Title: Strong Gravity Lecture - 230228

Speakers: William East

Collection: Strong Gravity (2022/2023)

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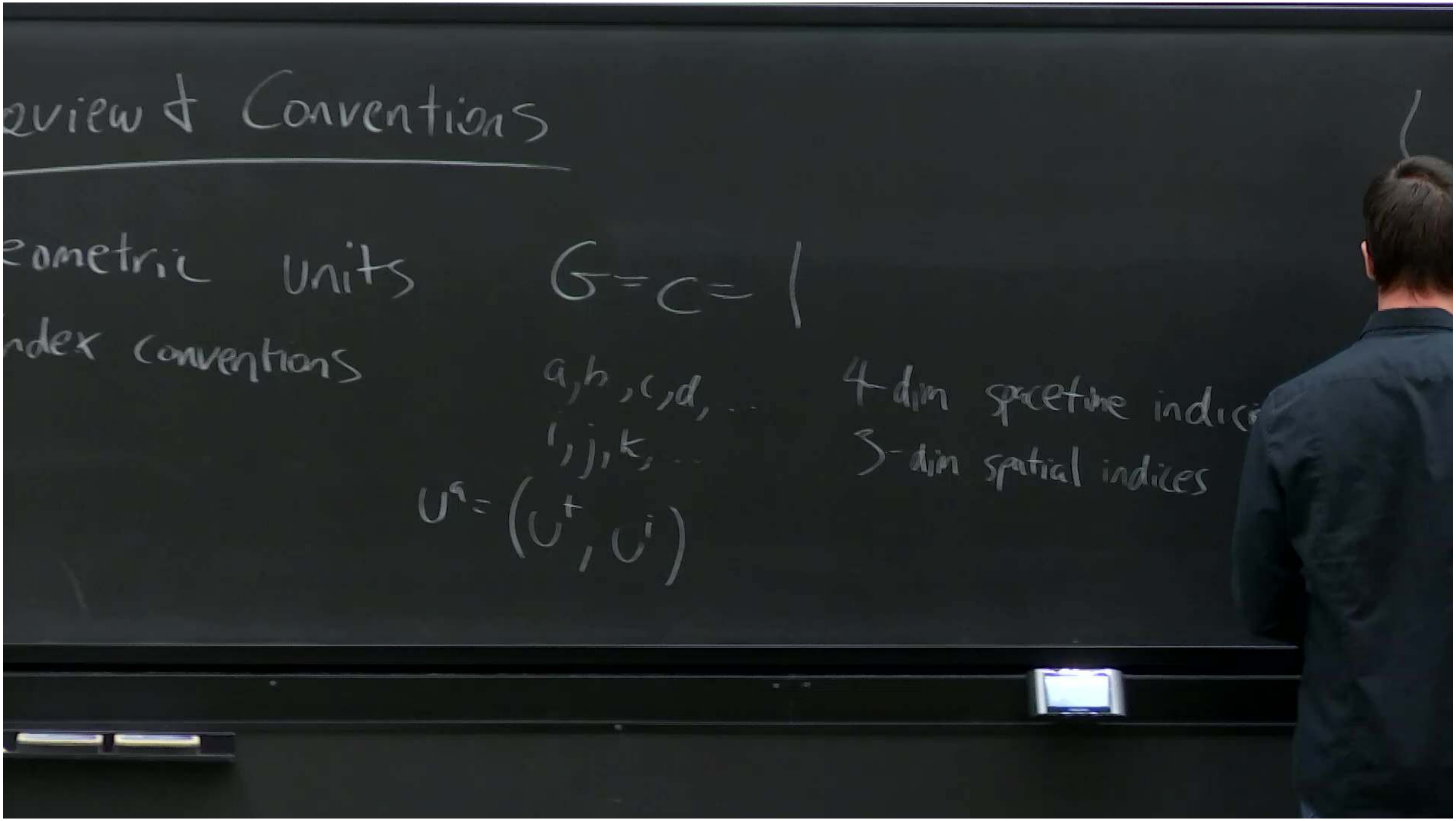
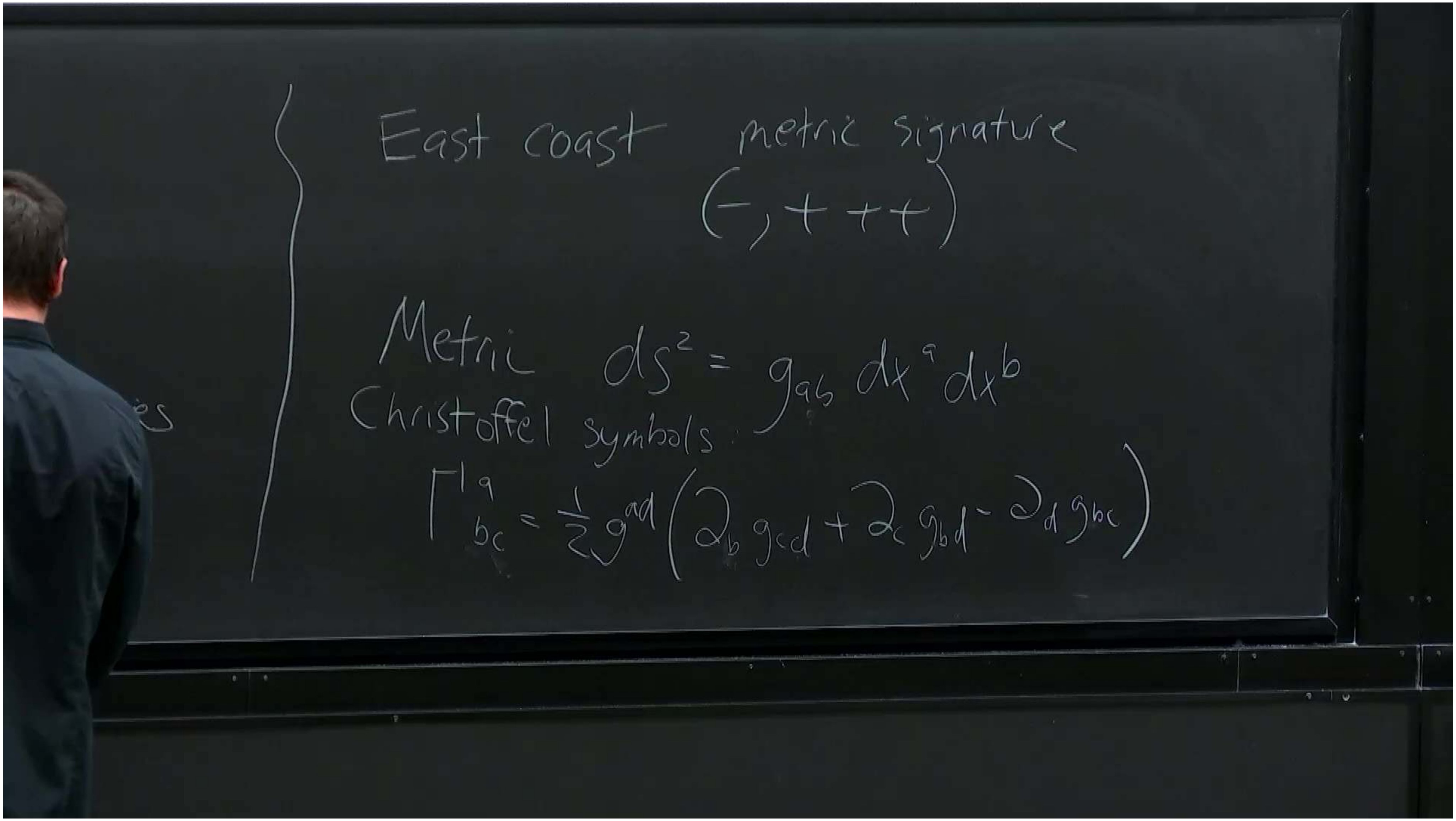
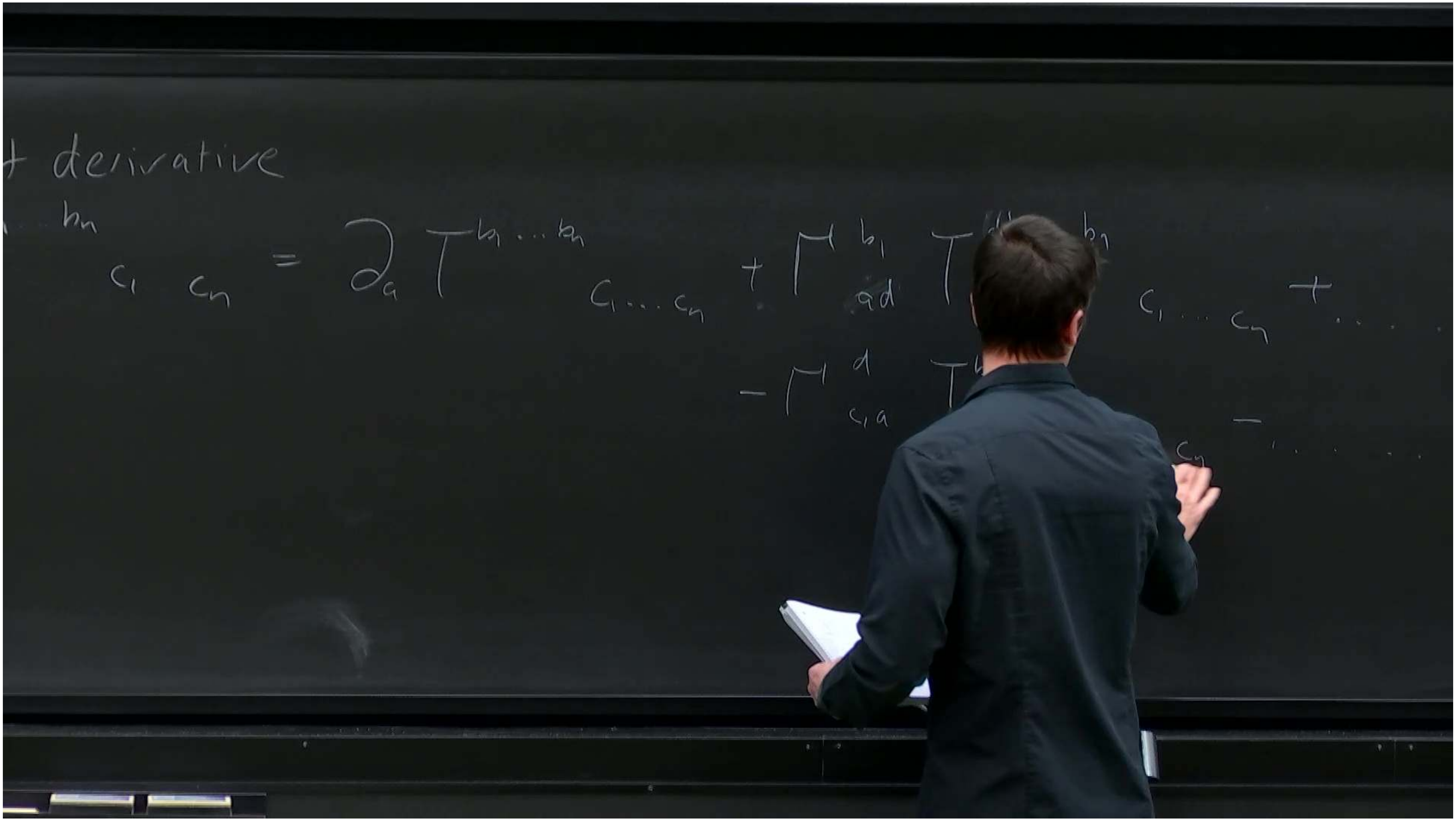
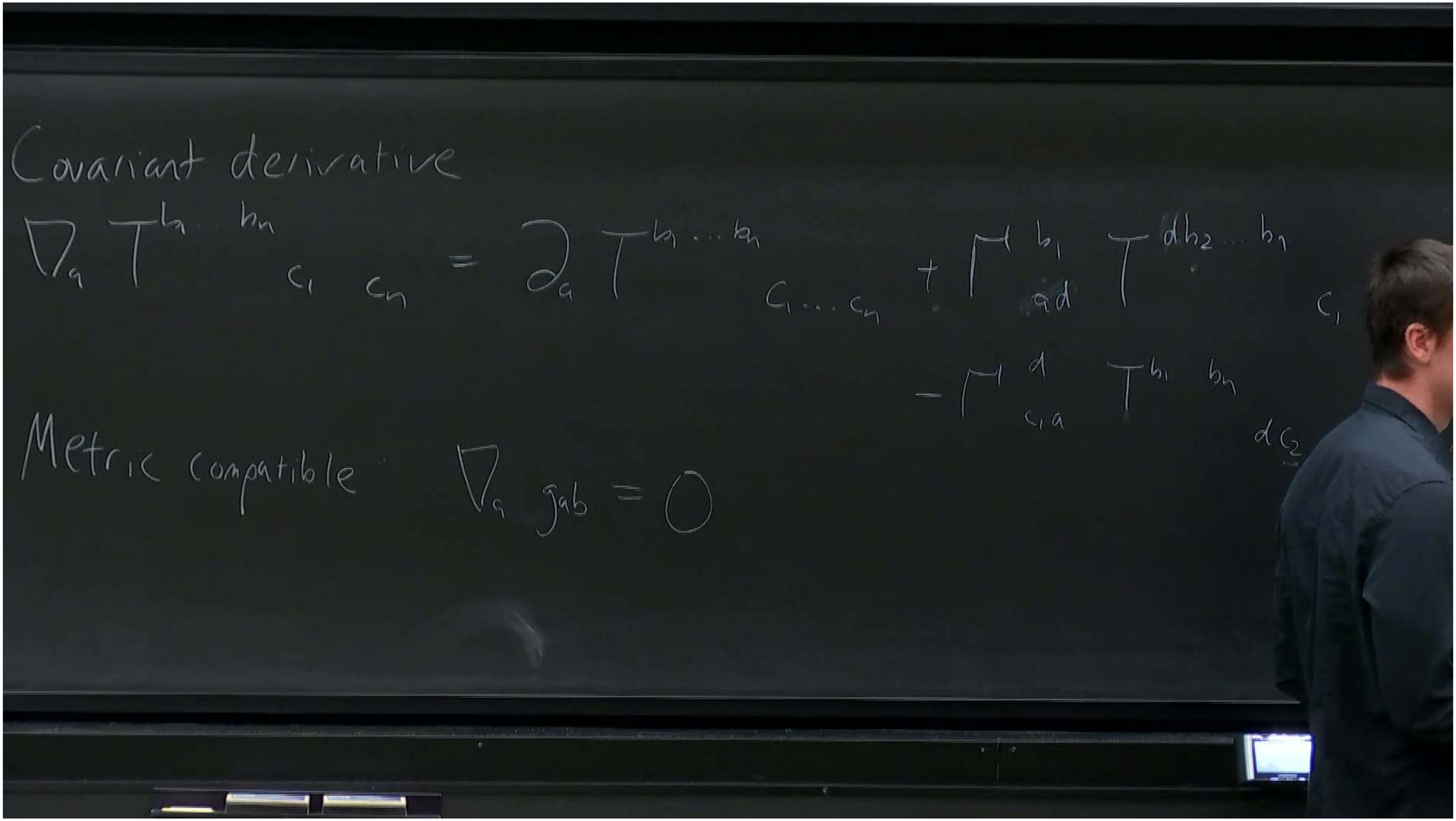
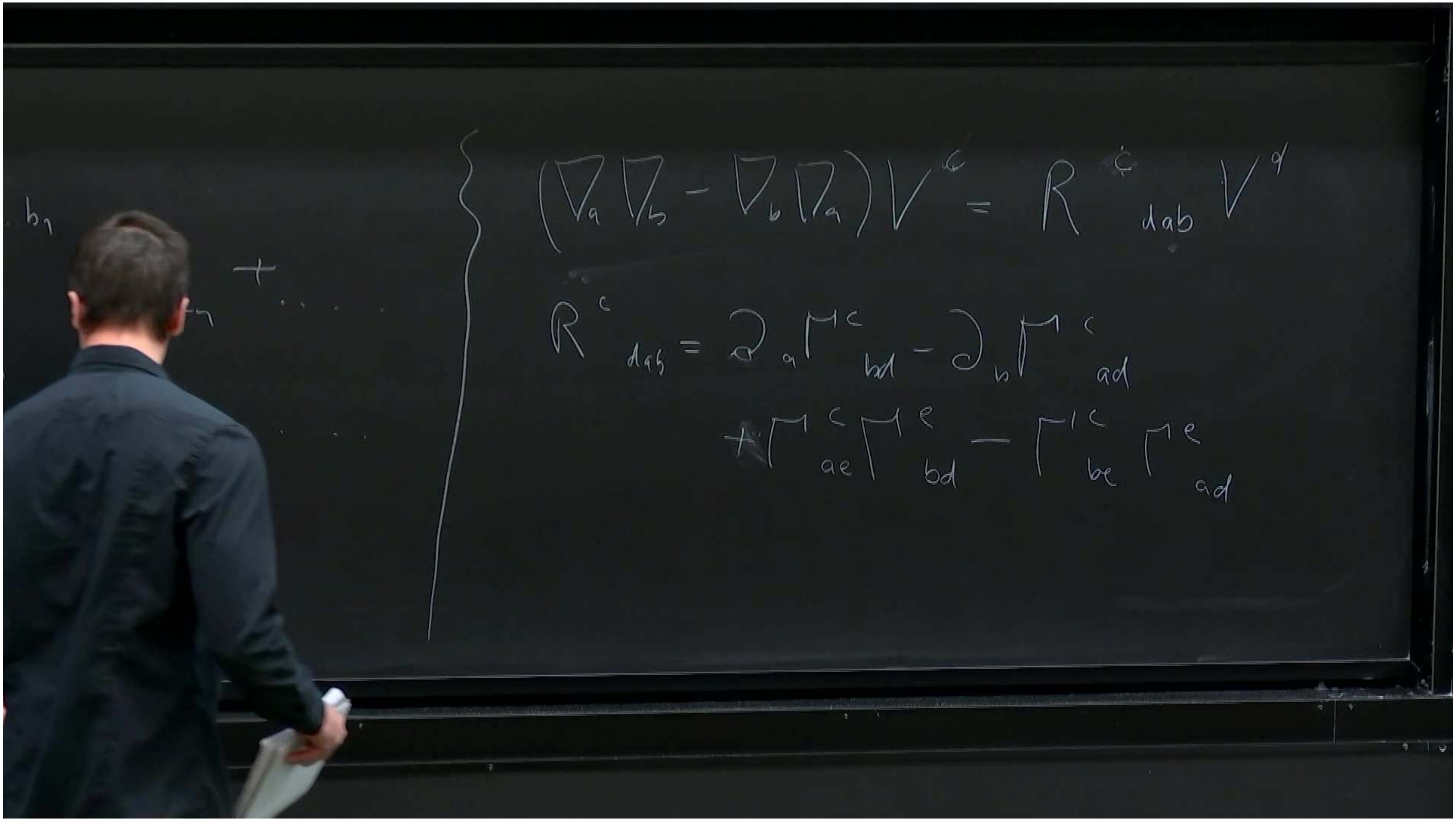
URL: https://pirsa.org/23020020

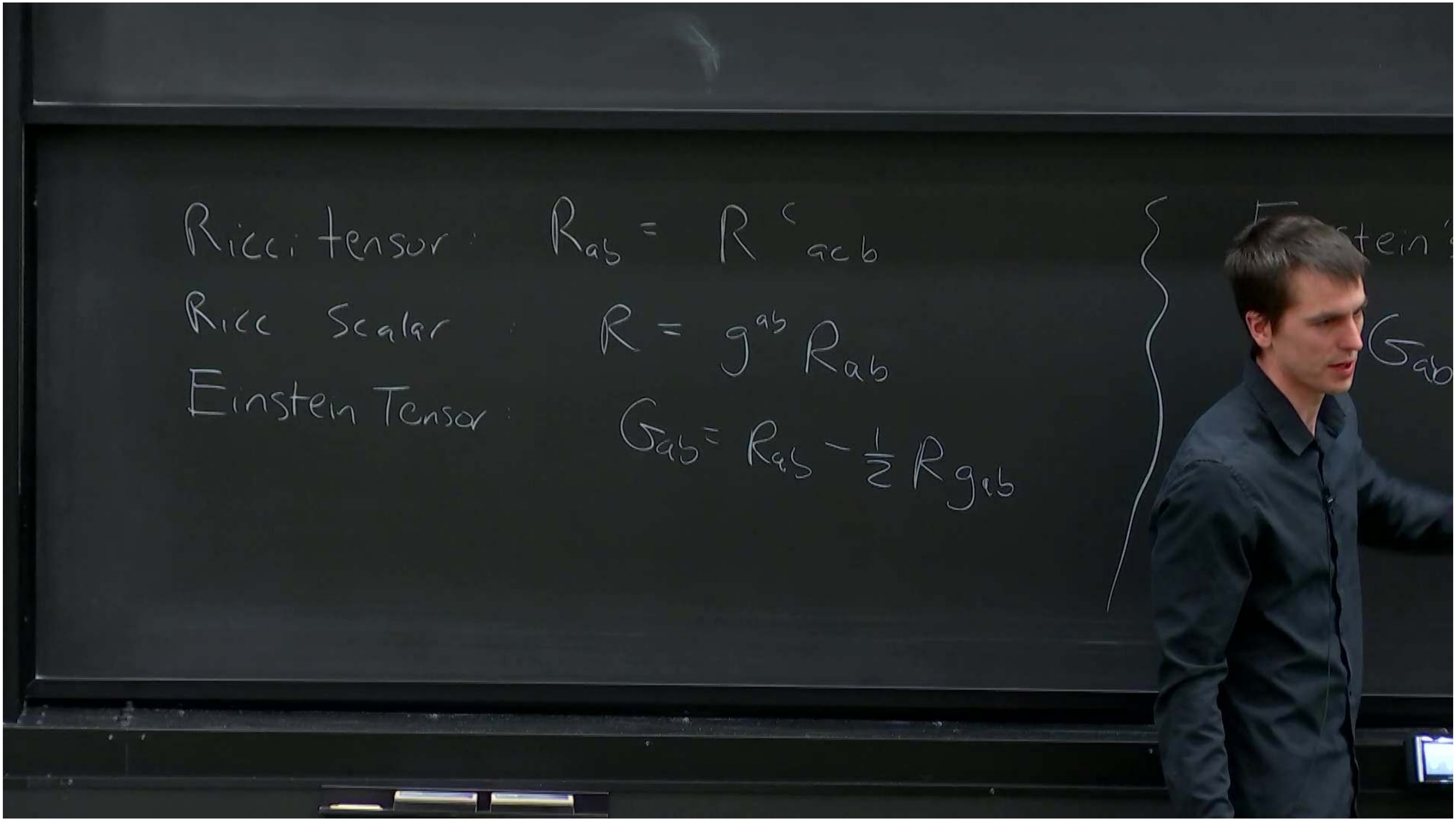

eview & Conventions
eometric Units G=c=1
ndex conventions
a, b, c, d, ... 4-dim spacetime indic
i, j, k, ... 3-dim spatial indices
u^a = (u^t, u^i)

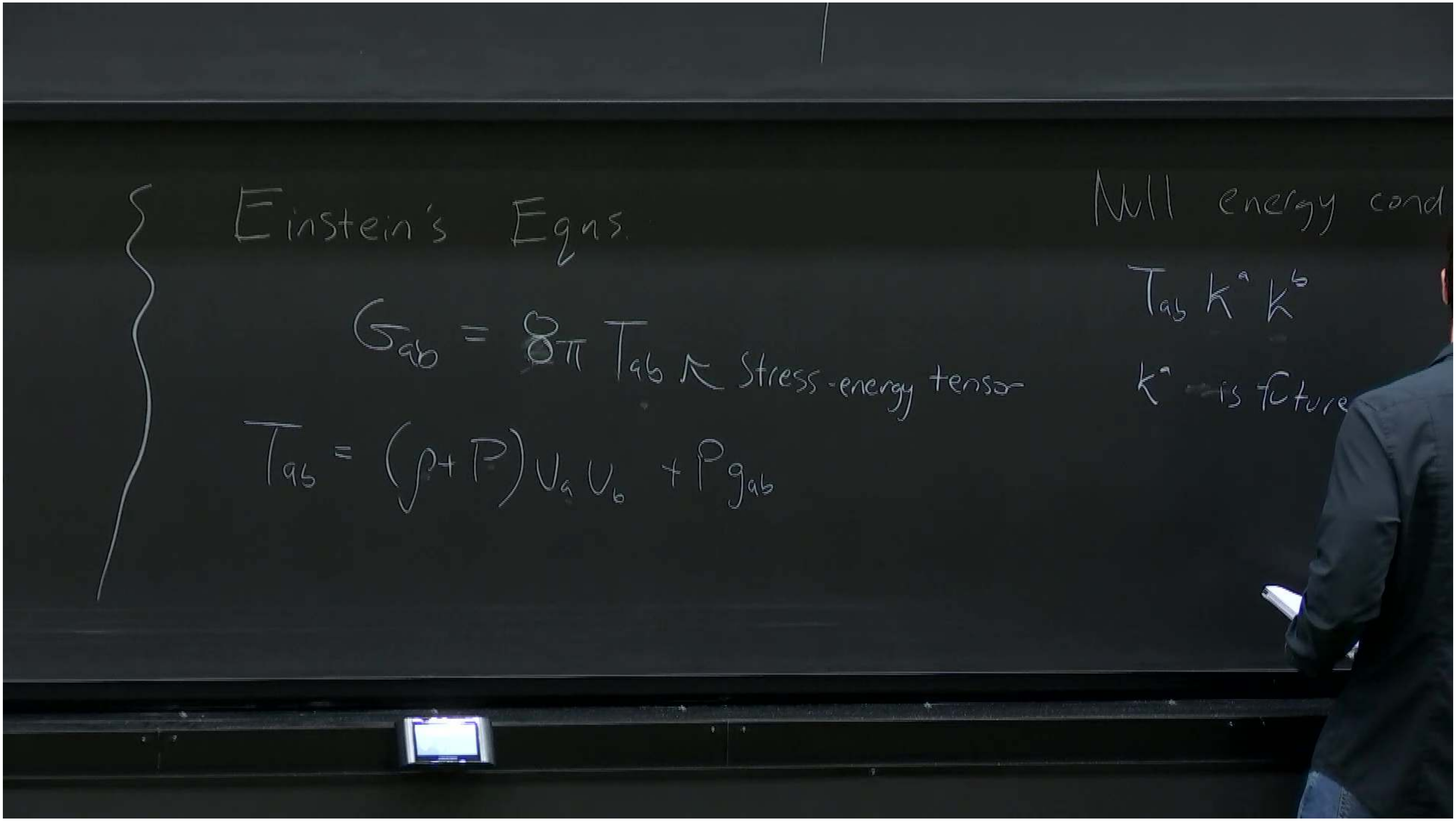

East coast metric signature
(−, + + +)
Metric $ds^2 = g_{ab}\, dx^a dx^b$
Christoffel symbols:
$\Gamma^a_{bc} = \frac{1}{2} g^{ad} \left(\partial_b g_{cd} + \partial_c g_{bd} - \partial_d g_{bc} \right)$


derivative


Covariant derivative
$\nabla_a T^{b_1 \dots b_n}{}_{c_1 \dots c_n} = \partial_a T^{b_1 \dots b_n}{}_{c_1 \dots c_n} + \Gamma^{b_1}{}_{ad} T^{d b_2 \dots b_n}{}_{c_1 \dots} - \Gamma^{d}{}_{c_1 a} T^{b_1 \dots b_n}{}_{d c_2 \dots}$
Metric compatible: $\nabla_a g_{ab} = 0$


$(\nabla_a \nabla_b - \nabla_b \nabla_a) V^c = R^c{}_{dab} V^d$
$R^c{}_{dab} = \partial_a \Gamma^c{}_{bd} - \partial_b \Gamma^c{}_{ad}$
$+ \Gamma^c{}_{ae} \Gamma^e{}_{bd} - \Gamma^c{}_{be} \Gamma^e{}_{ad}$


Ricci tensor: $R_{ab} = R^{c}{}_{acb}$
Ricci Scalar: $R = g^{ab} R_{ab}$
Einstein Tensor: $G_{ab} = R_{ab} - \frac{1}{2} R g_{ab}$


Einstein's Eqns.
Gab = 8π Tab ↖ Stress-energy tensor
Tab = (ρ+P) Ua Ub + P gab
Null energy cond
Tab k^a k^b
k^a is future

Einstein's Eqns.

$$G_{ab} = 8\pi T_{ab} \nwarrow \text{Stress-energy tensor}$$

$$T_{ab} = (\rho + P) U_a U_b + P g_{ab}$$

$$\nabla_a T^{ab} = 0$$

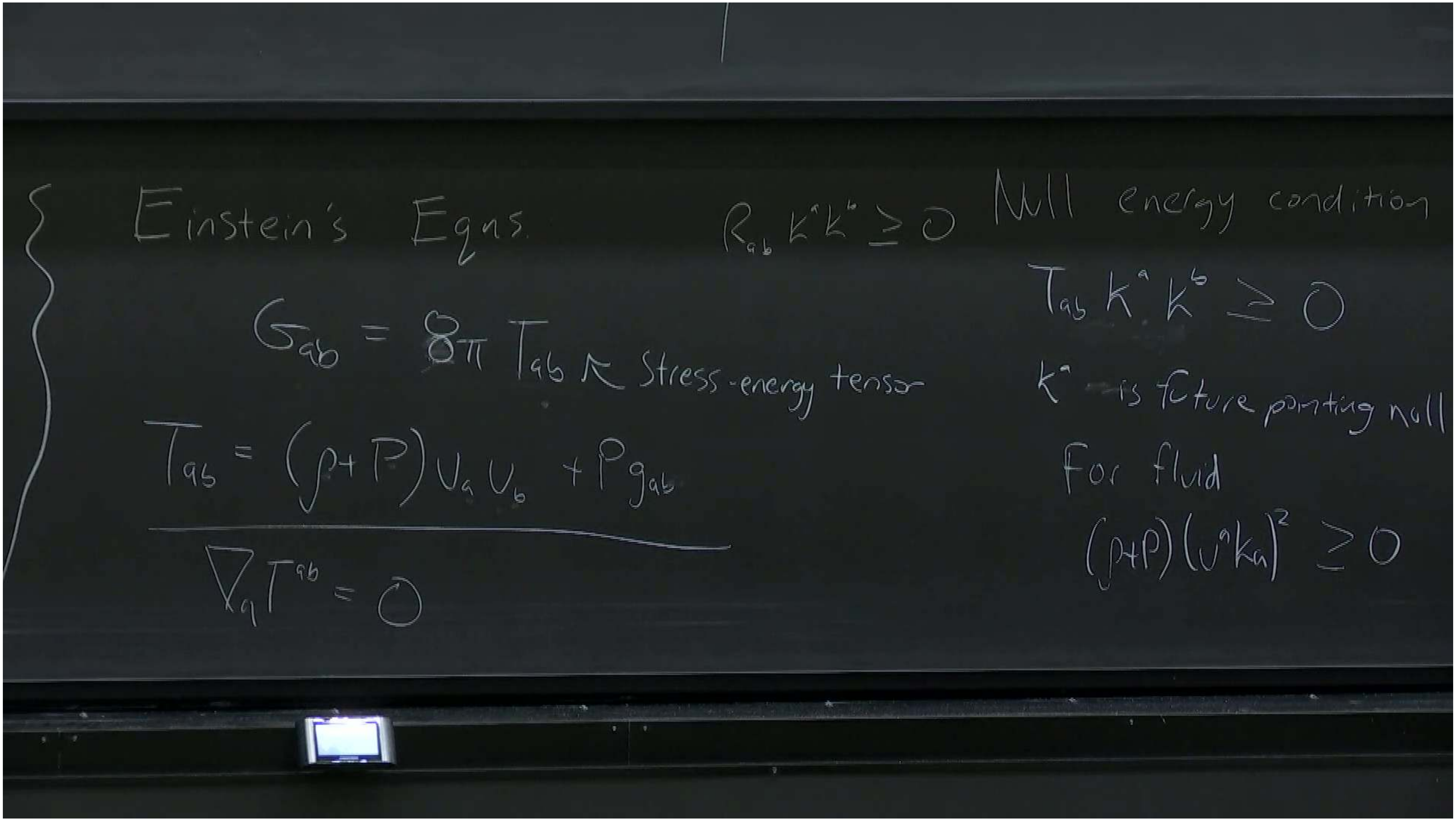
Null energy condition

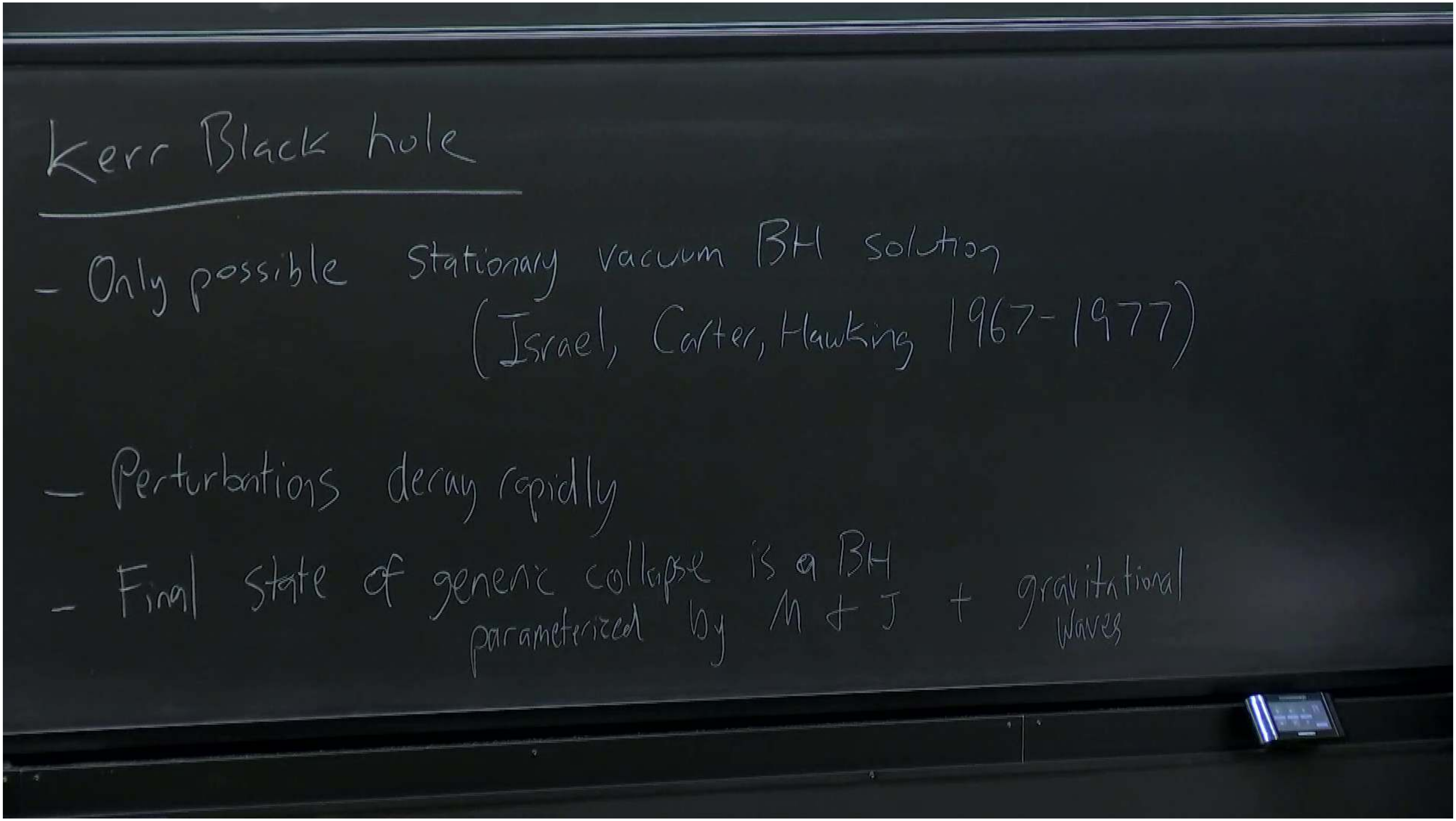
$$T_{ab} k^a k^b \geq 0$$

k^a is future pointing null

For fluid

$$(\rho + P)(U^a k_a)^2 \geq 0$$


Einstein's Eqns.
$G_{ab} = 8\pi T_{ab}$ ← Stress-energy tensor
$T_{ab} = (\rho + P) U_a U_b + P g_{ab}$
$\nabla_a T^{ab} = 0$
$R_{ab} k^a k^b \geq 0$ Null energy condition
$T_{ab} k^a k^b \geq 0$
k^a is future pointing null
For fluid
$(\rho + P)(U^a k_a)^2 \geq 0$


Kerr Black hole
- Only possible stationary vacuum BH solution
(Israel, Carter, Hawking 1967-1977)
- Perturbations decay rapidly
- Final state of generic collapse is a BH parameterized by M + J + gravitational waves

Kerr Black hole

- Only possible stationary vacuum BH solution (Israel, Carter, Hawking 1967-1977)

"No hair"

- Perturbations decay rapidly
- Final state of generic collapse is a BH parameterized by M + J + gravitational waves

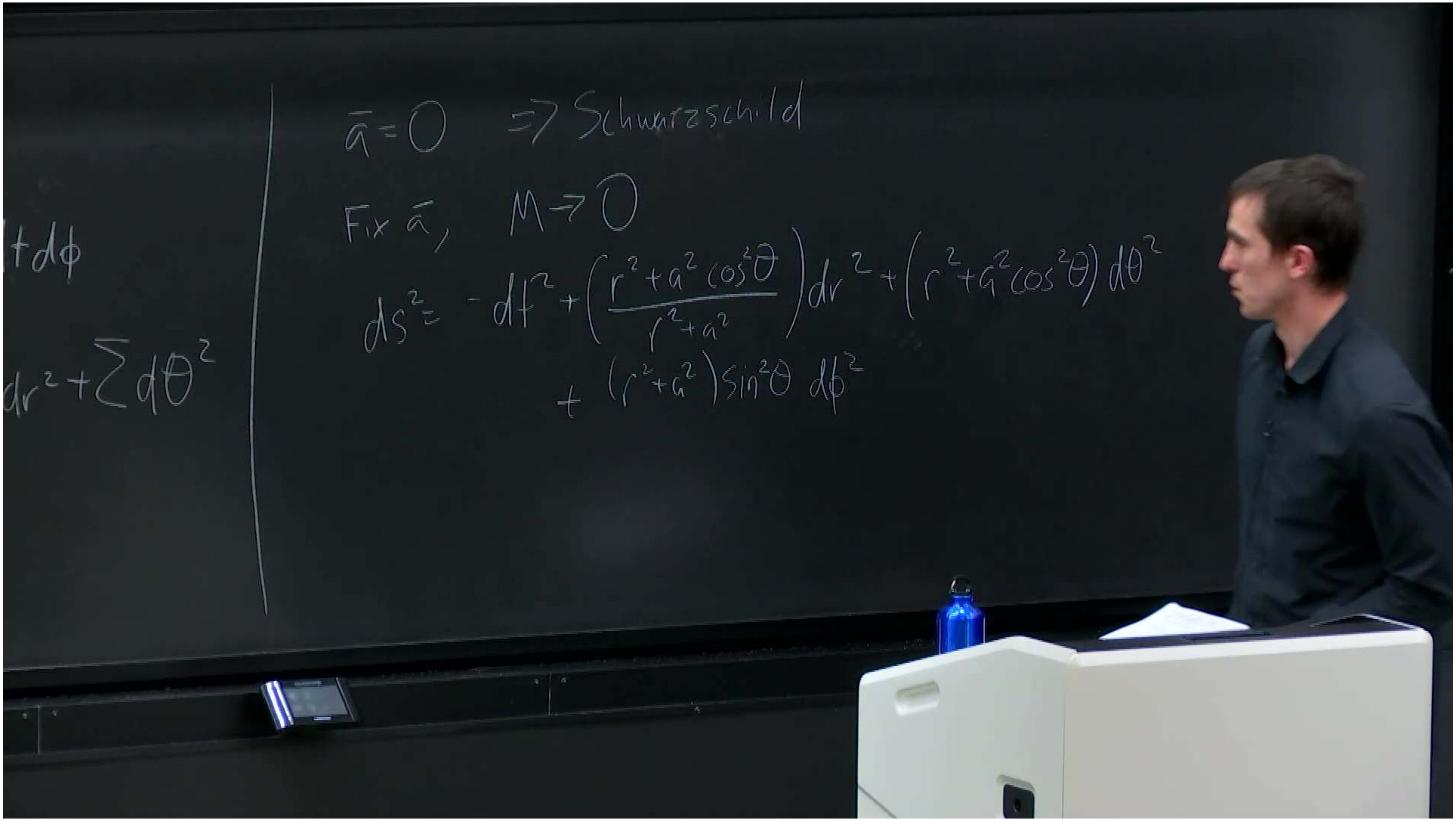
Metric in Boyer-Lindquist Coördinates:

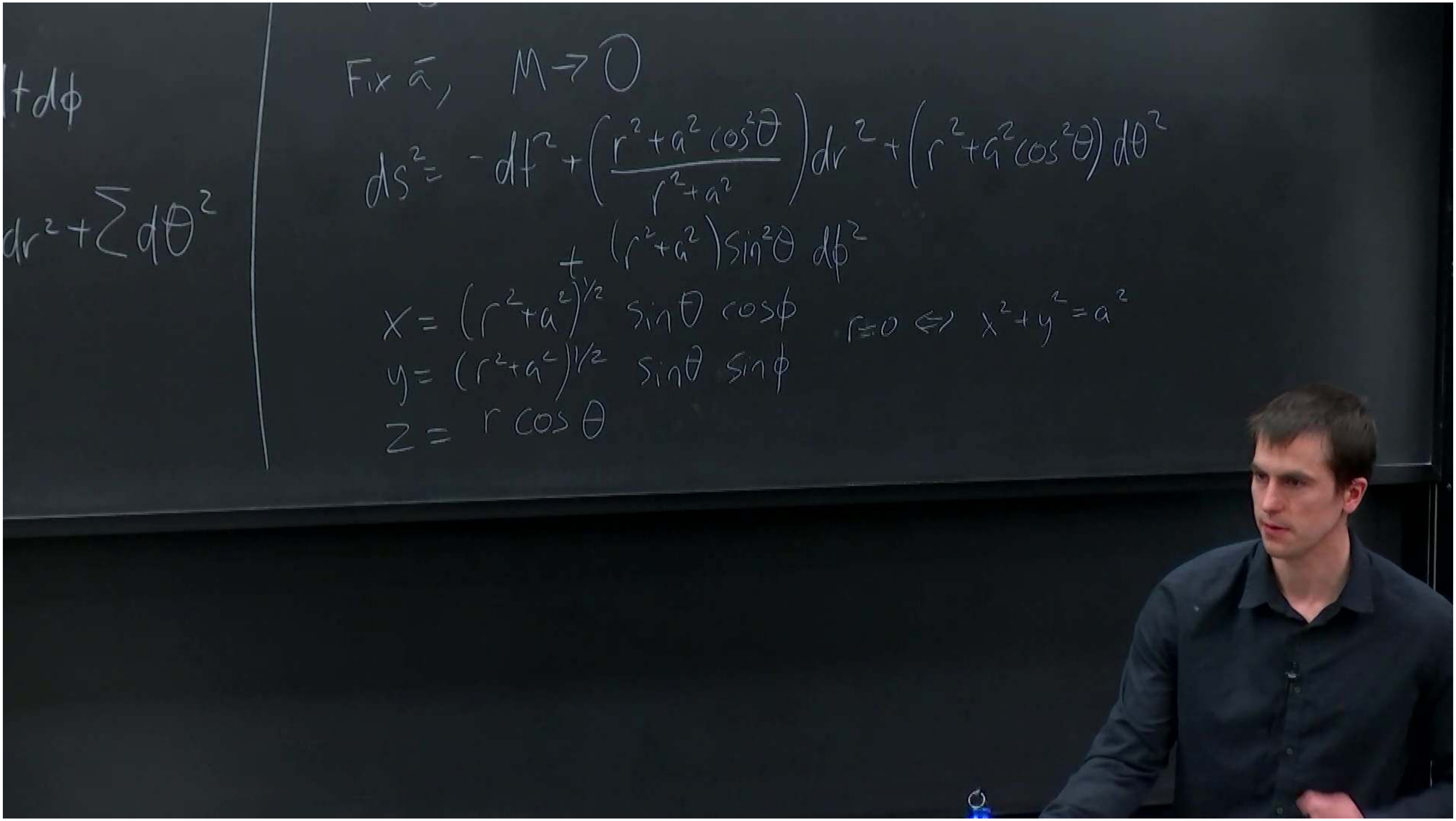
$$ds^2 = -\left(\frac{\Delta - a^2\sin^2\theta}{\Sigma}\right)dt^2 - \frac{2a\sin^2\theta\,(r^2+a^2-\Delta)}{\Sigma}\,dt\,d\phi$$

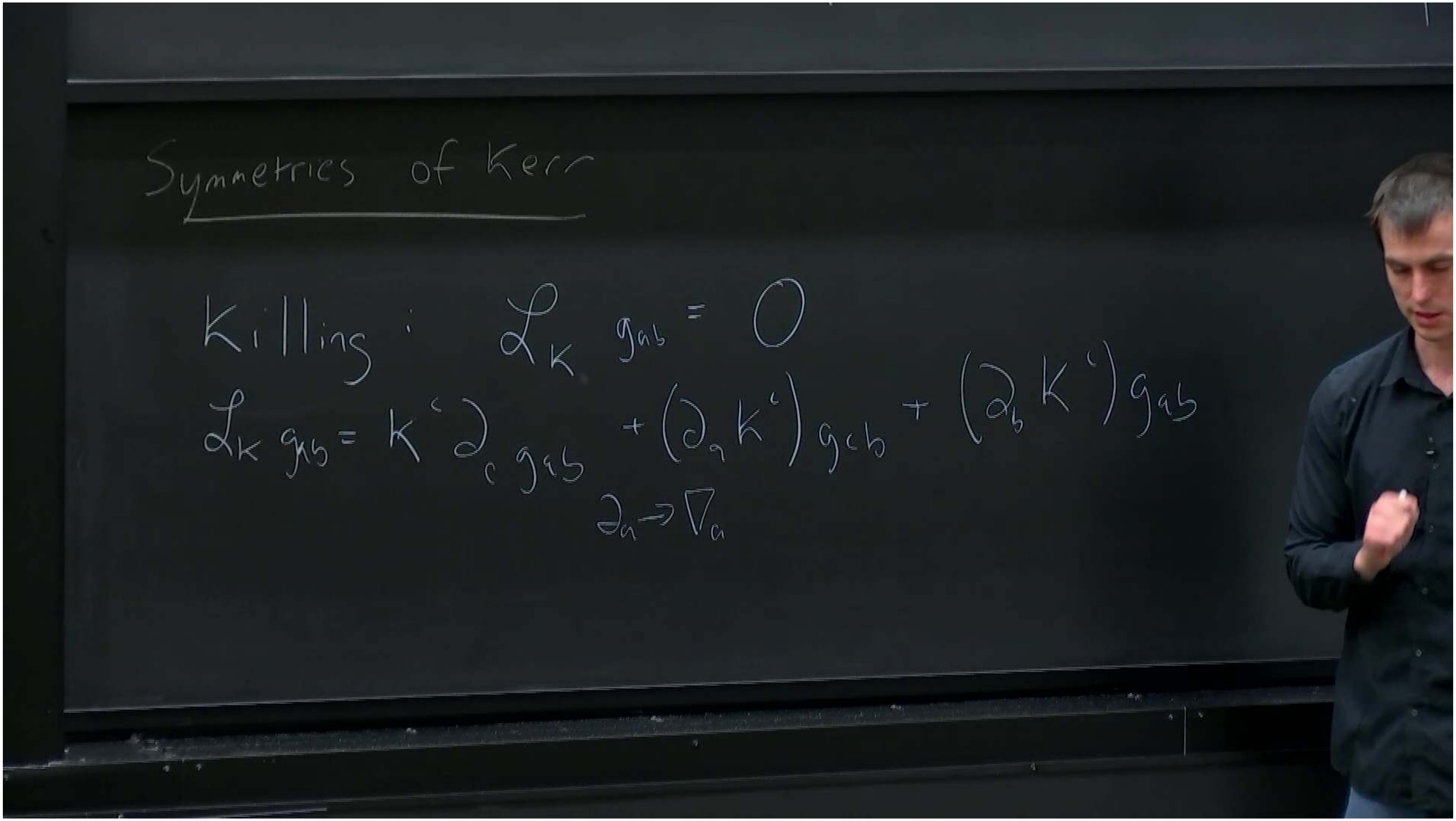
$$+\left[\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\Sigma}\right]\sin^2\theta\,d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma\,d\theta^2$$

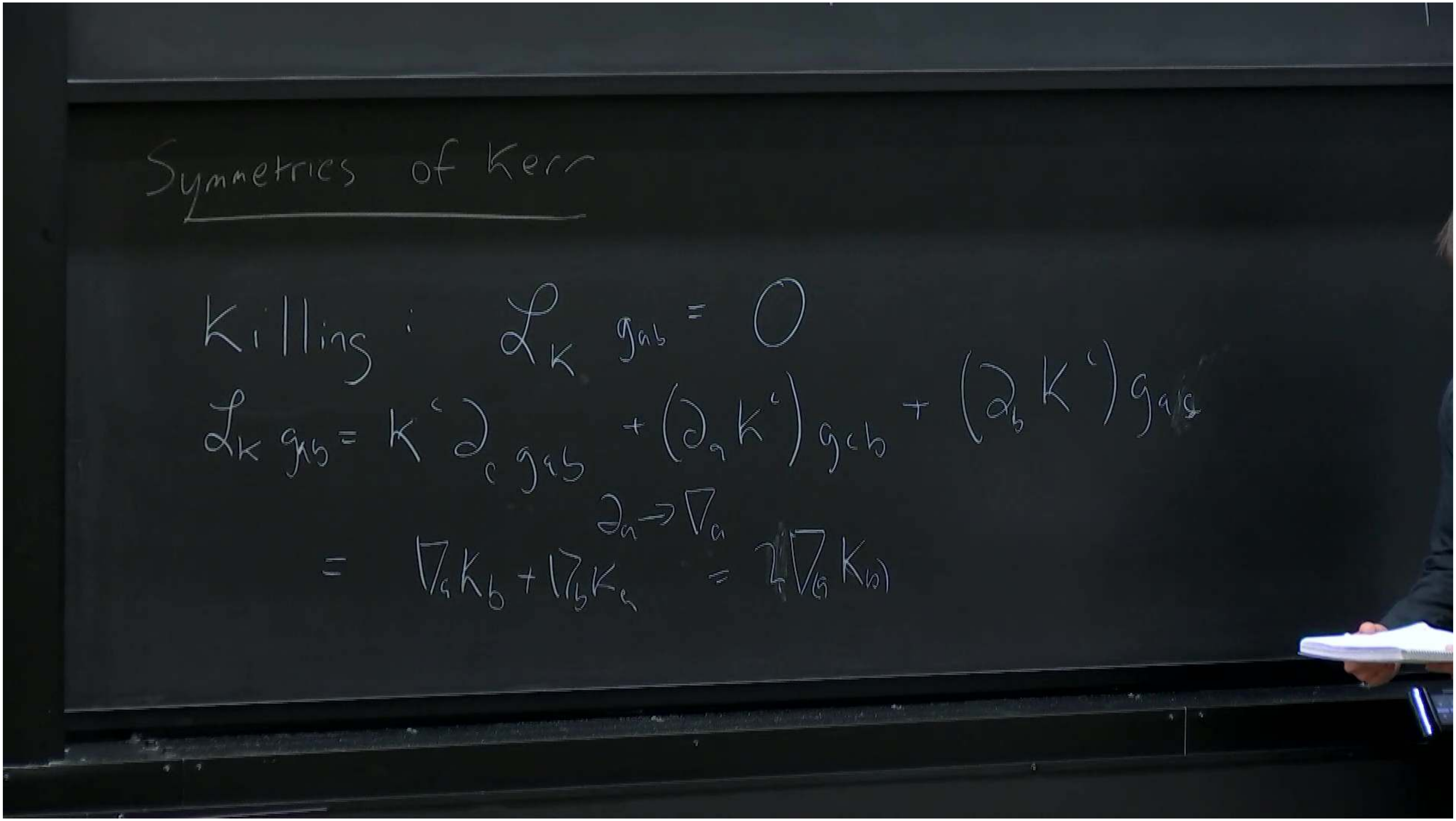
$$\Sigma = r^2 + a^2\cos^2\theta \qquad a = J/M$$

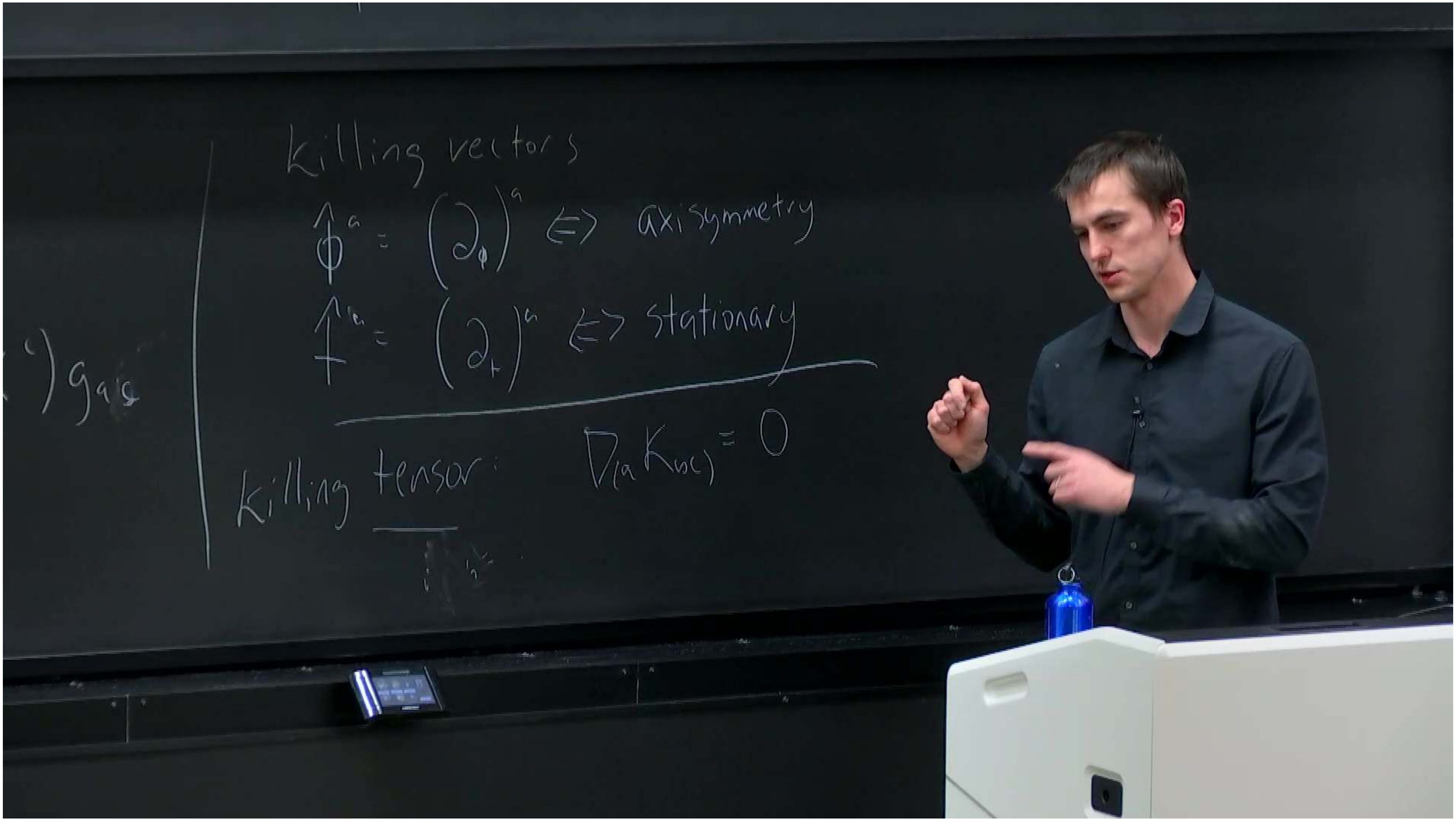
$$\Delta = r^2 + a^2 - 2Mr \qquad \bar{a} = J/M^2$$

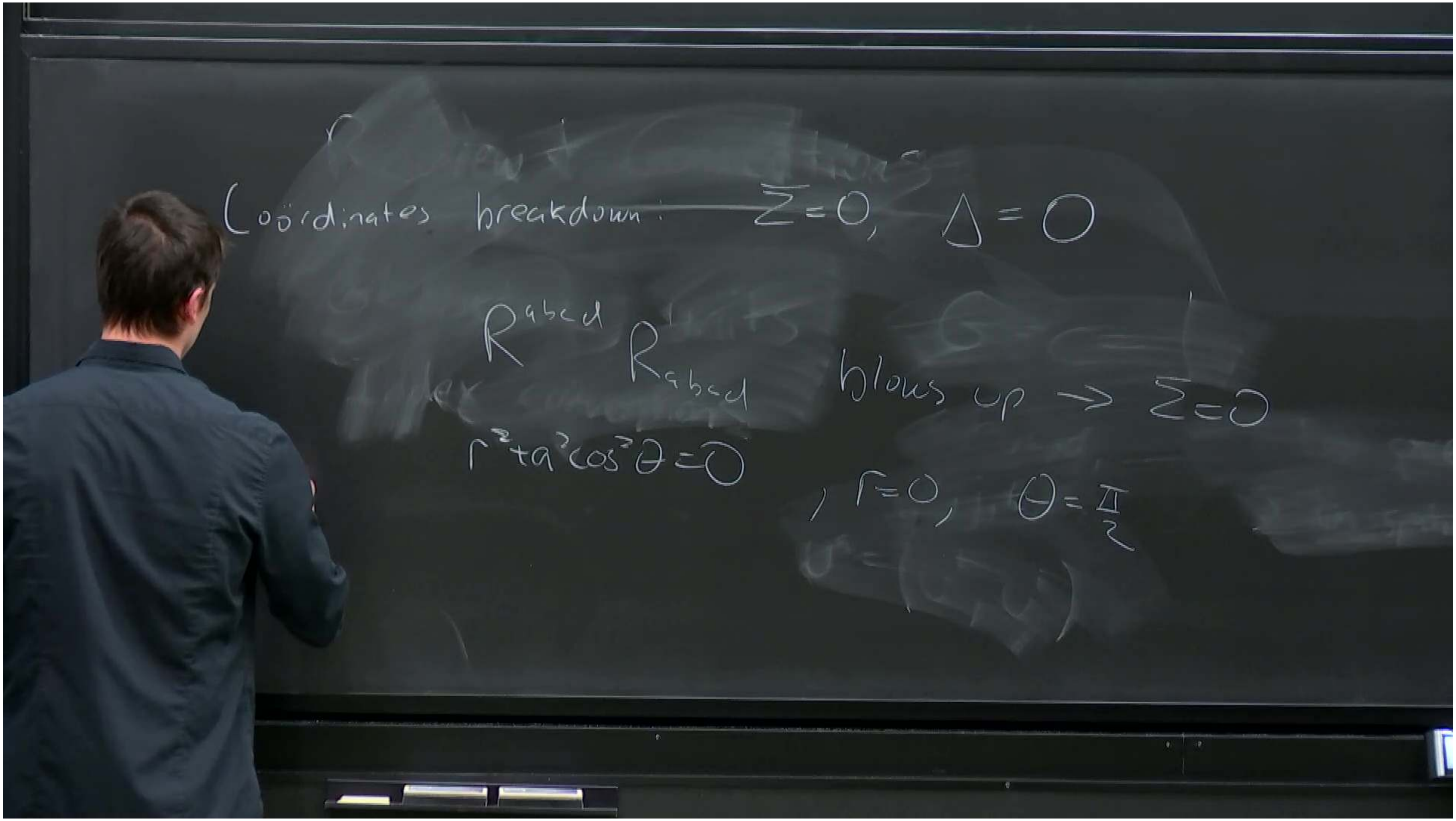

ā = 0 ⇒ Schwarzschild
Fix ā, M → 0
ds² = −dt² + ((r² + a² cos²θ)/(r² + a²)) dr² + (r² + a² cos²θ) dθ²
+ (r² + a²) sin²θ dφ²
+dφ
dr² + Σ dθ²

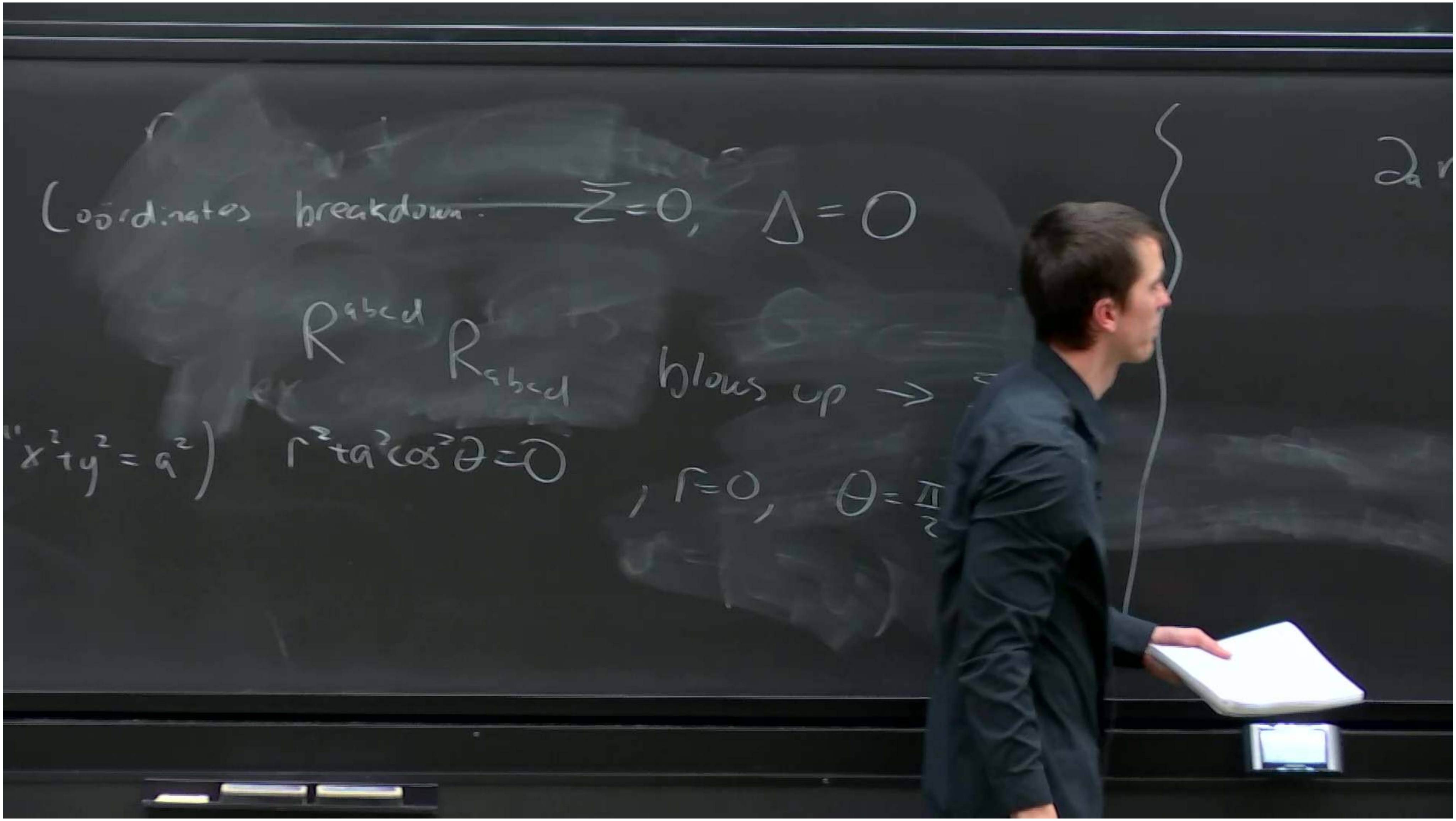

Fix a, M → 0
ds² = -dt² + ((r² + a² cos²θ)/(r² + a²)) dr² + (r² + a² cos²θ) dθ²
+ (r² + a²) sin²θ dφ²
x = (r² + a²)^1/2 sinθ cosφ
y = (r² + a²)^1/2 sinθ sinφ
z = r cos θ
r = 0 ⇔ x² + y² = a²

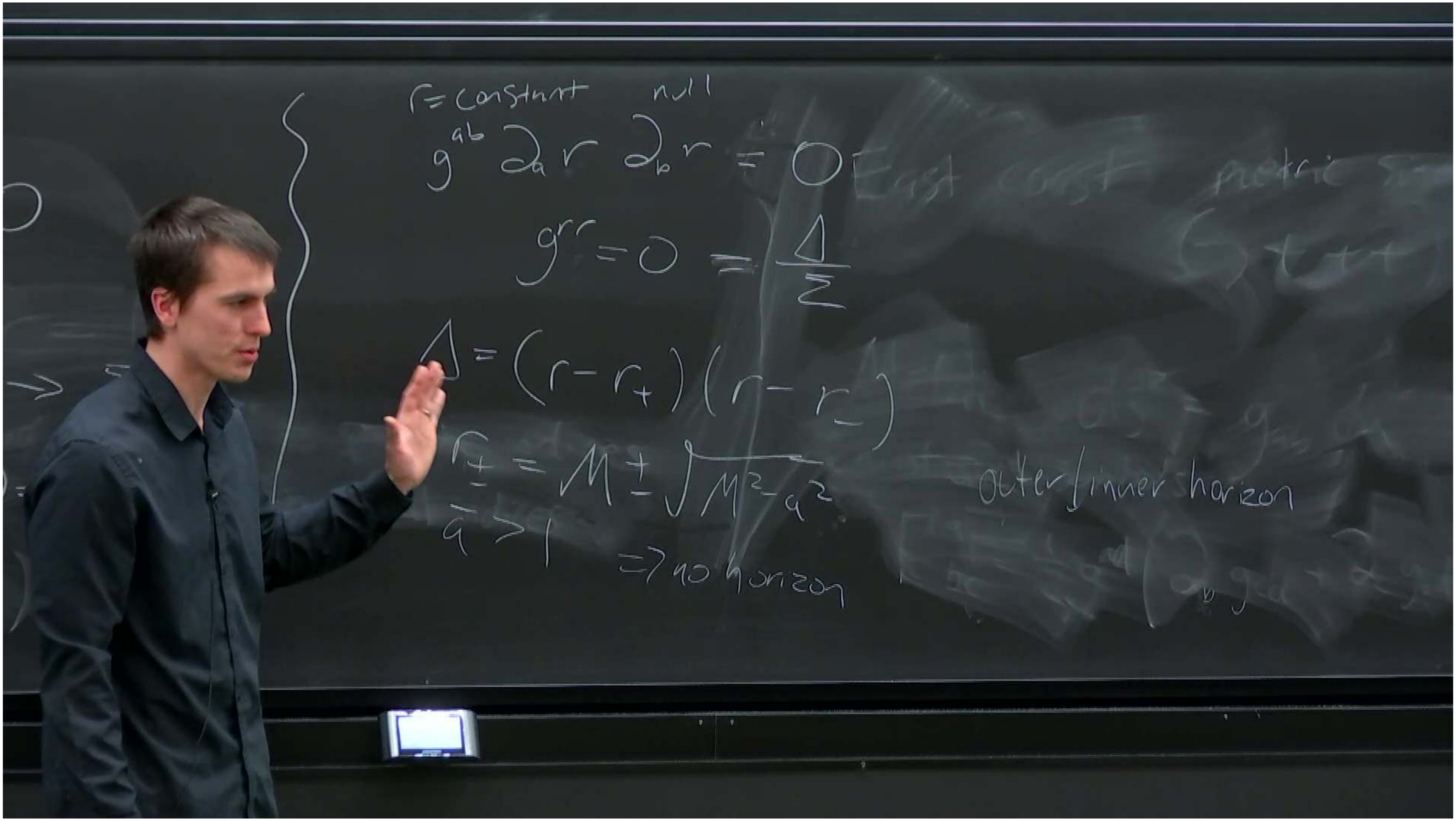

Symmetries of Kerr
Killing : $\mathcal{L}_K g_{ab} = 0$
$\mathcal{L}_K g_{ab} = K^c \partial_c g_{ab} + (\partial_a K^c) g_{cb} + (\partial_b K^c) g_{ac}$
$\partial_a \to \nabla_a$


Symmetries of Kerr
Killing: $\mathcal{L}_K g_{ab} = 0$
$\mathcal{L}_K g_{ab} = K^c \partial_c g_{ab} + (\partial_a K^c) g_{cb} + (\partial_b K^c) g_{ac}$
$\partial_a \to \nabla_a$
$= \nabla_a K_b + \nabla_b K_a = 2 \nabla_{(a} K_{b)}$


killing vectors
$\phi^a = (\partial_\phi)^a \Leftrightarrow$ axisymmetry
$t^a = (\partial_t)^a \Leftrightarrow$ stationary
killing tensor: $D_{(a}K_{bc)} = 0$


Coördinates breakdown: Σ=0, Δ=0
R^abcd R_abcd blows up ⇒ Σ=0
r²+a²cos²θ=0
r=0, θ=π/2


Coordinates breakdown: Σ=0, Δ=0
R^abcd R_abcd blows up →
x²+y²=a²) r²+a²cos²θ=0
, r=0, θ=π/2


r = constant null
g^ab ∂_a r ∂_b r = 0
g^rr = 0 = Δ/Σ
Δ = (r − r_+)(r − r_−)
r_± = M ± √(M² − a²)
a > 1 ⇒ no horizon
outer/inner horizon
East coast