Title: Standard Model Lecture - 230201

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Collection: Standard Model (2022/2023)

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# Standard Model

## Lecture # 11

### Renormalization Group Flow

EFT, Running Couplings

QCD, Asymptotic Safety, Quark Confinement

## Pati-Salam: $SU(4) \times SU(2)_L \times SU(2)_R$

| | $SU(4)$ | $SU(2)_L$ | $SU(2)_R$ |
|---|---|---|---|
| $Q_L$ | 4 | 2 | 1 |
| $Q_R$ | 4 | 1 | 2 |

$Q_L : \mathbb{C}^4 \otimes \mathbb{C}^2 \otimes \mathbb{C}^1 \quad Q_L^c : \mathbb{C}^{4*} \otimes \mathbb{C}^2 \otimes \mathbb{C}$

$Q_R : \mathbb{C}^4 \otimes \mathbb{C}^1 \otimes \mathbb{C}^2 \quad Q_R^c : \mathbb{C}^{4*} \otimes \mathbb{C} \otimes \mathbb{C}^2$

- leptons as 4th color of quark
- left-right symmetric

$$SU(3) \times SU(2) \times U(1) \longrightarrow SU(4) \times SU(2)_L \times SU(2)_R$$

$$(U_3, U_2, \alpha) \longrightarrow \left[ \begin{pmatrix} \alpha^p U_3 & \\ & \alpha^{-3p} \end{pmatrix}, U_2, \begin{pmatrix} \alpha^q & \\ & \alpha^{-q} \end{pmatrix} \right]$$

If $p = 1,\ q = 3 \Rightarrow Q_L \oplus Q_R^c \longrightarrow F$

$$\left.\begin{array}{l} Spin(4) = SU(2) \times SU(2) \\ Spin(6) = SU(4) \end{array}\right\} \to \text{Pati-Salam} = \begin{pmatrix} Spin(4) & \\ & Spin(6) \end{pmatrix} \subset Spin(10)$$

# Summary

$$\begin{tikzcd}
 & \mathrm{Spin}(10) \arrow[dl] \arrow[dr] & \\
SU(5) \arrow[dr] & & SU(4)\times SU(2)_L \times SU(2)_R \arrow[dl] \\
 & {[SU(3)\times SU(2)\times U(1)]/\mathbb{Z}_6} &
\end{tikzcd}$$

Pati-Salam: $SU(4)\times SU(2)_L \times SU(2)_R$

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- leptons as 4th color of quark
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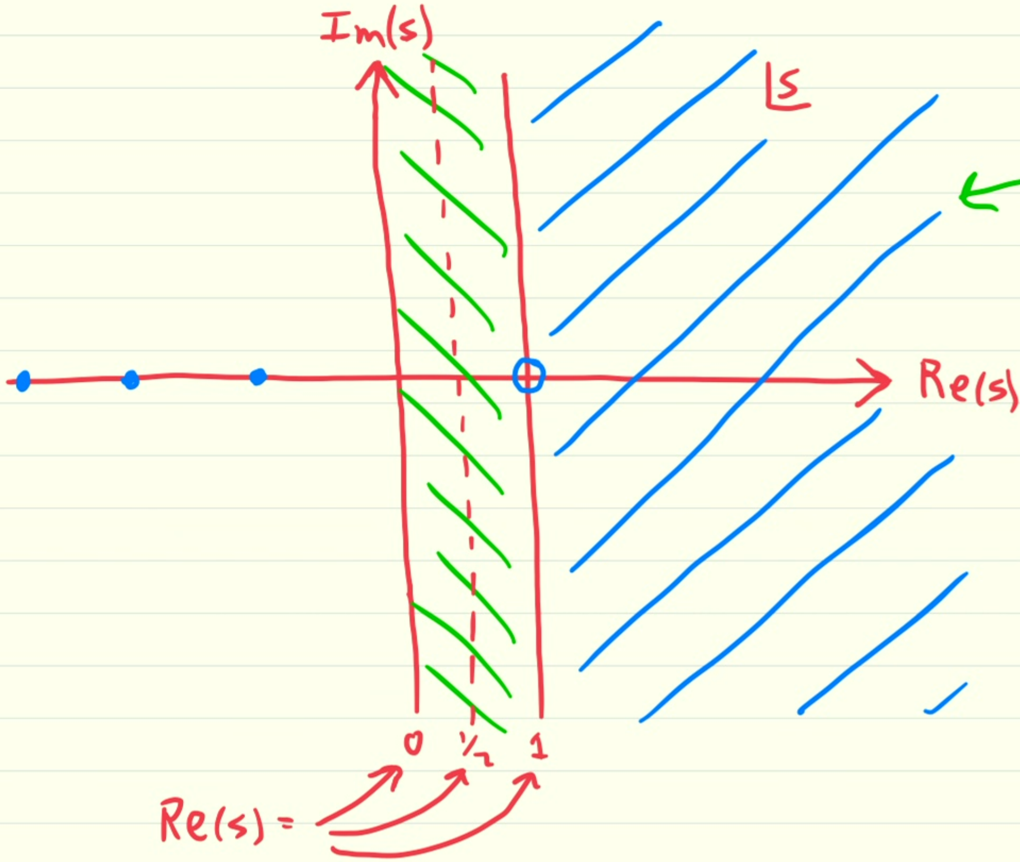
- $SU(3)\times SU(2)\times U(1) \longrightarrow SU(4)\times SU(2)_L\times SU(2)_R$

$$(U_3,\ U_2,\ \alpha) \longrightarrow \left[\begin{pmatrix}\alpha^p U_3 & \\ & \alpha^{-3p}\end{pmatrix},\ U_2,\ \begin{pmatrix}\alpha^q & \\ & \alpha^{-q}\end{pmatrix}\right]$$

If $p=1,\ q=3 \Rightarrow Q_L\oplus Q_R^c \longrightarrow F$

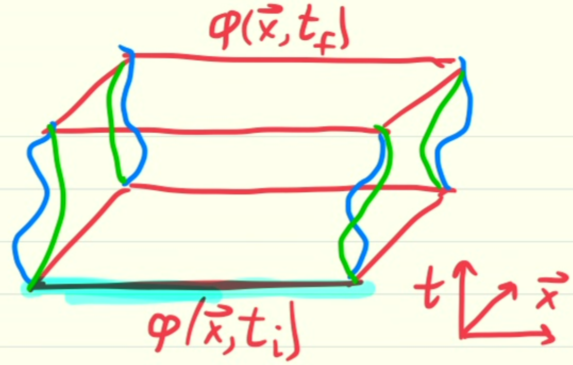
$$\left.\begin{aligned}Spin(4)&=SU(2)\times SU(2)\\ Spin(6)&=SU(4)\end{aligned}\right\} \to \text{Pati-Salam} = \left(\frac{Spin(4)}{}\Big|\,Spin(6)\right)\subset Spin(10)$$

# Riemann Conjecture




$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

(Riemann zeta function)




# Euclidean Path Integral

$$Z[J] = \int \mathcal{D}[\varphi]\, e^{iS[\varphi, J]}$$

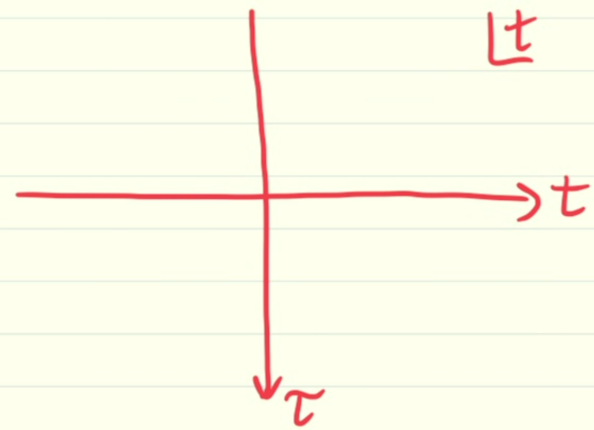
$$S[\varphi, J] = \int d^3x\, dt \left[ \tfrac{1}{2}(\partial_t \varphi)^2 - \tfrac{1}{2}(\vec{\nabla}\varphi)^2 - V(\varphi) - J\varphi \right]$$

$t = -i\tau$

$$= i \int d^3x\, d\tau \left[ \tfrac{1}{2}(\partial_\tau \varphi)^2 + \tfrac{1}{2}(\vec{\nabla}\varphi)^2 + V(\varphi) + J\varphi \right]$$

$$= iS_E[\varphi, J]$$

$$\Rightarrow Z[J] = \int \mathcal{D}[\varphi]\, e^{-S_E[\varphi, J]}$$



$$Z = \int \mathcal{D}[\varphi]\, e^{-S[\varphi]} \qquad \left[\varphi(x) = \int \frac{d^4p}{(2\pi)^4} \tilde{\varphi}(p)\, e^{ip\cdot x},\ \mathcal{D}[\varphi] = \prod_p d\tilde{\varphi}(p)\right]$$

$$= \int \mathcal{D}[\varphi_\Lambda]\, e^{-S_\Lambda[\varphi_\Lambda]} \qquad \left[\varphi_\Lambda(x) = \int_{p<\Lambda} \frac{d^4p}{(2\pi)^4} \tilde{\varphi}(p)\, e^{ip\cdot x},\ \mathcal{D}[\varphi_\Lambda] = \prod_{p<\Lambda} d\tilde{\varphi}(p)\right]$$

$$\underbrace{\int_{p<\Lambda} \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} \tilde{\varphi}(p)}_{\varphi_\Lambda(x)} = \underbrace{\int_{p<\Lambda'} \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} \tilde{\varphi}(p)}_{\varphi_{\Lambda'}(x)} + \underbrace{\int_{\Lambda'<p<\Lambda} \frac{d^4p}{(2\pi)^4} e^{ip\cdot x} \tilde{\varphi}(p)}_{\chi(x)}$$

$$\mathcal{D}[\varphi_\Lambda] = \mathcal{D}[\varphi_{\Lambda'}]\,\mathcal{D}[\chi]$$

$$S_\Lambda[\varphi_{\Lambda'} + \chi] = S_\Lambda[\varphi_{\Lambda'}] + \hat{S}_\Lambda[\varphi_{\Lambda'}, \chi]$$

$$= \int \mathcal{D}[\varphi_{\Lambda'}] \int \mathcal{D}[\chi]\, e^{-S_\Lambda[\varphi_{\Lambda'}]}\, e^{-\hat{S}[\varphi_{\Lambda'}, \chi]} =$$

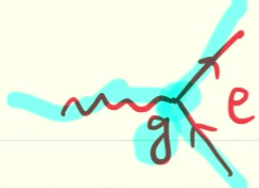
"Wilsonian Effective Action"

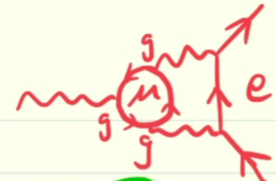
$$= \int \mathcal{D}[\varphi_{\Lambda'}]\, e^{-S_{\Lambda'}[\varphi_{\Lambda'}]} \qquad e^{-S_{\Lambda'}[\varphi_{\Lambda'}]} = e^{-S_\Lambda[\varphi_{\Lambda'}]} \underbrace{\int \mathcal{D}[\chi]\, e^{-\hat{S}[\varphi_{\Lambda'}; \chi]}}$$

$$= e^{-S_\Lambda[\varphi_{\Lambda'}]}\, e^{-\delta S_\Lambda[\varphi_{\Lambda'}]}$$

"RG flow"

## Running gauge couplings:




$$\alpha_i \equiv \frac{g_i^2}{4\pi} \qquad \beta(\alpha_i) \equiv \Lambda \frac{d\alpha_i}{d\Lambda} = \frac{d\alpha_i}{d(\ln\Lambda)} \qquad \left(\text{length} \sim \frac{1}{\Lambda}\right)$$

$$\beta(\alpha_i) = \frac{b_i}{2\pi}\alpha_i^2 \implies \alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi}\ln\left(\frac{\Lambda}{M_Z}\right)$$

$\alpha_1^{-1}(M_Z) = 59.2$ $\quad \alpha_2^{-1}(M_Z) = 29.6$ $\quad \alpha_3^{-1}(M_Z) = 8.5$

$b_1 = 41/10$ $\quad b_2 = -19/6$ $\quad b_3 = -7$


'74 (Georgi, Quinn, Weinberg): couplings unify at $M_{GUT} \sim 10^{15}$ GeV, $\Gamma_{p+} \simeq m_p^5 / M_{GUT}^4$

$$\alpha_i(\Lambda) = \frac{\alpha_i(M_Z)}{1 - \frac{b_i}{2\pi}\alpha_i(M_Z)\ln(\Lambda/M_Z)}$$

$b_i > 0 \Rightarrow \alpha_i(\Lambda) \to \infty$ at $\Lambda = M_W \exp\left[\frac{2\pi}{b_i \alpha_i(M_Z)}\right]$ ("Landau pole")

$b_i < 0 \Rightarrow \alpha_i(\Lambda) \to 0$ as $\Lambda \to \infty$ ("Asymptotic

**QCD:** quarks interacting via $SU(3)_c$ "strong force" (gluons)

Since $SU(3)$ $\beta$-function is **negative**:

- $\alpha_3(\Lambda) \to 0$ as $\Lambda \to \infty$ ("Asymptotic freedom")
- when $\Lambda$ gets **low** enough, Landau pole ($\Lambda_{QCD} \leftarrow$ the QCD or quark confinement scale)
- For $\Lambda < \Lambda_{QCD}$, QCD is strongly coupled/non-perturbative
- Quark are "confined": $q \quad \bar{q} \leftarrow$ "meson" e.g. $\pi^+ = \bar{d}u$, $\pi^- = \bar{u}d$

$q_1 \; q_2 \; q_3$ "baryon" $q_1^A q_2^B q_3^C \epsilon_{ABC}$ — $p = uud$ (proton), $n = udd$ (neutron)

$\bar{q}_1 \; \bar{q}_2 \; \bar{q}_3$ "anti-baryon" $\bar{q}_1^A \bar{q}_2^B \bar{q}_3^C \epsilon_{ABC}$ — $\bar{p} = \bar{u}\bar{u}\bar{d}$ (anti-proton), $\bar{n} = \bar{u}\bar{d}\bar{d}$ (anti-neutron)

String theory of confinement