Title: Conformal Regge theory predicts the existence of analytically continued CFT data for complex spin

Speakers: Alexandre Homrich

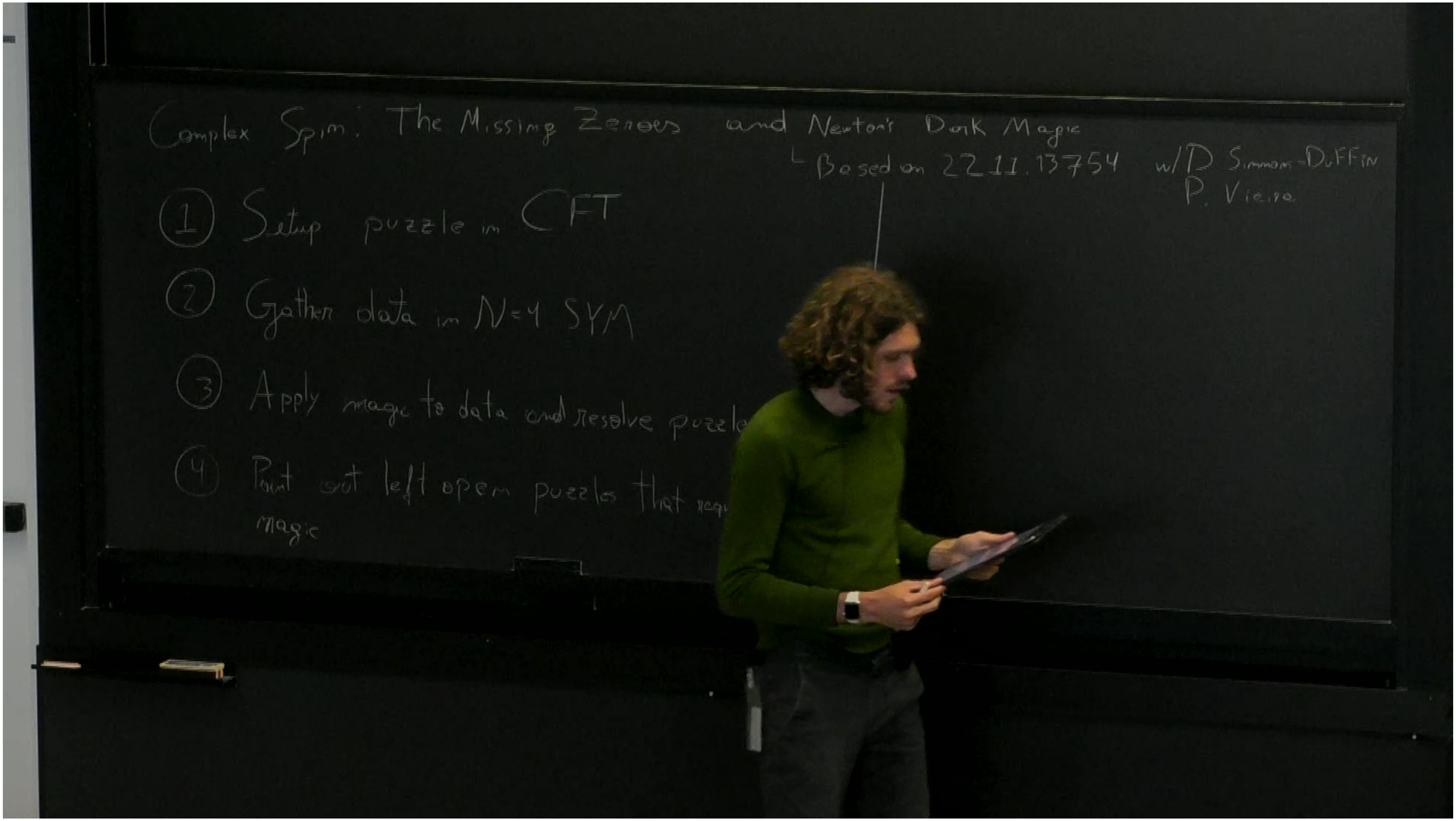
Series: Quantum Fields and Strings

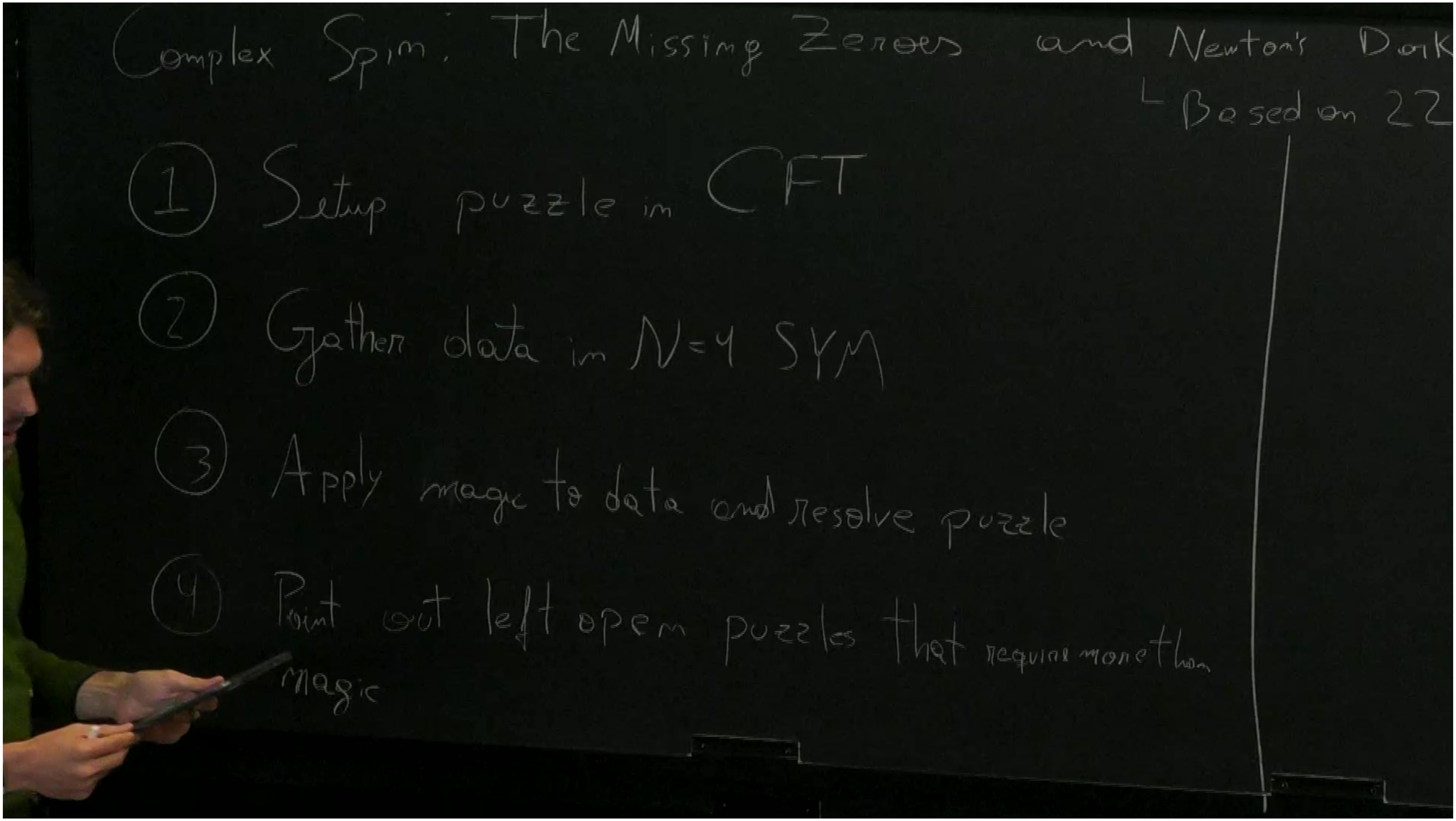
Date: January 31, 2023 - 2:00 PM

URL: https://pirsa.org/23010115

Abstract: Conformal Regge theory predicts the existence of analytically continued CFT data for complex spin. How could this work when there are so many more local operators with large spin compared to small spin? Using planar N=4 SYM as a testground we find a simple physical picture. Local operators do organize themselves into analytic families but the continuation of the higher families have zeroes in their structure OPE constants for lower integer spins. They thus decouple. Newton's interpolation series technique is perfectly suited to this physical problem and will allow us to explore the right complex spin half-plane.

Zoom link: https://pitp.zoom.us/j/93721160114?pwd=bWd3b09yYkRwRytjeTBKMUN6aTNsQT09


Complex Spin: The Missing Zeroes and Newton's Dark Magic
Based on 2211.13754 w/ D. Simmons-Duffin
P. Vieira
1 Setup puzzle in CFT
2 Gather data in N=4 SYM
3 Apply magic to data and resolve puzzl
4 Point out left open puzzles that requ
magic


Complex Spin: The Missing Zeroes and Newton's
Based on 22
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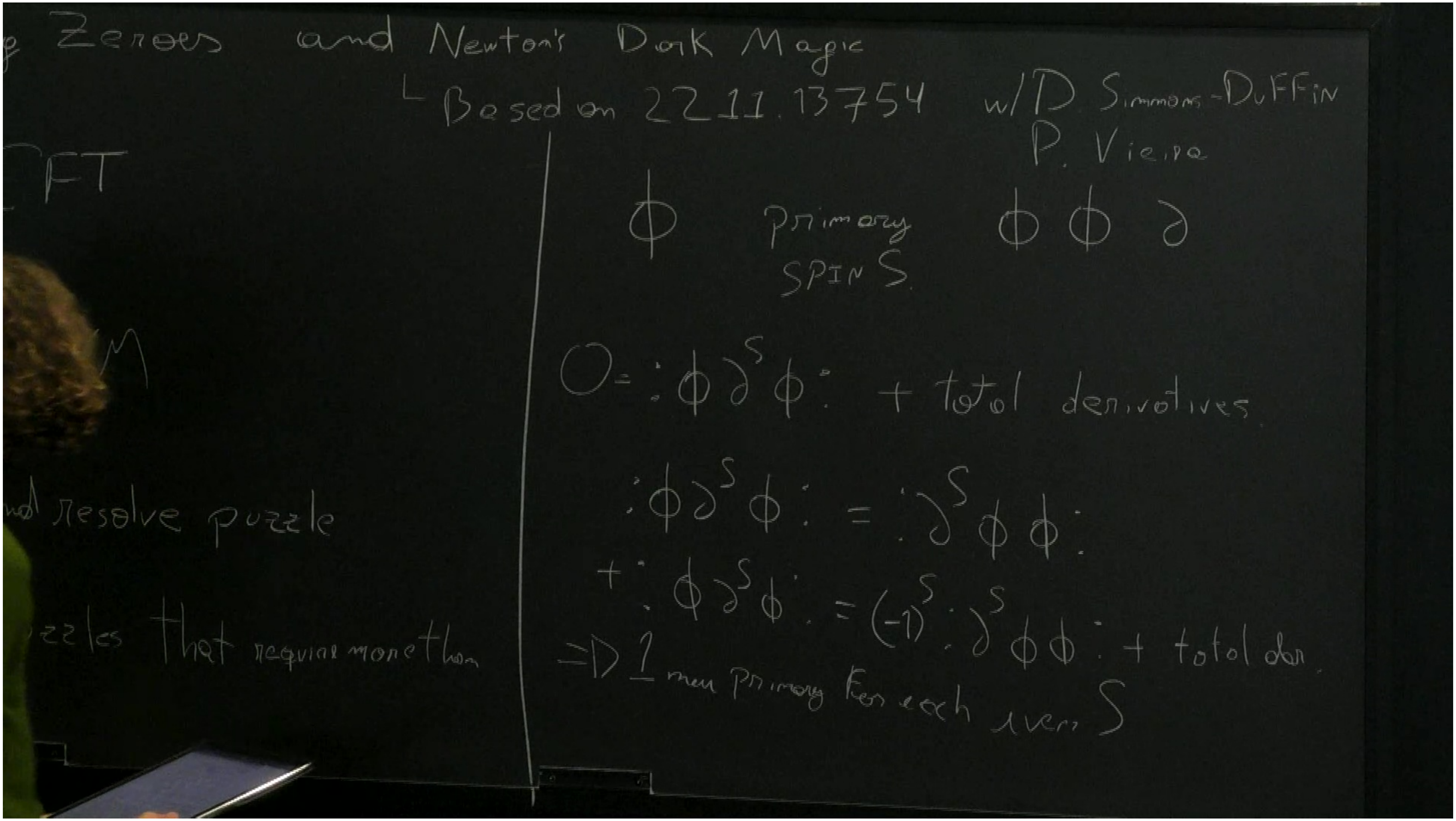
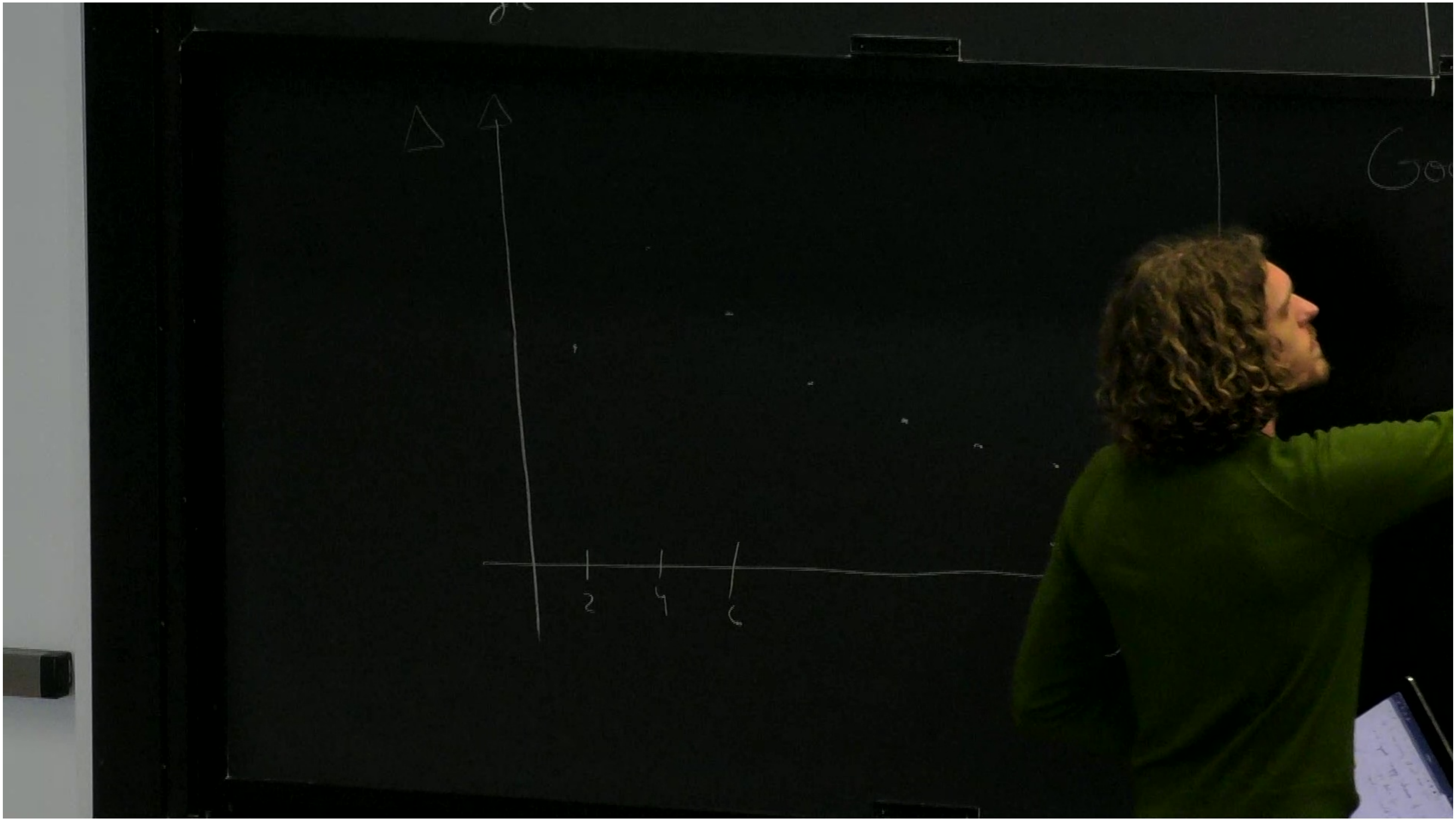
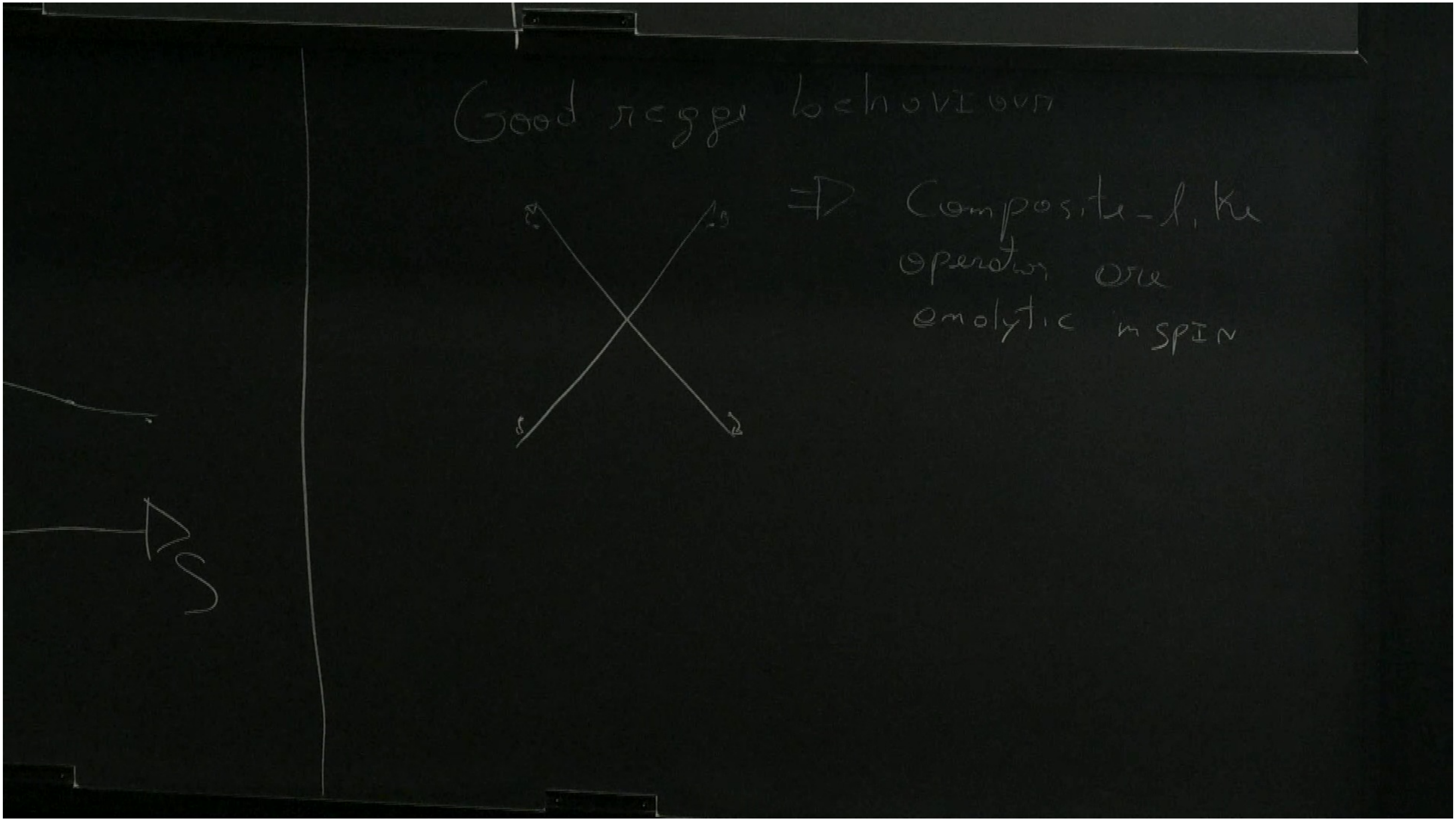
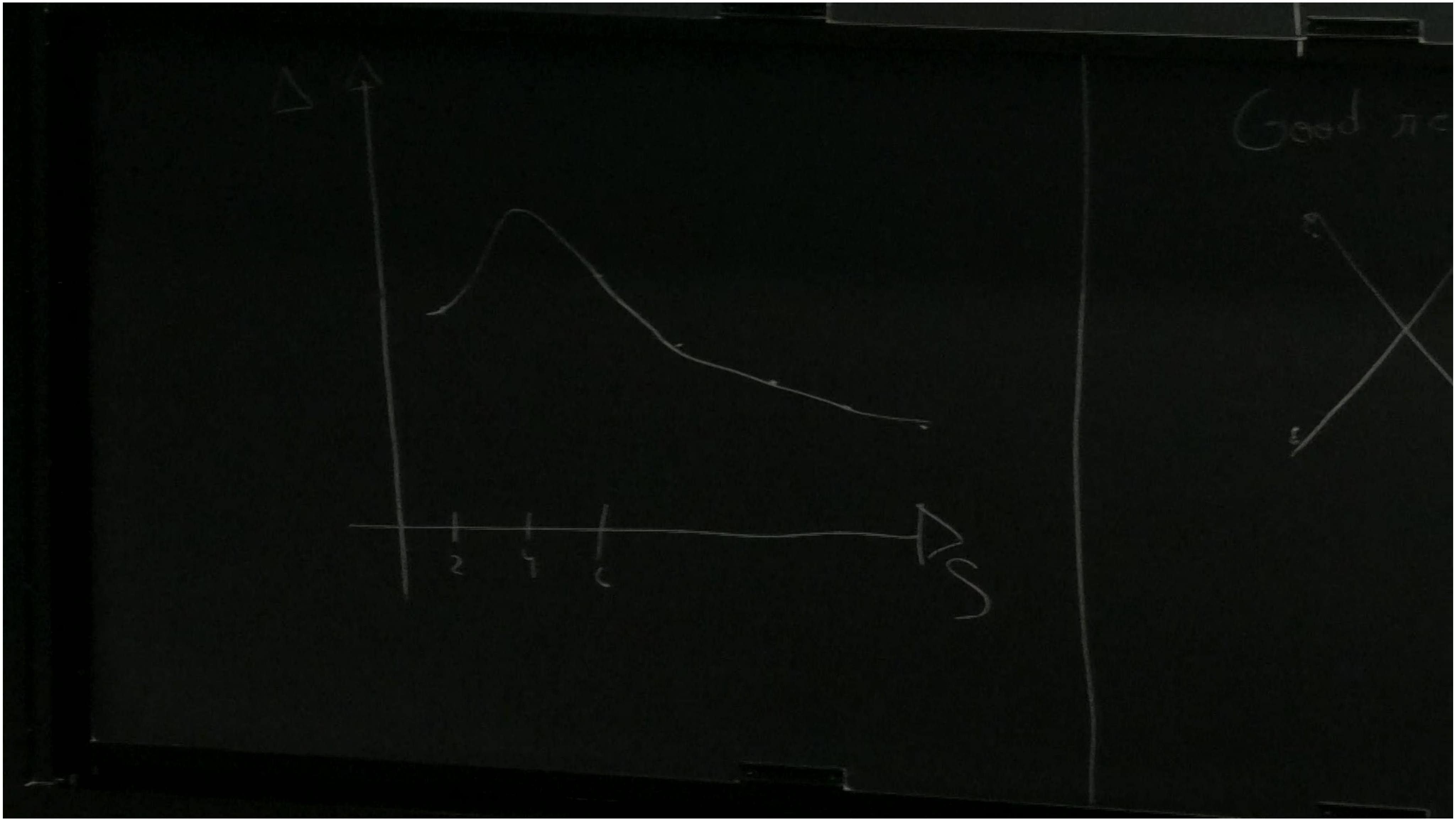
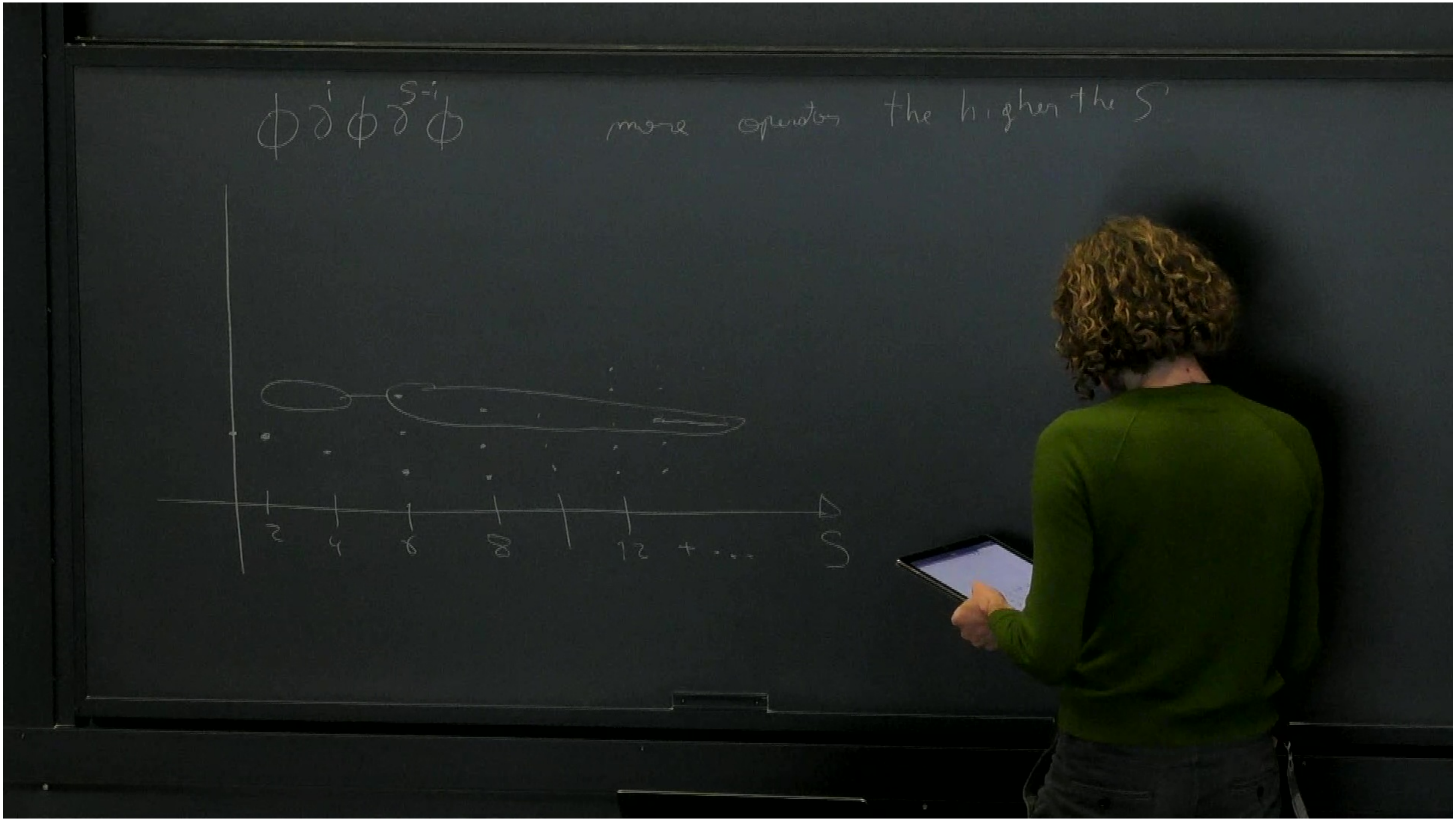
3 Apply magic to data and resolve puzzle
4 Point out left open puzzles that require more than magic


Zeroes and Newton's Dark Magic
Based on 2211.13754 w/ D. Simmons-Duffin
P. Vieira
FT
resolve puzzle
zzles that require more than
φ primary
SPIN S
φ φ ∂
O = :φ ∂^S φ: + total derivatives
:φ ∂^S φ: = :∂^S φ φ:
+ :φ ∂^S φ: = (-1)^S :∂^S φ φ: + total der.
⇒ 1 new primary for each even S

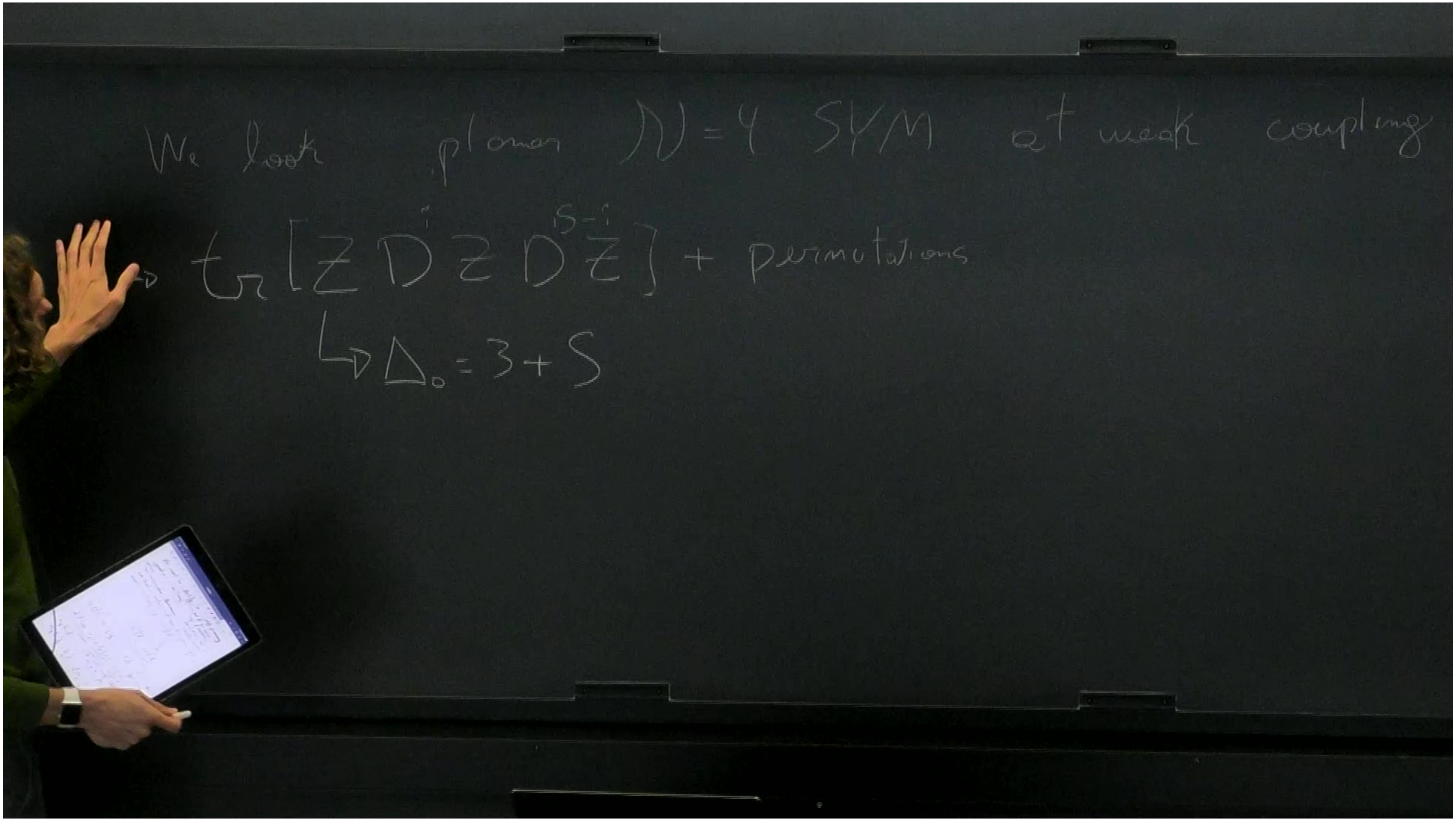

2
4
6

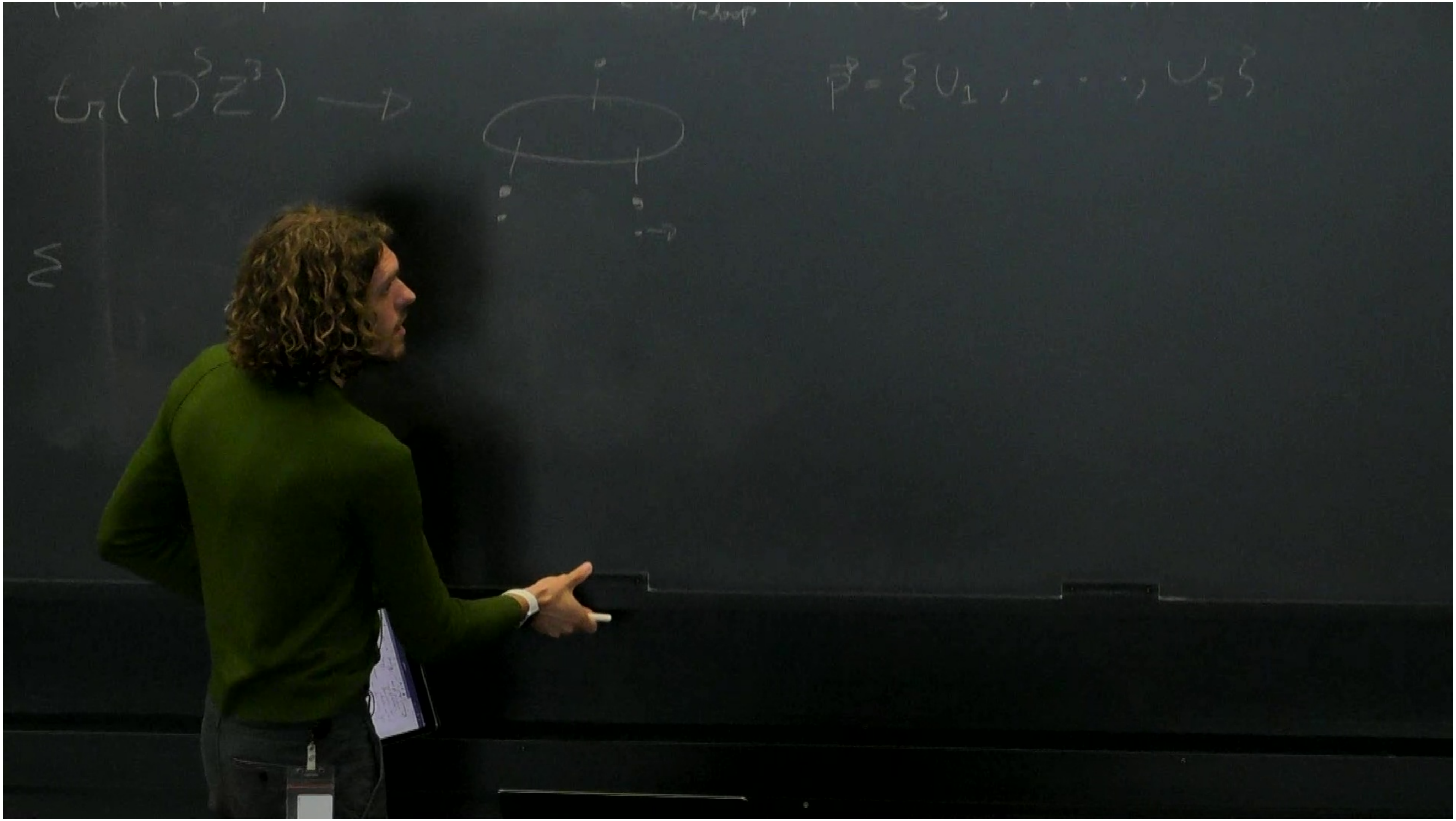

Good regge behaviour
⇒ Composite-like
operators are
analytic in SPIN

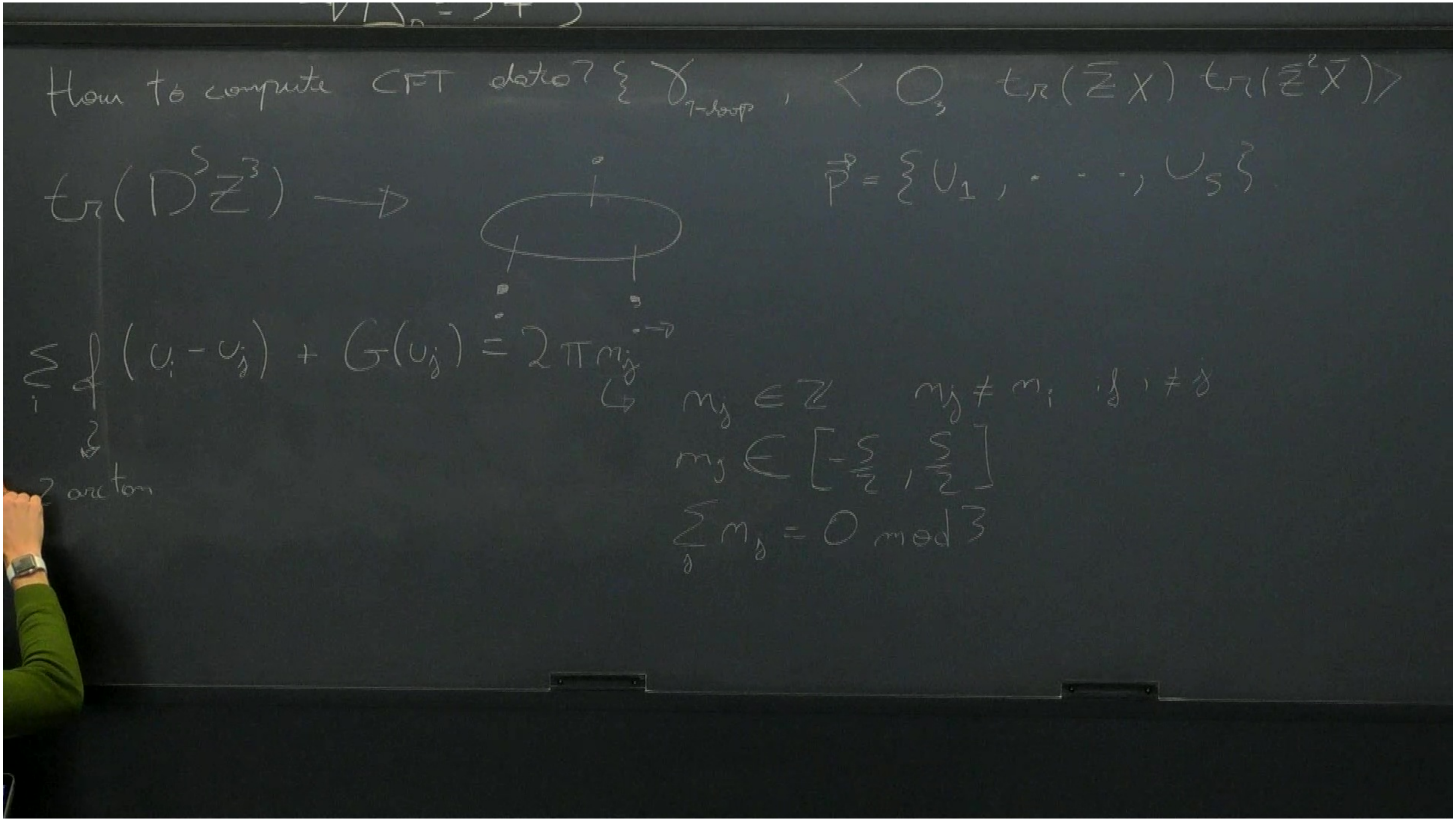

Δ
2
4
S
Good



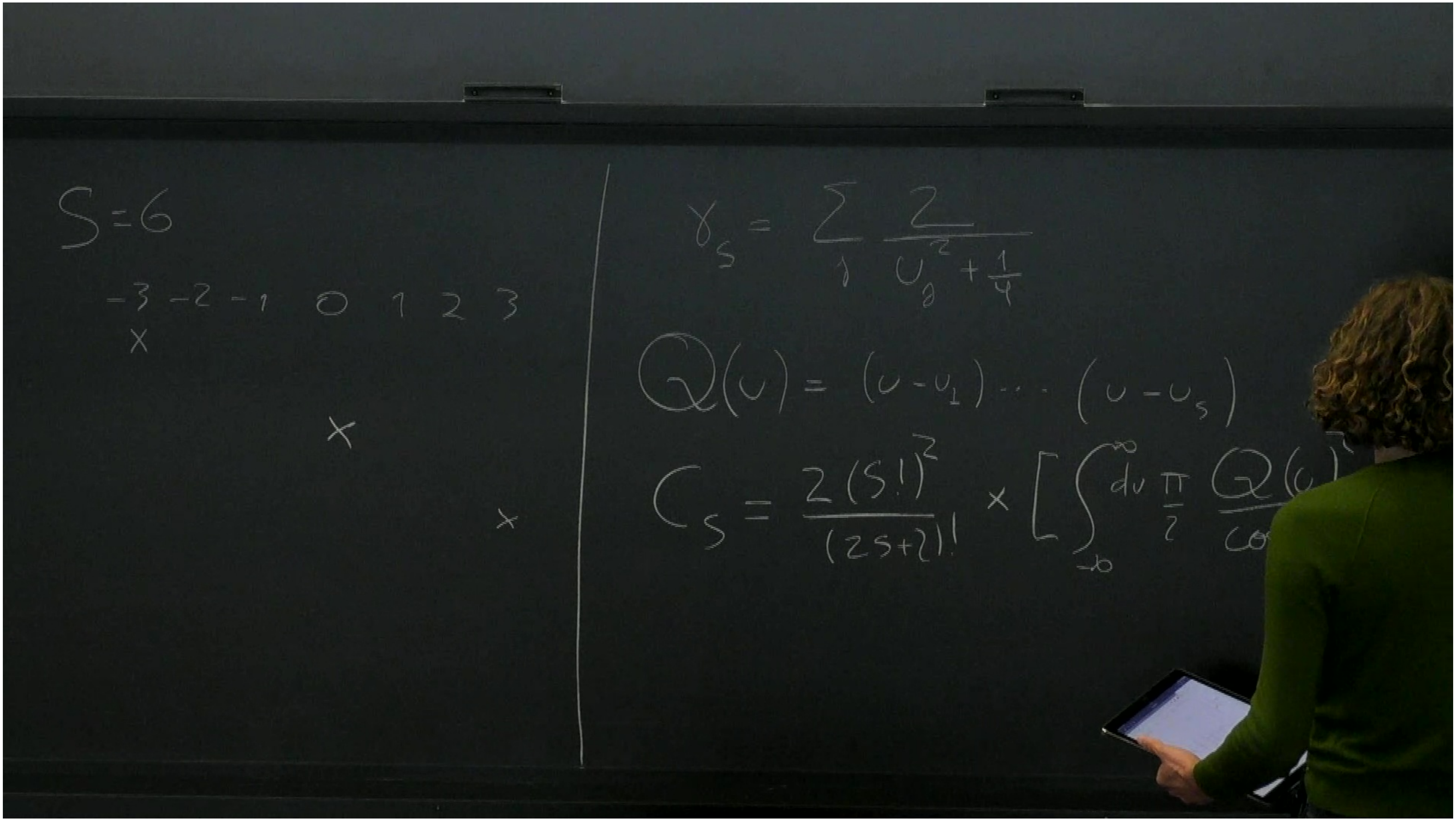
$\phi \partial^i \phi \partial^{S-i} \phi$
more operator the higher the S
2 4 6 8 12 + ...
S


We look planar $\mathcal{N}=4$ SYM at weak coupling
$\to \mathrm{tr}[Z D^{i} Z D^{S-i} Z]$ + permutations
$\hookrightarrow \Delta_0 = 3+S$





How to compute CFT data? { $\gamma_{1\text{-loop}}$, $\langle O_3 \; tr(\bar{Z}X)\; tr(\bar{Z}^2\bar{X}) \rangle$
$tr(D^5 Z^3) \longrightarrow$
$\vec{p} = \{u_1, \ldots, u_5\}$
$\sum_i f(u_i - u_j) + G(u_j) = 2\pi m_j$
2 arctan
$m_j \in \mathbb{Z}$ $m_j \neq m_i$ if $i \neq j$
$m_j \in [-\frac{5}{2}, \frac{5}{2}]$
$\sum_j m_j = 0 \bmod 3$


S=6
-3 -2 -1 0 1 2 3
γs = Σj 2 / (uj² + 1/4)
Q(u) = (u - u1) . . . (u - us)
Cs = 2(S!)² / (2S+2)! × [∫ du π/2 Q(u)

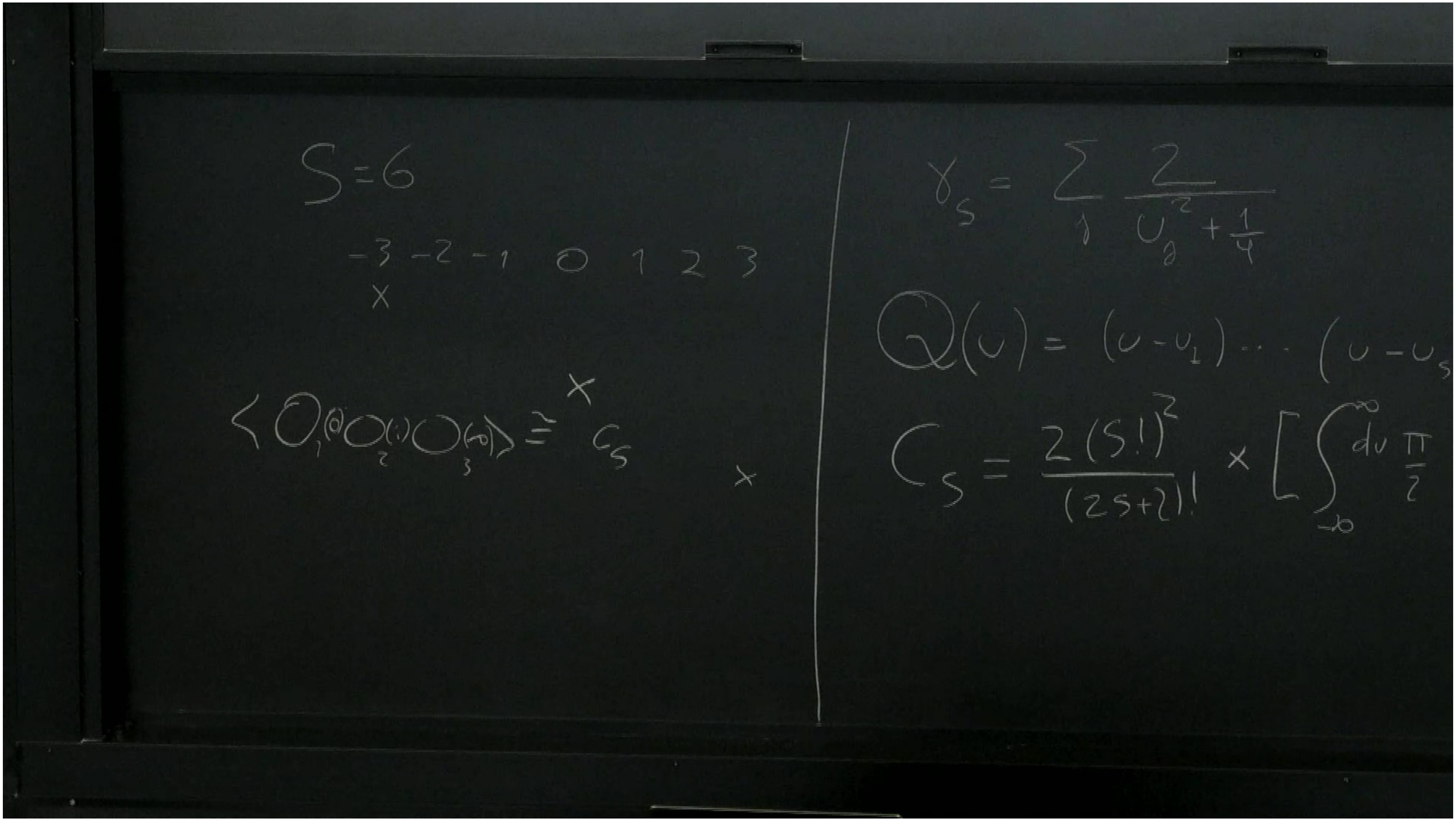
1 2 3

$$\gamma_s = \sum_j \frac{2}{u_j^2 + \frac{1}{4}}$$

$$Q(u) = (u - u_1) \cdots (u - u_s)$$

×

$$C_s = \frac{2(s!)^2}{(2s+2)!} \times \left[\int_{-\infty}^{\infty} du \, \frac{\pi}{2} \frac{Q(u)^2}{\cosh(\pi u)^2} \right]^{-1}$$


S=6
-3 -2 -1 0 1 2 3
X
<O1(0) O2(1) O3(∞)> = Cs
γs = Σj 2 / (uj² + 1/4)
Q(u) = (u - u1) ... (u - us
Cs = 2(S!)² / (2S+2)! × [∫ du π/2

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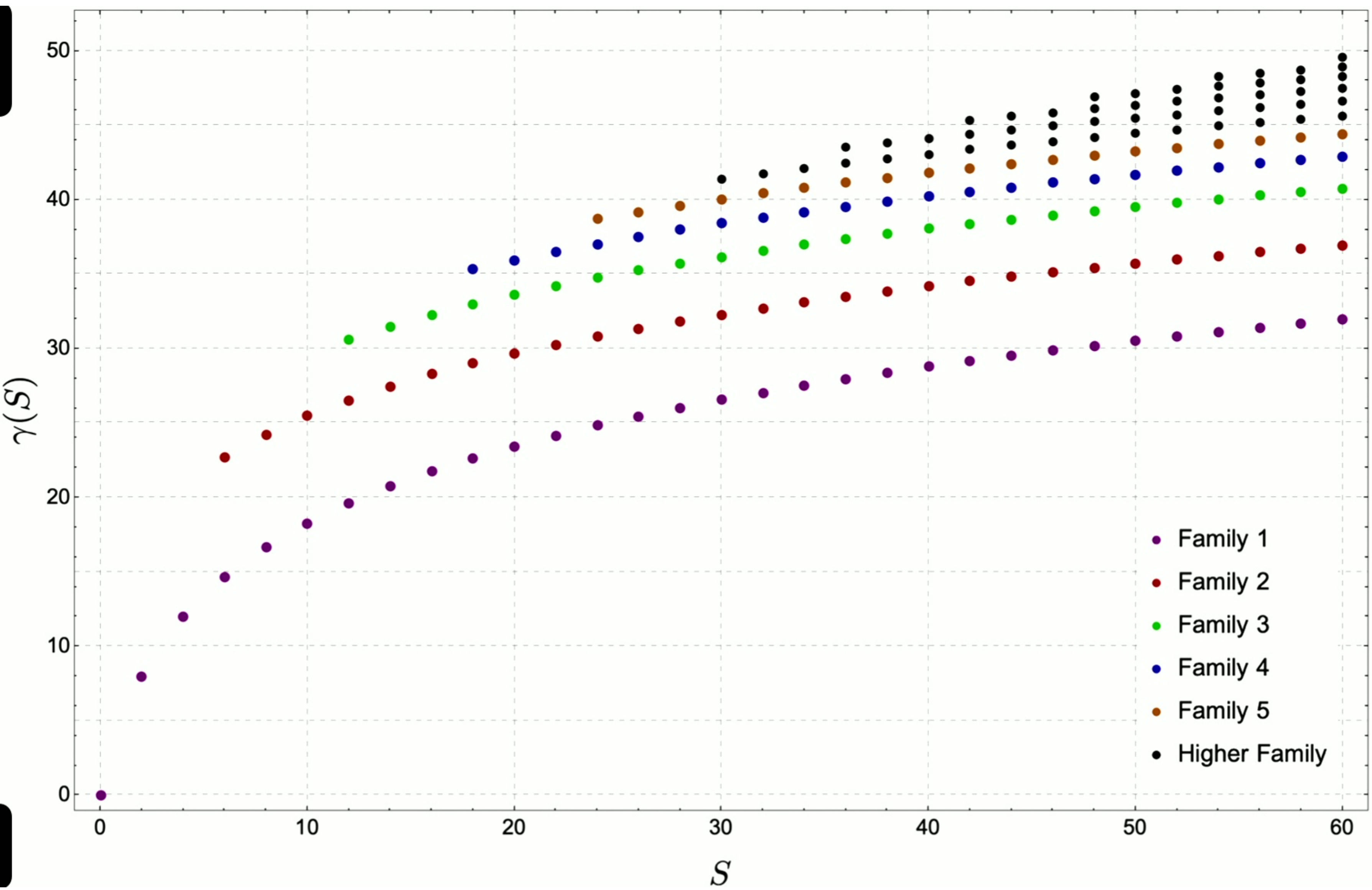
based on 2211.13754 with D. Simmons-Duffin and P. Vieira

Anomalous dimension data



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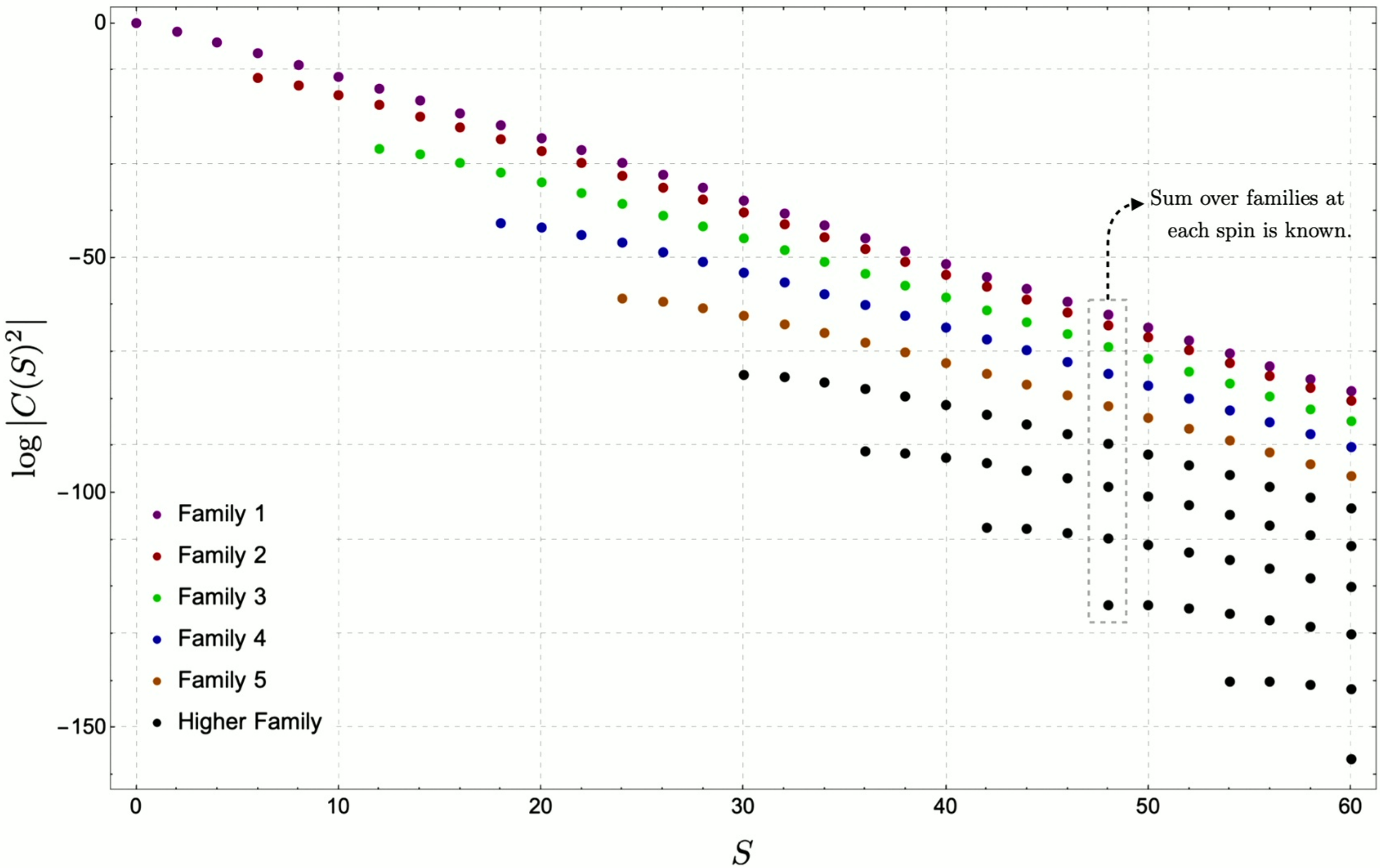
Structure constants data



3/8

- We have computed the CFT data for spin up to two hundred. What do you we do with that?
- Check Newton's Principia, book 3:

LEMMA V.

To find a curve line of the parabolic kind which shall pass through any given number of points.

Let those points be A, B, C, D, E, F, &c., and from the same to any right line HN, given in position, let fall as many perpendiculars AH, BI, CK, DL, EM, FN, &c.

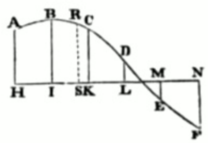
$b \quad 2b \quad 3b \quad 4b \quad 5b$

$c \quad 2c \quad 3c \quad 4c$

$d \quad 2d \quad 3d$

$e \quad 2e$

f



CASE 1. If HI, IK, KL, &c., the intervals of the points H, I, K, L, M, N, &c., are equal, take b, $2b$, $3b$, $4b$, $5b$, &c., the first differences of the perpendiculars AH, BI, CK, &c.; their second differences c, $2c$, $3c$, $4c$, &c.; their third, d, $2d$, $3d$, &c., that is to say, so as AH — BI may be $= b$, BI — CK $= 2b$, CK — DL $= 3b$, DL + EM $= 4b$, — EM + FN $= 5b$, &c.; then $b - 2b = c$, &c., and so on to the last difference, which is here f. Then, erecting any perpendicular RS, which may be considered as an ordinate of the curve required, in order to find the length of this ordinate, suppose the intervals HI, IK, KL, LM, &c., to be units, and let AH $= a$, — HS $= p$, $\frac{1}{2}p$ into — IS $= q$, $\frac{1}{3}q$ into + SK $= r$, $\frac{1}{4}r$ into + SL $= s$, $\frac{1}{5}s$ into + SM $= t$; proceeding, to wit, to ME, the last perpendicular but one, and prefixing negative signs before the terms HS, IS, &c., which lie from S towards A; and affirmative signs before the terms SK, SL, &c., which lie on the other side of the point S; and, observing well the signs, RS will be $= a + bp + cq + dr + es + ft$, + &c.

4/8

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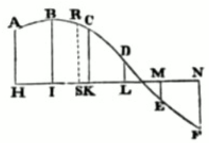

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Modern English →

Suppose f is holomorphic and exponentially decaying in the right half plane. Then, the sequence of polynomials

$$f_N(z) \equiv \sum_{j=0}^{N} \binom{z}{j} \sum_{i=0}^{j} \binom{j}{i} (-1)^{j-i} f(i)$$

converges to f.

- We can reconstruct f from its values at the integers.

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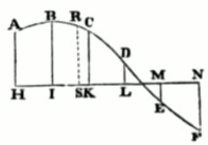

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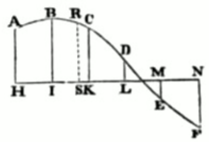

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- We can reconstruct f from its values at the integers.
- Convergence will hold to the right of the first singularity.
- The function can in fact grow exponentially as $e^{a|z|}$ provided a is not too big.

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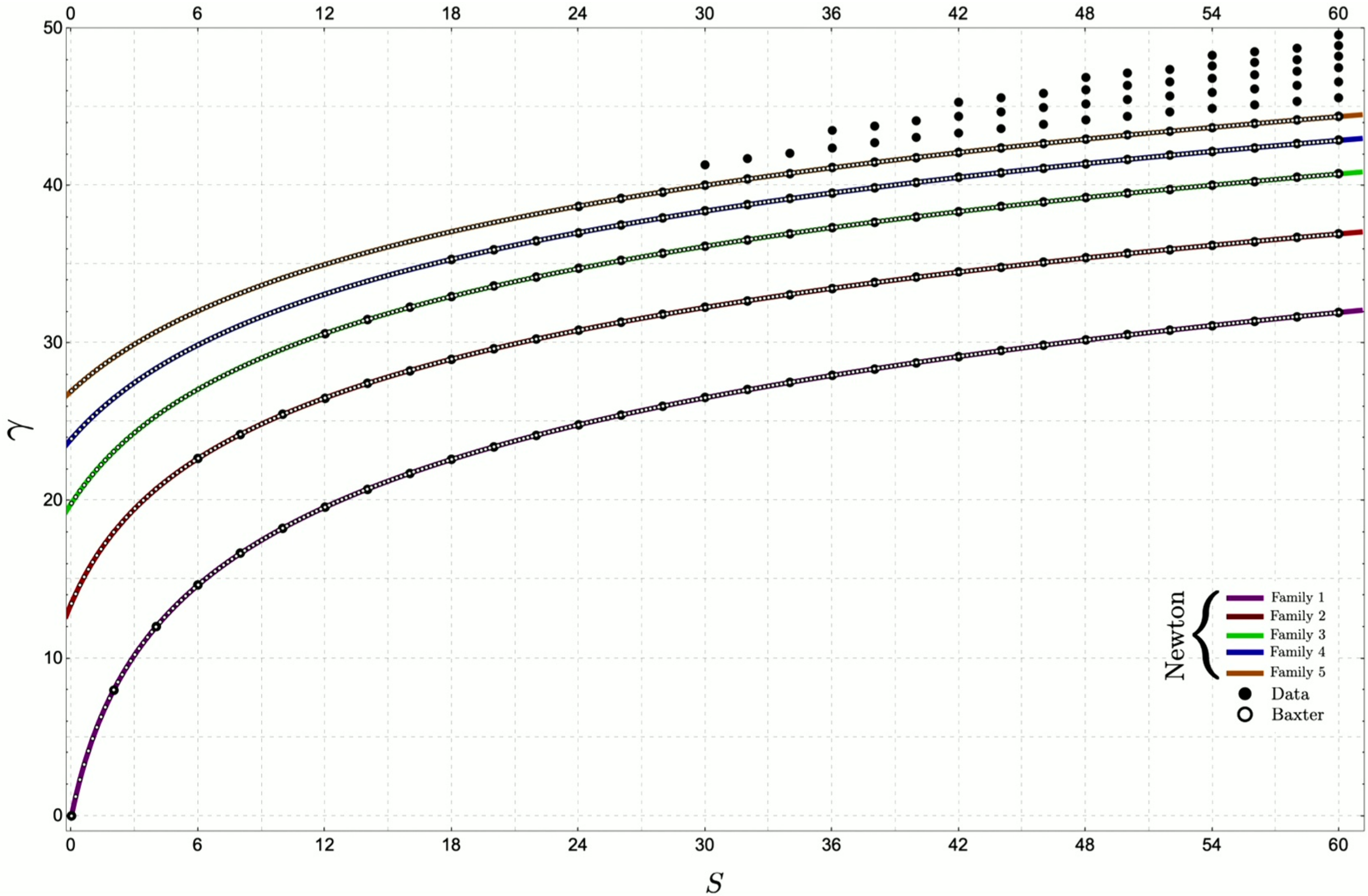
Anomalous dimension Newton series



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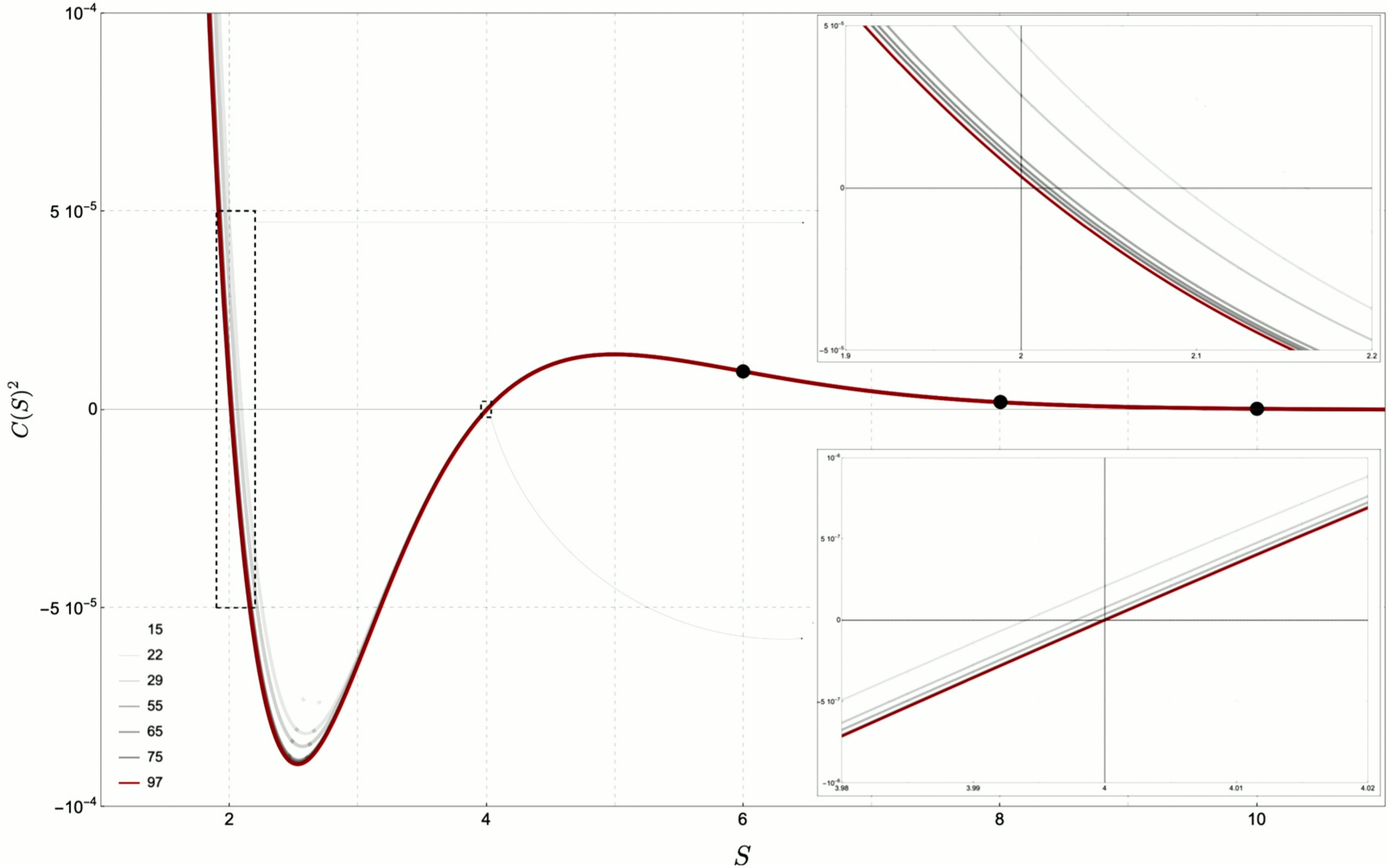
2nd family structures Newton series



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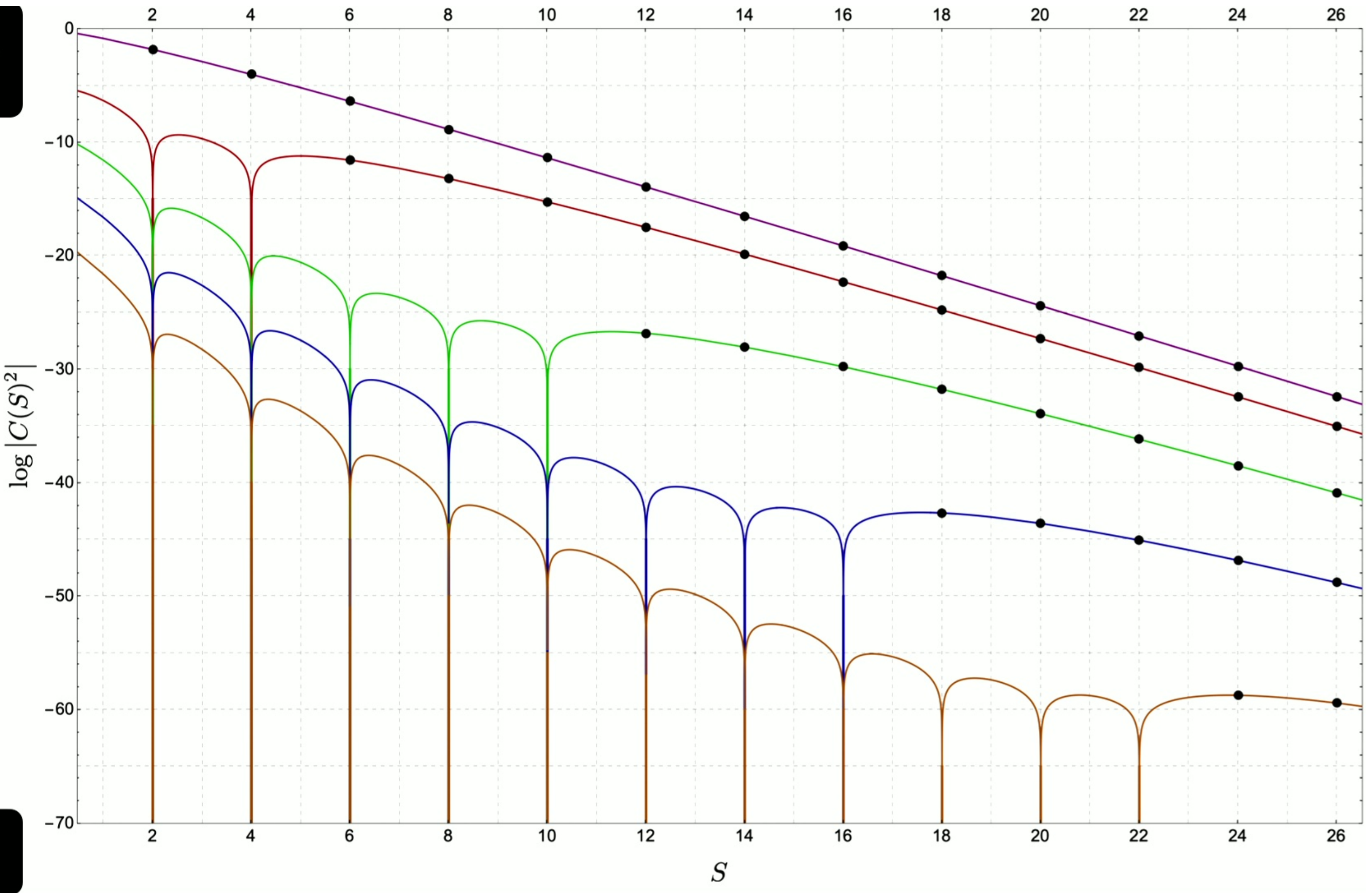
Structure constants Newton series



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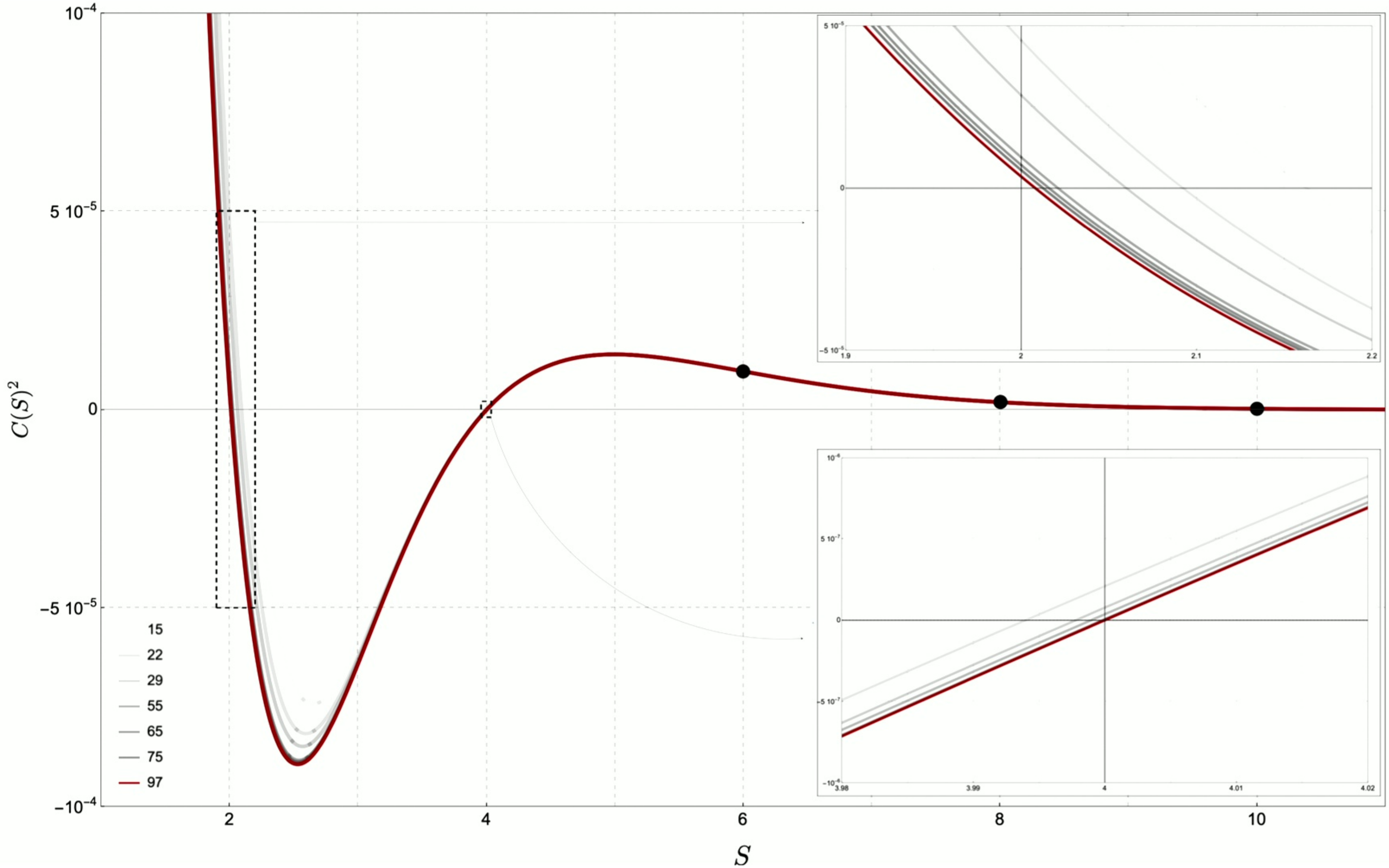
2nd family structures Newton series



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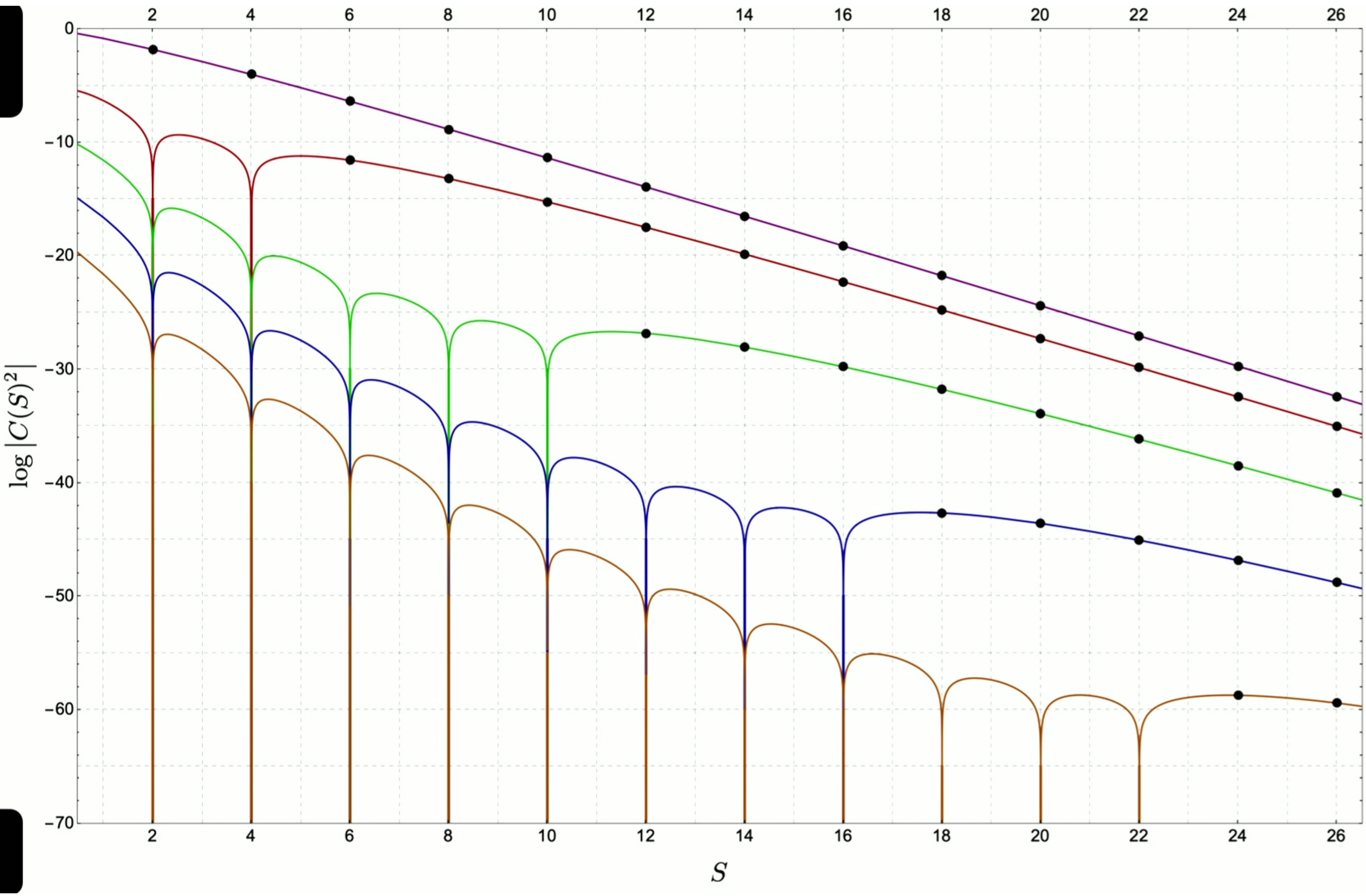
Structure constants Newton series



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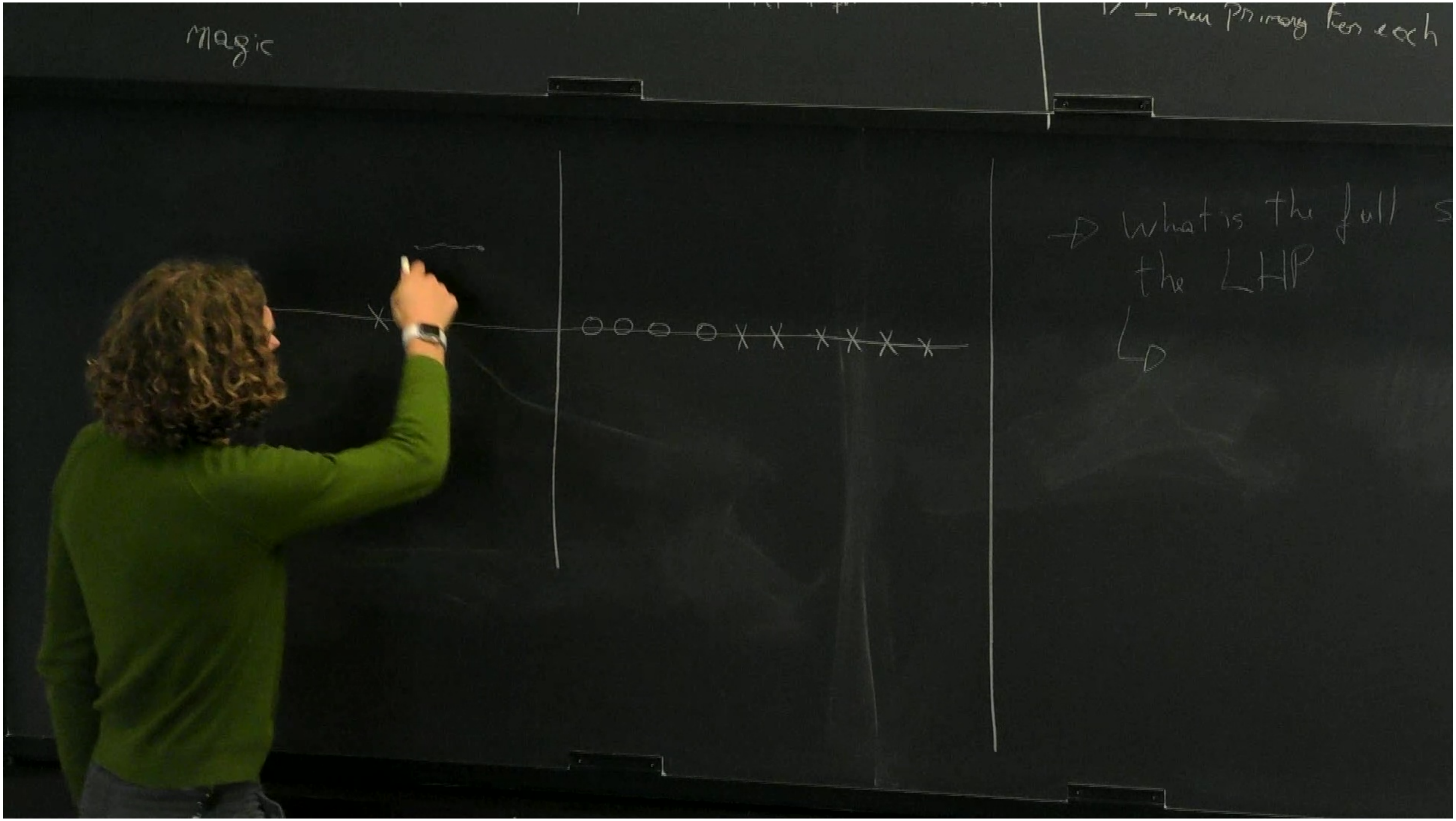
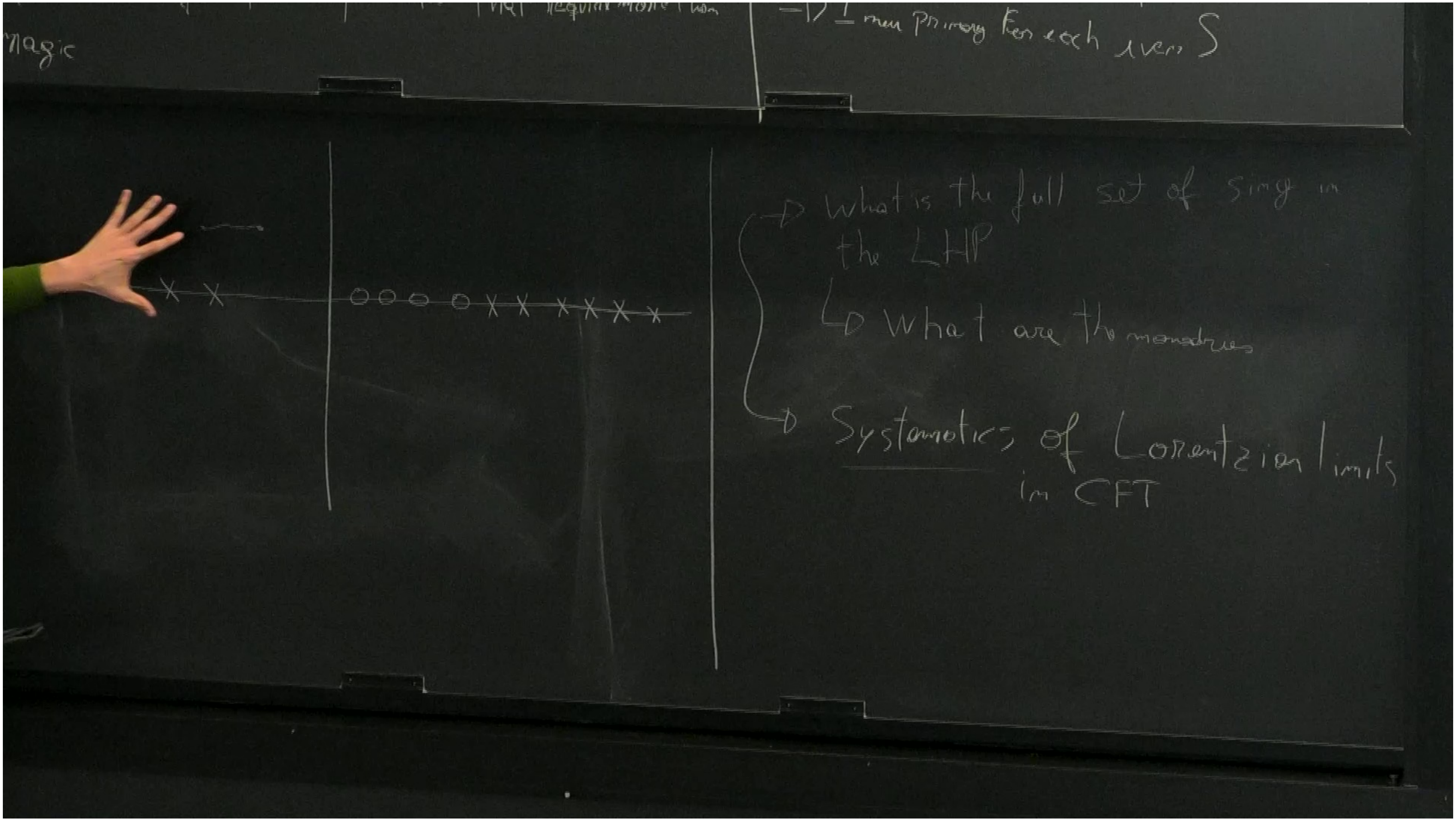

magic
What is the full
the LHP


magic
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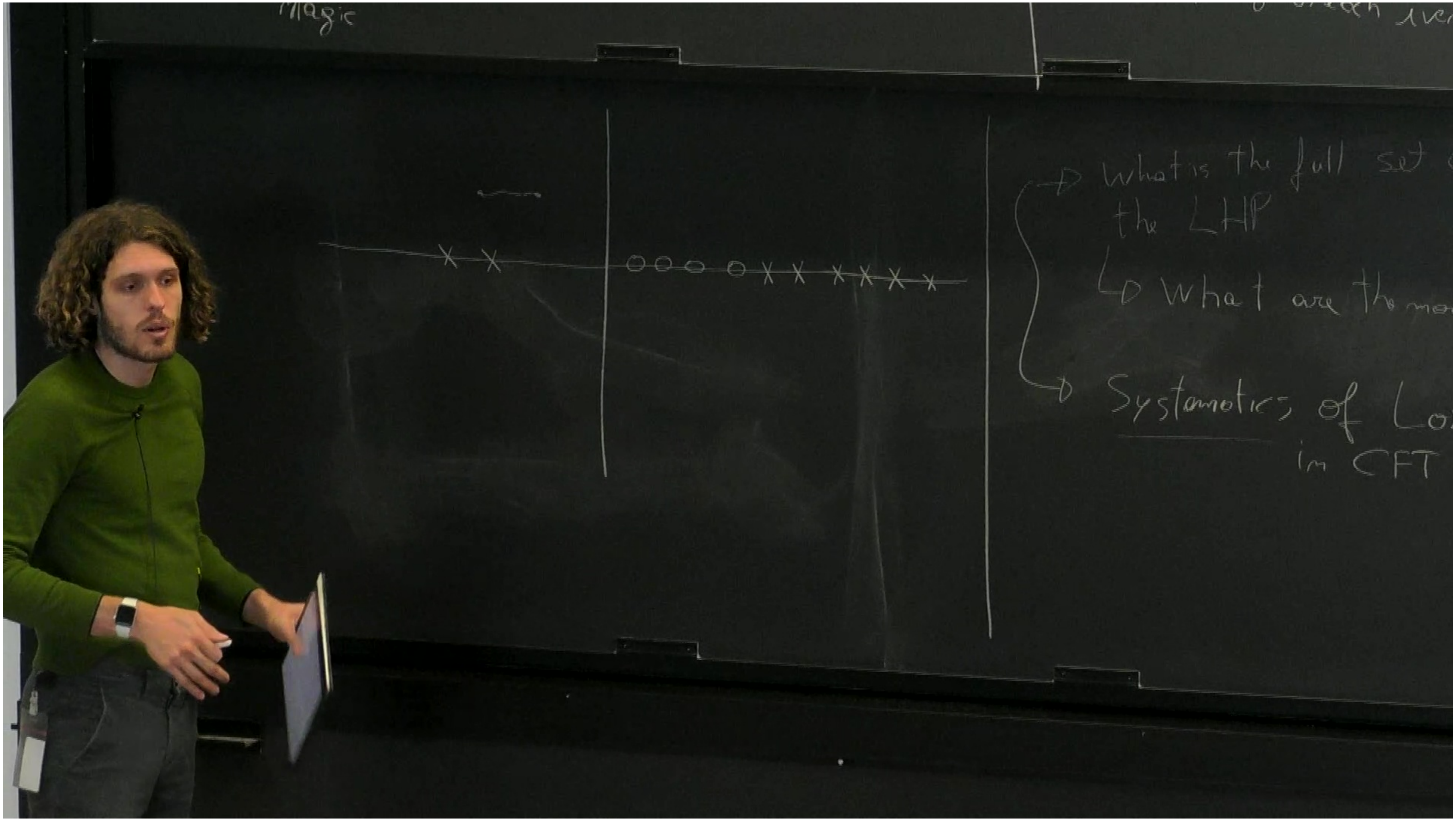
What is the full set of sing in the LHP
What are the monodromies
Systematics of Lorentzian limits in CFT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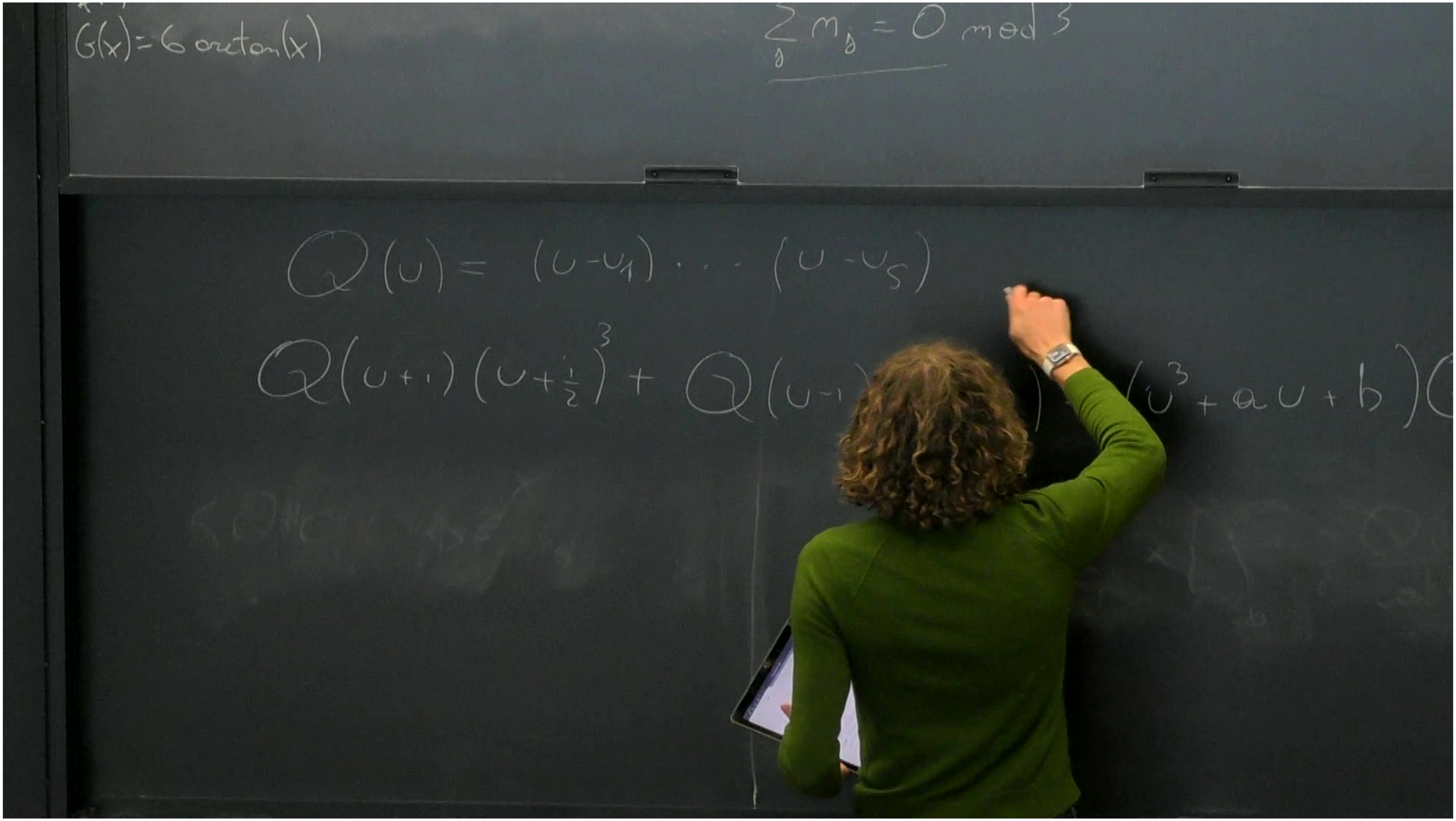
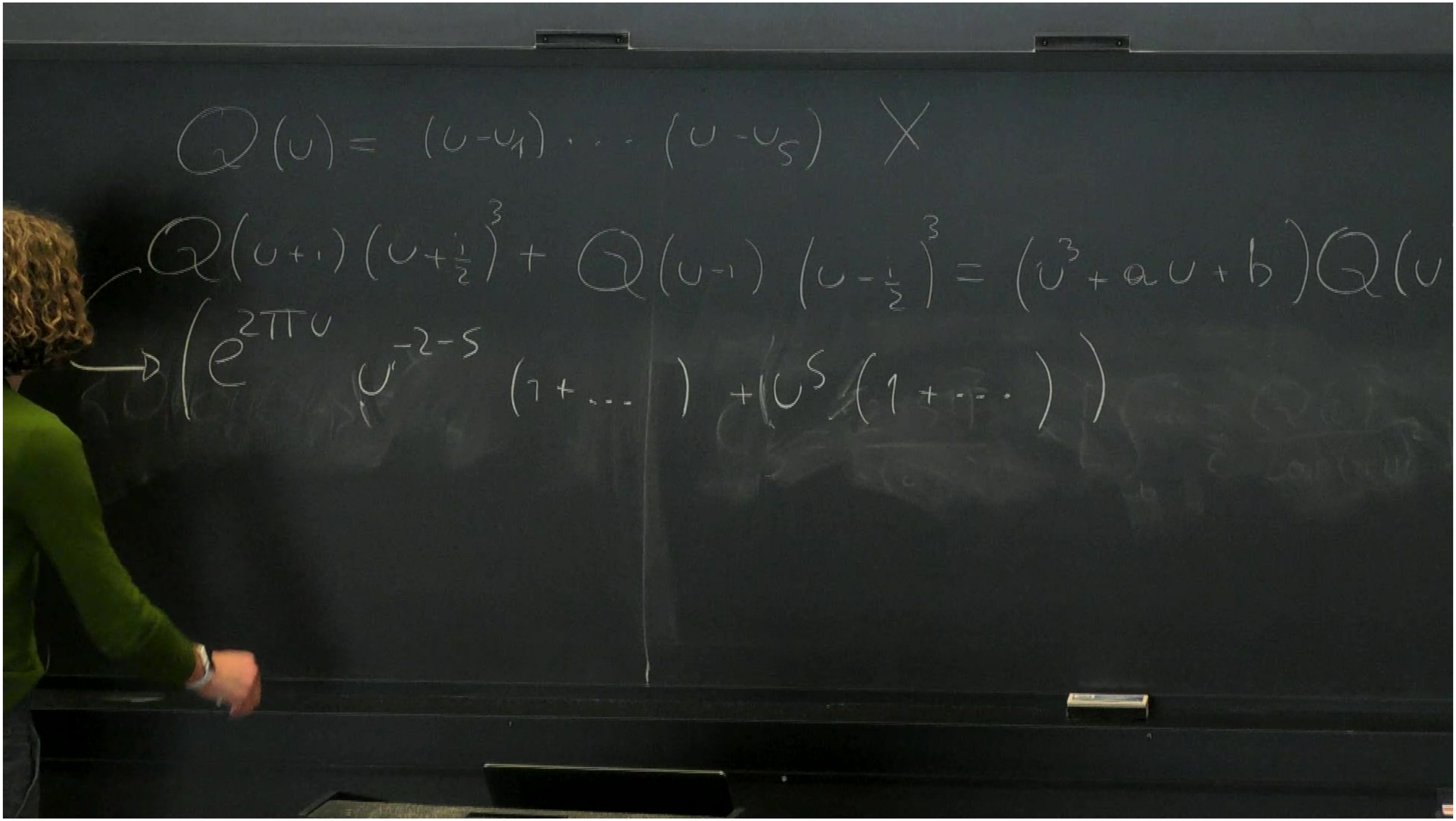
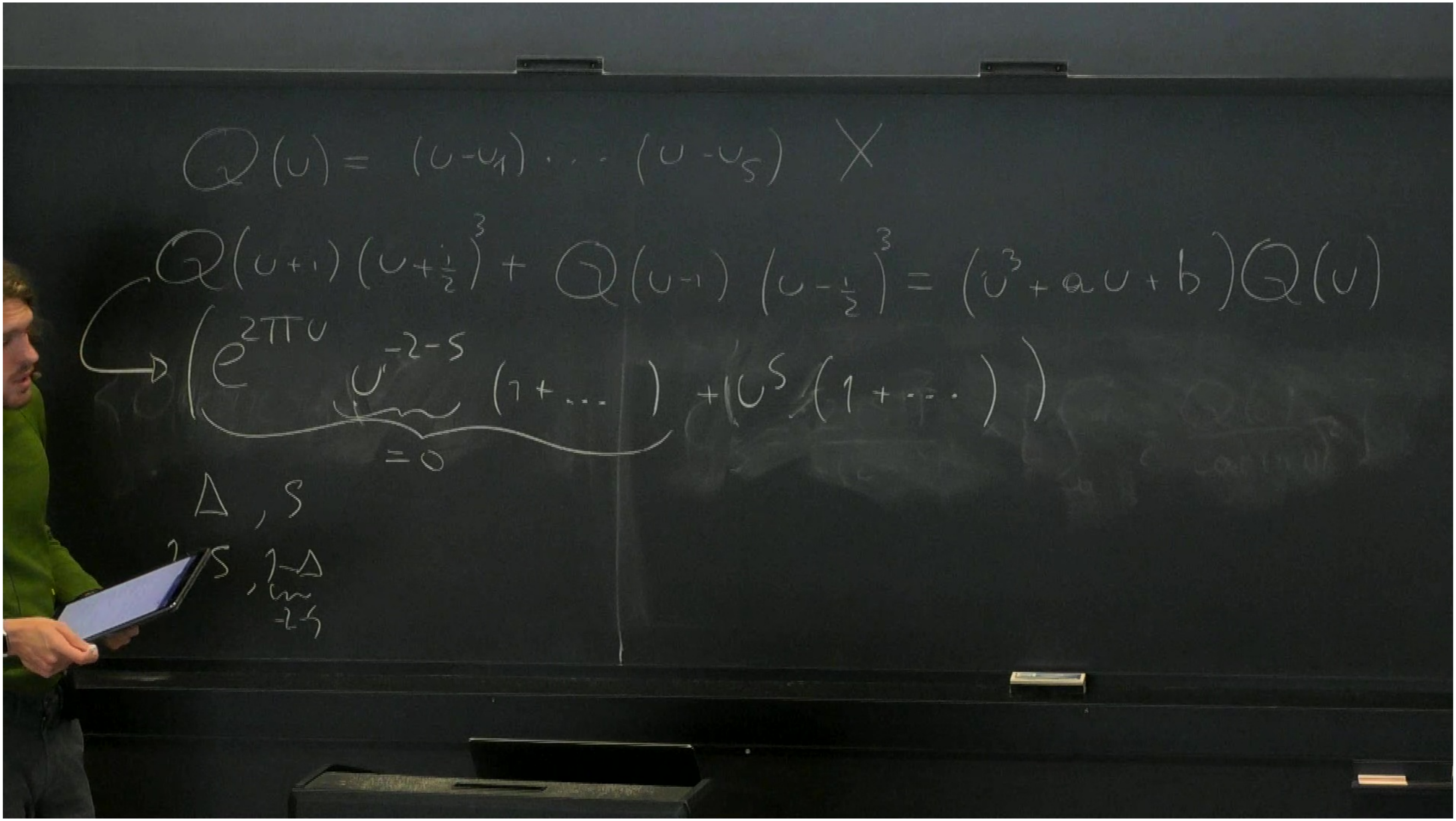
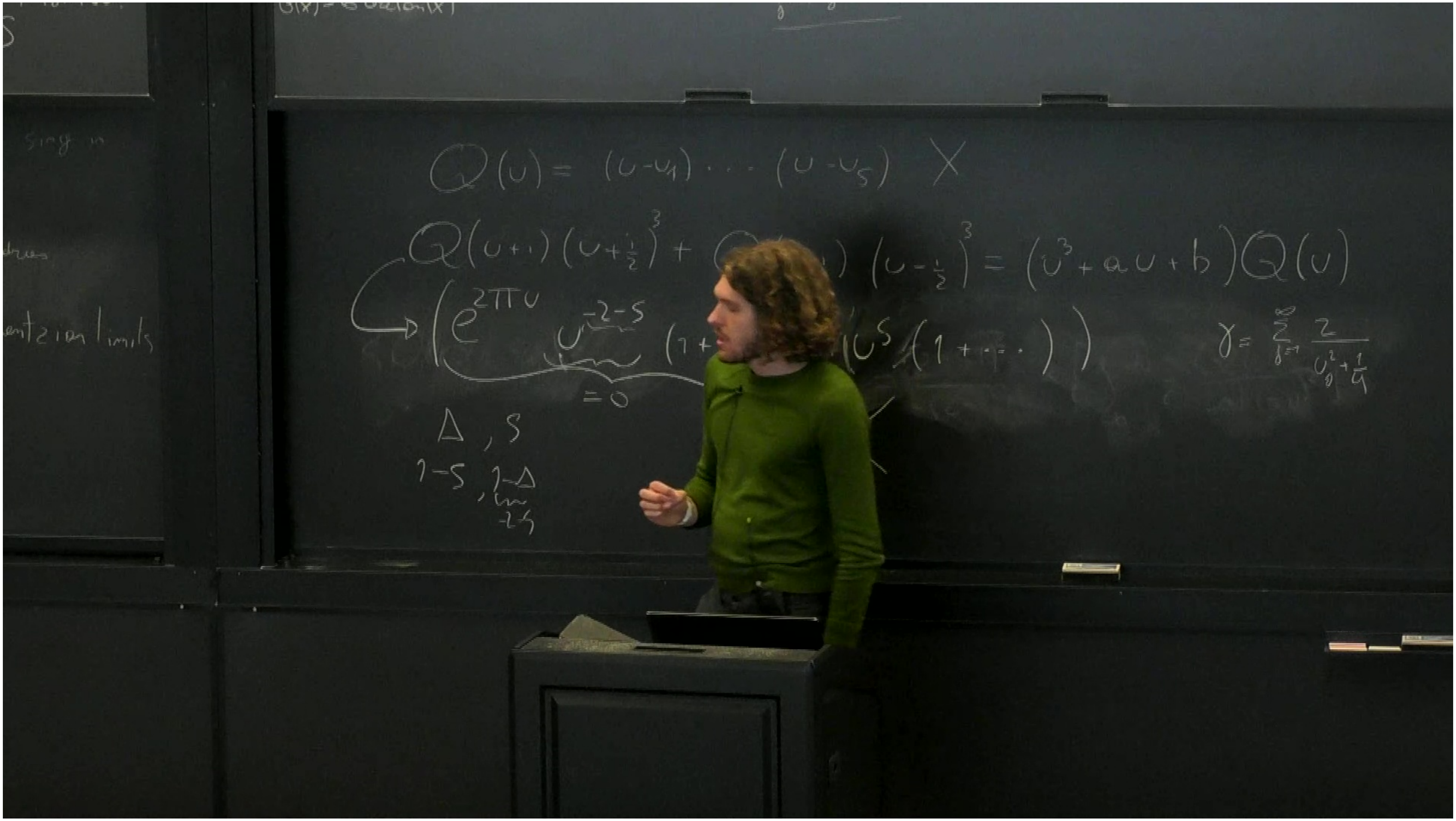

G(x)=6 arctan(x)
Σ m = 0 mod 3
Q(u) = (u-u1) . . . (u-us)
Q(u+i)(u+i/2)^3 + Q(u-i)
(u^3 + au + b)


$Q(u) = (u-u_1)\cdots(u-u_5)$ X
$Q(u+1)(u+\frac{1}{2})^3 + Q(u-1)(u-\frac{1}{2})^3 = (u^3+au+b)Q(u$
$\rightarrow (e^{2\pi u} u^{-2-S}(1+\dots) + u^S(1+\dots))$


Q(u) = (u - u1) . . . (u - uS) X
Q(u+1) (u+1/2)^3 + Q(u-1) (u-1/2)^3 = (u^3 + a u + b) Q(u)
(e^2πu u^-2-S (1 + . . .) + u^S (1 + . . .))
= 0
Δ , S
S , 1-Δ
-2-S



Q(u) = (u-u1) . . . (u-u5)
= (u³ + a u + b) Q(u)
= 0
Δ , S
1-S , 1-Δ