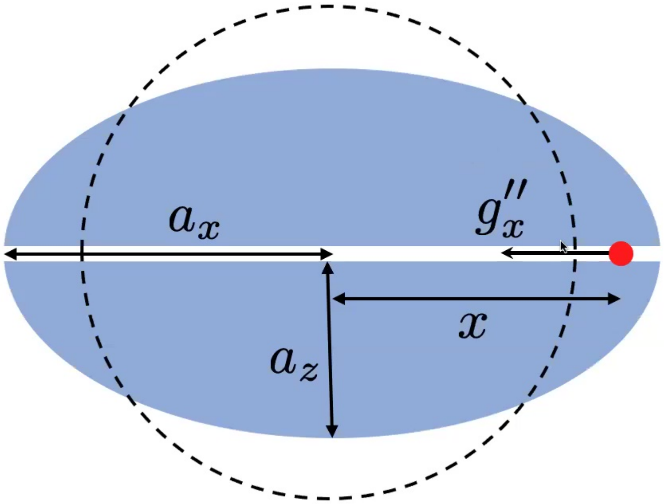
- Harmonic oscillator
- ~ eigenfrequency of the f-mode

Toy model to demonstrate the frequency shift

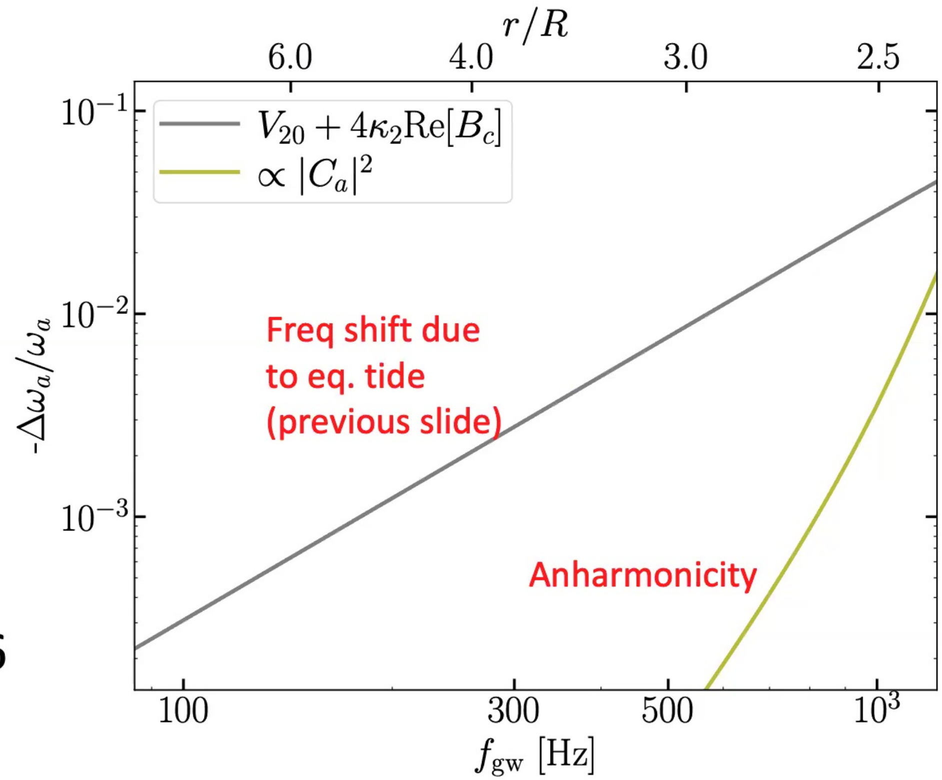


- Companion's gravity reduces inward acc. along x
- Fractional frequency decrease ~ (R/r)^3~(another) mode's amplitude
- $(\Delta\omega_y = -\Delta\omega_x/2,\ (\Delta\omega_x + \Delta\omega_y)/2 = \Delta\omega_x/4 < 0.)$

Different from anharmonicity (Yu, Weinberg & Arras 21, 22)



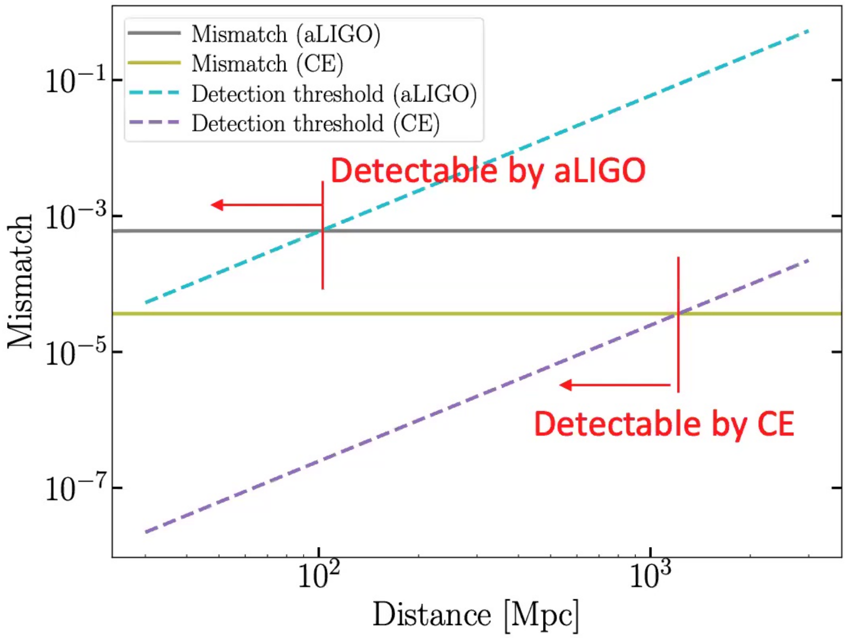
- Background deformation due to f-mode
- Frequency shift ~ mode energy ~ (R/r)^6
- Anharmonicity: *free* oscillator
- Our case: continuously forced by M'

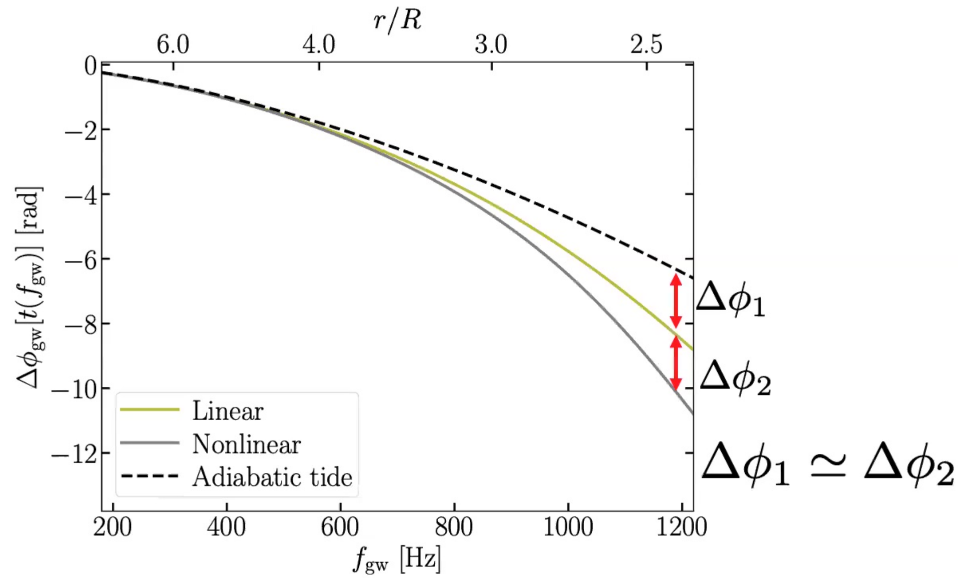


(The pg-instability is a special case of anharmonicity; frequency shift > original frequency – instability!)

- Detectability?

Detectability



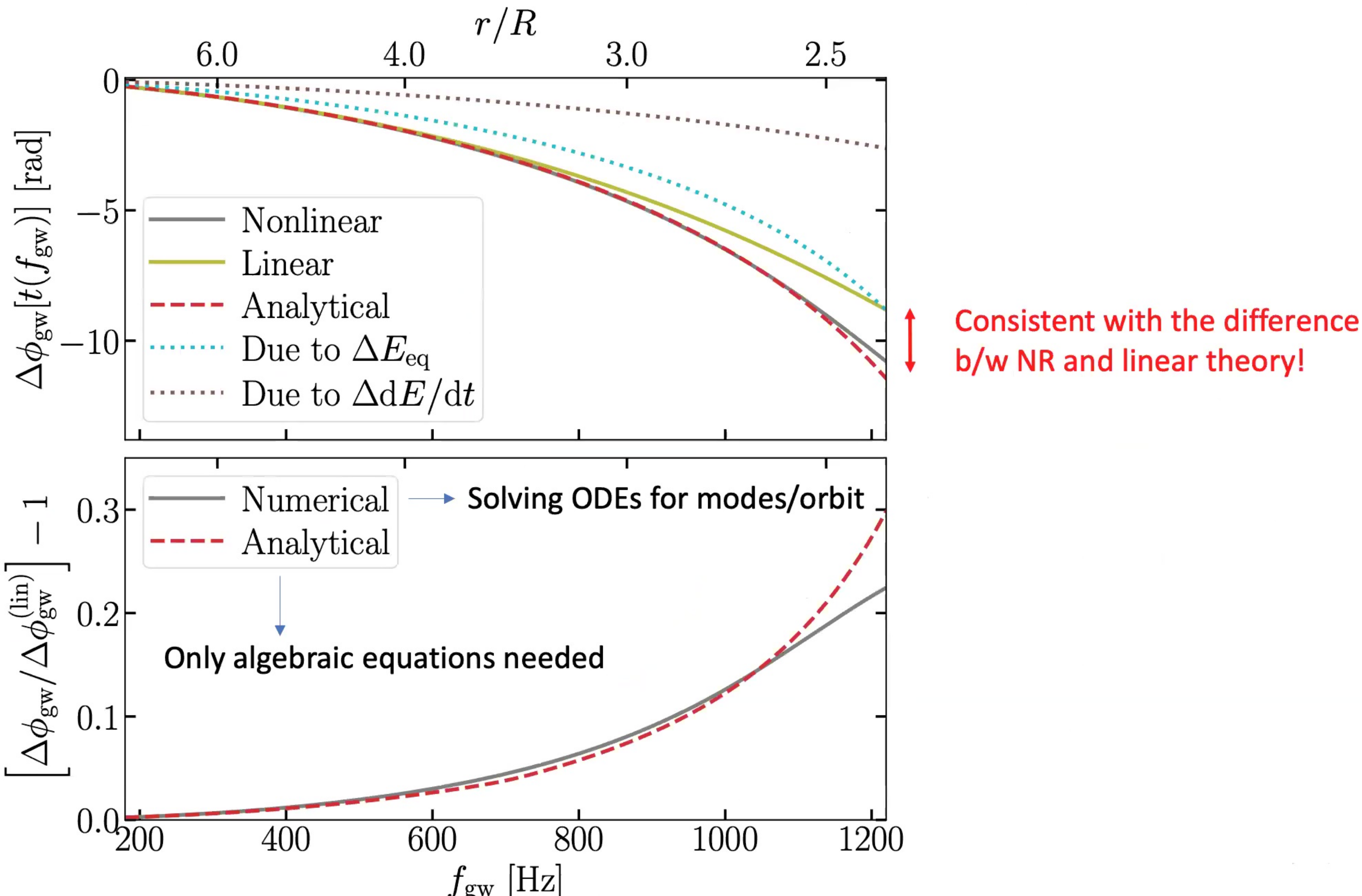


- ~ 1 rad phase difference at 1000 Hz
- Nonlinear tide detectable by aLIGO!
- ALL BNSs experience the nonlinear tide
- Not accounted, bias EoS!

(e.g., Pratten+ 22)

Conclusion

- Nonlinear tide: ~ 1 rad extra phase shift at 1000 Hz
- Consistent with the discrepancy between the linear theory and NR
- Nonlinear tide -> lowering mode frequency -> enhancing mode resonance
- Phase shift detectable by aLIGO!



r/R
6.0
4.0
3.0
2.5
0
-5
-10
$\Delta\phi_{\rm gw}[t(f_{\rm gw})]$ [rad]
Nonlinear
Linear
Analytical
Due to $\Delta E_{\rm eq}$
Due to $\Delta {\rm d}E/{\rm d}t$
Consistent with the difference
b/w NR and linear theory!
Numerical
Analytical
Solving ODEs for modes/orbit
Only algebraic equations needed
0.3
0.2
0.1
0.0
$\left[\Delta\phi_{\rm gw}/\Delta\phi_{\rm gw}^{\rm (lin)}\right] - 1$
200
400
600
800
1000
1200
$f_{\rm gw}$ [Hz]

Significance of 4-mode interaction

- Analytical solution requires only 3-mode terms with first-order perturbation
- Second-order perturbation due to 3-mode (partially) cancels with 4-mode term
- Numerical solution requires both to avoid run-away instabilities!

(Wu 98)