Title: Standard Model Lecture - 230202

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Collection: Standard Model (2022/2023)

Date: February 02, 2023 - 3:45 PM

URL: https://pirsa.org/23020013

Standard Model

Lecture # 12

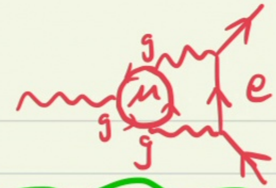
Brief glimpses:

i) SM EFT

ii) Neutrinos

iii) Anomalies

Running gauge couplings:

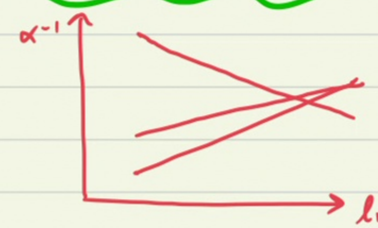


$$\alpha_i \equiv \frac{g_i^2}{4\pi} \qquad \beta(\alpha_i) \equiv \Lambda \frac{d\alpha_i}{d\Lambda} = \frac{d\alpha_i}{d(\ln\Lambda)} \qquad \left(\text{length} \sim \frac{1}{\Lambda}\right)$$

$$\beta(\alpha_i) = \frac{b_i}{2\pi}\alpha_i^2 \implies \alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\left(\frac{\Lambda}{M_Z}\right)$$

$\alpha_1^{-1}(M_Z) = 59.2$ $\qquad \alpha_2^{-1}(M_Z) = 29.6$ $\qquad \alpha_3^{-1}(M_Z) = 8.5$

$b_1 = 41/10$ $\qquad b_2 = -19/6$ $\qquad b_3 = -7$



'74 (Georgi, Quinn, Weinberg): couplings unify at $M_{GUT} \sim 10^{15}$ GeV, $\Gamma_{p+} \simeq m_p^5 / M_{GUT}^4$

$$\alpha_i(\Lambda) = \frac{\alpha_i(M_Z)}{1 - \frac{b_i}{2\pi}\alpha_i(M_Z)\ln(\Lambda/M_Z)}$$

$b_i > 0 \Rightarrow \alpha_i(\Lambda) \to \infty$ at $\Lambda = M_W \exp\left[\frac{2\pi}{b_i \alpha_i(M_Z)}\right]$ ("Landau pole")

$b_i < 0 \Rightarrow \alpha_i(\Lambda) \to 0$ as $\Lambda \to \infty$ ("Asymptotic

QCD: quarks interacting via $SU(3)_c$ "strong force" (gluons)

Since $SU(3)$ β-function is negative:

- $\alpha_3(\Lambda) \to 0$ as $\Lambda \to \infty$ ("Asymptotic freedom")
- when Λ gets low enough, Landau pole ($\Lambda_{QCD} \leftarrow$ the QCD or quark confinement scale)
- For $\Lambda < \Lambda_{QCD}$, QCD is strongly coupled/non-perturbative
- Quark are "confined": $q \; \bar{q}$ $\leftarrow$ "meson" e.g. $\pi^+ = \bar{d}u$, $\pi^- = \bar{u}d$

$q_1 \; q_2 \; q_3$ "baryon" $q_1^A q_2^B q_3^C \epsilon_{ABC}$

$p = uud$ (proton)

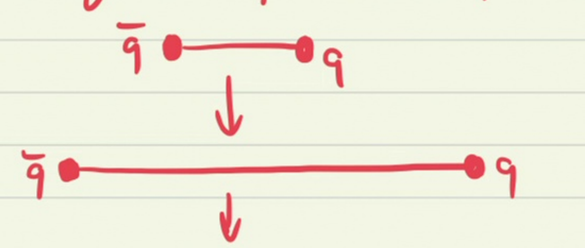
$n = udd$ (neutron)

$\bar{q}_1 \; \bar{q}_2 \; \bar{q}_3$ "anti-baryon" $\bar{q}_1^A \bar{q}_2^B \bar{q}_3^C \epsilon_{ABC}$

$\bar{p} = \bar{u}\bar{u}\bar{d}$ (anti-proton)

$\bar{n} = \bar{u}\bar{d}\bar{d}$ (anti-neutron)

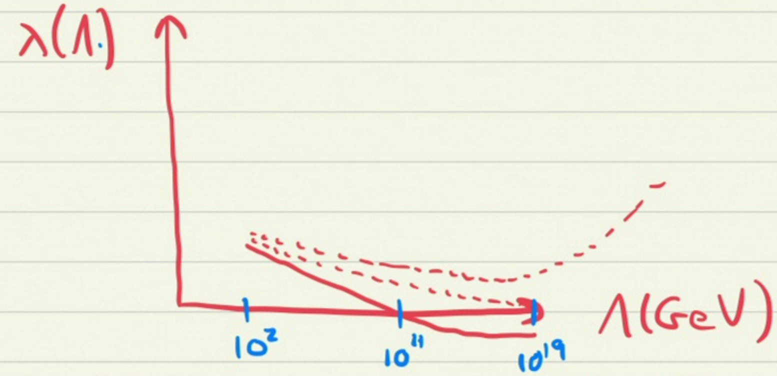
String theory of confinement



Higgs (In)Stability/Criticality

In Wilsonian action S_Λ:

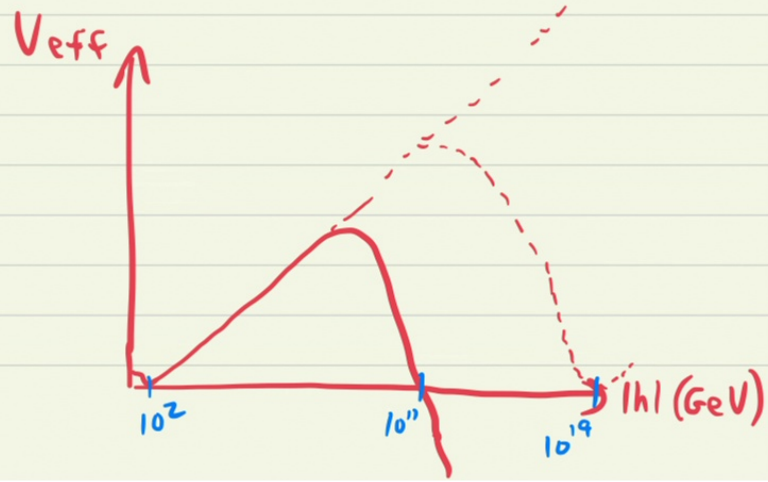
$$V_\Lambda(h) = -m^2(\Lambda)(h^\dagger h) + \lambda(\Lambda)(h^\dagger h)^4$$



In Effective Action Γ: $\Lambda \to |h|$

$$V_{eff}(h) = -m^2(|h|)(h^\dagger h) + \lambda(|h|)(h^\dagger h)^2$$

VEV $\langle h \rangle$ is minimum of V_{eff}
(not V_Λ)



	SU(3)	SU(2)	U(1)
q_L	3	2	1/6
u_R	3	1	2/3
d_R	3	1	-1/3
l_L	1	2	-1/2
ν_R	1	1	0
e_R	1	1	-1
h	1	2	1/2

3 observational puzzles:

1: ν masses + oscillations

2: matter/anti-matter asym.

3: DM

3 models:

- SM:
  omit ν_R row + omit NR terms
  → doesn't explain 1, 2, or 3
- SMEFT:
  omit ν_R row + include NR terms
  → explains 1, but not 2 or 3
  → more incomplete (NR even w/out grav)
- νSM:
  include ν_R row + omit NR terms
  → explains 1, 2, and 3.
  → more complete (renorm. w/out grav)
  → and ν_R required by all unification schemes we discussed

SMEFT:

- define $\Psi_L^i = \tilde{h}^{+} \ell_L^i$ ← LH spinor, gauge invariant, mass dim 5/2.

$\Rightarrow$ Majorana-like mass term: $\frac{-1}{\Lambda}\left[\overline{\Psi}_{L,c}^{\,i}\, y^{ij}\, \Psi_L^j + h.c.\right]$ ← Weinberg term (only dim 5 term in SMEFT)

- using $\tilde{h} = \frac{1}{\sqrt{2}}\begin{pmatrix} v_h + H \\ 0 \end{pmatrix} \Longrightarrow \Psi_L^i = \frac{1}{\sqrt{2}} v_h \nu_L^i + \ldots$

$$\Rightarrow -\frac{1}{\Lambda}\left[\overline{\Psi}_{L,c}^{\,i}\, y^{ij}\, \Psi_L^j + h.c.\right] = -\frac{1}{2}\left[\overline{\nu}_{L,c}^{\,i}\, M^{ij}\, \nu_L^j + h.c.\right] \qquad M^{ij} = \frac{v_h^2}{2\Lambda} y^{ij}$$

Majorana mass matrix

- Recall for quarks: u_L^i emits W_μ^+ to become $d_L^i = \underbrace{V_{CKM}{}^i{}_j\, d_L'^j}_{\text{lin. comb. of mass eigenstates}}$

weak eigenstate

- Similarly for leptons: e_L^i absorbs W_μ^+ to become ν_L^i ← weak eigenstate,

lin. comb. of mass eigenstates (eigenvectors of M^{ij})

νSM (part 1)

$$\mathcal{L}_{(\nu)SM} = -\frac{1}{2}\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}] - \frac{1}{2}\mathrm{Tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$- (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \lambda (h^\dagger h)^2$$

$$+ i[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} \bar{u}_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + (\bar{\nu}_R \not{D} \nu_R) + \bar{e}_R \not{D} e_R]$$

$$- [\bar{q}_L \tilde{h} Y_u u_R + \bar{q}_L h Y_d d_R + (\bar{\ell}_L \tilde{h} Y_\nu \nu_R) + \bar{\ell}_L h Y_e e_R + h.c.]$$

$$- (\bar{\nu}_R^c M \nu_R) \quad \underline{+ \theta \,\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]}$$

All Lorentz-invariant, gauge-invariant, renormalizable terms built from fields in SM table.

νSM (part 2)

- If $\bar{\nu}_R^c M \nu_R$ term is **zero** then fermionic action becomes:

$$= i\left[\bar{u}\not{\partial}u + \bar{d}\not{\partial}d + \bar{e}\not{\partial}e + \cancel{\bar{\nu}_L\not{\partial}\nu_L} + \bar{\nu}\not{\partial}\nu\right] - \left[\bar{u}m_u u + \bar{d}m_d d + \bar{e}m_e e + \bar{\nu}m_\nu \nu\right] + \ldots$$

3 Dirac ν's = 3×4 = 12 states

(3 Dirac masses, 4 states/mass)

- If $\bar{\nu}_R^c M \nu_R$ term is non-zero

6 Majorana ν's = 6×2 = 12 states

(6 Majorana masses, 2 states/mass)

Mass matrix: $\begin{pmatrix} M & m_\nu \\ m_\nu & 0 \end{pmatrix}$

eigenvalues: "seesaw mechanism"

$\sim M \leftarrow$ 3 heavy $\equiv$

$\sim \frac{m_\nu^2}{M} \leftarrow$ 3 light $\equiv$

- "Leptogenesis": If $\not{CP}$, 1 heavy ν can decay to more matter than anti-matter.

- Dark matter: 1 heavy ν can be stable (or long-lived compared to 10^{10} yrs)
  - Two ways:
    - → produced by primordial plasma ("warm DM", $m_{dm} \sim$ keV, unstable)
    - → Hawking radiated from Bang ("cold DM", $m_{dm} = 4.8 \times 10^8$ GeV, stable) ("CPT-symm. Universe" 2018)

prediction: lightest ν is massless (testable w/ cosmology, 0νββ decay)

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$$- [\bar{u}m_u u + \bar{d}m_d d + \bar{e}m_e e + \bar{\nu}m_\nu \nu] + \ldots \leftarrow$$

3 Dirac ν's $= 3 \times 4 = 12$ states
(3 Dirac masses, 4 states/mass)

- If $\bar{\nu}_R^c M \nu_R$ term is non-zero

6 Majorana ν's $= 6 \times 2 = 12$ states
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"seesaw mechanism"

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$$- (D_\mu h)^\dagger (D^\mu h) + m^2 (h^\dagger h) - \lambda (h^\dagger h)^2$$
$$+ i[\bar{q}_L \not{D} q_L + \bar{u}_R \not{D} \bar{u}_R + \bar{d}_R \not{D} d_R + \bar{\ell}_L \not{D} \ell_L + (\bar{\nu}_R \not{D} \nu_R) + \bar{e}_R \not{D} e_R]$$
$$- [\bar{q}_L \tilde{h} Y_u u_R + \bar{q}_L h Y_d d_R + (\bar{\ell}_L \tilde{h} Y_\nu \nu_R) + \bar{\ell}_L h Y_e e_R + h.c.]$$
$$- (\bar{\nu}_R^c M \nu_R) \quad + \theta\, \mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]$$

All Lorentz-invariant, gauge-invariant, renormalizable terms built from fields in SM table.

Anomalies:

- Classical sym: $\varphi \to \varphi + \delta\varphi$ leaves $S[\varphi]$ invariant
- Quantum sym: $\varphi \to \varphi + \delta\varphi$ leaves $\int \mathcal{D}[\varphi]\, e^{iS[\varphi]}$ invariant
- Anomaly: $\varphi \to \varphi + \delta\varphi$ leaves $S[\varphi]$ but not $\mathcal{D}[\varphi]$ invariant
- Global sym: $\varphi(x) \to e^{i\alpha}\varphi(x)$ ← anomaly OK!

  Gauge sym: $\varphi(x) \to e^{i\alpha(x)}\varphi(x)$ ← anomaly = inconsistency!
- Example: "Chiral anomaly"

$Z = \int \mathcal{D}A\, \mathcal{D}\psi_R\, \mathcal{D}\bar{\psi}_R\, e^{iS}$ $\qquad S = \int d^4x \left(-\frac{1}{4} \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}_R \not{\partial} \psi_R \right)$

classical sym: $\psi_R \to e^{i\alpha}\psi_R \Rightarrow S \to S$

but $\mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \to \mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R\, e^{i\int d^4x \frac{\alpha g^2}{32\pi^2} \mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}}$ $\qquad \left(\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}\right)$

if we replace $\psi_R \to \psi_L$, then $i \to -i$