Title: Standard Model Lecture - 230206

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Collection: Standard Model (2022/2023)

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Standard Model

Lecture # 13

Is the Standard Model Exceptional?

Anomalies:

- Classical sym: $\varphi \to \varphi + \delta\varphi$ leaves $S[\varphi]$ invariant
- Quantum sym: $\varphi \to \varphi + \delta\varphi$ leaves $\int \mathcal{D}[\varphi] e^{iS[\varphi]}$ invariant
- Anomaly: $\varphi \to \varphi + \delta\varphi$ leaves $S[\varphi]$ but not $\mathcal{D}[\varphi]$ invariant
- Global sym: $\varphi(x) \to e^{i\alpha}\varphi(x)$ $\leftarrow$ anomaly OK!

  Gauge sym: $\varphi(x) \to e^{i\alpha(x)}\varphi(x)$ $\leftarrow$ anomaly = inconsistency!
- Example: "Chiral anomaly"

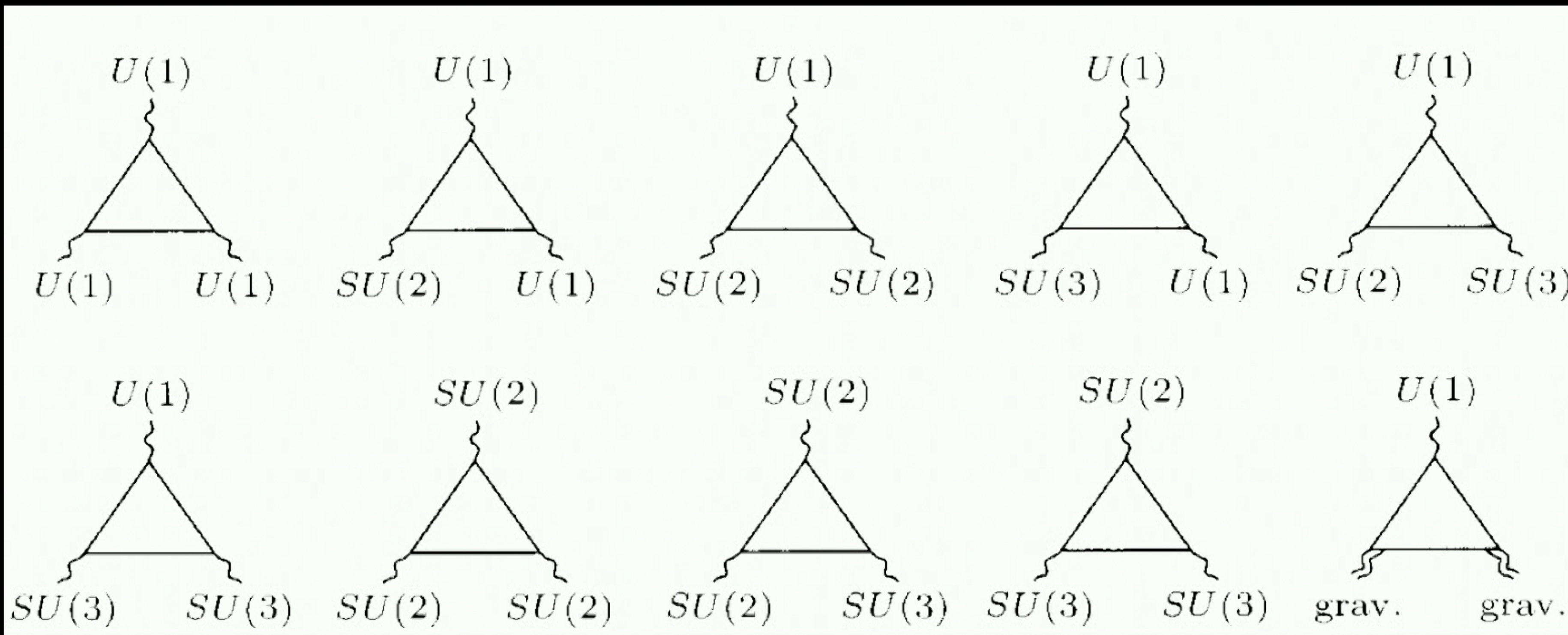
$$Z = \int \mathcal{D}A\, \mathcal{D}\psi_R\, \mathcal{D}\bar{\psi}_R\, e^{iS} \qquad S = \int d^4x \left(-\tfrac{1}{4} \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}_R \not{D} \psi_R\right)$$

classical sym: $\psi_R \to e^{i\alpha}\psi_R \Rightarrow S \to S$

but $\mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R \to \mathcal{D}\psi_R \mathcal{D}\bar{\psi}_R\, e^{i\int d^4x \frac{\alpha g^2}{32\pi^2} \mathrm{Tr}\, F_{\mu\nu}\tilde{F}^{\mu\nu}}$ $\quad \left(\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}\right)$

if we replace $\psi_R \to \psi_L$, then $i \to -i$

In Standard Model, gauge and gravitational anomalies must cancel:



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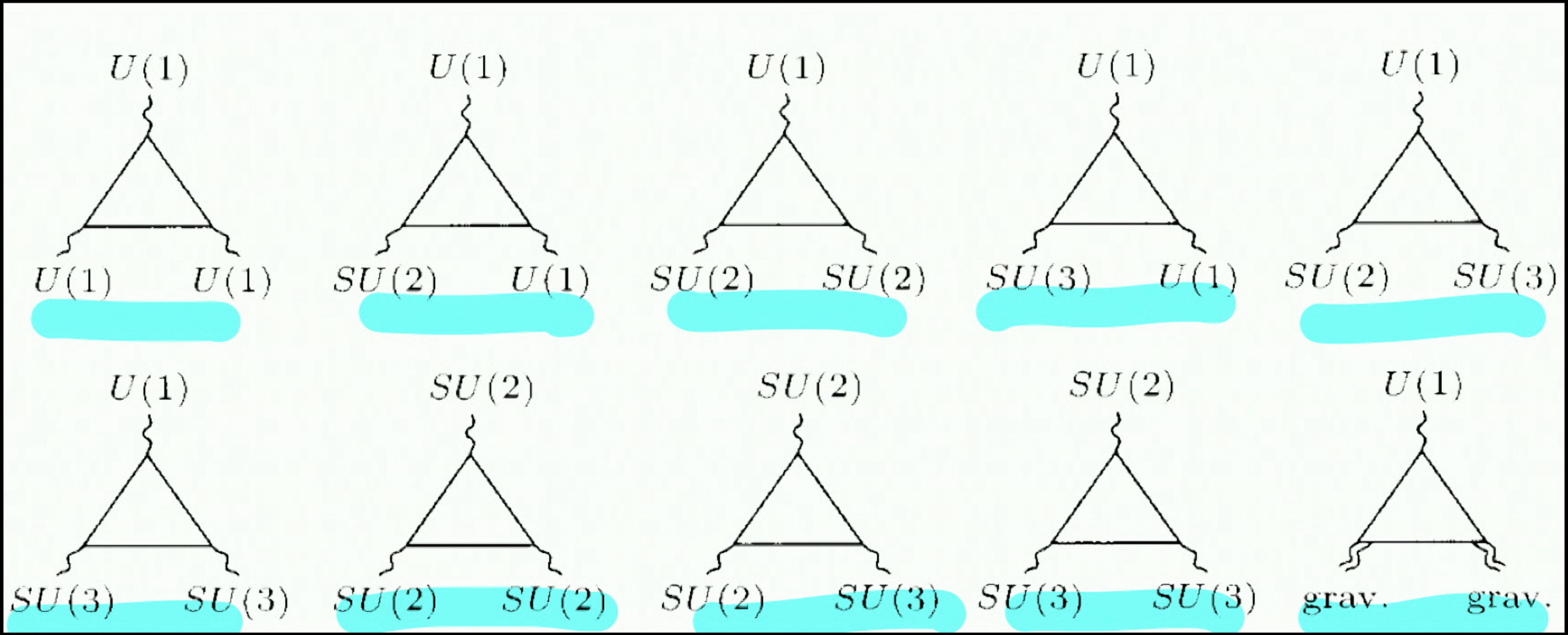

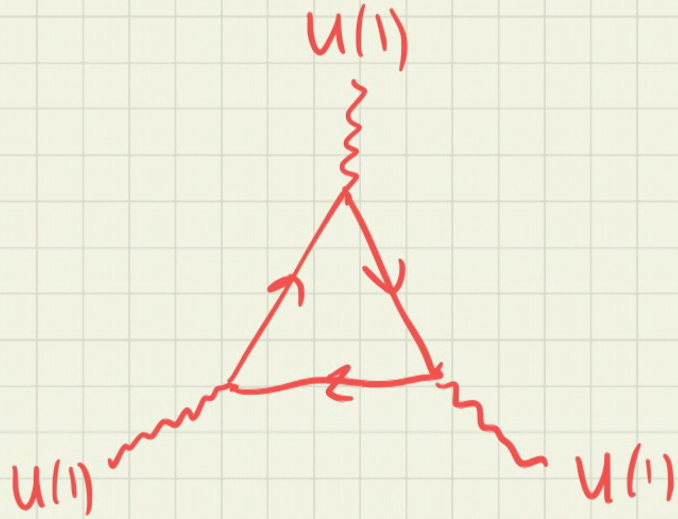

$$3\times 2\times\left(\frac{1}{6}\right)^3+3\times 1\times\left(\frac{-2}{3}\right)^3+3\times 1\times\left(\frac{1}{3}\right)^3$$
$$+\,1\times 2\times\left(-\frac{1}{2}\right)^3+1\times 1\times(1)^3=0$$

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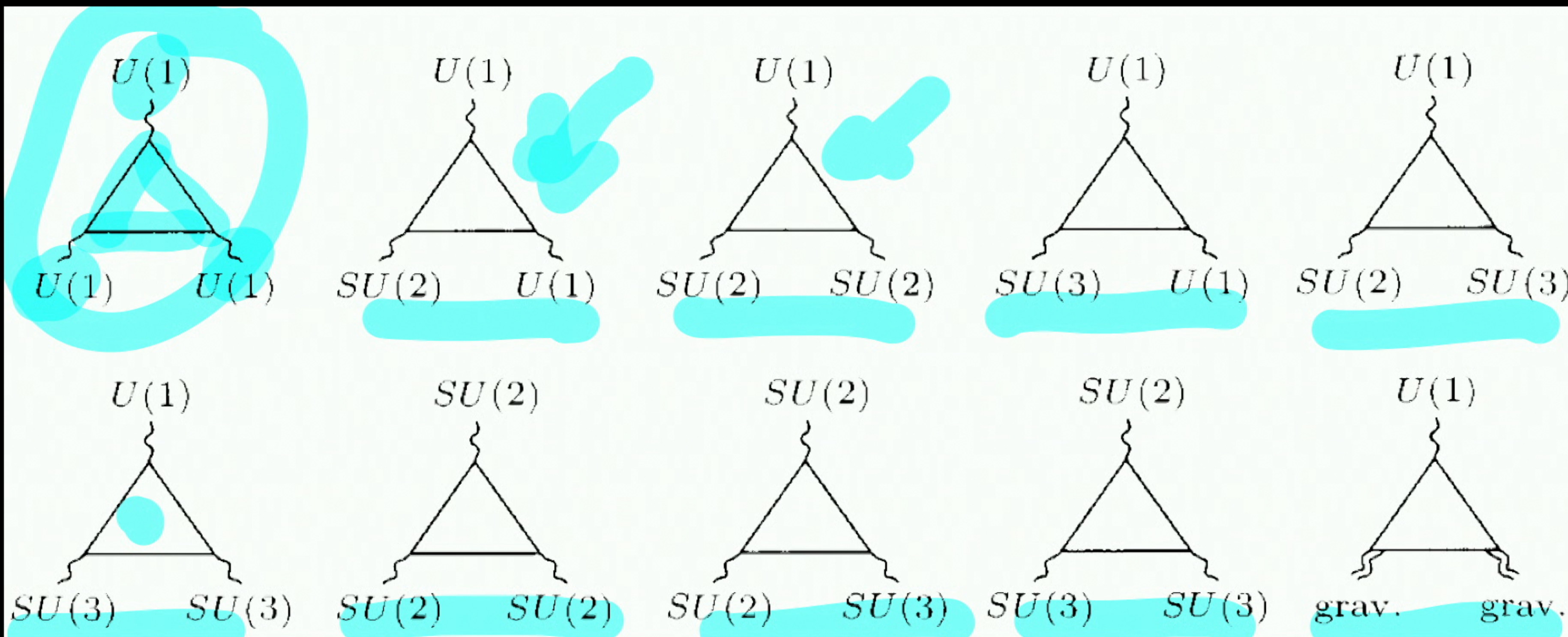

Anomaly cancellation: very constraining!

Two fun exercises:

	$SU(3)$	$SU(2)$	$U(1)_Y$
q_L	3	2	Y_q
u_R	3	1	Y_u
d_R	3	1	Y_d
ℓ_L	1	2	Y_ℓ
e_R	1	1	Y_e

	$SU(3)$	$SU(2)$	$U(1)_Y$	$U(1)_X$
q_L	3	2	Y_q	X_q
u_R	3	1	Y_u	X_u
d_R	3	1	Y_d	X_d
ℓ_L	1	2	Y_ℓ	X_ℓ
ν_R	1	1	Y_ν	X_ν
e_R	1	1	Y_e	X_e

What about local scale ("Weyl") invariance?

$g_{\mu\nu}(x) \to \Omega^2(x)\, g_{\mu\nu}(x)$

- Emphasized by Weyl, Dirac, Dicke,…, 't Hooft:
  - Natural generalization of diff invariance (gen. covariance)
- Ignoring Higgs, Standard Model is *classically* Weyl invariant.
- But Weyl symmetry is anomalous:

$$\langle T^{\mu}_{\mu} \rangle = c\, C^2 - a\, E$$

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- But Weyl symmetry is anomalous:

$$\langle T^\mu_\mu \rangle = c\, C^2 - a\, E$$

Cancelling Weyl anomalies and Vacuum Energy?

$$
\begin{aligned}
a &= \frac{1}{360(4\pi)^2}\left[\, n_0 + \frac{11}{2} n_{1/2} + 62 n_1 - 28 n_0' \,\right] \\
c &= \frac{1}{120(4\pi)^2}\left[\, n_0 + 3\, n_{1/2} + 12 n_1 - 8\, n_0' \,\right]
\end{aligned}
$$

$$
E_{\mathbf{k}} = \frac{\hbar\omega}{2}\left[\, n_0 - 2n_{1/2} + 2n_1 + 2n_0' \,\right]
$$

$$
S_4[\varphi] = \frac{1}{2}\int d^4x \sqrt{g}\, \varphi\, \Delta_4\, \varphi
$$

$$
\Delta_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\,\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu
$$

See Fradkin and Tseytlin, Nucl. Phys. B203 (1982), 157.

Cancelling Weyl anomalies and Vacuum Energy?

$$a = \frac{1}{360(4\pi)^2}[\, n_0 + \frac{11}{2} n_{1/2} + 62 n_1 - 28 n_0' \,]$$
$$c = \frac{1}{120(4\pi)^2}[\, n_0 + 3\, n_{1/2} + 12 n_1 - 8\, n_0' \,]$$

$$E_{\mathbf{k}} = \frac{\hbar\omega}{2}[\, n_0 - 2n_{1/2} + 2n_1 + 2n_0' \,]$$

$$n_{1/2} = 4n_1, \quad n_0' = 3n_1, \quad n_0 = 0.$$

Matches standard model!

$$n_1 = 8 + 3 + 1 = 12$$
$$n_{1/2} = 3 \times 16 = 48$$

Dimension-zero scalars: notable features

$$\langle \varphi(t,\mathbf{x})\varphi(t,\mathbf{x}')\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \frac{1}{4k^3}$$

Scale invariant!

seem to explain spectrum of primordial perturbations in cosmology: arXiv:2302.00344

No new local degrees of freedom.

See Bogoliubov, Logunov, Oksak & Todorov (1990) and V.O. Rivelles (2003).

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Cancelling Weyl anomalies and Vacuum Energy?

$$a = \frac{1}{360(4\pi)^2}\left[n_0 + \frac{11}{2} n_{1/2} + 62 n_1 \right]$$

$$c = \frac{1}{120(4\pi)^2}\left[n_0 + 3 n_{1/2} + 12 n_1 \right]$$

$$E_{\mathbf{k}} = \frac{\hbar\omega}{2}\left[n_0 - 2 n_{1/2} + 2 n_1 \right]$$

Standard Model

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Is the Standard Model Exceptional?

Questions:

1) Why $G_{SM} = [SU(3)_c \times SU(2)_L \times U(1)_Y]/\mathbb{Z}_6$?

2) Why
$$\rho^{(1)}_{SM} = \underset{q_L}{(3, 2, \tfrac{1}{6})} \oplus \underset{d^c_R}{(\bar{3}, 1, \tfrac{1}{3})} \oplus \underset{u^c_R}{(\bar{3}, 1, -\tfrac{2}{3})}$$
$$\oplus \underset{\ell_L}{(1, 2, -\tfrac{1}{2})} \oplus \underset{e^c_R}{(1, 1, 1)} \oplus \underset{\nu^c_R}{(1, 1, 0)} \ ?$$

3) Why 3 generations:

$$\rho_{SM} = \rho^{(1)}_{SM} \oplus \rho^{(1)}_{SM} \oplus \rho^{(1)}_{SM}$$

Old Hints:

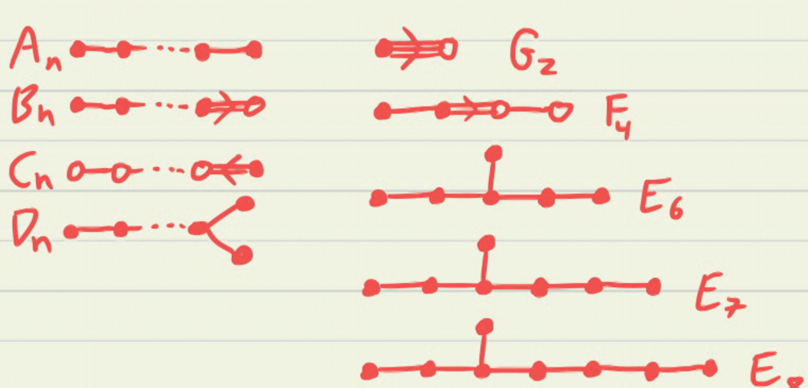
1) Grand Unification

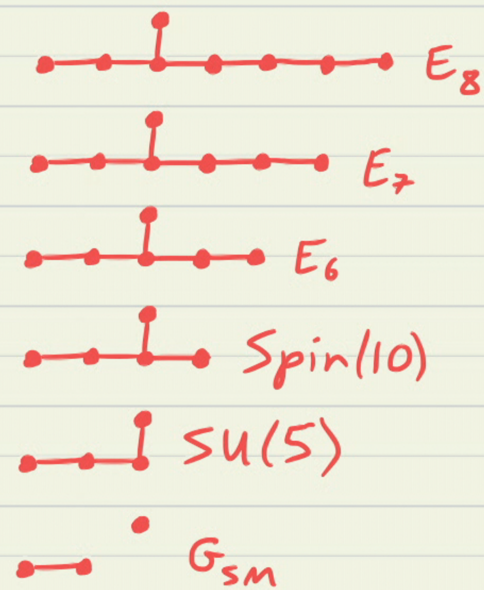
i) $SU(5) \longrightarrow G_{SM}$ ii) $Spin(10) \longrightarrow G_{SM}$

$\Lambda \mathbb{C}^5 \longrightarrow \rho_{SM}^{(1)}$ $16 \longrightarrow \rho_{SM}^{(1)}$

2) Exceptional?

Killing-Cartan:





Magic Square

(Freudenthal, Tits, Gursey, Ramond, Adams)

$\mathbb{K}$ \ $\tilde{\mathbb{K}}$	$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{O}$
$\mathbb{R}$	so(3)	su(3)	sp(3)	f_4
$\mathbb{C}$	su(3)	su(3) + su(3)	su(6)	e_6
$\mathbb{H}$	sp(3)	su(6)	so(12)	e_7
$\mathbb{O}$	f_4	e_6	e_7	e_8

$$m(\mathbb{K},\tilde{\mathbb{K}}) = \underbrace{tri(\mathbb{K}) \oplus tri(\tilde{\mathbb{K}}) \oplus (\mathbb{K}\otimes\tilde{\mathbb{K}})_1}_{\text{bosonic}} \oplus \underbrace{(\mathbb{K}\otimes\tilde{\mathbb{K}})_2 \oplus (\mathbb{K}\otimes\tilde{\mathbb{K}})_3}_{\text{fermionic}}$$

bosonic $\oplus$ fermionic ← superalgebra

$$\underbrace{m(\mathbb{C},\mathbb{O})}_{e_6} = \underbrace{tri(\mathbb{C}) \oplus tri(\mathbb{O}) \oplus (\mathbb{C}\otimes\mathbb{O})_1}_{spin(10)\oplus u(1)} \oplus \underbrace{(\mathbb{C}\otimes\mathbb{O})_2 \oplus (\mathbb{C}\otimes\mathbb{O})_3}_{16:\ \text{Weyl spinor of } spin(10)}$$

$$\underbrace{m(\mathbb{O},\mathbb{O}')}_{e_8} = tri(\mathbb{O}) \oplus tri(\mathbb{O}') \oplus (\mathbb{O}\otimes\mathbb{O}')_1 \oplus (\mathbb{O}\otimes\mathbb{O}')_2 \oplus (\mathbb{O}\otimes\mathbb{O}')_3$$

see K. Krasnov arXiv: 2104.01786

right counting for 3 gen (internal + spacetime)

Another Hint:

Dubois-Violette + Todorov (+Baez) (2018)

$h_3(\mathbb{O}) \longleftrightarrow G_{SM}$

What about $\mathfrak{g}_{SM}$?