Title: Gravitational Physics Lecture - 230203

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Collection: Gravitational Physics (2022/2023)

Date: February 03, 2023 - 9:00 AM

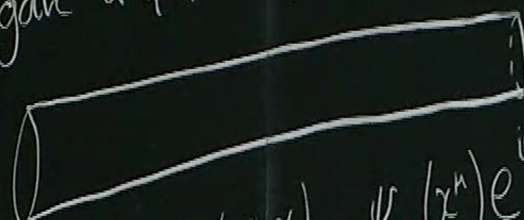
URL: https://pirsa.org/23020007

Lecture 11 Extra Dimensions

Concept almost as old as GR!

Kaluza (19–21, 1919) suggested a 5th dim. unify GR + EM

Klein (1926?) gave a QM interpretation & compactification



$\updownarrow L \quad \Delta\chi = L$

$$\Psi(x^\mu,\chi) = \Psi_0(x^\mu)\, e^{in\chi 2\pi/L}$$

$$ds_5^2 = ds_4^2 - d\chi^2$$

$$D^2\Psi = e^{in\chi 2\pi/L}\left(\Box_4\Psi_0 + \left(\frac{2\pi}{L}\right)^2 n^2\Psi_0\right) \leftarrow \text{massive ptcle } m = n\frac{2\pi}{L}$$

KK – "nothing" depends on χ

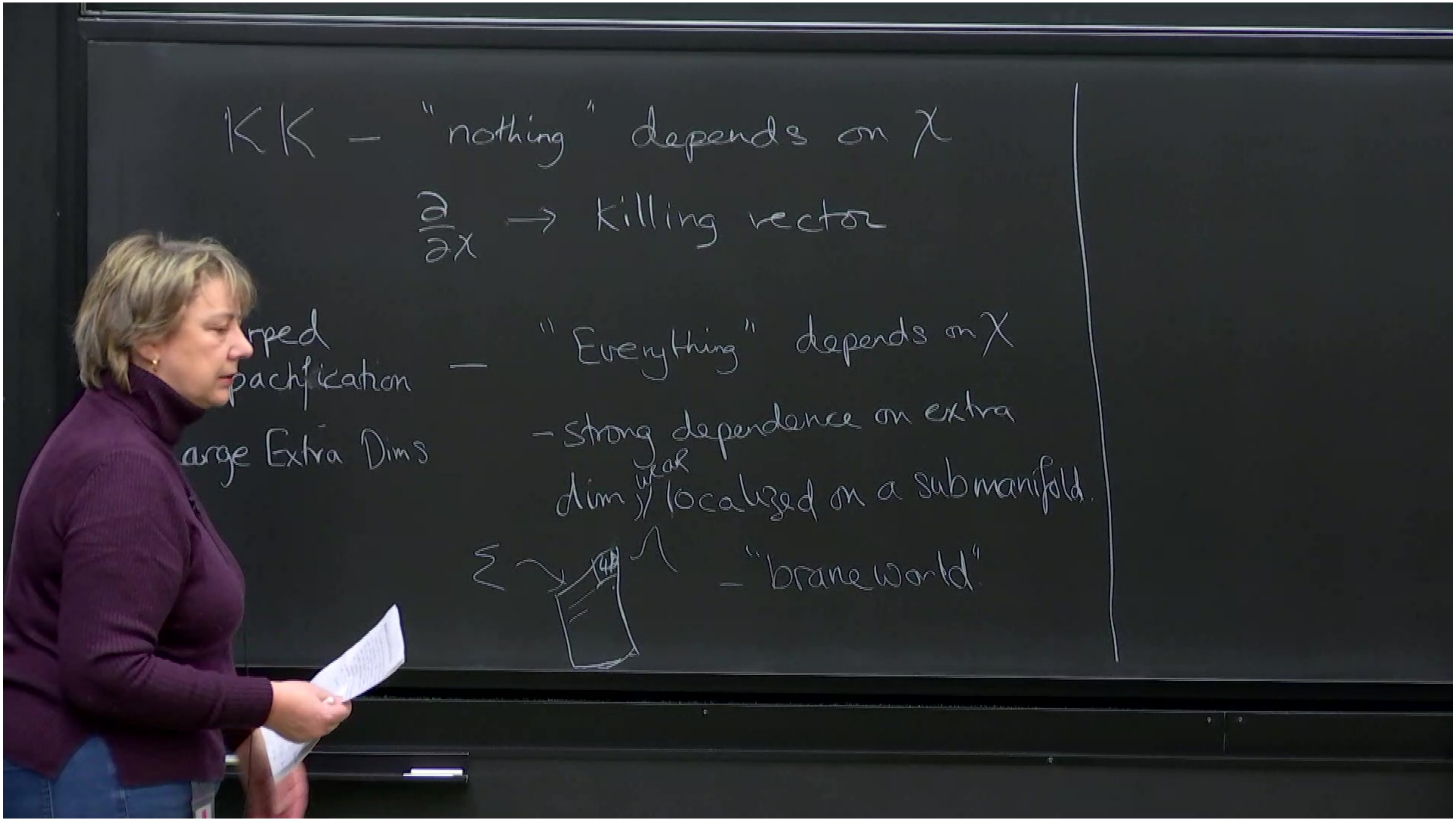
$\frac{\partial}{\partial \chi} \rightarrow$ Killing vector

Warped compactification – "Everything" depends on χ

"Large Extra Dims

– strong dependence on extra dim w/ near localized on a submanifold.

Σ  – "braneworld"



KK – "nothing" depends on X
$\frac{\partial}{\partial x}$ → Killing vector
rped
pactification – "Everything" depends on X
– strong dependence on extra
dim / weak localized on a submanifold.
arge Extra Dims
Σ → 4d
– "braneworld"

on X

extra

submanifold.

ld"

$$g_{5\,ab} = \begin{array}{cc} 10 & 4 \\ \left[\begin{array}{c|c} \tilde{g}_{\mu\nu} & g_{\mu 5} \\ \hline g_{5\nu} & g_{55} \end{array}\right] & \\ & 1 \end{array} \qquad g_{ab,\chi} = 0$$

Package as:

$$ds^2 = g_{0\,\mu\nu}\, dx^\mu dx^\nu - e^{2\sigma}\left[d\chi + A_\mu dx^\mu\right]$$

on X

extra

submanifold.

ld"

$$g_{5\,ab} = \begin{array}{cc} 10 & 4 \\ \left[\begin{array}{c|c} \hat{g}_{\mu\nu} & g_{\mu 5} \\ \hline g_{5\nu} & g_{55} \end{array}\right] & \\ & 1 \end{array} \qquad g_{ab,\chi}=0$$

Package as:

$$ds^2 = g_{0\,\mu\nu}\, dx^\mu dx^\nu - e^{2\sigma}\left[d\chi + A_\mu dx^\mu\right]^2$$

So $\det g_5 = e^{2\sigma} \det g_0$

$g_{ab,x} = 0$.

$\ldots - e^{2\sigma}\left[dX + A_\mu dx^\mu\right]^2$

$\det g_0$

Fields are σ, A_μ, $g_{0\mu\nu}$.

scalar spin 0 — vector spin 1 — tensor spin 2

from 4D perspective.

To compute curvature - see handout.

$$S_5 = -\frac{1}{16\pi G_5}\int d^5x \sqrt{g_5}\, R_5$$

$$= -\frac{L}{16\pi G_5}\int d^4x \sqrt{g_4}\left[e^{\sigma} R_4 - 2\,\Box e^{\sigma} + \frac{1}{4}e^{3\sigma}F^2\right]$$

Note – have $e^{\sigma} R_4$ not R_4

Remove e^{σ} by a conformal transfn

$$\hat{g}_{\mu\nu} = \Omega^2 g_{0\,\mu\nu}$$

$$\hat{R} = \Omega^{-2}\left(R - 6\Omega^{-1}\Box\Omega\right)$$

Choose $\Omega = e^{-\sigma/2}$ & redefine $\sigma = \varphi/\sqrt{3}$.

gives $S = \frac{1}{16\pi G_N} \int d^4x \left(R + \frac{1}{2}(\partial\phi)^2 - \frac{e^{\sqrt{3}\phi}}{4} F^2 \right)$

"Einstein" canonical scalar em

where $G_N = G_5/L$ - the 4D Newton's const.

— gives a potential hierarchy of Planck scales

if $L \gg L_s$ $\quad G_N \sim \frac{G_{4+d}}{L^d}$

$$x\left(-R+\frac{1}{2}(\partial\phi)^2-\frac{e^{\sqrt{3}\phi}}{4}F^2\right)$$

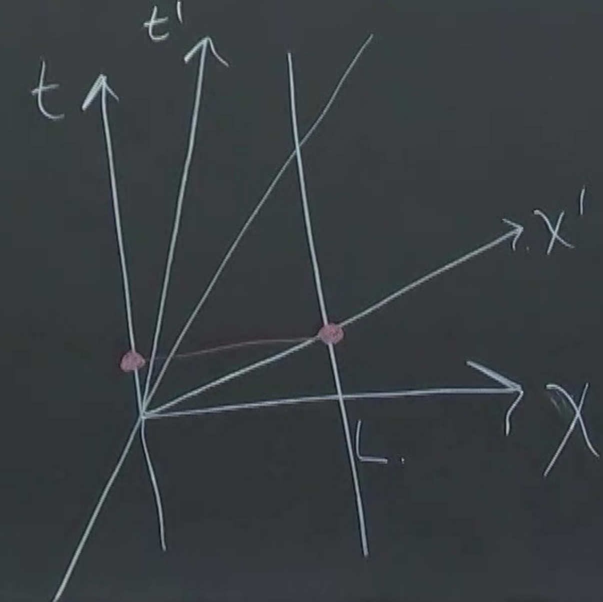
"...nstein" — canonical scalar — em

→ Newton's const.

→ Planck scales

$$ds^2=e^{-\sqrt{3}\phi}\,\hat{g}_{\mu\nu}dx^\mu dx^\nu-e^{2\sqrt{3}\phi}\left[d\chi+A_\mu dx^\mu\right]^2.$$

· Note compactification "picks" a frame.



Compactify
$\chi \sim \chi + L$
at constant t!

$$\left.\begin{aligned} t' &= \gamma(t - vx) \\ x' &= \gamma(x - vt) \end{aligned}\right\} \quad \begin{aligned} t' &\sim t' - v\gamma L \\ x' &\sim x + \gamma L \end{aligned}$$

Apply to black string

$$ds^2 = \underbrace{\left(1 - \frac{2GM}{r}\right) dt^2}_{dt^2 \to \frac{(dt + v\,dz)^2}{1-v^2}} - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_{II}^2 - \underbrace{dz^2}_{dz^2 \to \frac{(dz + v\,dt)^2}{1-v^2}}$$

Then identify $z \sim z + L$

$$-g_{zz} = e^{2\sqrt{3}\phi} = \frac{1}{1-v^2}\left[1 - v^2\left(1 - \frac{2GM}{r}\right)\right]$$

$$= 1 + \frac{2GMv^2}{(1-v^2)r}$$

Complete square

$$v^2\left(1-\frac{2GM}{r}\right)\Big]$$

$$\frac{GMv^2}{-v^2)r}$$

$$ds^2 = \left(1 - \frac{2GM}{(1-v^2)r + 2GMv^2}\right)dt^2 - \frac{dr^2}{1-\frac{2GM}{r}}$$

$$- r^2 d\Omega_{II}^2$$

$$- \left(1 + \frac{2GMv^2}{(1-v^2)r}\right)\left[dz + \frac{2GMv\,dt}{(1-v^2)r + 2GMv}\right]^2.$$

Now write $\hat{r} = r + \frac{2GMv^2}{(1-v^2)}$

$$\hat{M} = \frac{M}{1-v^2}$$

$$Q = \frac{2GMv}{1-v^2}$$

So that $\underline{A} = \frac{Q}{\hat{r}}\, \underline{dt}$

$$\ldots t^2 - \frac{dr^2}{1-\frac{2GM}{r}}$$

$$\ldots \left[\frac{2GMv\,dt}{(1-v^2)r+2GMv}\right]^2$$

Now write $\hat{r} = r + \dfrac{2GMv^2}{(1-v^2)}$

$$\hat{M} = \frac{M}{1-v^2}$$

$$Q = \frac{2GMv}{1-v^2}$$

So that $\underline{A} = \dfrac{Q}{\hat{r}}\,\underline{dt}$ (electric charge) w write $\hat{r} = r + \frac{2GMv^2}{(1-v^2)}$

$$\hat{M} = \frac{M}{1-v^2}$$

$$Q = \frac{2GMv}{1-v^2}$$

o that $\underline{A} = \frac{Q}{\hat{r}}\,dt$ (electric charge)

$$e^{-2\sqrt{3}} = \left(1 - \frac{vQ}{r}\right)$$

$$\hat{g}_{\mu\nu}dx^\mu dx^\nu = \left(1-\frac{2G\hat{M}}{\hat{r}}\right)\left(1-\frac{vQ}{\hat{r}}\right)^{-1} dt^2 - \left(1-\frac{2G\hat{M}}{\hat{r}}\right)^{-1}\left(1-\frac{vQ}{\hat{r}}\right)^{+1/2} d\hat{r}^2 - \left(1-\frac{vQ}{\hat{r}}\right)^{3/2}\hat{r}^2\, d\Omega_{II}^2$$

ELECTRIC KK BLACK HOLE

E-m has a duality symmetry under $F \Leftrightarrow \star F$

$$\frac{Q}{r^2}\, dt \wedge dr \longleftrightarrow Q \sin\theta \, d\theta \wedge d\varphi$$

$d\underline{F} = 0$ but is $\underline{F} = \underline{d}\,\underline{A}$?

$A = -Q\cos\theta \, d\varphi$?. singular on axis

$\rightarrow \quad A_N = Q(1-\cos\theta)\, d\varphi$ — regular on NP
still singular on SP.

but $\underline{A}_S = -Q(1+\cos\theta)d\varphi$ is regular on SP.

We have just geometry & $A_\mu \longleftrightarrow g_{\mu 5}$.

$$\underline{A}_S = \underline{A}_N - 2Q\,\underline{d}\varphi.$$

From a KK perspective

$$[d\psi_N + Q(1-\cos\theta)d\varphi] \longleftrightarrow [d\psi_S - Q(1+\cos\theta)d\varphi]$$

is regular on SP.

$A_\mu \longleftrightarrow g_{\mu s}$.

$] \longleftrightarrow [d\psi_S - Q(1+\cos\theta)d\varphi]$

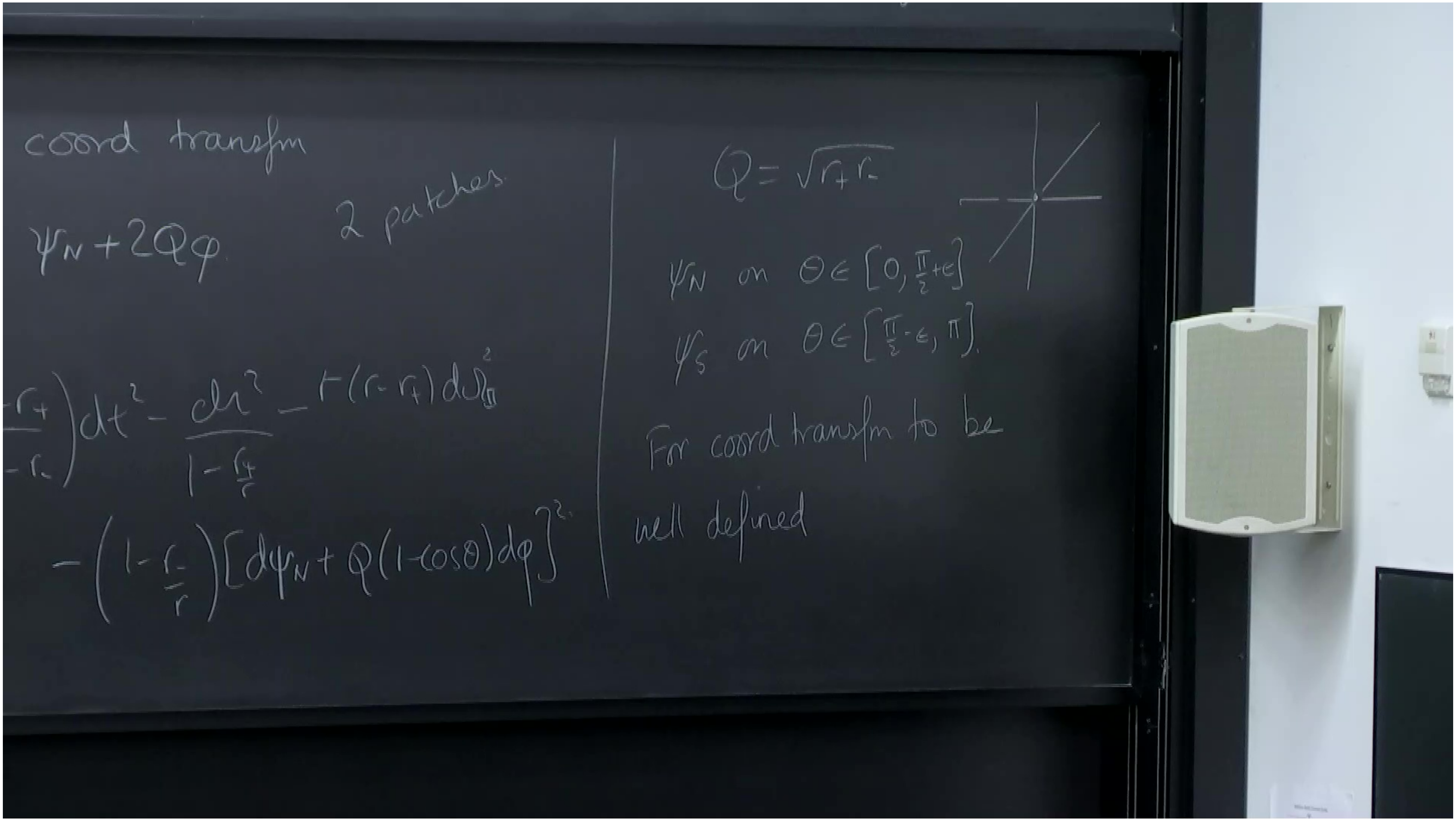
This is a coord transfm

$$\psi_S = \psi_N + 2Q\varphi.$$

Soln is

$$ds^2 = \left(\frac{r-r_+}{r-r_-}\right)dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - r(r-r_+)\,d\Omega_{II}^2$$

$$-\left(1-\frac{r_-}{r}\right)\left[d\psi_N + Q(1-\cos\theta)d\varphi\right]^2.$$


coord transfm
$\psi_N + 2Q\varphi$
2 patches
$Q = \sqrt{r_+ r_-}$
ψ_N on $\theta \in [0, \frac{\pi}{2}+\epsilon]$
ψ_S on $\theta \in [\frac{\pi}{2}-\epsilon, \pi]$
For coord transfm to be
well defined

$$\psi_S = \psi_N + 2Q\varphi$$

$$= \psi_N + 2Q(\varphi + 2\pi)$$

$$= \psi_S + 4\pi Q$$

$$\Delta\psi_S = \frac{4\pi Q}{n}$$

Charge is qua

Extremal limit

$$ds^2 = dt^2 - \frac{dr^2}{(1-Q/}$$

Charge is quantised

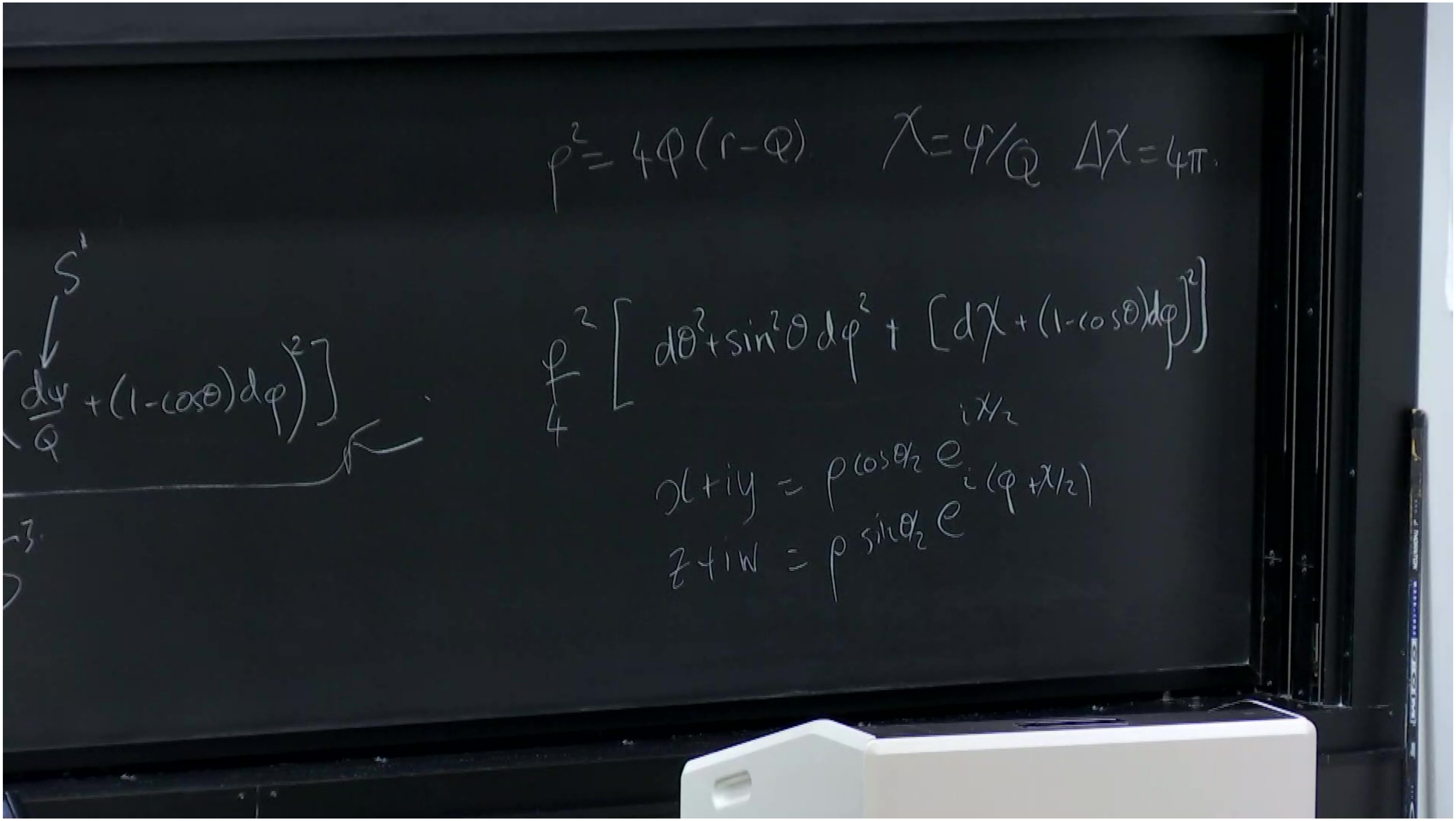
Extremal limit $r_+ = r_-$

$$ds^2 = dt^2 - \frac{dr^2}{(1-Q/r)} - \left(1-\frac{Q}{r}\right)\underbrace{\left[r^2 d\Omega_{\mathrm{II}}^2 + Q^2\left(\frac{d\psi}{Q} + (1-\cos\theta)\,d\varphi\right)^2\right]}_{S^3}$$

Charge is quantised

Extremal limit $r_+ = r_-$

$$ds^2 = dt^2 - \frac{dr^2}{(1-Q/r)} - \left(1-\frac{Q}{r}\right)\underbrace{\left[r^2 \overset{S^2}{d\Omega_{II}^2} + Q^2\left(\overset{S^1}{\frac{d\psi}{Q}} + (1-\cos\theta)\,d\varphi\right)^2\right]}_{S^3}$$



$\rho^2 = 4Q(r-Q)$. $\chi = \psi/Q$ $\Delta\chi = 4\pi$.
S^1
$\left(\frac{d\psi}{Q} + (1-\cos\theta)d\varphi\right)^2\Big]\Big]$
$\frac{\rho^2}{4}\left[d\theta^2 + \sin^2\theta\, d\varphi^2 + \left[d\chi + (1-\cos\theta)d\varphi\right]^2\right]$
$x + iy = \rho\cos\theta/2\; e^{i\chi/2}$
$z + iw = \rho\sin\theta/2\; e^{i(\varphi+\chi/2)}$
S^3