Title: Gravitational Physics Lecture - 230201

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Collection: Gravitational Physics (2022/2023)

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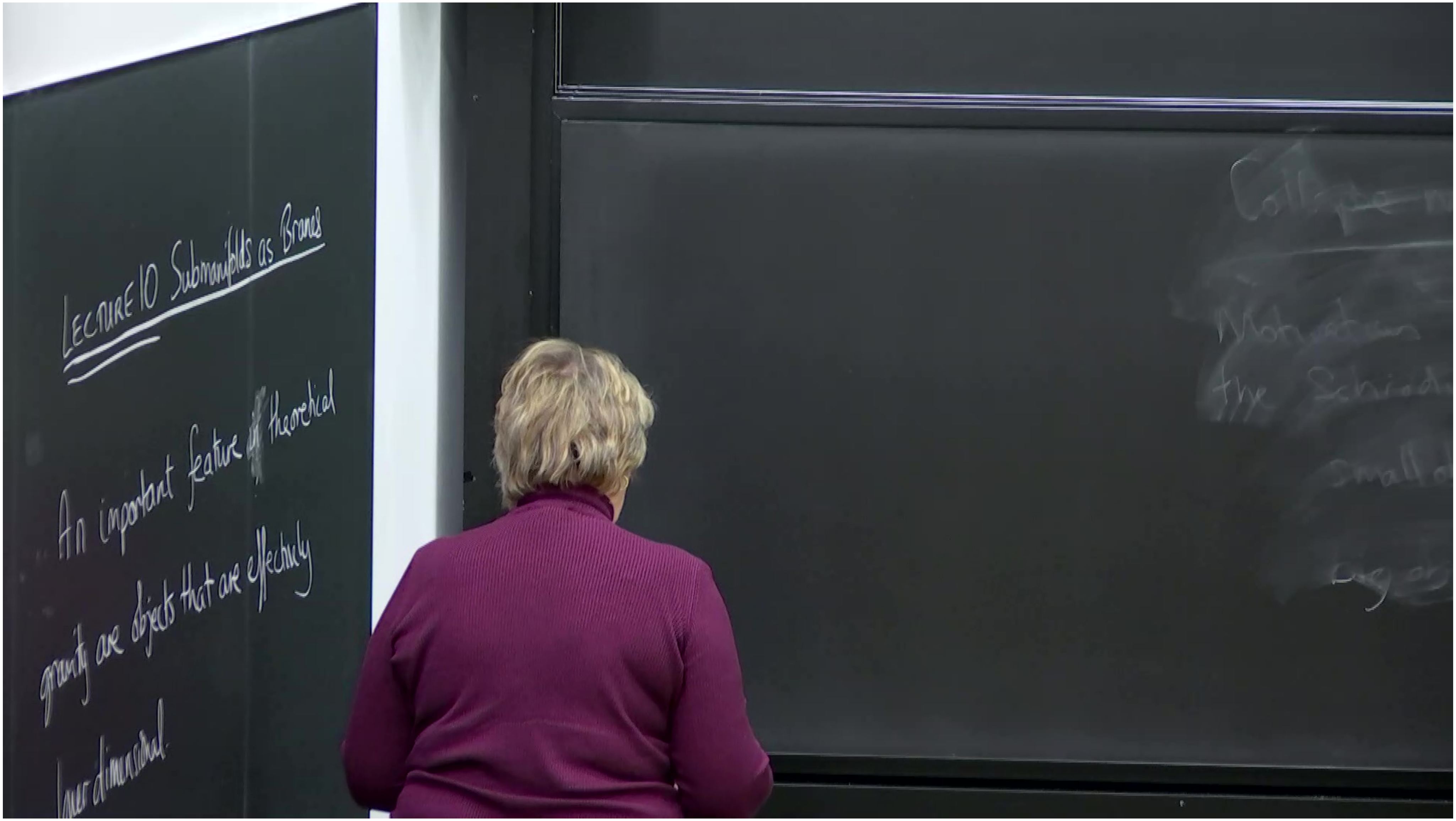
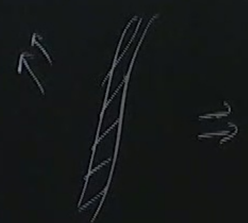
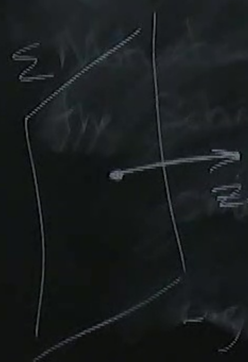

LECTURE 10 Submanifolds as Branes
An important feature theoretical
gravity are objects that are effectively
lower dimensional.

Codim 1 : domain wall – an object effectively a "wall" in spacetime

e.g. ferromagnet





Choose "Gaussian Normal" coords

z = proper distance from wall

$$ds^2 = A^2(z)\, \gamma_{\mu\nu} dx^\mu dx^\nu - dz^2$$

$\mu = 0, 1 \ldots D-2.$

Take $\gamma_{\mu\nu}(x)$ & compute curvature "wall" in spacetime.

rmal" coords.

n wall

$-dz^2$

te curvature.

• $\underline{\omega}^{\hat{z}} = \underline{d}z \qquad \underline{\omega}^{\hat{a}} = A\,\underline{\omega}_0{}^{\hat{a}} = A\,\omega_0{}^a{}_\mu\, dx^\mu$

• $\underline{d}\underline{\omega}^{\hat{z}} = 0 \qquad \underline{d}\underline{\omega}^{\hat{a}} = A'\,\underline{d}z\wedge\underline{\omega}_0{}^{\hat{a}} + A\,\underline{d}\underline{\omega}_0{}^{\hat{a}}$

$- \underline{\Theta}_0{}^{\hat{a}}{}_{\hat{b}}\wedge\underline{\omega}_0{}^{\hat{b}}$

$\Rightarrow \underline{\Theta}^{\hat{a}}{}_{\hat{z}} = \frac{A'}{A}\underline{\omega}^{\hat{a}} \qquad \underline{\Theta}^{\hat{a}}{}_{\hat{b}} = \underline{\Theta}_0{}^{\hat{a}}{}_{\hat{b}}$

• $\underline{R}^{\hat{a}}{}_{\hat{z}} = A''\,\underline{d}z\wedge\underline{\omega}_0{}^{\hat{a}} - A'\,\cancel{\underline{\Theta}_0{}^{\hat{a}}{}_{\hat{b}}\wedge\underline{\omega}_0{}^{\hat{b}}} + \cancel{\underline{\Theta}^{\hat{a}}{}_{\hat{b}}\wedge\underline{\Theta}^{\hat{b}}{}_{\hat{z}}}$

$A'\underline{\omega}_0{}^{\hat{b}}$

Take $\gamma_{\mu\nu}(x)$ & compute curvature.

$$R^{\hat a}{}_{\hat b} = d\,\Theta_0{}^{\hat a}{}_{\hat b} + \Theta^{\hat a}{}_{\hat z}\wedge\Theta^{\hat z}{}_{\hat b} + \Theta_0{}^{\hat a}{}_{\hat c}\wedge\Theta_0{}^{\hat c}{}_{\hat b}$$

$$= \underbrace{R_0{}^{\hat a}{}_{\hat b}}_{\frac{1}{2}R_0{}^{\hat a}{}_{\hat b\hat c\hat d}\,\omega_0^{\hat c}\wedge\omega_0^{\hat d}} + \left(\frac{A'}{A}\right)^2 \omega^{\hat a}\wedge\omega^{\hat c}\,\eta_{\hat b\hat c}$$

$$\Rightarrow R^{\hat a}{}_{\hat b\hat c\hat d} = \frac{1}{A^2}R_0{}^{\hat a}{}_{\hat b\hat c\hat d} + \left(\frac{A'}{A}\right)^2\left[\delta^{\hat a}{}_{\hat c}\,\eta_{\hat b\hat d} - \delta^{\hat a}{}_{\hat d}\,\eta_{\hat b\hat c}\right]$$

$R^{\hat z}{}_{\hat z} = (0-0$

- $R^z{}_z = (D-1)\dfrac{A''}{A}$

$$R^\mu{}_\nu = \frac{1}{A^2} R_0{}^\mu{}_\nu + \left[\frac{A''}{A} + (D-2)\left(\frac{A'}{A}\right)^2\right]\delta^\mu_\nu$$

Einstein eqns:

$$\frac{(D-1)(D-2)}{2}\left(\frac{A'}{A}\right)^2 + \frac{R_0}{2A^2} = 8\pi G T_{zz}$$

$$\check{R}^{z}{}_{z} = (D-1)\frac{A''}{A}$$

$$R^{\mu}{}_{\nu} = \frac{1}{A^2} R_0{}^{\mu}{}_{\nu} + \left[\frac{A''}{A} + (D-2)\left(\frac{A'}{A}\right)^2\right]\delta^{\mu}_{\nu}$$

Einstein eqns:

$$\frac{(D-1)(D-2)}{2}\left(\frac{A'}{A}\right)^2 + \frac{R_0}{2A^2} = 8\pi G T_{zz}$$

$$G_{0\mu\nu} - \frac{(D-2)}{2} g_{\mu\nu}\left[2\frac{A''}{A} + (D-3)\left(\frac{A'}{A}\right)^2\right] = 8\pi G T_{\mu\nu}$$

$$R^z{}_z = (D-1)\frac{A''}{A}$$

$$R^\mu{}_\nu = \frac{1}{A^2} R_0{}^\mu{}_\nu + \left[\frac{A''}{A} + (D-2)\left(\frac{A'}{A}\right)^2\right]\delta^\mu_\nu$$

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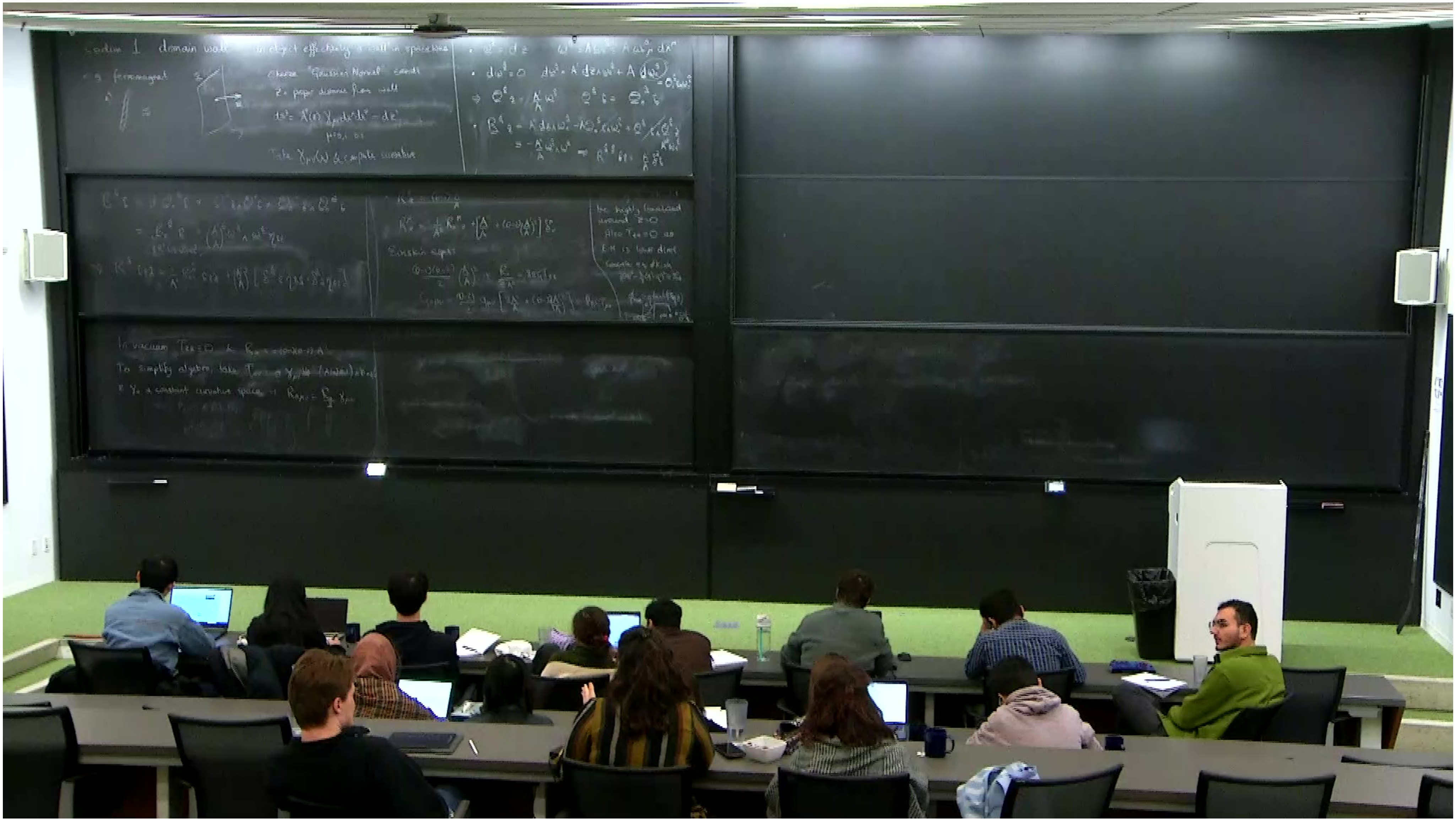
E-m of Σ should be highly localized around $z=0$.

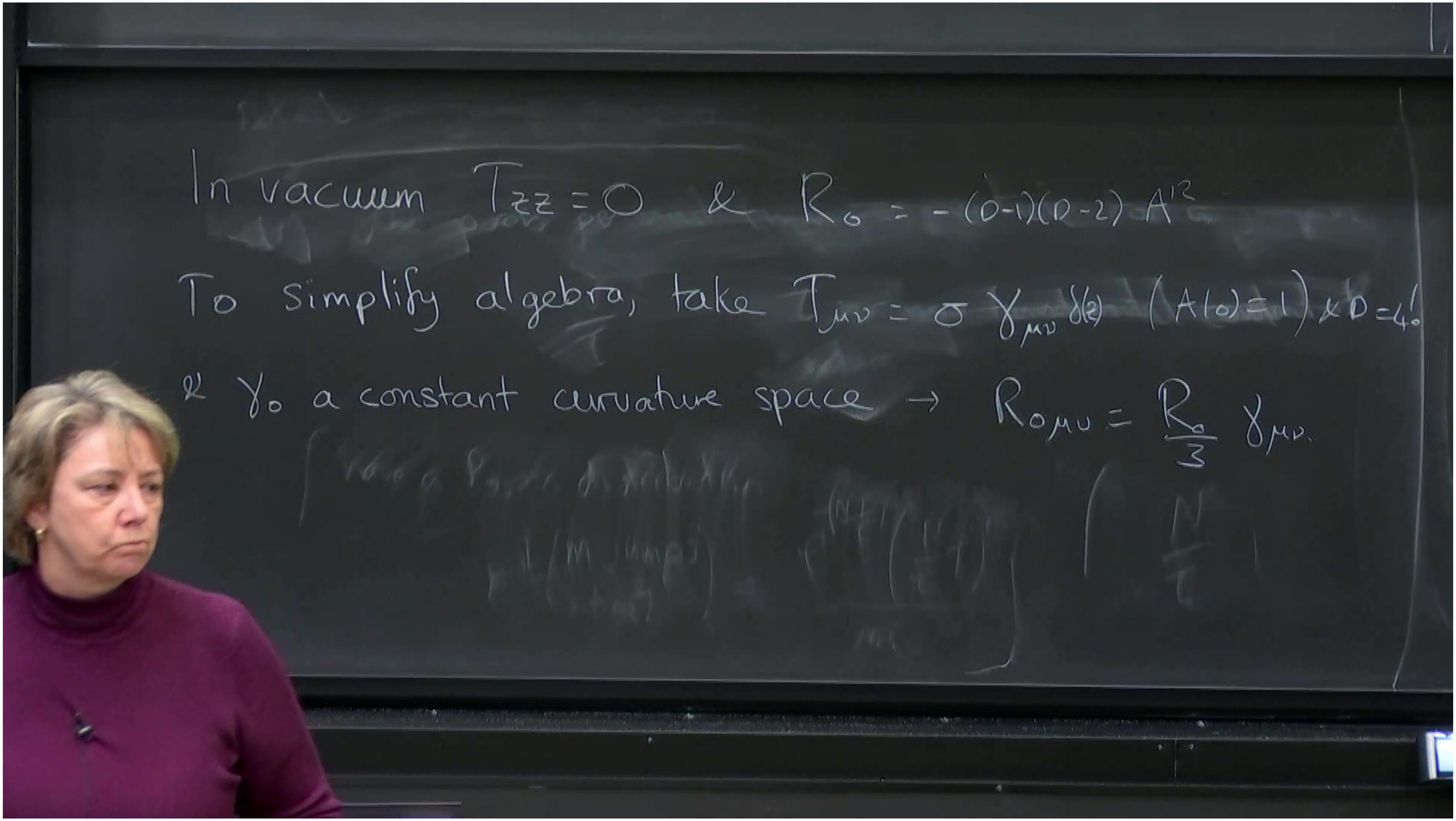
Also $T_{zz}=0$ as E-M is lower dim.

Concrete e.g. ϕ kink

$$\frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{8}(\phi^2-\eta^2)^2 = \mathcal{L}_\phi$$

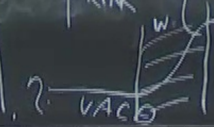
$$\phi_{kink} \sim \eta \tanh\left(\frac{\sqrt{\lambda}\,\eta z}{2}\right)$$




In vacuum $T_{zz} = 0$ & $R_0 = -(D-1)(D-2)A'^2$
To simplify algebra, take $T_{\mu\nu} = \sigma\, \gamma_{\mu\nu}\, \delta(z)$ $(A(0)=1)$ & $D=4$!
& γ_0 a constant curvature space → $R_{0\mu\nu} = \frac{R_0}{3}\gamma_{\mu\nu}$.

$$G_{0\mu\nu} - \frac{(D-2)}{2} g_{\mu\nu}\left[2\frac{A''}{A} + (D-3)\left(\frac{A'}{A}\right)^2\right] = 8\pi G T_{\mu\nu}$$

$\phi_{KINK} \sim \eta \tanh(\sqrt{\lambda}\eta z)$



vac ⊕ $w \sim \frac{1}{\sqrt{\lambda}\,\eta}$

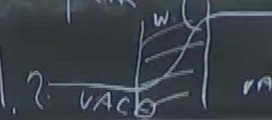
$$\frac{A''}{A} = -4\pi G\sigma(z)$$ for δ-fn - a jump in A'

(flat dir^n)

$$A = 1 - 2\pi G\sigma|z|.$$

$$G_{0\mu\nu} - \frac{(D-2)}{2} g_{\mu\nu}\left[2\frac{A''}{A} + (D-3)\left(\frac{A'}{A}\right)^2\right] = 8\pi G T_{\mu\nu}$$

$\phi_{KINK} \sim \eta \tanh(\sqrt{\lambda}\,\eta z)$



$w \sim \frac{1}{\sqrt{\lambda}\,\eta}$

$$\frac{A''}{A} = -4\pi G \sigma(z)$$ for δ-fn – a jump in A'

$$A = 1 - 2\pi G\sigma |z|$$
$$= 1 - |z|/\ell$$

→ gives R_0 & $\gamma_{\mu\nu}dx^\mu dx^\nu = dt^2 - \ell^2\cosh^2\!\left(\frac{t}{\ell}\right) d\Omega_{II}^2$

– a 3D de Sitter universe

Geometry of Σ & M

$z>0$: $\tau = (\ell - z)\sinh(t/\ell)$

$\rho = (\ell - z)\cosh(t/\ell)$

$$d\tau^2 - d\rho^2 = \left[\frac{(\ell - z)}{\ell}\cosh\left(\frac{t}{\ell}\right)dt - dz\sinh\left(\frac{t}{\ell}\right)\right]^2 - \left[\frac{(\ell - z)}{\ell}\sinh\left(\frac{t}{\ell}\right)dt - dz\cosh\left(\frac{t}{\ell}\right)\right]^2$$

$$= (1 - z/\ell)^2 dt^2 - dz^2$$

Geometry of Σ & M

$z>0$: $\quad \tau = (\ell - z)\sinh(t/\ell)$

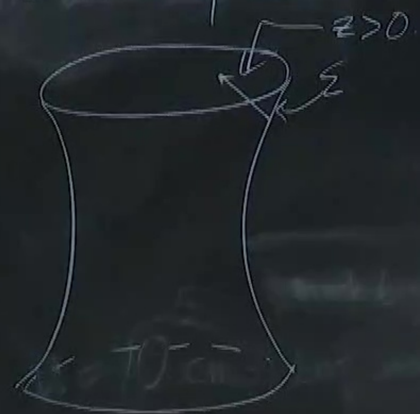
$\qquad\quad \rho = (\ell - z)\cosh(t/\ell)$

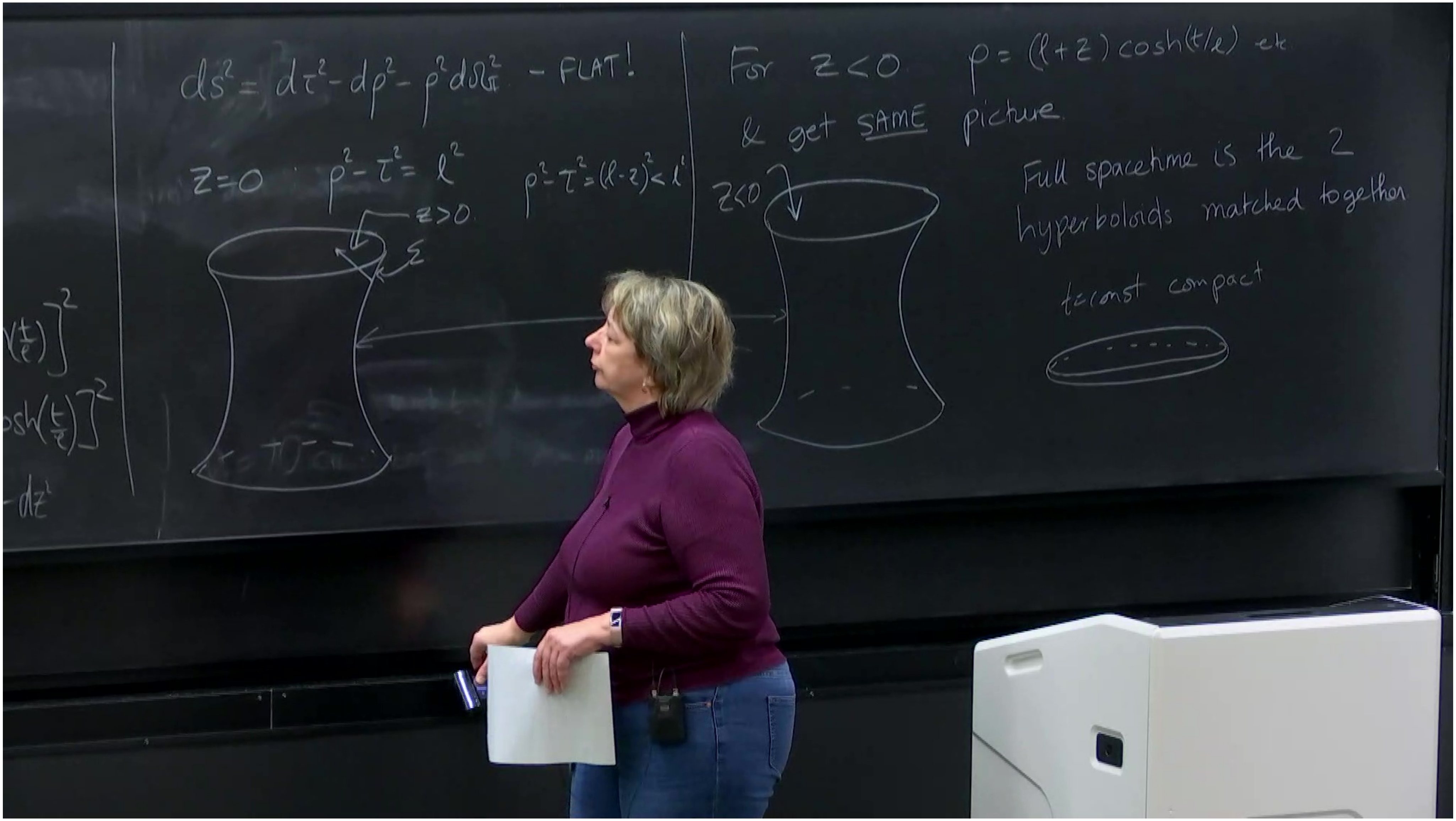
$$d\tau^2 - d\rho^2 = \left[\frac{(\ell - z)}{\ell}\cosh\left(\tfrac{t}{\ell}\right)dt - dz\sinh\left(\tfrac{t}{\ell}\right)\right]^2 - \left[\frac{(\ell - z)}{\ell}\sinh\left(\tfrac{t}{\ell}\right)dt - dz\cosh\left(\tfrac{t}{\ell}\right)\right]^2$$

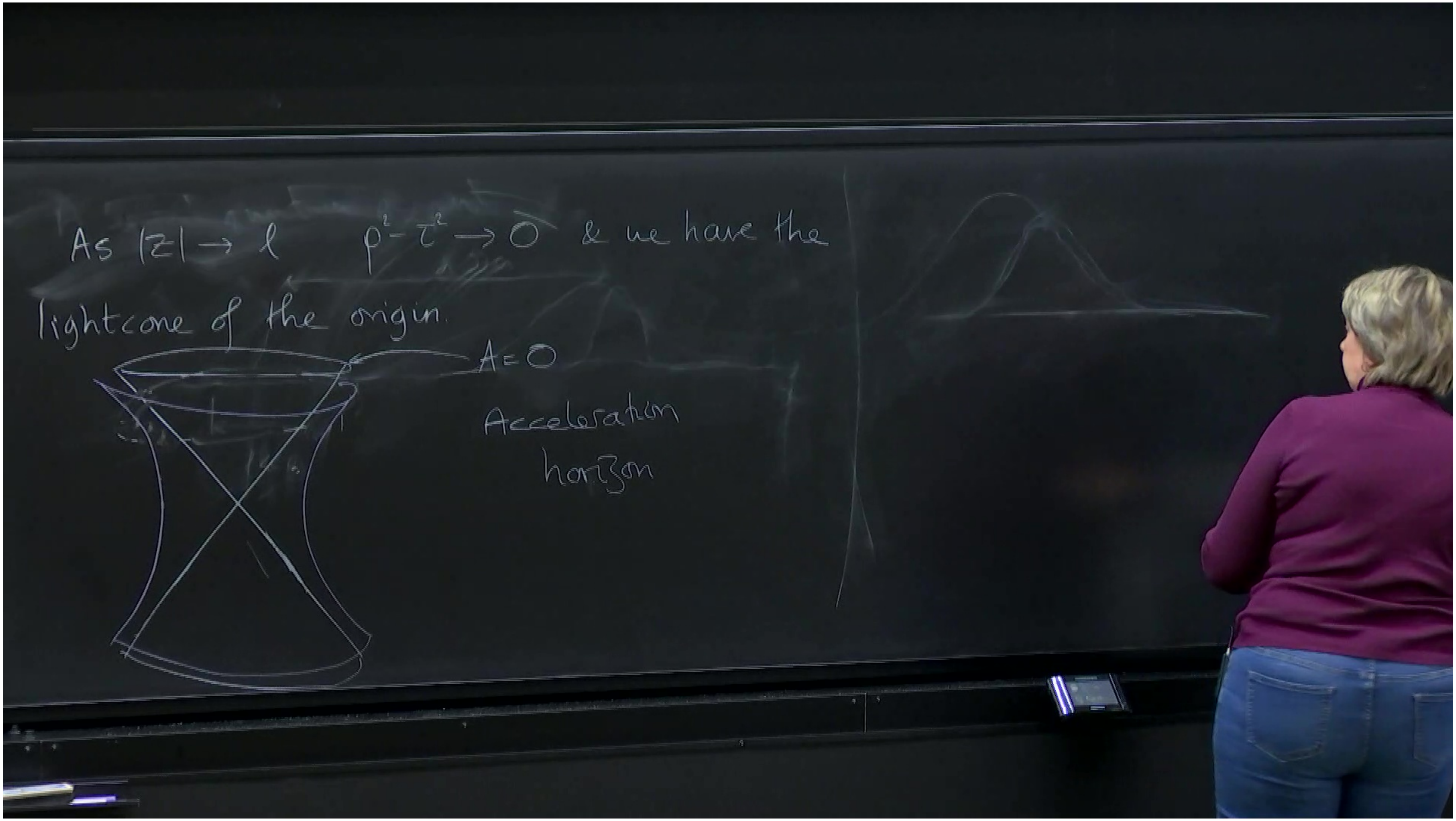
$$= (1 - z/\ell)^2 dt^2 - dz^2 = A^2 dt^2 - dz^2$$

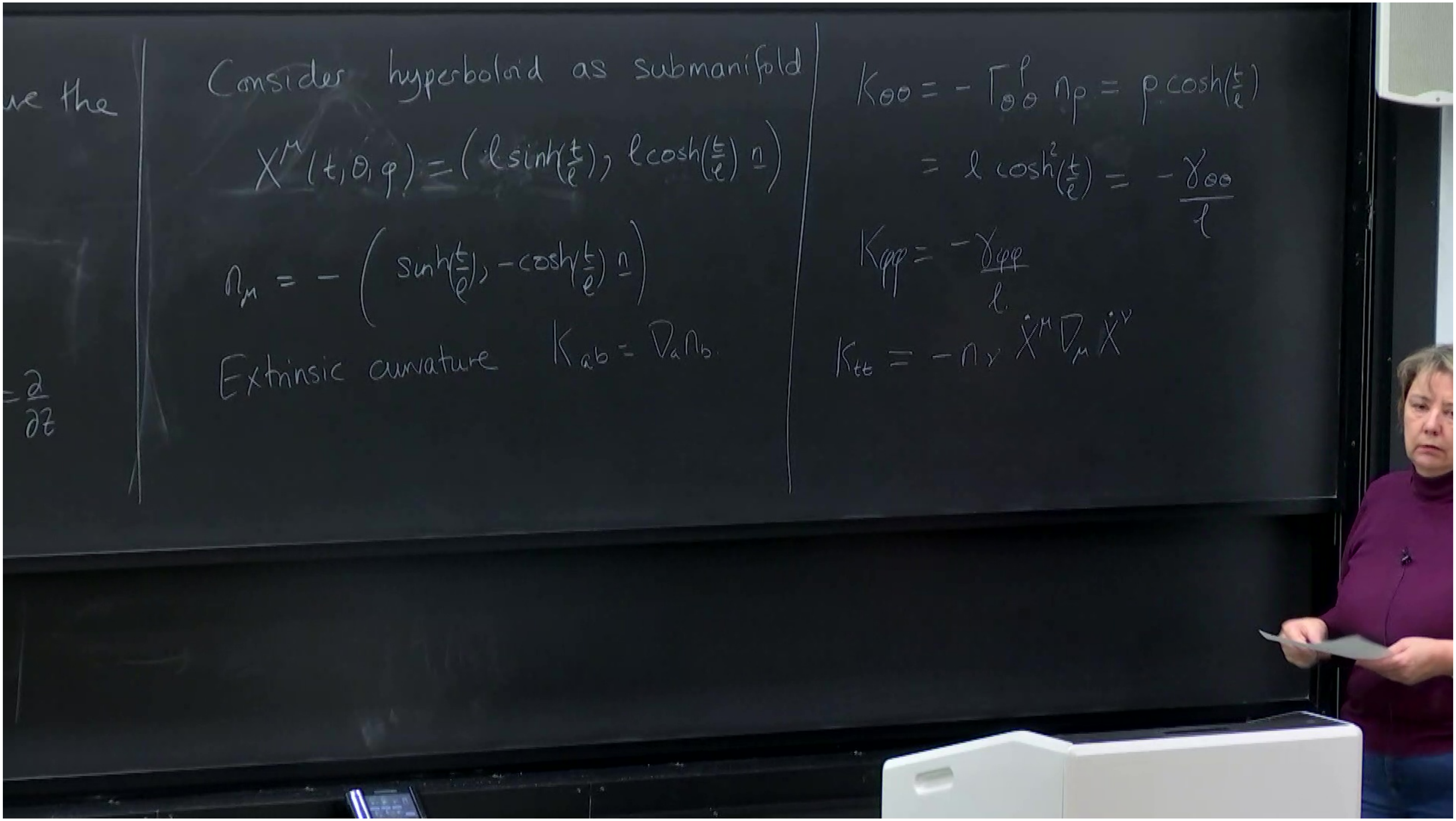
$ds^2 = d\tau^2 - d\rho^2 - \rho^2 d\Omega_{II}^2$ — FLAT!

$z=0$: $\rho^2 - \tau^2 = \ell^2 \qquad \rho^2 - \tau^2 = (\ell - z)^2 < \ell^2$




ds² = dτ² − dρ² − ρ²dΩ² − FLAT!
z=0
ρ² − τ² = ℓ²
ρ² − τ² = (ℓ−z)² < ℓ²
z>0
Σ
For z<0 ρ = (ℓ+z) cosh(t/ℓ) etc
& get SAME picture
z<0
Full spacetime is the 2
hyperboloids matched together
t=const compact


As $|z| \to \ell$ $\rho^2 - \tau^2 \to 0$ & we have the
lightcone of the origin.
$A = 0$
Acceleration
horizon

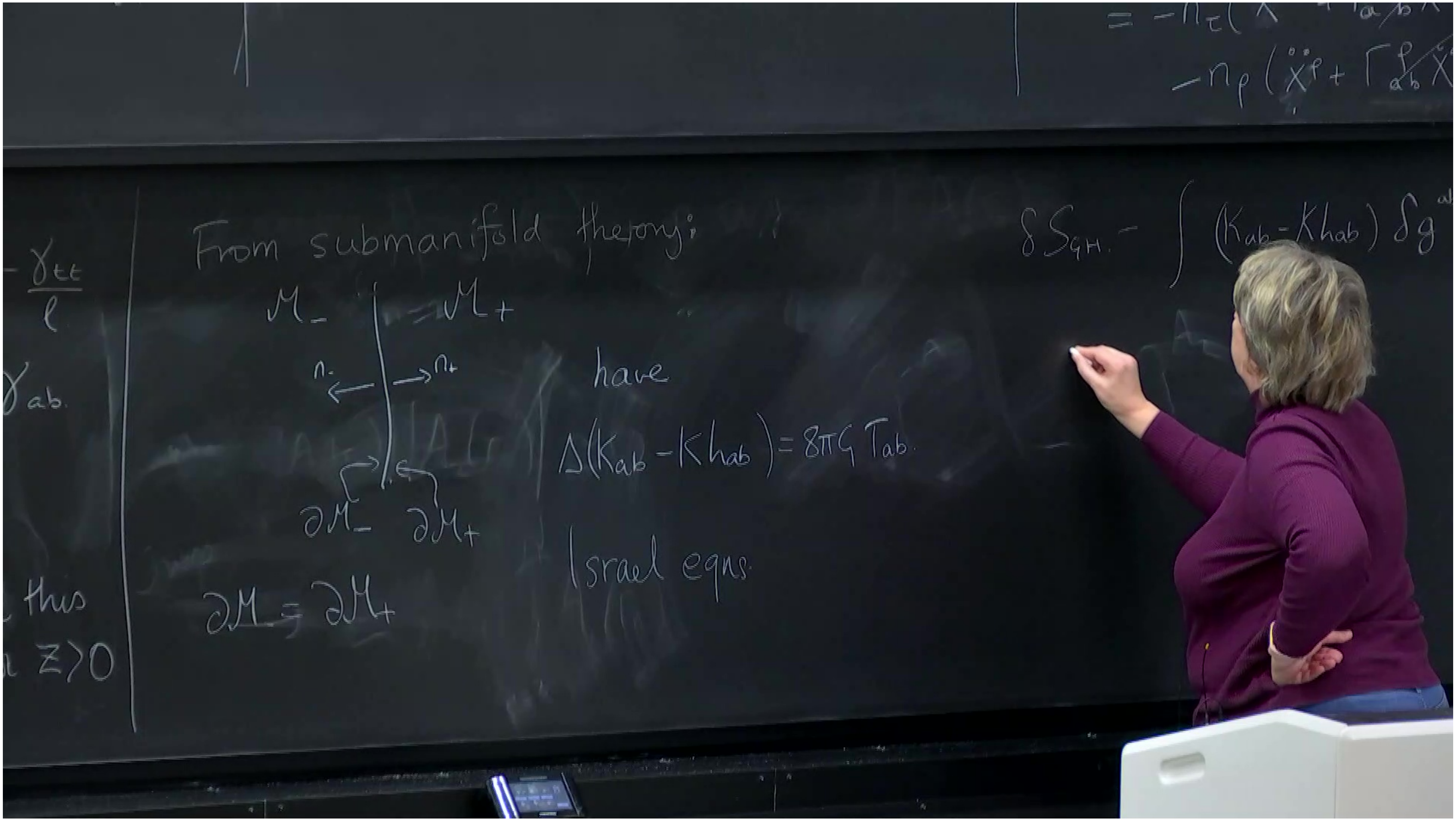

Consider hyperboloid as submanifold
$X^\mu(t,\theta,\varphi) = (\ell \sinh(\frac{t}{\ell}), \ell \cosh(\frac{t}{\ell})\, \underline{n})$
$n_\mu = -(\sinh(\frac{t}{\ell}), -\cosh(\frac{t}{\ell})\, \underline{n})$
Extrinsic curvature $K_{ab} = D_a n_b$
$K_{\theta\theta} = -\Gamma^{\rho}_{\theta\theta} n_\rho = \rho \cosh(\frac{t}{\ell})$
$= \ell \cosh^2(\frac{t}{\ell}) = -\frac{\gamma_{\theta\theta}}{\ell}$
$K_{\varphi\varphi} = -\frac{\gamma_{\varphi\varphi}}{\ell}$
$K_{tt} = -n_\nu \dot{X}^\mu D_\mu \dot{X}^\nu$

$$\frac{1}{\ell}\sinh^2\left(\frac{t}{\ell}\right) - \frac{1}{\ell}\cosh^2\frac{t}{\ell} = -\frac{1}{\ell} = -\frac{\gamma_{tt}}{\ell}$$

i.e. $$K_{ab} = -\frac{1}{\ell}\gamma_{ab} = -2\pi G\sigma\gamma_{ab}$$

$$K = -6\pi G\sigma$$

$$\Rightarrow K_{ab} - K\gamma_{ab} = 4\pi G\sigma\gamma_{ab}$$ ← note this is for $z>0$


From submanifold theory:
$\mathcal{M}_-$ $\mathcal{M}_+$
n_- n_+
$\partial\mathcal{M}_-$ $\partial\mathcal{M}_+$
$\partial\mathcal{M}_- = \partial\mathcal{M}_+$
have
$\Delta(K_{ab} - K h_{ab}) = 8\pi G\, T_{ab}$.
Israel eqns.
$\delta S_{GH} = \int (K_{ab} - K h_{ab})\, \delta g^{ab}$

$$= -n_\tau\left(\ddot{X}^\tau + \cdots\right)$$

$$-n_\rho\left(\ddot{X}^\rho + \Gamma^\rho_{ab}\dot{X}^a\dot{X}^b\right)$$

m submanifold theory:

$\mathcal{M}_-$ $\mathcal{M}_+$

n_- n_+

$\partial\mathcal{M}_-$ $\partial\mathcal{M}_+$

$\ldots = \partial\mathcal{M}_+$

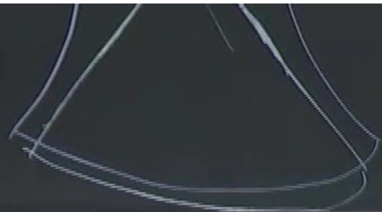
have

$$\Delta(K_{ab} - K h_{ab}) = 8\pi G\, T_{ab}$$

Israel eqns.

$$\delta S_{GH} \propto -\frac{1}{4\pi G}\int (K_{ab} - K h_{ab})\,\delta g^{ab}$$

$$\delta S_{matter} \sim \int_\Sigma T_{ab}\,\delta h^{ab}$$



∂t

$$\frac{1}{\ell}\sinh^2\left(\frac{t}{\ell}\right) - \frac{1}{\ell}\cosh^2\frac{t}{\ell} = -\frac{1}{\ell} = -\frac{\gamma_{tt}}{\ell}$$

i.e. $$K_{ab} = -\frac{1}{\ell}\gamma_{ab} = -2\pi G\sigma\gamma_{ab}$$

$$K = -6\pi G\sigma$$

$$\Rightarrow \boxed{K_{ab} - K\gamma_{ab}} = 4\pi G\sigma\gamma_{ab}$$ ← note this is for $z>0$

From submanifold theory:

$\mathcal{M}_-$ | $\mathcal{M}_+$

n_- ← | → n_+

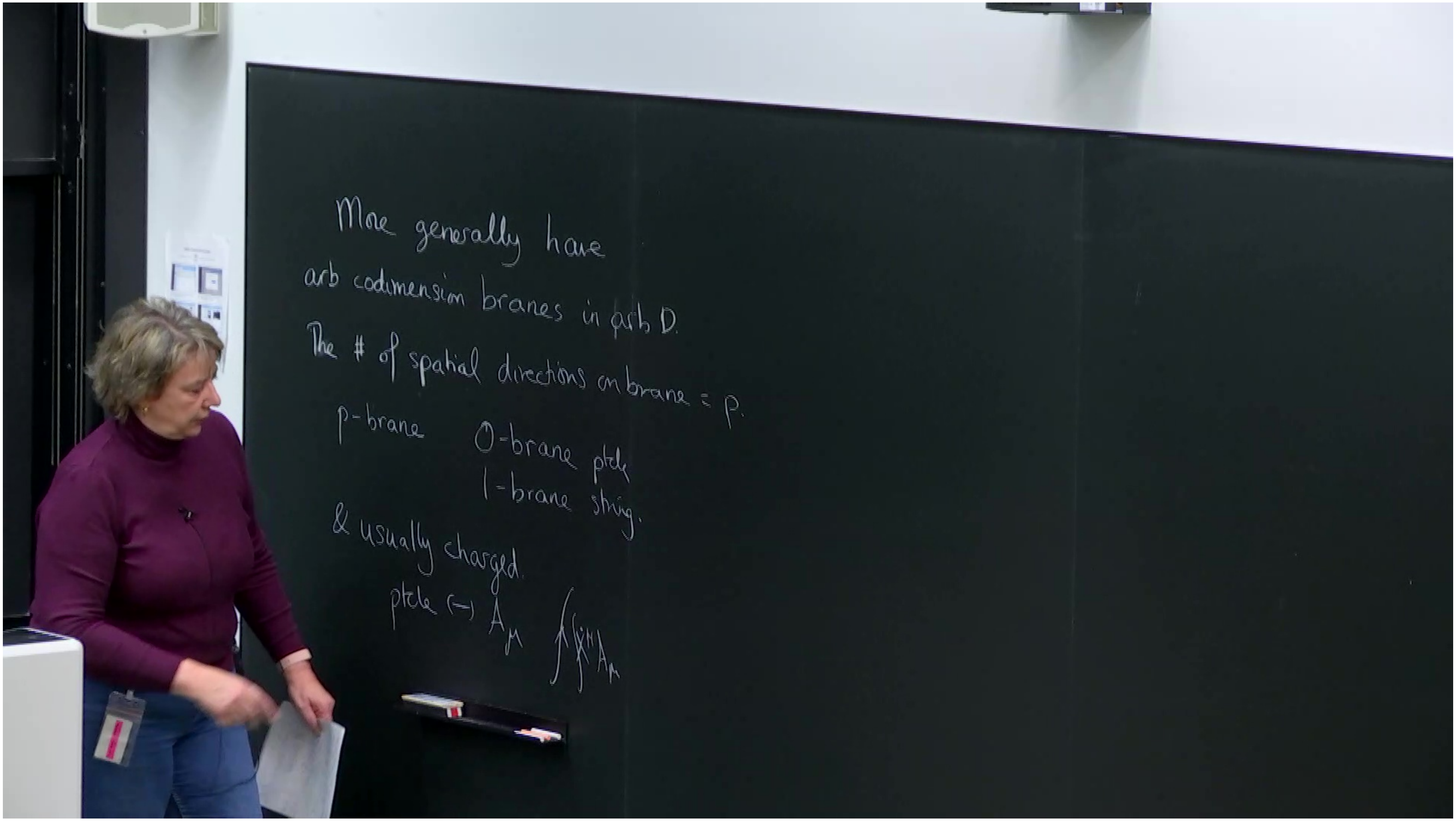
$K^-_{ab} = -K(n^-)$ (ISRAEL) | K^+_{ab}

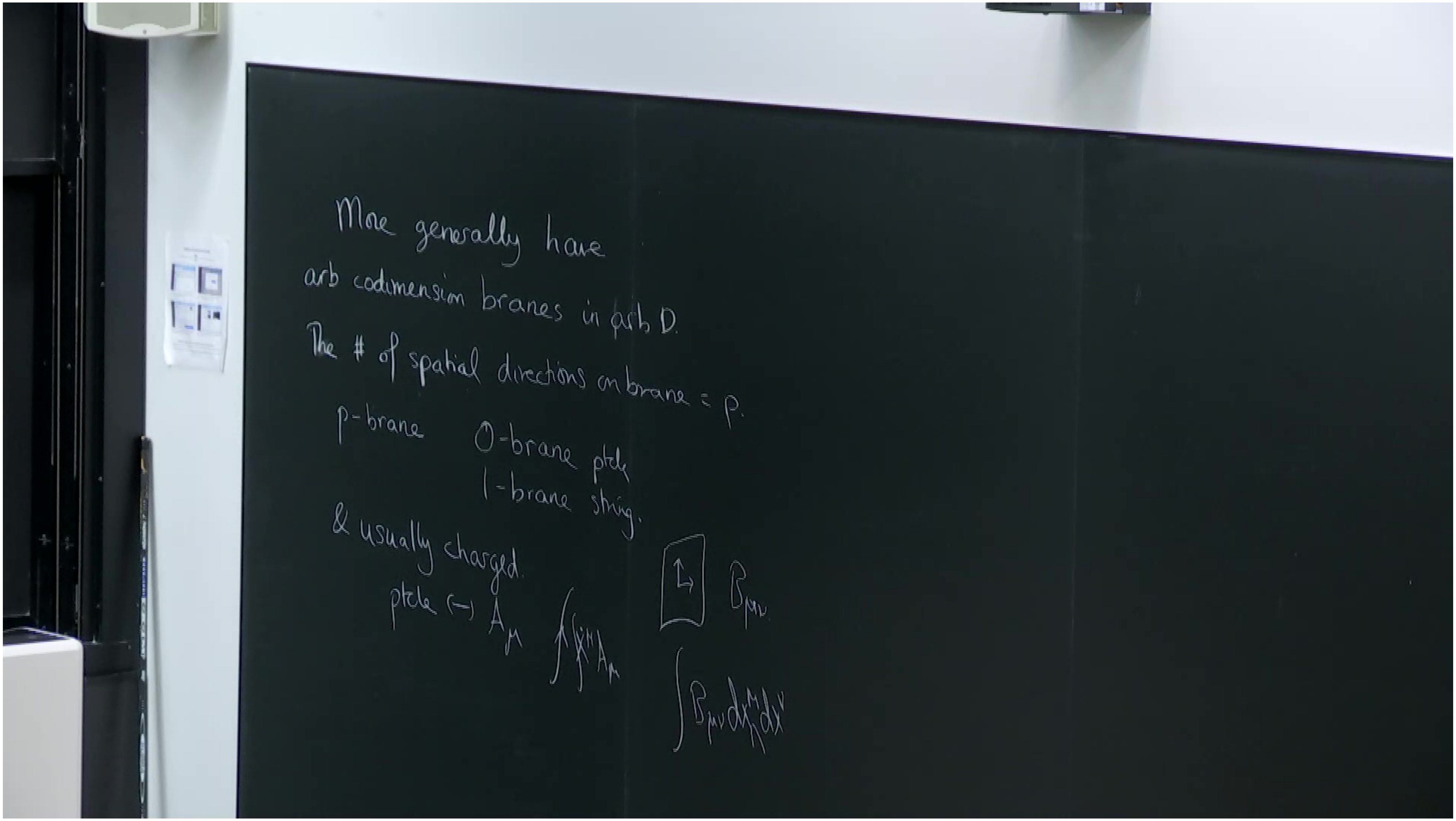
$\partial\mathcal{M}_-$ $\partial\mathcal{M}_+$

$\partial\mathcal{M} = \partial\mathcal{M}_+$

have $$\Delta(K_{ab} - K h_{ab}) = 8\pi G T_{ab}$$

Israel eqns


More generally have
arb codimension branes in arb D.
The # of spatial directions on brane = p.
p-brane
0-brane ptcle
1-brane string.
& usually charged.
ptcle ⟷ A_μ
$\int \dot{x}^\mu A_\mu$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$B_{\mu\nu}$
$\int B_{\mu\nu} dx^\mu \wedge dx^\nu$