Title: Effective Field Theory and Symmetries

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Collection: Symmetries Graduate School 2023

Date: February 01, 2023 - 11:00 AM

URL: https://pirsa.org/23020022

Effective Field Theory #2

From Last time:

1) Quantization

$$Z[J] = N \int [d\phi]\, e^{i \int d^4x \left[\mathcal{L}(\phi, \partial\phi) + J\phi \right]}$$

"call field ϕ at each point"

"source"

2) Free field $\mathcal{L} = \mathcal{L}_0 = \frac{1}{2}(\partial_\mu \phi)^2 - m^2\phi^2$

$$Z[J] = N\, e^{-i \int d^4x \, d^4y \, J(x)\, i D_F(x-y) J(y)}$$

3) Quadratic - gaussian

$$C = \int d^4x \, \phi \, \Box \phi$$

2) Free field $\mathcal{L} = \mathcal{L}_0 = \frac{1}{2}(\partial_\mu \phi)^2 - m^2\phi^2$

$Z[J] = N \, e^{-i \int d^4x \, d^4y \, J(x) \, i D_F(x-y) J(y)}$

3) Quadratic - gaussian

$$\int [d\phi] \, e^{-i \int d^4x \, \phi \mathcal{D} \phi} = \# [\det \mathcal{D}]^{-\frac{1}{2}}$$

$$= \# e^{-\frac{i}{2} \int d^4x \langle x | \ln \mathcal{D} | x \rangle}$$

P.I. language - integrating out

$$Z = \int [dA][d\psi d\bar{\psi}]\, e^{i\int d^4x\, \mathcal{L}_{QE}(A,\psi)}$$

$$= \int [dA]\, e^{i\int_{\Lambda} d^4x\, \mathcal{L}_{eff}(A)}$$

$\Lambda = m$

↳ limits of PI

(1) QED has limits $\Lambda = M_W$

$A_\mu \to W^3_\mu, B_\mu, A$

$$= \int [dA]\, e^{i \int d^4x\, \mathcal{L}_{eff}(A)}$$

$\Lambda = m$

↳ limits of PI

(1) QED has limits $\Lambda = M_W$

$A_\mu \to W^3_\mu, B_\mu, \ldots$

pure QED = EFT

pure QED = EFT

We say Λ = cutoff or limit of EFT

We do dim reg ← all energies

OK, high scales are local

⟹ local terms

matching or measuring

EFT 1.0 — Effective Lagrangian

- New physics

$$\mathcal{L} = \frac{c'}{\Lambda^2}(\phi^+\phi)\,\vec{W}_{\mu\nu}\vec{W}^{\mu\nu} \xrightarrow{} \frac{c'}{v^2}$$

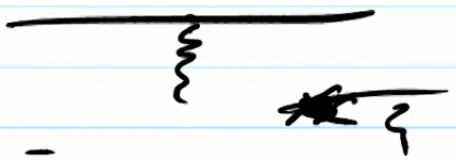
$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} = 1 - \frac{c'}{\Lambda^2}v^2 = 1.00039 \pm 0.00019$$

$M_Z^2 \cos^2\theta_W$ Λ^2 ± 0.0019

$$\frac{c'}{\Lambda^2} \sim \left(\frac{1}{15\ T_9\ V}\right)^2$$

Caveat: some operators can be removed

$\mu \to e\gamma$



$\mathcal{L}_{\not\mu} = \bar{\mu}\, i\not{D}\, e + h.c \qquad + \lambda \bar{\mu} q$

$D_\mu + i g A_\mu$

Matrix elements vanish on shell !

Reason removed by field redefinitions

$$e' = e + \# \mu$$
$$\mu' = \mu + \# e$$

- loops

Linear σ model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma + \frac{1}{2}\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi} - V(\sigma, \vec{\pi})$$

$$V(\sigma, \vec{\pi}) = -\frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \sigma\, \partial^\mu \sigma + \frac{1}{2}\partial_\mu \vec{\pi}\, \partial^\mu \vec{\pi} - V(\sigma, \vec{\pi})$$

$$V(\sigma, \vec{\pi}) = -\frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2$$

SSB $\cup\cap\cup$ $\qquad \langle \sigma \rangle \simeq v = \sqrt{\frac{\mu^2}{\lambda}}$

Expand $\sigma = v + \tilde{\sigma}$

$$m_{\tilde{\sigma}} = \sqrt{2}\mu = \sqrt{2\lambda}\, v$$

$$m_{\vec{\pi}} = 0$$

SSB  $\langle \sigma \rangle = v = \sqrt{\frac{\mu^2}{\lambda}}$

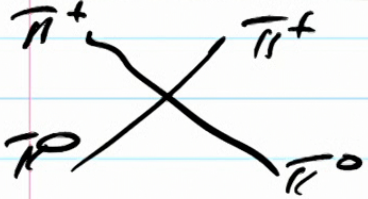
Expand $\sigma = v + \tilde{\sigma}$

$m_{\tilde{\sigma}} = \sqrt{2}\mu = \sqrt{2\lambda}\, v$

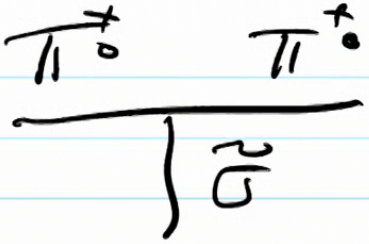
$m_{\pi} = 0$

$V(\tilde{\sigma}, \pi) = \lambda v \tilde{\sigma}(\tilde{\sigma}^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\tilde{\sigma}^2 + \vec{\pi}^2)^2$

$\pi^+\pi^0$ scattering



$= -2i\lambda$

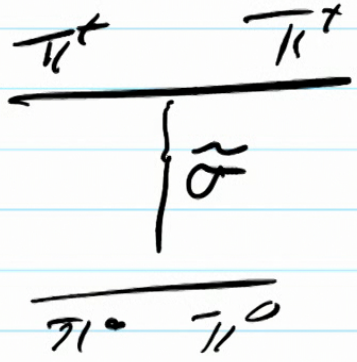


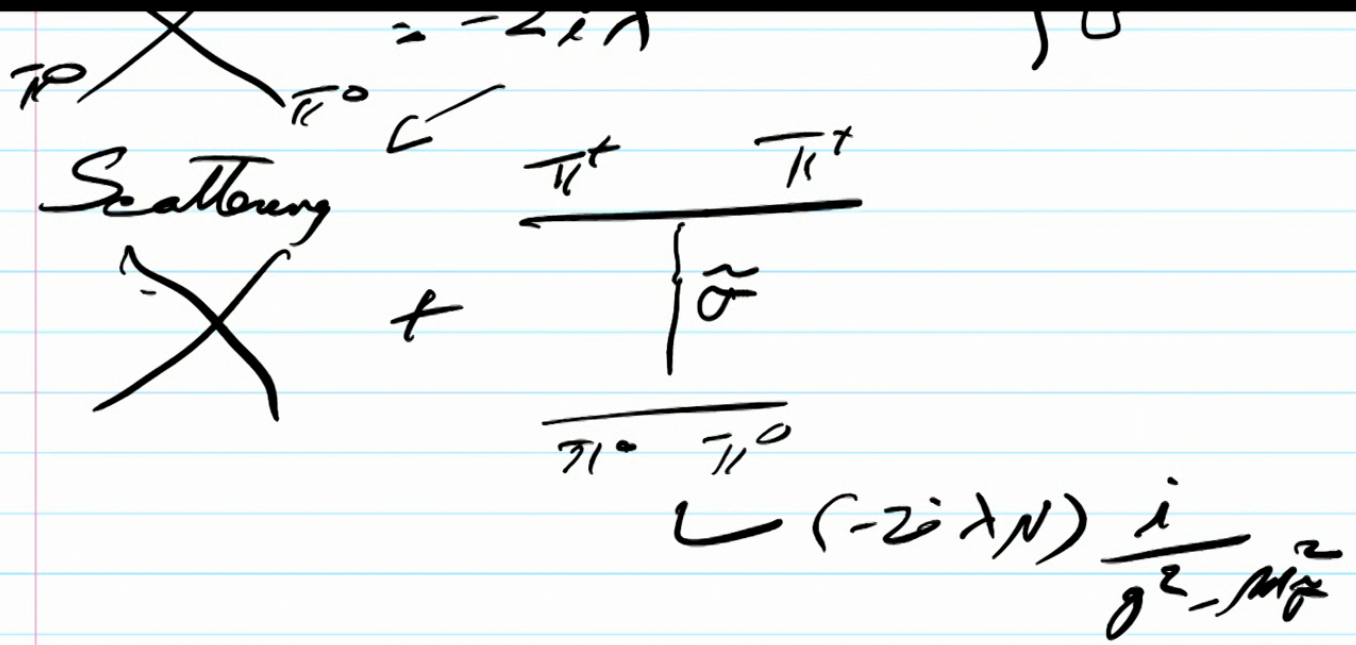
$= -2i\lambda v$

Scattering



$+$





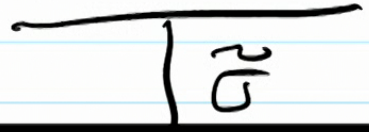
$$\hookrightarrow (-2i\lambda_N)\frac{i}{q^2 - m_{\tilde{\sigma}}^2}$$

$$\mathcal{M} = -2i\lambda\left[1 + \frac{2\lambda v^2}{q^2 - 2\lambda v^2}\right]$$

$\pi^+\pi^0$ scattering



$= -2i\lambda$



$= -2i\lambda v$

$$V(\sigma,\pi) = \lambda v^2 \sigma(\sigma^2+\pi^2) + \frac{\lambda}{4}(\sigma^2+\pi^2)^2$$

$$\mathcal{M} = -2i\lambda\left[1 + \frac{2\lambda v^2}{q^2 - 2\lambda v^2}\right]$$

$$= i\frac{q^2}{v^2}\left[1 + \frac{q^2}{2\lambda v^2} + \dots\right]$$ Cancel!

$\pi^+\pi^0$ scattering π^+ π^+

Forming $\mathcal{L}_{eff}$

Notation $\Sigma = \sigma + i \vec{\tau} \cdot \vec{\pi}$ 2×2 matrix

~ Pauli matrices

$Tr(\tau_i) = 0$

$Tr(\tau_i \tau_j) = 2\delta_{ij}$

No approx:

$$\mathcal{L} = \frac{1}{4} Tr(\partial_\mu \Sigma \, \partial^\mu \Sigma) + \frac{\mu^2}{4} Tr(\Sigma^+ \Sigma) - \frac{\lambda}{16} \left[Tr(\Sigma^+ \Sigma) \right]^2$$

$$Tr(\tau_i \tau_j) = 2\delta_{ij}$$

No approx:

$$\mathcal{L} = \frac{1}{4} Tr(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \frac{\mu^2}{4} Tr(\Sigma^\dagger \Sigma) - \frac{\lambda}{16}\left[Tr[\Sigma^\dagger \Sigma]\right]^2$$

Symmetry $\Sigma \to L \Sigma R$

L: unitary 2×2 matrix $\in SU(2)$

R: another unitary 2×2 matrix

unitary 2×2 matrix

$SU(2)$ matrix 2×2

$SU(2)_L \times SU(2)_R$

$\nu + \tilde{\sigma}$

$U = e^{\quad}$

$$\tilde{\sigma} = S + \cdots$$
$$\tilde{\pi} = \pi' + \cdots$$

$$\mathcal{L} = \frac{(\nu+s)^2}{4} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) -$$

Change names:

$$\Sigma = \underbrace{\sigma}_{\nu + \tilde{\sigma}} + i\vec{\tau}\cdot\vec{\pi} = (\nu + s)\,U$$

$$U = e^{i\frac{\vec{\tau}\cdot\vec{\pi}'}{\nu}}$$

$$\tilde{\sigma} = s + \cdots$$

$$\vec{\pi} = \vec{\pi}' + \cdots$$

$$U^\dagger U = 1$$

$$\mathcal{L} = \frac{(\nu+s)^2}{4} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) -$$

$\tilde{\sigma} = S + \cdots -$

$\vec{\tilde{\pi}} = \vec{\pi}' + \cdots -$

$U^\dagger U = 1$

$$\mathcal{L} = \frac{(v+S)^2}{4} \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) - \lambda v S^2 - \frac{\lambda}{4} S^4$$

no approx

$$+\frac{1}{2}\left[(\partial_\mu S)^2 - m_\sigma^2 S^2\right]$$

Pauli matrices

$$Tr(\tau_i) = 0$$

$$Tr(\tau_i \tau_j) = 2\delta_{ij}$$

No approx:

$$\mathcal{L} = \frac{1}{4} Tr(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \frac{\mu^2}{4} Tr(\Sigma^\dagger \Sigma) - \frac{\lambda}{16}\left[Tr[\Sigma^\dagger \Sigma]\right]^2$$

Symmetry $\Sigma \to L \Sigma R$

L: unitary 2×2 matrix, $SU(2)$

R: another unitary 2×2 matrix

$$SU(2)_L \times SU(2)_R$$

no approx, $+\frac{1}{2}\left[(\partial_\mu s)^2 - m_\sigma^2 s^2\right]$

Symmetry obvious (after SSB)

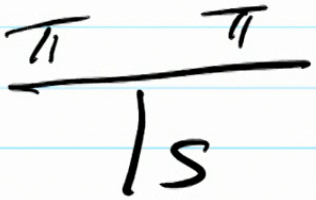
$U \to L\, U\, R^+$

$U \to L\, U\, R^+$

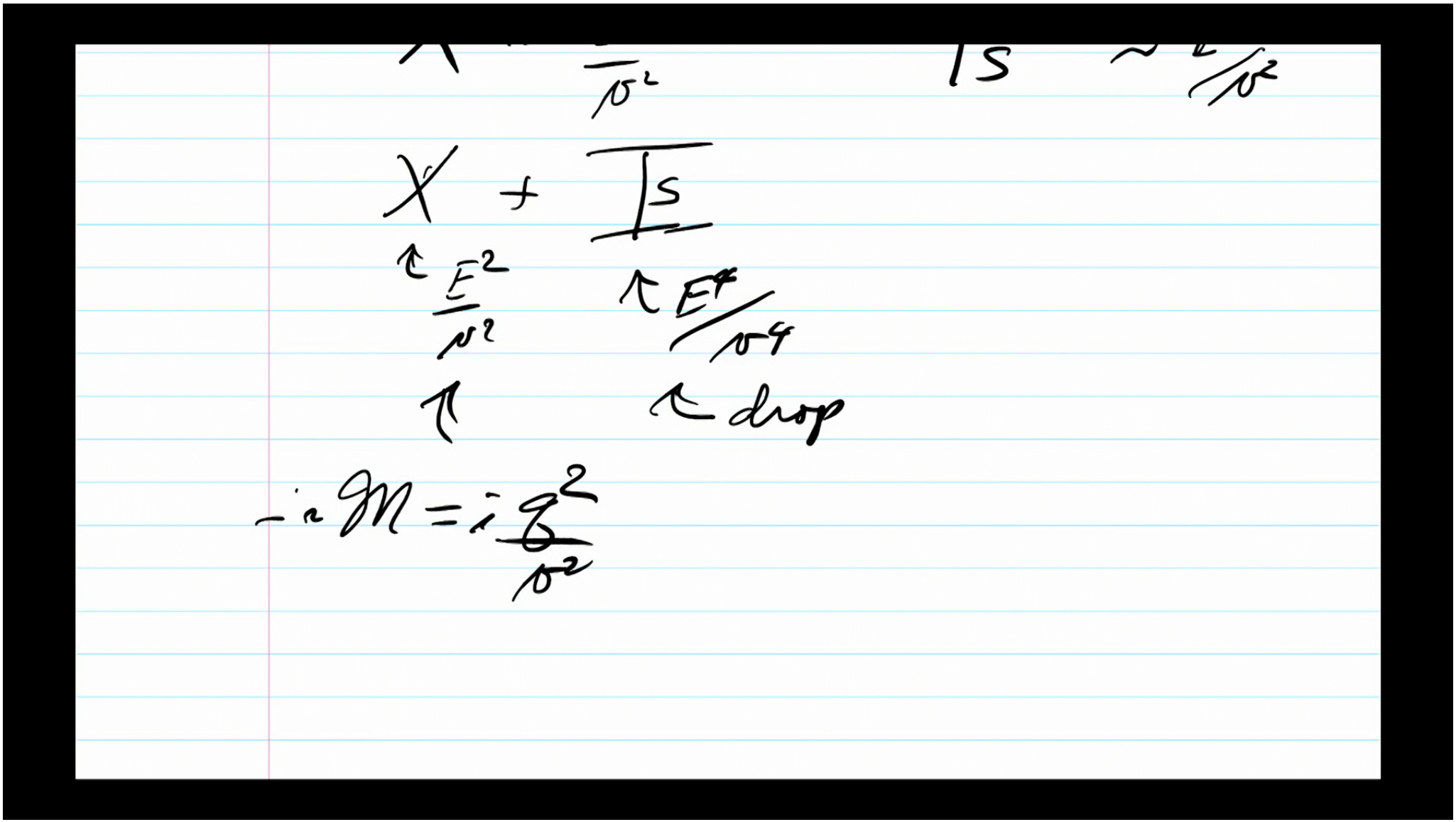
Renorm. hidden

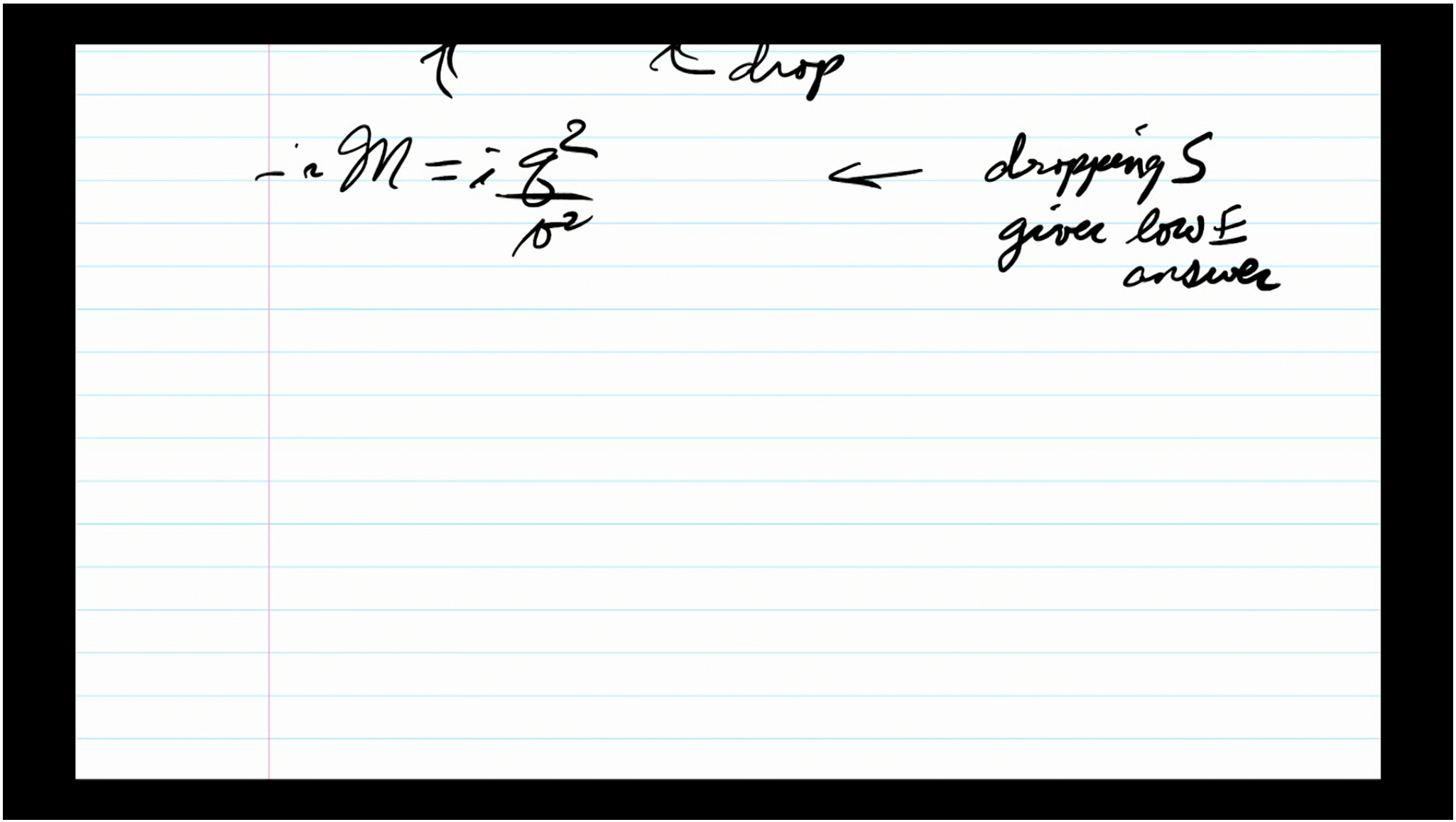
Scattering

$\chi \sim \frac{E^2}{\Lambda^2}$



$\sim \frac{E^2}{\Lambda^2}$


$\frac{1}{t^2}$
$s \sim \frac{1}{t^2}$
$X + \overline{\overline{s}}$
$\uparrow \frac{E^2}{t^2}$
$\uparrow \frac{E^4}{t^4}$
$\uparrow$
$\leftarrow$ drop
$-i\mathcal{M} = i\frac{g^2}{t^2}$


drop
$-i\mathcal{M} = i\frac{g^2}{q^2}$
dropping S gives low E answer

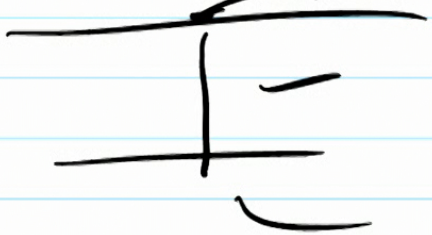
$\frac{\phi}{v^2}$

given low E answer

To this order

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$$

Integrating out



$\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^\dagger$

$\mathrm{Tr}(\partial_\mu U\,\partial_\mu U^\dagger)$

$\uparrow J(x)$

$J(x)\;D_F(x-y)\;J(y) \rightarrow -\frac{1}{m_\sigma^2}\,JJ$

$$\mathcal{L}_{eff} = \frac{v^2}{4}\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger) + \frac{v^2}{8m_\sigma^2}\left[\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^\dagger\right]^2$$

$\uparrow J(x)$

$J(x)\ D_F(x-y)\ J(y) \rightarrow -\frac{i}{m^2_\sigma}\ JJ$

$$\mathcal{L}_{eff} = \frac{v^2}{4} Tr\left(\partial_\mu U\, \partial^\mu U^\dagger\right) + \underbrace{\frac{v^2}{8 m^2_\sigma}\left[Tr\, \partial_\mu U\, \partial^\mu U^\dagger\right]^2}_{E^4}$$

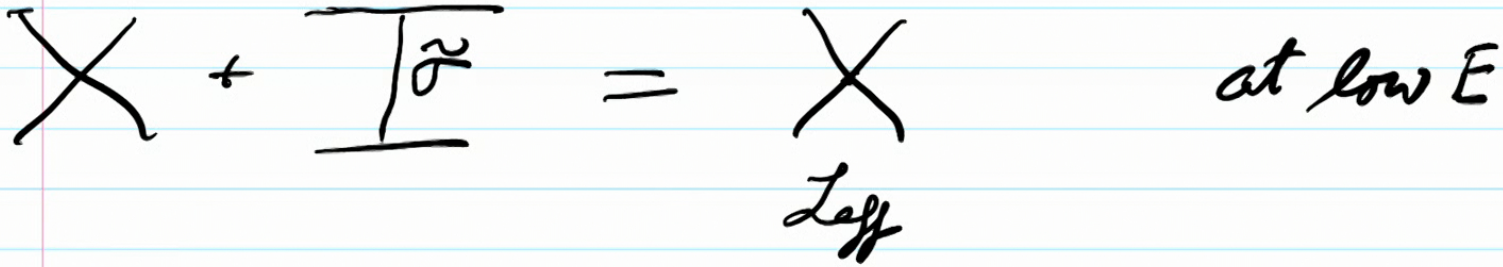
first 2 orders in scattering

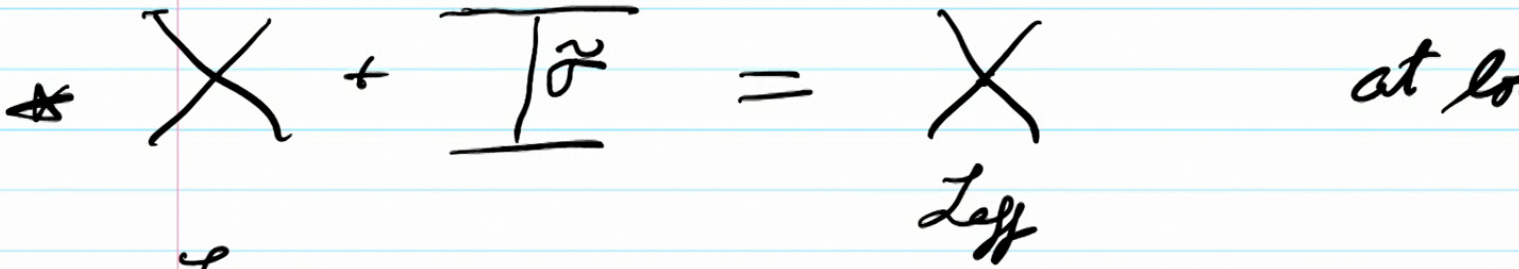
Symmetry obeyed
"Non renorm."

Haag's Theorem → "Names don't count"

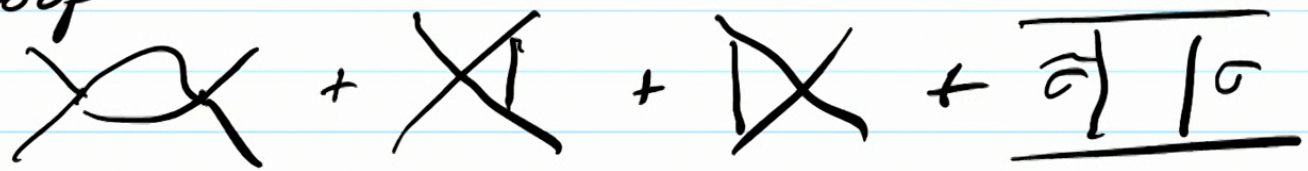
$$\phi' = F(\phi) = \phi + \dots$$

"on-shell"

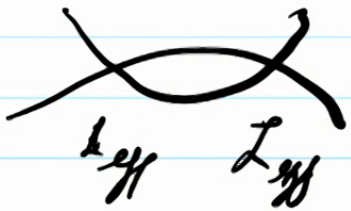

at low E
$\mathcal{L}_{eff}$



Loop



EFT



Logic

Logic: equal by κ at low E

diff at high E

High E is local

looks like Z_{eff}

diff at high E

High E is local

looks like Z_{eff} ⇒ local term

$$\mathcal{M}_{\text{full}} = \frac{t}{v^2} + \left[\frac{1}{m_\sigma^2 v^2} - \frac{11}{96\pi^2 v^4}\right] t^2 - \frac{1}{144\pi^2 v^4}[s(s-u)+u(u-s)] - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{m_\sigma^2} + s(s-u)\ln\frac{-s}{m_\sigma^2} + u(u-s)\ln\frac{-u}{m_\sigma^2}\right]. \quad (3.7)$$

The effective theory result [Le 72, GaL 84] has a very similar form but does not know about the existence of the σ,

$$\mathcal{M}_{\text{eff}} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4} + \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u)+u(u-s)]/v^4 - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right], \quad (3.8)$$

$$\ell_1^r = \ell_1 + \frac{1}{384\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

Combining eqs. (36) and (38) the final result for real r_{13} is

$$\left(\prod_{i=1}^{4} m_i\right) a(x_1 - x_2)\, D_0$$

$$= \sum_{j=1}^{4} \sum_{k=1}^{2} (-1)^{j+k} \left\{ \mathrm{Li}_2(1 + s_j x_{kj}) + \eta(-x_{kj},\, s_j)\, \ln(1 + s_j x_{kj}) \right.$$

$$\left. + \mathrm{Li}_2\left(1 + \frac{x_{kj}}{s_j}\right) + \eta\left(-x_{kj},\, \frac{1}{s_j}\right) \ln\left(1 + \frac{x_{kj}}{s_j}\right) \right\}$$

$$+ \sum_{k=1}^{2} (-1)^{k+1} \left[\tilde{\eta}(-x_k,\, \tilde{r}_{13}) \left\{ \ln(r_{13} x_k) + \ln\left(Q\left(\frac{1}{x_k^0}, 0, 0, 1 \right) - i\epsilon \right) \right. \right.$$

$$+\tilde{\eta}\left(-x_k,\ \frac{1}{\tilde{r}_{24}}\right)\left\{\ln\left(\frac{x_k}{r_{24}}\right)+\ln\left(Q\left(\frac{r_{24}}{x_k^0},\ 1,\ 0,\ 0\right)-i\epsilon\right)\right.$$

$$\left.+\ln\left(\frac{\bar{Q}(1,\ 0,\ 0,\ x_k^0)}{d}+i\epsilon\gamma_{k,3-k}\ \mathrm{sgn}(\mathrm{Im}\ \tilde{r}_{24})\right)\right\}$$

$$-\left(\tilde{\eta}\left(-x_k,\ \frac{\tilde{r}_{13}}{\tilde{r}_{24}}\right)+\eta\left(\tilde{r}_{13},\ \frac{1}{\tilde{r}_{24}}\right)\right)\left\{\ln\left(\frac{r_{13}x_k}{r_{24}}\right)+\ln\left(Q\left(\frac{r_{24}}{x_k^0},\ 1,\ 0,\ 0\right)-i\epsilon\right)\right.$$

$$\left.+\ln\left(\frac{\bar{Q}(0,\ 0,\ 1,\ r_{13}x_k^0)}{d}+i\epsilon\gamma_{k,3-k}\ \mathrm{sgn}(r_{13}\ \mathrm{Im}\ \tilde{r}_{24})\right)\right\}$$

$$\left.+\eta\left(\tilde{r}_{13},\ \frac{1}{\tilde{r}_{24}}\right)\tilde{\eta}\left(-x_k,\ -\frac{\tilde{r}_{13}}{\tilde{r}_{24}}\right)\right]. \tag{35}$$

For the numerical evaluation of the $i\epsilon$-prescription we note that the $\bar{Q}(\ldots)/d$-terms are real if and only if $(1/r_{\ldots}-r_{\ldots})\sqrt{\Delta}$ is real

Nuclear Physics B 367 (1991) 637–656
North-Holland

PHYSICS B

A compact expression for the scalar one-loop four-point function *

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Received 1 July 1991
Accepted for publication 23 August 1991

We present an expression for the scalar one-loop four-point function involving only 16 Spence functions. In its present form it is valid for real inner masses and real external momenta.

1. Introduction

G+L

$$\mathcal{M}_{\text{full}} = \frac{t}{v^2} + \left[\frac{1}{m_\sigma^2 v^2} - \frac{11}{96\pi^2 v^4}\right] t^2 - \frac{1}{144\pi^2 v^4}[s(s-u)+u(u-s)] - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{m_\sigma^2} + s(s-u)\ln\frac{-s}{m_\sigma^2} + u(u-s)\ln\frac{-u}{m_\sigma^2}\right]. \quad (3.7)$$

The effective theory result [Le 72, GaL 84] has a very similar form but does not know about the existence of the σ,

Eff Th

$$\mathcal{M}_{\text{eff}} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4} + \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u)+u(u-s)]/v^4 - \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right], \quad (3.8)$$

$$\ell_1^r = \ell_1 + \frac{1}{384\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

$$\ell_2^r = \ell_2 + \frac{1}{192\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

$$+\left[2\ell_2^r+\frac{7}{576\pi^2}\right][s(s-u)+u(u-s)]/v^4 \qquad (3.8)$$
$$-\frac{1}{96\pi^2v^4}\left[3t^2\ln\frac{-t}{\mu^2}+s(s-u)\ln\frac{-s}{\mu^2}+u(u-s)\ln\frac{-u}{\mu^2}\right],$$

$$\ell_1^r=\ell_1+\frac{1}{384\pi^2}\left[\frac{2}{4-d}-\gamma+\ln 4\pi\right]$$
$$\ell_2^r=\ell_2+\frac{1}{192\pi^2}\left[\frac{2}{4-d}-\gamma+\ln 4\pi\right]$$

Match

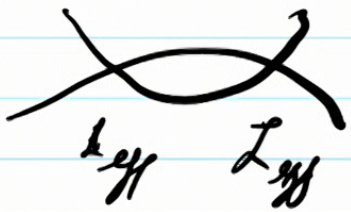
$$\ell_1^r=\frac{v^2}{8m_\sigma^2}+\frac{1}{192\pi^2}\left[\ln\frac{m_\sigma^2}{\mu^2}-\frac{35}{6}\right]$$
$$\ell_2^r=\frac{1}{384\pi^2}\left[\ln\frac{m_\sigma^2}{\mu^2}-\frac{11}{6}\right].$$

$$i\mathcal{M}_{\text{full}}=\int\frac{d^4k}{(2\pi)^4}\left[-2i\lambda+(-2i\lambda v)^2\frac{i}{(k+\quad)^2-\quad^2}\right]\frac{i}{(k+\quad+\quad)^2-\quad^2}\frac{i}{k^2}$$

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + l_1 \left[\mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger\right]^2$$

$$+ l_2 \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)$$
$$\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)$$

EFT



$\leftarrow$ keeping external p low

Logic: equal by $\leftarrow$ at low E in loop

diff at high E in loop

High E is <u>local</u>

looks like $\mathcal{L}_{\text{eff}} \Rightarrow$ <u>local term</u>

$$\mathcal{L} = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + l_1 \left[\mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger\right]^2$$

$$+ l_2 \mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)$$

$$\times \mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)$$

$$\mathcal{M}_{\rm full} = \frac{t}{v^2} + \left[\frac{1}{m_\sigma^2 v^2} - \frac{1}{96\pi^2 v^4}\right] t^2$$
$$- \frac{1}{144\pi^2 v^4}[s(s-u) + u(u-s)]$$
$$- \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{m_\sigma^2} + s(s-u)\ln\frac{-s}{m_\sigma^2} + u(u-s)\ln\frac{-u}{m_\sigma^2}\right]. \quad (3.7)$$

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Eff Th

$$\mathcal{M}_{\rm eff} = \frac{t}{v^2} + \left[8\ell_1^r + 2\ell_2^r + \frac{5}{192\pi^2}\right]\frac{t^2}{v^4}$$
$$+ \left[2\ell_2^r + \frac{7}{576\pi^2}\right][s(s-u) + u(u-s)]/v^4 \quad (3.8)$$
$$- \frac{1}{96\pi^2 v^4}\left[3t^2 \ln\frac{-t}{\mu^2} + s(s-u)\ln\frac{-s}{\mu^2} + u(u-s)\ln\frac{-u}{\mu^2}\right],$$

$$\ell_1^r = \ell_1 + \frac{1}{384\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$
$$\ell_2^r = \ell_2 + \frac{1}{192\pi^2}\left[\frac{2}{4-d} - \gamma + \ln 4\pi\right]$$

renorm

Match

Locality

Energy expansion

Matching

Power counting theorem (Weinberg)

Tree level E^2

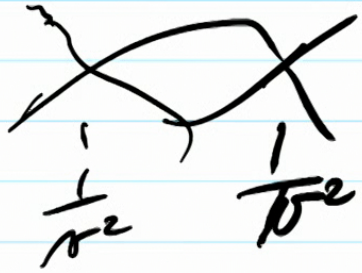
One loop E^4

Two loop E^6

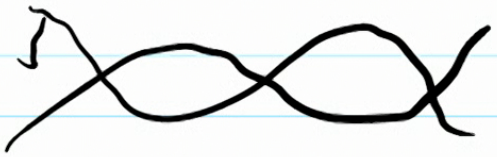
Tree level

One loop E^4

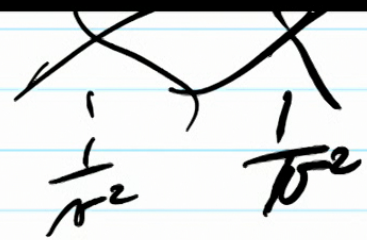
Two loop E^6



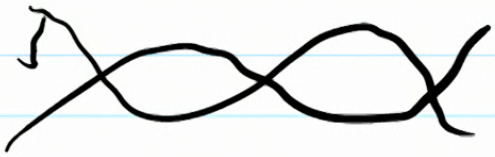
$\frac{E^4}{p^4}$



$\frac{E^6}{p^6}$



$\frac{1}{\Lambda^2}$ $\frac{1}{\Lambda^2}$ $\frac{E^4}{\Lambda^4}$



$\frac{E^6}{\Lambda^6}$

EFT 2.0 $\Rightarrow$ full QFT

QED

$m = 0$ along the way

$$S = \int d^4x \, D_\mu \phi \, D^\mu \phi$$

$$\hookrightarrow \partial_\mu + ie A_\mu$$

$$\int [d\phi] \, e^{iS} = Z[A] = e^{i \int d^4x \mathcal{L}_{eff}}$$

$$\int [d\phi]\, e^{iS} = Z[A] = e^{iS d^4x \mathcal{L}_{eff}}$$

$$\mathcal{L} = D_\mu \phi\, D^\mu \phi \rightarrow \phi \Box \phi$$

$\Box \leftarrow D_\mu D^\mu$

$\mathcal{O} \sim D_\mu D^\mu$

$$\int [d\phi]\, e^{i \int d^4x\, \phi \mathcal{O} \phi}$$

$$= e^{i\, \mathrm{Tr} \ln \mathcal{O}}$$

$$\propto e^{\int d^4x}$$

$$\mathcal{D} = \partial^2 + 2ie A^\mu \partial_\mu + ie(\partial_\mu A^\mu) - e^2 A^2$$

$$= \partial^2 + \mathcal{O}(A)$$

$$\ln \mathcal{D} = \ln\left[\partial^2 \left(1 + \frac{1}{\partial^2} \mathcal{O}\right)\right]$$

$$= \ln \partial^2 + \frac{1}{\partial^2}\mathcal{O} + \frac{1}{2}\frac{1}{\partial^2}\mathcal{O}\frac{1}{\partial^2}\mathcal{O} + \cdots$$

$$= \partial^2 + \mathcal{O}(\Delta)$$

$$\ln \mathcal{D} = \ln\left[\partial^2 \left(1 + \frac{1}{\partial^2} v\right)\right]$$

$$= \ln \partial^2 + \frac{1}{\partial} v - \frac{1}{2}\frac{1}{\partial} v \frac{1}{\partial} v + \cdots$$

$\ln \partial^2$: const

$$= \ln \sigma^2 + \frac{1}{\Box}\sigma - \frac{1}{2}\frac{1}{\Box}\sigma\frac{1}{\Box}\sigma + \cdots$$

↑ const

$$\langle x|\frac{1}{\Box}\sigma|x\rangle =$$

$$\Box D_F(x-y) = \delta(x-y)$$

$$= \ln \mathcal{J}^2 + \frac{1}{\Box} \varphi - \frac{1}{2} \frac{1}{\Box} \varphi \frac{1}{\Box} \varphi + \cdots$$

↑ const

$$\langle x | \frac{1}{\Box} \varphi | x \rangle = D_F(x-x)\, \varphi(x) \qquad \Delta \bigcirc 1$$

$$\Box D_F(x-y) = \delta(x-y)$$

const

$$\langle x | \frac{1}{\Box} v | x \rangle = D_F(x-x)\, v(x)$$

$$\Box\, D_F(x-y) = \delta(x-y)$$

$$\int |y\rangle\langle y|$$

$$\langle x | \frac{1}{\Box} v \frac{1}{\Box} v | x \rangle = D_F(x-y)\, v(y)\, D_F(y-x)\, v(x)$$



const

$$\langle x|\frac{1}{\Box}v|x\rangle = D_F(x-x)\,v(x)$$

$$\Box D_F(x-y) = \delta(x-y)$$

$$\langle x|\frac{1}{\Box}v\frac{1}{\Box}v|x\rangle = \int dy\, D_F(x-y)\,v(y)\,D_F(y-x)\,v(x)$$



$$\langle x | \frac{1}{\Box} \nu \frac{1}{\Box} \nu | x \rangle = D_F(x-y)\,\nu(y)\,D_F(y-x)\,\nu(x)$$



Work:

$$S = -i \int d^d x\, d^d y\; F_{\mu\nu}(x)\, \frac{D_F(x-y)}{4(d-1)}\, F^{\mu\nu}(y)$$

Work:

$$S = -i \int d^4x\, d^4y\; F_{\mu\nu}(x)\, \frac{D_+^2(x-y)}{4(d-1)}\, F^{\mu\nu}(y)$$

$$\left[\frac{1}{\varepsilon} + \frac{Q^2}{5M^2} \text{ or } \log Q^2 \right] \times \times$$

ε $\frac{1}{\varepsilon}$ or $\log q^2$]

BFM for single field

$$\phi = \phi_B + \delta\phi$$

integrate