Title: Effective Field Theory and Symmetries

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Collection: Symmetries Graduate School 2023

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Effective Field Theory #3

Last time:

A) EFT from Linear Sigma Model

1) Lowest order: $\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^+)$ $\qquad U = e^{i \frac{\vec{\tau} \cdot \vec{\pi}}{v}}$

2) Integrating out σ (S) at tree level $\leftarrow S$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^+) + \frac{v^2}{8 M_\sigma^2} \left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^+) \right]^2$$

3) Most general $\mathcal{L}$ at this order

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3) Most general $\mathcal{L}$ at this order

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + l_1 \left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \right]^2$$

$$+ l_2 \, \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger) \mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)$$

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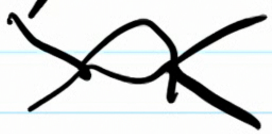
$\uparrow \sigma$

3) Most general $\mathcal{L}$ at this order

$$\mathcal{L} = \frac{v^2}{4}\,\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \ell_1\left[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)\right]^2 + \ell_2\,\mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)\,\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)$$

4) Loops and renormalization



$$\mathcal{M}_{\rm eff} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4} + \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u) + u(u-s)]/v^4 - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right],$$

where:

$$\ell_1^r = \ell_1 + \frac{1}{384\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

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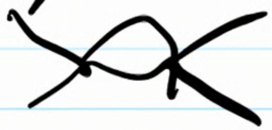

$$\mathcal{M}_{\text{eff}} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4}$$
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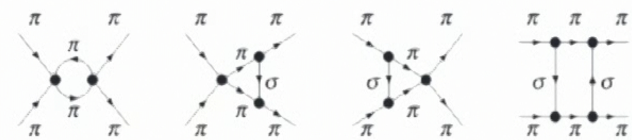
$\leftarrow$ divergences like local terms in $\mathcal{L}$

5) Matching to full theory



(a) (b) (c) (d)

- exact match if

$$\ell_1^r = \frac{v^2}{8m_\sigma^2} + \frac{1}{192\pi^2}\left[\ln\frac{m_\sigma^2}{\mu^2} - \frac{35}{6}\right]$$
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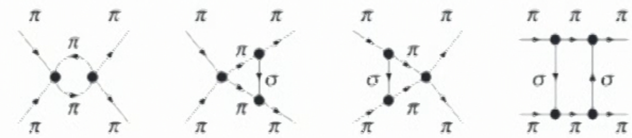

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(or you could measure these)

B) Background Field Method

- QED example

$$\mathcal{L} = \int d^4x \ \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) = -\frac{1}{2} \phi^* D_\mu D^\mu \phi$$

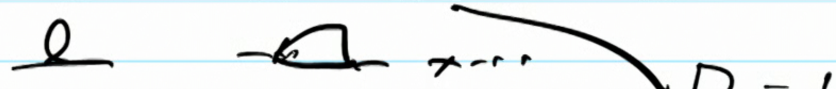
Integrating out ϕ

$$D_\mu = \partial_\mu + ieA_\mu$$

$$S_{eff} = \mathrm{Tr} \ln (D_\mu D) = \mathrm{Tr} \ln (\partial^2 + v)$$

$$= \mathrm{Tr} \ln \partial^2 + \mathrm{Tr} \ln \left(1 + \frac{1}{\partial^2} v \right)$$

$$= \text{const} + \int d^4x \left[\frac{1}{\Box} v + \frac{1}{\Box} v \frac{1}{\Box} v + \dots \right]$$



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$D_F = \frac{1}{\Box}$

After algebra (not shown)

$$S = \int d^4x\, d^4y\, F_{\mu\nu}(x)\, \frac{D_F^2(x-y)}{4(d-1)}\, F^{\mu\nu}(y)$$

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 $+ \dots$

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To complete this calculation:

$$D_F^2(x-y) = F.T.$$ 

$$= F.T.\left[\frac{1}{\epsilon} + \dots \begin{cases} -\log q^2/\mu^2 \\ -q^2/m^2 \end{cases} \right] \text{or} \begin{matrix} m^2 = 0 \\ m^2 \text{ large} \end{matrix}$$

$$F.T\ \frac{1}{\epsilon} \Rightarrow \delta^4(x-y)\,\frac{1}{\epsilon} \quad \text{(wavefunction renorm)}$$

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$$F.T.\ \log q^2 = L(x-y)$$ nonlocal effect if $m=0$

This reproduces QED as described earlier

$x \quad y \qquad D_F = \frac{1}{\Box}$

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Generalization - to be used soon

$$S = \int d^4x\, \phi^* \left[d_\mu d^\mu + \sigma \right] \phi$$

$$d_\mu = \partial_\mu + \Gamma_\mu$$

Same steps:

Divergence is:

$$S_{div} = \int d^4x\, \frac{1}{\varepsilon} \left[\frac{1}{12} [d_\mu, d_\nu] [d^\mu, d^\nu] + \frac{1}{2} \sigma^2 \right]$$

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Alternate method - "Heat Kernel"

Same steps.

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Alternate method – "Heat kernel"
(see DSM)

Background Field Renorm of NLSM

Logic : Break field into background + fluct.

$\phi = c$

$$\phi = \bar{\phi} + \delta\phi$$

Standard form $\mathcal{L}(\bar{\phi}) + \delta\phi\,[d_a d^a + \sigma]\,\delta\phi$

Integrate over $\delta\phi$  

Divergences $\frac{1}{\varepsilon}\left[\frac{1}{12}[d_a, d] \text{-----}\right.$

Standard for $\mathcal{L}(\phi) + \delta\phi\,[d_\mu, d^\mu + \sigma]\,\delta\phi$

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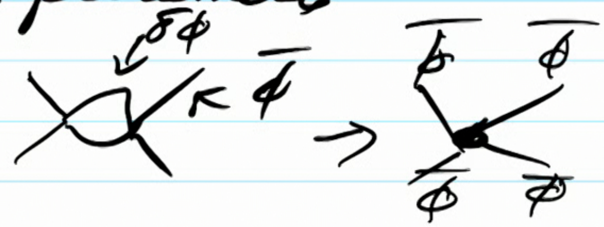
Standard for $\mathcal{L}(\phi) + \delta\phi\,[\partial_\mu \partial + U]\,\delta\phi$

Integrate over $\delta\phi$ 

Divergences $\frac{1}{\varepsilon}\left[\frac{1}{12}\{d_\mu, d\} \cdots\cdots \right]$

Local $\mathcal{L}$ – renorm. parameter

Apply at tree level 

For NLSM

$$U = e^{i\vec{\tau}\cdot\vec{\pi}/v}$$

$$= \bar{U}\, e^{i\vec{\tau}\cdot\vec{\Delta}} \leftarrow \delta\phi$$

$$\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) = \mathrm{Tr}(\partial_\mu \bar{U}\, \partial^\mu \bar{U}^\dagger)$$

$$+ \mathrm{Tr}[\partial_\mu \tau\cdot\Delta\, \partial^\mu \tau\cdot\Delta +$$

$$+ \bar{U}^\dagger \partial_\mu \bar{U}(\tau\cdot\Delta\, \partial_\mu(\tau\cdot\Delta) + \partial_\mu(\tau\cdot\Delta)\tau\cdot\Delta)$$

$$\Rightarrow \mathrm{Tr}[\;] = \Delta^a \left[d_a d^a + \sigma \right]^{ab} \Delta^b$$

$$+\,U^\dagger \partial_\mu U\,(\tau\cdot\Delta\,\partial_\mu(\tau\cdot\Delta)\quad (\partial_\mu(\tau\cdot\Delta))\,\tau\cdot\Delta$$

$$\Rightarrow \mathrm{Tr}[\;] = \Delta^a\,[d_\mu d^a + \sigma]^{ab}\,\Delta^b$$

$$\Gamma_\mu^{\cdot} = \frac{1}{4}\,\mathrm{Tr}\left([\tau^a,\tau^b]\,\bar{U}^\dagger \partial_\mu \bar{U}\right)$$

$$\sigma = \frac{1}{8} \mathrm{Tr}\left([\tau^a, \bar{u}^\dagger \partial_\mu \bar{u}] [\tau^b, \bar{u}^\dagger \partial^\mu \bar{u}] \right)$$

1

Work out

$$\frac{1}{16\pi^2 \varepsilon}\left[\frac{1}{12}[d, d][d, d] + \frac{1}{2}\sigma\right]$$

$$= -\frac{1}{192\pi^2}\frac{1}{\varepsilon}\left\{\frac{1}{2}\left[\mathrm{Tr}\,\partial_\mu \bar{u}\,\partial^\mu \bar{u}^\dagger\right]^2 + \mathrm{Tr}\,\partial_\mu \bar{u}\,\partial_\nu \bar{u}^\dagger \times \mathrm{Tr}\,\partial^\mu \bar{u}\,\partial^\nu \bar{u}^\dagger\right\}$$

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$\Rightarrow$ same renorm as before

for <u>all</u> interactions !

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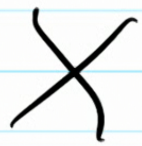
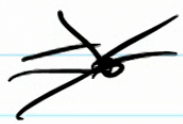
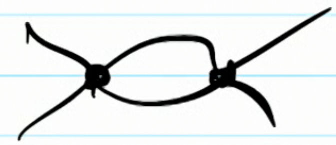
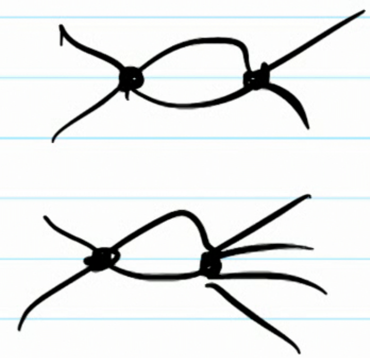
 $= \frac{E^2}{v^2}$

 $= \frac{E^2}{v^6}$



 $= \frac{\ell^2}{r^2}$

 $= \frac{\ell^2}{r^6}$



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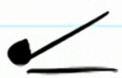
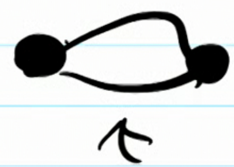
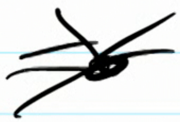


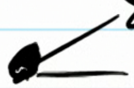
 $=$ 

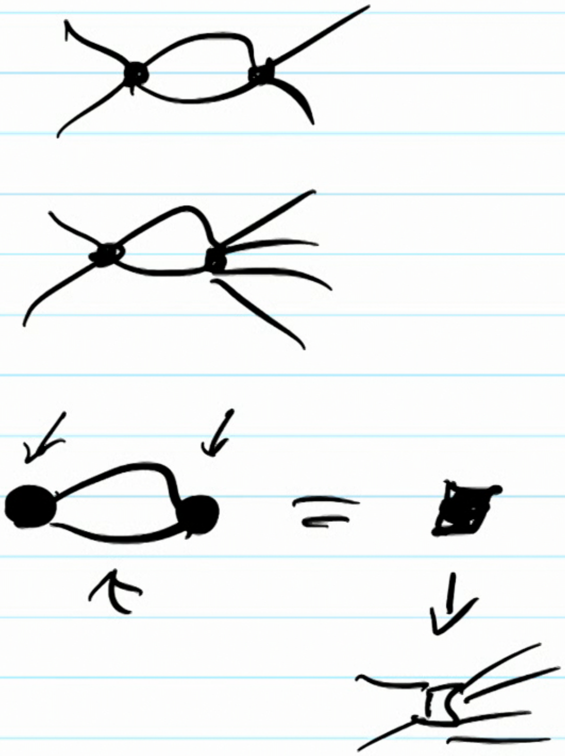
for <u>all</u> interactions!

$$\frac{p^2}{v^2}$$

 $= \dfrac{p^2}{v^6}$

Here

 $\delta\phi$ $\delta\phi$



QCD Story

- "Hydrogen atom" ?

– "Hydrogen atom" of EFT

– "Hydrogen atom" of EFT

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^2 + \bar{\psi}\, i \not{D} \psi - \bar{\psi} m \psi$$

$u, d \Longrightarrow \psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$

Chirality $\quad \psi_L = \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$

$$\psi_R = \frac{1}{2}(1-\gamma_5)\psi$$

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$$\psi\, i\not{D}\,\psi = \bar{\psi}_L\, i\not{D}\,\psi_L + \bar{\psi}_R\, i\not{D}\,\psi_R$$

$$\bar{\psi}\, m\, \psi =$$

Chirality $\psi_L = \frac{1}{2}(1+\gamma_5)\psi$ $\psi = \psi_L + \psi_R$

$$\psi_R = \frac{1}{2}(1-\gamma_5)\psi$$

$$\bar{\psi} i \not{D} \psi = \bar{\psi}_L i \not{D} \psi_L + \bar{\psi}_R i \not{D} \psi_R$$

$$\bar{\psi} m \psi = \bar{\psi}_L m \psi_R + \bar{\psi}_R m \psi_L$$

$$\psi = \psi_L + \psi_R$$

$$\psi_R = \frac{1}{2}(1-\gamma_5)\psi$$

$$\bar\psi\, i \not{D}\, \psi = \bar\psi_L\, i\not{D}\, \psi_L + \bar\psi_R\, i \not{D}\, \psi_R$$

$$\bar\psi\, m\, \psi = \bar\psi_L\, m\, \psi_R + \bar\psi_R\, m\, \psi_L$$

<u>if $m=0$</u> global symmetry $\psi_L \Rightarrow L\,\psi_L$

$$\psi_R \Rightarrow R\,\psi_R$$

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if $m \neq 0$ broken

if m is "small" - slightly broken

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Expt: L, R not there in spectrum

light pions

if $m = 0$ π Goldstone bosons

$m \neq 0$ π almost G. B.

$\Rightarrow$ F

Elaboration

- symmetry breaking
- γ, W ...

Result Gasser Leutwyler

Elaboration

- symmetry breaking
- γ, W ...

Result: Gasser Leutwyler

$$
\begin{aligned}
\mathcal{L}_4 &= \sum_{i=1}^{10} L_i O_i \\
&= L_1 \left[\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right)\right]^2 + L_2 \,\mathrm{Tr}\left(D_\mu U D_\nu U^\dagger\right) \cdot \mathrm{Tr}\left(D^\mu U D^\nu U^\dagger\right) \\
&\quad + L_3 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger\right) \\
&\quad + L_4 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger\right) \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \\
&\quad + L_5 \,\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger \left(\chi U^\dagger + U \chi^\dagger\right)\right) + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger\right)\right]^2 \\
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&\quad + i L_9 \,\mathrm{Tr}\left(L_{\mu\nu} D^\mu U D^\nu U^\dagger + R_{\mu\nu} D^\mu U^\dagger D^\nu U\right) + L_{10} \,\mathrm{Tr}\left(L_{\mu\nu} U R^{\mu\nu} U^\dagger\right),
\end{aligned}
$$

Elaboration

– symmetry breaking

– γ, W ...

Result: Gasser Leutwyler

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i O_i$$

$$= L_1 \left[\mathrm{Tr}\left(D_\mu U D^\mu U^\dagger \right) \right]^2 + L_2 \, \mathrm{Tr}\left(D_\mu U D_\nu U^\dagger \right) \cdot \mathrm{Tr}\left(D^\mu U D^\nu U^\dagger \right) \quad \leftarrow \text{previously}$$

$$+ L_3 \, \mathrm{Tr}\left(D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger \right) \quad \leftarrow SU(3)\,(u, d, s)$$

$$+ L_4 \, \mathrm{Tr}\left(D_\mu U D^\mu U^\dagger \right) \mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger \right)$$

$$+ L_5 \, \mathrm{Tr}\left(D_\mu U D^\mu U^\dagger \left(\chi U^\dagger + U \chi^\dagger \right) \right) + L_6 \left[\mathrm{Tr}\left(\chi U^\dagger + U \chi^\dagger \right) \right]^2$$

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$\chi = M B_0$

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L

$$
\begin{aligned}
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\end{aligned}
$$

← previously

← $SU(3)(u,d,s)$

$\chi = M B_0$

$W_{\mu\nu}$

$F_{\mu\nu} = L_{\mu\nu}$, $R_{\mu\nu} = F_{\mu}$

Table VII–1. *Renormalized coefficients in the chiral lagrangian $\mathcal{L}_4$ given in units of 10^{-3} and evaluated at renormalization point $\mu = m_\rho$ [BiJ 12].*

Coefficient	Value	Origin
L_1^r	1.12 ± 0.20	$\pi\pi$ scattering
L_2^r	2.23 ± 0.40	and
L_3^r	-3.98 ± 0.50	$K_{\ell 4}$ decay
L_4^r	1.50 ± 1.01	F_K/F_π
L_5^r	1.21 ± 0.08	F_K/F_π
L_6^r	1.17 ± 0.95	F_K/F_π
L_7^r	-0.36 ± 0.18	Meson masses
L_8^r	0.62 ± 0.16	F_K/F_π
L_9^r	7.0 ± 0.2	Rare pion
L_{10}^r	-5.6 ± 0.2	decays

Table VII–3. *Chiral predictions and data in the radiative complex of transitions.*

Reaction	Quantity	Theory	Experiment
$\gamma \to \pi^+\pi^-$	$\langle r_\pi^2 \rangle$ (fm^2)	0.45[a]	0.45 ± 0.01
$\gamma \to K^+K^-$	$\langle r_K^2 \rangle$ (fm^2)	0.45	0.31 ± 0.03
$\pi^+ \to e^+\nu_e\gamma$	$h_V (m_\pi^{-1})$	0.027	0.0254 ± 0.0017
	h_A/h_V	0.441[a]	0.441 ± 0.004
$K^+ \to e^+\nu_e\gamma$	$(h_V + h_A)(m_K^{-1})$	0.136	0.133 ± 0.008

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$\gamma\pi^+ \to \gamma\pi^+$	$(\alpha_E + \beta_M)(10^{-4}$ fm)	0	0.17 ± 0.02
	$(\alpha_E - \beta_M)(10^{-4}$ fm)	5.6	13.6 ± 2.8
$K \to \pi e^+ \nu_e$	$\xi = f_-(0)/f_+(0)$	-0.13	-0.17 ± 0.02
	λ_+ (fm^2)	0.067	0.0605 ± 0.001
	λ_0 (fm^2)	0.040	0.0400 ± 0.002

[a]Used as input.

} Prediction

Table VII–4. *The pion scattering lengths and slopes.*

	Experimental	Lowest order[a]	First two orders[a]
a_0^0	0.220 ± 0.005	0.16	0.20
b_0^0	0.25 ± 0.03	0.18	0.26
a_0^2	-0.044 ± 0.001	-0.045	-0.041
b_0^2	-0.082 ± 0.008	-0.089	-0.070
a_1^1	0.038 ± 0.002	0.030	0.036
b_1^1	—	0	0.043
a_2^0	$(17 \pm 3) \times 10^{-4}$	0	20×10^{-4}
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Expansion

$$\left(\frac{E}{\Lambda}\right)^n$$

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[a] Predictions of chiral symmetry.

Expansion

$$\left(\frac{E}{\Lambda}\right)^n$$

$\Lambda \sim 1$ GeV

Operator Product Expansion

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$$H_{int} = \sum c_i \, \mathcal{O}_i$$ ← local operators

Operator Product Expansion

$$\mathcal{H}_{int} = \sum c_i \, \mathcal{O}_i$$ ← local operators

Matched at one energy ← 0

Apply at another — $c_i(\mu)$

E.T. matching

Run

Apply

Series of EFT

Match

Run

Match

Run

Apply

E.F. matching

Run

Apply

Series of EFT

M_W Match

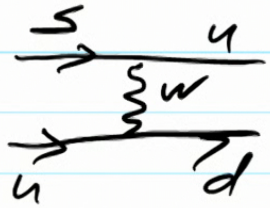
Run

m_b Match

Run

Λ_{QCD} Apply

Example (DSM)
Weak int + QCD



Match at M

Match at M_W $\bar{d}\,\gamma_\mu(1+\gamma_5)\,u\,\bar{u}\,\gamma^\mu(1+\gamma_5)\,s \times \frac{1}{k^2-M_W^2}$

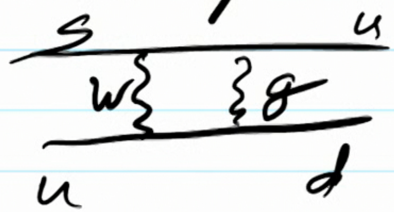
No running

Match at M_W $\quad \bar{d}\gamma_\mu(1+\gamma_5)u\,\bar{u}\gamma^\mu(1+\gamma_5)s \times \frac{1}{k^2-M_W^2}$

No running $\quad \mathcal{H} = \bar{d}\,\Gamma\,u\,\bar{u}\,\Gamma\,s\,\frac{1}{-M_W^2}$

One loop running

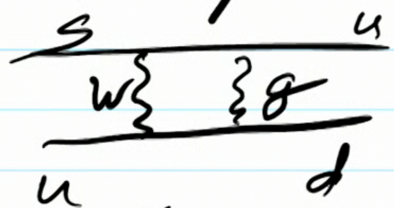
One loop running



+ perms

$\int dl$

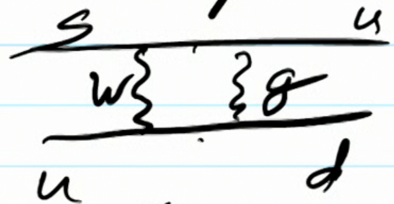
One loop running



+ perms

$$\int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{\ell^2 - M_W^2}$$

One loop running



+ perms

SU(3) matrix color

$$\int \frac{d^4\ell}{(2\pi)^4}\,\frac{1}{\ell^2 - M_W^2}\,\frac{1}{k^2}\,\frac{1}{k^2}\quad \underbrace{\bar{d}\,\gamma_\mu(1+\gamma_5)\,\lambda^A u\;\bar{u}\,\gamma^\mu(1+\gamma_5)\,\lambda^A s}_{O_2}$$

$$-\frac{1}{16\pi}\,\frac{1}{M^2}\,\ln\frac{\ell^2}{\ell^2+M^2}\Bigg|_{\ell=\mu}^{\ell=M_W}$$

$$\ln \mu^2/M_W^2$$

$/M_W$

Result

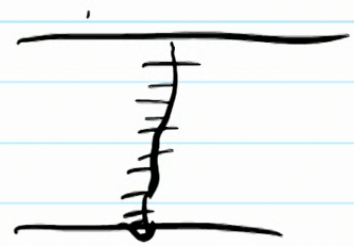
$\frac{1}{M_W^2}$

$$\mathcal{H}_W = G_W [CKM] \left[1 \cdot O_1 - \frac{g^2}{16\pi^2} \ln \frac{M_W^2}{\mu^2} O_2 \right]$$

$$\mathcal{H}_W = G_W [CKM] \left[1 \cdot O_1 - \frac{g^2}{16\pi^2} \ln \frac{M_W^2}{\mu^2} O_2 \right]$$

Renorm group summation

$$\Rightarrow C_1(\mu) O_1 + C_2(\mu) O_2$$



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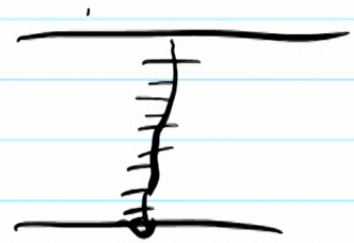

Wilsonian